

Filtres Passe-Haut

1^{er} ordre

$$T(\omega) = A_{\max} \frac{j\frac{\omega}{\omega_c}}{1 + j\frac{\omega}{\omega_c}}$$

$$|A_{\max} = A(\omega) \text{ en THF}|$$

Diagrammes de Bode Courbe de gain

$$A(\omega) = A_{\max} \frac{\frac{\omega}{\omega_c}}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

TBF

$$A(\omega) \xrightarrow{\omega \rightarrow 0} 0$$

$$G(\omega) \xrightarrow{\omega \rightarrow 0} -\infty \quad \text{limite infinie} \rightarrow \text{asymptote oblique}$$

THF

$$A(\omega) \xrightarrow{\omega \rightarrow \infty} A_{\max}$$

$$G(\omega) \xrightarrow{\omega \rightarrow \infty} \log(A_{\max}) \quad \text{limite finie} \rightarrow \text{asymptote horizontale}$$

Equation de l'asymptote oblique

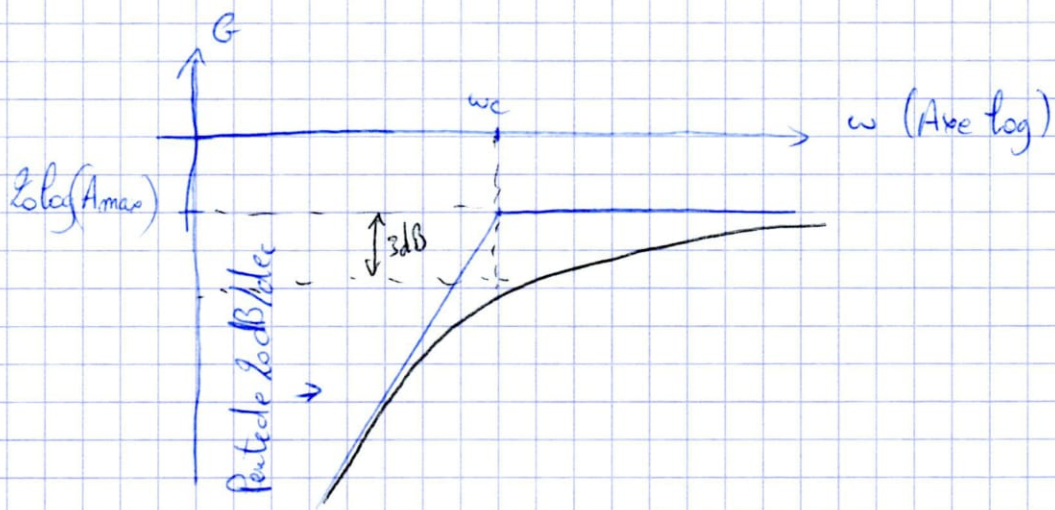
$$1 + \left(\frac{\omega}{\omega_c}\right)^2 \approx 1$$

$$\frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}} \approx 1$$

$$A(\omega) = A_{\max} \frac{\frac{\omega}{\omega_c}}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}} \approx A_{\max} \frac{\omega}{\omega_c}$$

$$G(\omega) \approx \log(A_{\max} \frac{\omega}{\omega_c}) = \log(\omega) + \log(A_{\max}/\omega_c)$$

Droite de pente 20 dB/dec



Courbe de phase

$$\begin{aligned}\varphi(\omega) &= \arg(A_{max} j \frac{\omega}{\omega_c}) - \arg(1 + j \frac{\omega}{\omega_c}) \\ &= \frac{\pi}{2} - \text{Arctan}\left(\frac{\omega}{\omega_c}\right)\end{aligned}$$

TBF

$$\varphi(\omega) \xrightarrow{\omega \rightarrow 0} \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

THF

$$\varphi(\omega) \xrightarrow{\omega \rightarrow \infty} \frac{\pi}{2} - \frac{\pi}{2} = 0$$



Filtre passe-bas

1^{er} ordre

$$T(\omega) = A_{\max} \frac{1}{1 + j \frac{\omega}{\omega_c}}$$

$$|A_{\max} = A(\omega) \text{ en TBF}|$$

Diagrammes de Bode

Courbe de Gain

$$A(\omega) = A_{\max} \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}}$$

TBF

$$A \xrightarrow{\omega \rightarrow 0} A_{\max}$$

$$G \xrightarrow{\omega \rightarrow 0} 20 \log(A_{\max}) \quad \text{Limite finie} \Rightarrow \text{asymptote horizontale}$$

THF

$$A \xrightarrow{\omega \rightarrow \infty} 0$$

$$G \xrightarrow{\omega \rightarrow \infty} -\infty \quad \text{Limite infinie} \Rightarrow \text{asymptote oblique}$$

Equation de l'asymptote oblique

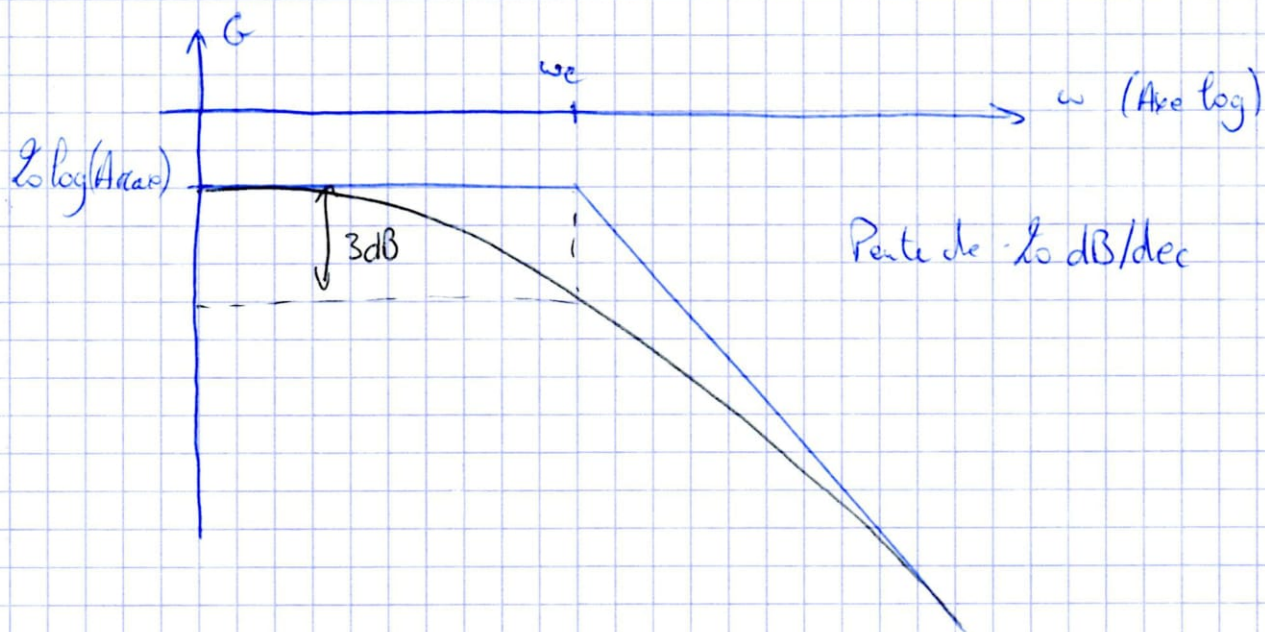
$$1 + \left(\frac{\omega}{\omega_c}\right)^2 \sim \left(\frac{\omega}{\omega_c}\right)^2$$

$$\frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_c}\right)^2}} \sim \frac{\omega_c}{\omega}$$

$$A_{\omega} \sim A_{\max} \frac{\omega_c}{\omega}$$

$$G \sim 20 \log\left(A_{\max} \frac{\omega_c}{\omega}\right) = -20 \log(\omega) + 20 \log A_{\max}$$

Pente de -20dB/dec



Courbe de phase

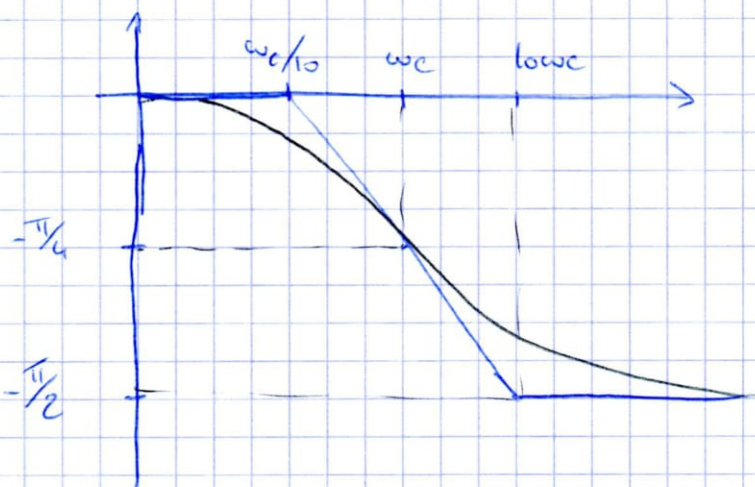
$$\begin{aligned}
 \varphi(\omega) &= \arg(A_{max}) - \arg\left(1 + j\frac{\omega}{\omega_c}\right) \\
 &= -\arctan\left(\frac{\omega}{\omega_c}\right)
 \end{aligned}$$

TBF

$$\varphi \xrightarrow[\omega \rightarrow 0]{} 0$$

THF

$$\varphi \xrightarrow[\omega \rightarrow \infty]{} -\frac{\pi}{2}$$



passé Bas 1^{er} ordre

Filtres Passe-Bas

$$T(\omega) = A_0 \frac{1}{1 + j2\frac{\omega}{\omega_0} - (\frac{\omega}{\omega_0})^2}$$

$$|A_0 = A(\omega) \text{ en TBF}|$$

Diagrammes de Bode :

Courbe de Gain :

$$A(\omega) = A_0 \frac{1}{\sqrt{(1 - (\frac{\omega}{\omega_0})^2)^2 + (2\frac{\omega}{\omega_0})^2}}$$

TBF

$$A(\omega) \xrightarrow{\omega \rightarrow 0} A_0$$

Rq : A_{TBF} = Amplification en continu

TBF

$$G \xrightarrow{\omega \rightarrow 0} 20 \log(A_0)$$

\rightarrow Limite fixe

\rightarrow Asymptote horizontale

THF

$$A \xrightarrow{\omega \rightarrow \infty} 0$$

$$G \rightarrow -\infty \rightarrow \text{Asymptote oblique}$$

Equation de l'asymptote

$$\frac{1}{\sqrt{(1 - (\frac{\omega}{\omega_0})^2)^2 + (2\frac{\omega}{\omega_0})^2}} \sim \frac{\omega_0^2}{\omega^2}$$

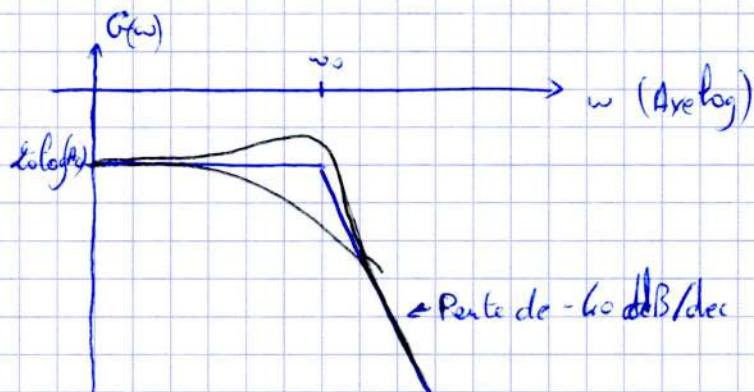
$$A_0 \sim A_0 \frac{\omega_0^2}{\omega^2}$$

\hookrightarrow asymptote de THF des passe bande

$$G \sim 20 \log \left(A_0 \frac{\omega_0^2}{\omega^2} \right) = -20 \log \omega^2 + 20 \log (A_0 \omega_0^2)$$

$$= -40 \log (\omega) + 20 \log (A_0 \omega_0^2)$$

Droite de pente -40 dB / decade .



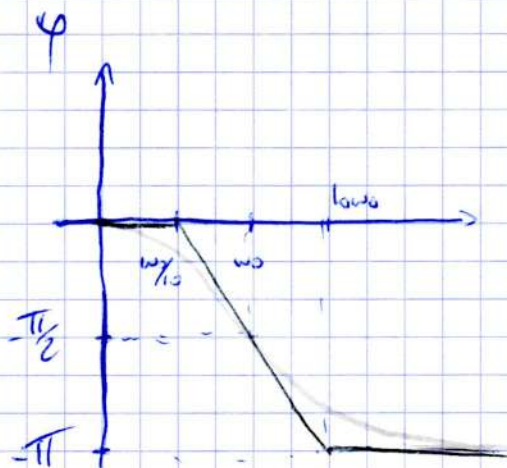
Courbe de phase

$$\begin{aligned}\varphi(\omega) &= \text{Arg}(H_0) + \arg\left(1 + 2j\zeta \frac{\omega}{\omega_0} - \left(\frac{\omega}{\omega_0}\right)^2\right) \\ &= -\arg\left(1 + 2j\zeta \frac{\omega}{\omega_0} - \left(\frac{\omega}{\omega_0}\right)^2\right) = -\text{Arctan}\left(\frac{2\zeta \frac{\omega}{\omega_0}}{1 - \left(\frac{\omega}{\omega_0}\right)^2}\right)\end{aligned}$$

TBF $\varphi(\omega) \xrightarrow{\omega \rightarrow 0} 0$

$$\varphi(\omega) \xrightarrow{\omega \gg \omega_0} -\frac{\pi}{2}$$

TBF $\varphi(\omega) \xrightarrow{\omega \rightarrow \infty} -\pi$



Filtres Passe-Haut

$$T(\omega) = A_0 \frac{-\left(\frac{\omega}{\omega_0}\right)^2}{1 + j 2 \frac{\omega}{\omega_0} - \left(\frac{\omega}{\omega_0}\right)^2}$$

$$A_0 = A(\omega) \text{ en THF}$$

Diagrammes de BodeCourbe de Gain:

$$A(\omega) = A_0 \frac{\left(\frac{\omega}{\omega_0}\right)^2}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2 \frac{\omega}{\omega_0}\right)^2}}$$

TBF

$$A \xrightarrow{\omega \rightarrow 0} 0$$

$$G \xrightarrow{\omega \rightarrow 0} -\infty \quad \text{asymptote oblique}$$

THF

$$A \xrightarrow{\omega \rightarrow \infty} A_0 \quad \text{Rq: } A_0 = A_{THF}$$

$$G \xrightarrow{\omega \rightarrow \infty} 20 \log(A_0) \quad \text{limite finie asymptote horizontale}$$

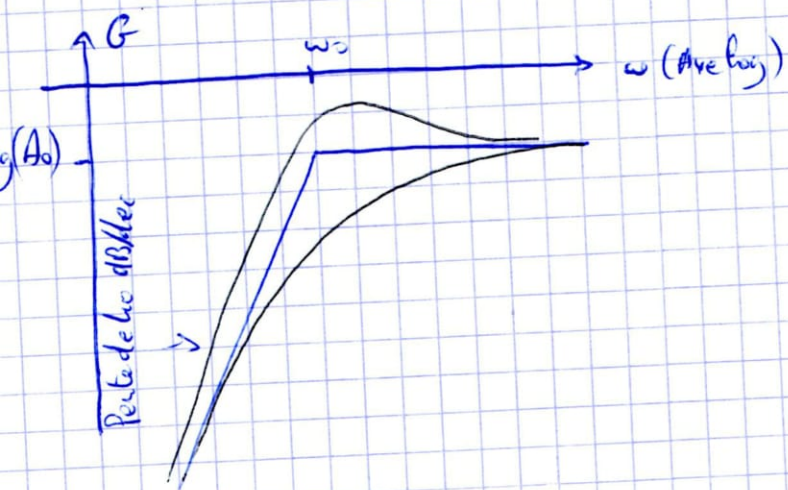
Equation de l'asymptote

$$\frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2 \frac{\omega}{\omega_0}\right)^2}} \approx 1$$

$$A_0 \left(\frac{\omega}{\omega_0}\right)^2 \approx A_0 \frac{\omega^2}{\omega_0^2}$$

$$G \approx 20 \log \left(A_0 \frac{\omega^2}{\omega_0^2} \right) = 20 \log(\omega^2) + 20 \log \left(\frac{A_0}{\omega_0^2} \right) \\ = 40 \log(\omega) + 20 \log \left(\frac{A_0}{\omega_0^2} \right)$$

Droite de pente 40 dB/dec



Courbe de phase

$$\begin{aligned}\varphi(\omega) &= \text{Arg} \left(A_0 \cdot \left(\frac{\omega}{\omega_0} \right)^2 \right) - \text{arg} \left(1 + j 2 \frac{\omega}{\omega_0} - \left(\frac{\omega}{\omega_0} \right)^2 \right) \\ &= \pi - \text{Arctan} \left(\frac{2 \frac{\omega}{\omega_0}}{1 - \left(\frac{\omega}{\omega_0} \right)^2} \right)\end{aligned}$$

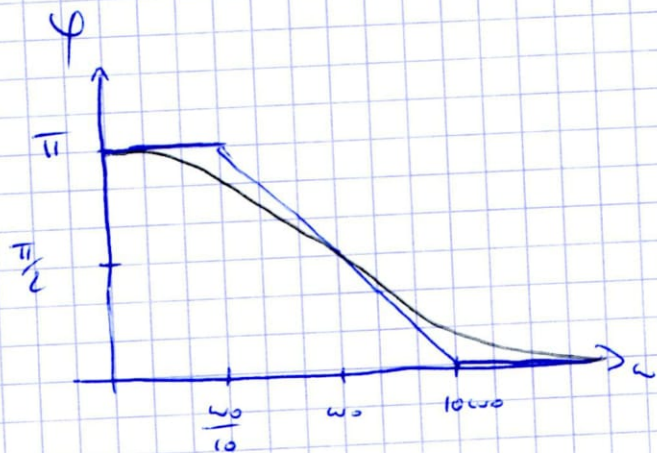
TBF

$$\varphi_{\omega \rightarrow 0} \rightarrow \pi - 0 \Rightarrow \pi$$

TBF

$$\varphi_{\omega \rightarrow \infty} \rightarrow \pi - \pi = 0$$

$$\varphi_{\omega \rightarrow \omega_0} \rightarrow \pi - \frac{\pi}{2} = \frac{\pi}{2}$$



Passé Haut 2^{ème} ordre

Filtres Passe-Bande

2^{ème} ordre

$$T(\omega) = A_{\max} \frac{2z \frac{\omega}{\omega_0}}{1 + 2z \frac{\omega}{\omega_0} + \left(\frac{\omega}{\omega_0}\right)^2}$$

Diagrammes de Bode

Courbe de gain:

$$A(\omega) = A_{\max} \frac{2z \frac{\omega}{\omega_0}}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2z \frac{\omega}{\omega_0}\right)^2}}$$

TBF

$$A \rightarrow 0$$

$$G \rightarrow -\infty \Rightarrow \text{asymptote oblique}$$

THF

$$A \rightarrow 0$$

$$G \rightarrow -\infty \Rightarrow \text{asymptote oblique}$$

TBF:

$$\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2z \frac{\omega}{\omega_0}\right)^2 \approx 1$$

$$\frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2z \frac{\omega}{\omega_0}\right)^2}} \approx 1$$

$$A(\omega) \approx A_{\max} 2z \frac{\omega}{\omega_0}$$

* Pente de 20 dB/dec.

Equation de l'asymptote:

$$G(\omega) \approx 20 \log(A_{\max} 2z \frac{\omega}{\omega_0})$$

$$\approx 20 \log(\omega) + 20 \log(A_{\max} 2z \frac{1}{\omega_0})$$

THF

$$\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 \approx \left(\frac{\omega}{\omega_0}\right)^4$$

$$\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2z \frac{\omega}{\omega_0}\right)^2 \approx \left(\frac{\omega}{\omega_0}\right)^4$$

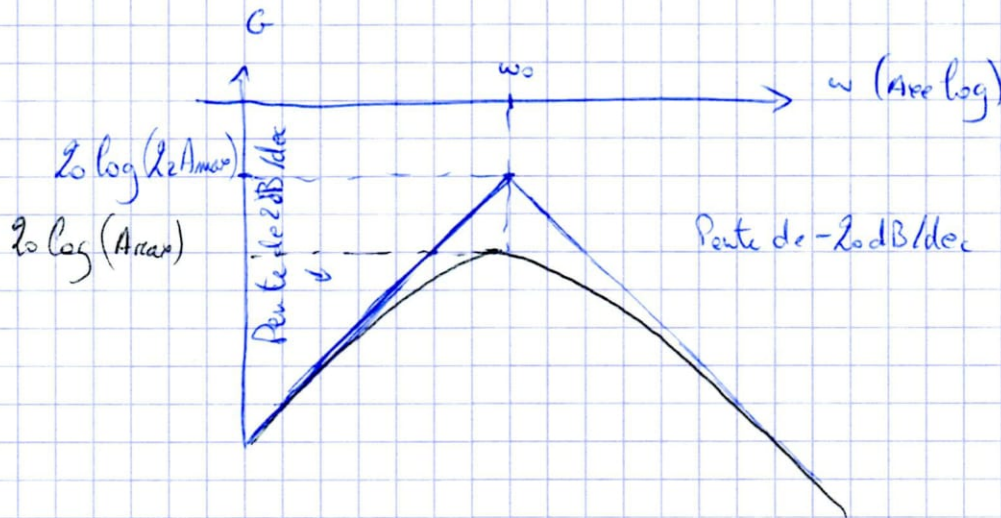
$$\frac{1}{\sqrt{\left(1 - \left(\frac{\omega}{\omega_0}\right)^2\right)^2 + \left(2z \frac{\omega}{\omega_0}\right)^2}} \approx \left(\frac{\omega_0}{\omega}\right)^2$$

$$A(\omega) \sim \begin{cases} \text{Arax} & \omega < \omega_0 \\ \text{Arax} \frac{\omega_0}{\omega} & \omega > \omega_0 \end{cases}$$

Equation de l'asymptote

$$G(\omega) \sim 20 \log(\text{Arax} \frac{\omega_0}{\omega})$$

$$\sim -20 \log(\omega) + 20 \log(\text{Arax} \omega_0) \quad \text{Pente de } -20 \text{ dB/dec}$$



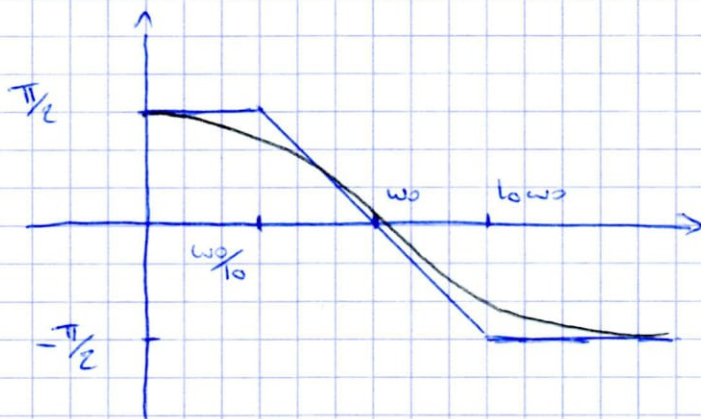
Courbe de phase

$$\begin{aligned} \varphi(\omega) &= \arg(\text{Arax} \frac{\omega_0}{\omega}) - \arg(1 + j \frac{\omega}{\omega_0} - (\frac{\omega}{\omega_0})^2) \\ &= \frac{\pi}{2} - \arctan\left(\frac{2 \frac{\omega}{\omega_0}}{1 - (\frac{\omega}{\omega_0})^2}\right) \end{aligned}$$

$$\text{IBF: } \varphi \xrightarrow{\omega \rightarrow 0} \frac{\pi}{2} - 0 = \frac{\pi}{2}$$

$$\text{THF: } \varphi \xrightarrow{\omega \rightarrow \infty} \frac{\pi}{2} - \pi = -\frac{\pi}{2}$$

$$\varphi \xrightarrow{\omega \rightarrow \omega_0} \frac{\pi}{2} - \frac{\pi}{2} = 0$$



Passé Bande $\varphi_{\text{emp}} \approx 0$

En TBF, un condensateur se comporte comme un interrupteur ouvert
une bobine se comporte comme un fil

En THF, un condensateur se comporte comme un fil
une bobine se comporte comme un interrupteur ouvert.

$$A = \frac{V_s}{V_e} \quad G = 20 \log(A)$$

$$\underline{T(\omega)} = \frac{\underline{V_s}}{\underline{V_e}}$$