Exercise Session 1

1. Consider the following discrete-time system

$$x(t+1) = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(t)$$

(a) Let $u(t) = 0$ for all t and $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Find the value of $x_2(t)$ with respect to t.

(b) Let $u(t) = 5$ for all t and $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Compute $y(t)$ with respect to t.

2. Using the sample-and-hold method, discretize the systems: (a) $\dot{x}(t) = 3u(t)$; (b) $\ddot{x}(t) = u(t)$.

3. Investigate stability of the following systems

(a) $x(t+1) = \begin{bmatrix} 3 & -1 \\ -6 & 4 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 84 \end{bmatrix} u(t)$ (b) $x(t+1) = \begin{bmatrix} 0.25 & -0.25 \\ -0.25 & 0.25 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 2 \end{bmatrix} u(t)$

$y(t) = \begin{bmatrix} 3 & 3 \end{bmatrix} x(t)$ $\qquad\qquad\qquad\qquad$ $y(t) = \begin{bmatrix} 42 & -221.7 \end{bmatrix} x(t)$

4. Investigate reachability and observability of the following system

$$x(t+1) = \begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} x(t)$$

Reachability should be investigated using reachability matrix, and observability using PBH test.

5. Consider the system

$$x(t+1) = \begin{bmatrix} 0.5 & 0 \\ 0.8 & 0.2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

The goal is to design a state feedback $u(t) = -Lx(t)$ such as to place the poles at suitable locations.

(a) Are we able to place the poles of the system at arbitrary locations?

(b) Are we able to place the poles at $\lambda_1 = 0.5$ $\lambda_2 = 0$?