Predicting Deformation of Compliant Assemblies Using Covariant Statistical Tolerance Analysis

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Abstract: In assemblies with compliant parts, dimensional variation causes misalignment between mating parts. To correct the misalignment, the compliant parts are deformed before being fastened. The resultant springback and residual stress can hinder performance. A new method uses statistical tolerance analysis and stochastic finite element analysis to predict the probable range of deformation caused by dimensional variation. To account for surface variation, a hybrid method models the surface covariance in which Legendre polynomials are used to model long wavelengths and the frequency spectrum is used to model shorter wavelengths. The hybrid covariance model accurately predicts the covariance of simulated parts and the covariance calculated from sheet-metal part data. The hybrid covariance method is an important part of an effective system for statistical analysis of variation in compliant assemblies.
Keywords: Assembly Variation; Tolerance Analysis; Stochastic Finite Element Method

1. INTRODUCTION

Thin, compliant parts, whether sheet metal, plastic, or composite, are subject to warping, distortion, misalignment and surface waviness due to manufacturing variation. The variation can cause assembled parts to spring back to a new equilibrium position, with attendant residual stresses. The final shape may have objectionable appearance or aerodynamics and the residual stresses could shorten the fatigue life. A tool that predicts the springback and residual stress in the assembled parts would be of great value to industries that rely on flexible assemblies: aerospace, automotive, electronics, etc.

Tolerance and variational analysis methods predict the amount of misalignment that will occur between parts, but cannot predict the amount of springback and stress. Compliant Statistical Tolerance Analysis (CSTA) methods have been developed to predict the amount of springback and residual stress that will occur within assemblies containing compliant parts due to manufacturing variation.

Many CSTA methods exist in the literature, employing various variational methods but universally employing finite element analysis (FEA) to account for part compliance. [Liu and Hu, 1997] combine FEA with Monte Carlo simulation and lower the

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computational cost using influence coefficients. [Chang and Gossard, 1997] simulate the assembly and measurement processes, along with the part stiffness matrix, to find the variation introduced into the assembly. [Sellem and Riviere, 1998] combine the results of Liu and Hu and Chang and Gossard to create a linear method that uses influence coefficients to find the part variation. [Camelio *et al*., 2003] use the method of Liu and Hu in a multi-station assembly method that accurately represents the assembly process and [Camelio *et al*., 2004] express the part covariance matrix in terms of its eigenvectors to identify the critical variation modes. [Merkley and Chase, 1996] develop a set of linear equations that account for part covariance in conjunction with part stiffness matrices with only two finite element solutions. [Bihlmaier, 1999] includes surface variation using spectral analysis in Merkley and Chase's method. In a similar method, [Huang and Ceglarek, 2002] use a discrete-cosine-transformation to model the part form error. [Soman, 1999] shows that the surface variation of compliant parts often have significant variation with wavelengths longer than the part-length, which cannot be accurately modeled using spectral analysis, and [Stout, 2002] presents a polynomial-based method to model such variation. [Tonks and Chase, 2004] develop a method to model long wavelength variation using a series of orthogonal polynomials.

In this work, the CSTA method of [Merkley and Chase, 1996] is summarized, the need to model the effects of surface variation is explained, and typical surface variation is investigated. A hybrid geometric covariance model that combines the work of [Bihlmaier, 1999] and [Tonks and Chase, 2004] is presented. The hybrid geometric covariance model is used to predict the geometric covariance of a set of simulated parts and the covariance calculated from measured data taken from a set of sheet-metal parts.

2. LINEAR CSTA METHOD

The CSTA method, first developed by [Merkley and Chase, 1996], finds the residual stress and springback after assembly due to dimensional and surface variation. The solution can be divided into three sections, solving for the total misalignment in the assembly; finding the covariance of the misalignment of each part, and determining the mean and standard deviation of the springback and stress in each assembled compliant part.

2.1. Calculating the Total Misalignment

The total misalignment in the assembly is due to dimensional variations from each part, as well as fixture and tooling error. To account for the sources of dimensional variation, all parts are considered rigid and statistical tolerance analysis (STA) is used to find the mean and variance of the misalignment for each set of mating parts. Many STA methods exist and are summarized in [Chase and Parkinson, 1991]. An additional STA method is presented in [Shen *et al*., 2004]. A description of this process using a vectorized STA method is found in [Mortensen, 2002].

2.2. Calculating the Misalignment Covariance

STA gives the mean and variance of the misalignment between each set of mating parts, but to solve for the springback and stress in the compliant parts, the covariance of the misalignment at the closure points (rivet/spot weld locations) is needed as is the portion of the total misalignment absorbed by each part on assembly. In the misalignment covariance matrix $\Sigma_{\delta_{\mathbf{0}}}$ the diagonal terms are the variance of the variation at each point and the off-diagonal terms are a measure of the interaction between the variations at two points. The off-diagonal terms are obtained using part data or a covariance model that accounts for part surface variation. Because $\Sigma_{\delta_{\mathbf{0}}}$ is affected only by geometric variation, it is called the *geometric covariance*. The methods used to model the geometric covariance are presented later in this work.

When two compliant parts are joined together and released, the joint moves to a force equilibrium position (see Fig 1). To find the mean and covariance of the equilibrium position for each part a stochastic finite element method (SFEM) is used, as shown in [Tonks and Chase, 2004]. This problem varies from typical SFEM, because the displacements applied to assemble the model are analyzed rather than an external load applied to a fully assembled model. Various SFEM methods exist and are summarized in [Tonks and Chase, 2004]. The method used by [Merkley and Chase, 1996] requires that each part be meshed and an equivalent stiffness matrix be created according to

$$\begin{aligned}\mathbf{K}_{eq,a} &= \left(\mathbf{K}_a + \mathbf{K}_b\right)^{-1}\mathbf{K}_b \\ \mathbf{K}_{eq,b} &= \left(\mathbf{K}_a + \mathbf{K}_b\right)^{-1}\mathbf{K}_a\end{aligned} \quad . \tag{1}$$

With the equivalent stiffness matrices and the mean of the misalignment between the parts μ_{δ_0}, the means of the misalignment of the individual parts are found from

$$\begin{aligned}\mu_{\delta_a} &= \mathbf{K}_{eq,a}\mu_{\delta_{\mathbf{0}}} \\ \mu_{\delta_b} &= \mathbf{K}_{eq,b}\mu_{\delta_{\mathbf{0}}}\end{aligned} \quad , \tag{2}$$

where $\mu_{\delta_0} = \mu_{\delta_a} + \mu_{\delta_b}$. For a linear relationship such as in Eq. (2), the covariance of the part misalignment is found from the equivalent stiffness matrix and $\Sigma_{\delta_{\mathbf{0}}}$ according to

$$\begin{aligned}\Sigma_{\delta_a} &= \mathbf{K}_{eq,a}\Sigma_{\delta_{\mathbf{0}}}\mathbf{K}_{eq,a}^T \\ \Sigma_{\delta_b} &= \mathbf{K}_{eq,b}\Sigma_{\delta_{\mathbf{0}}}\mathbf{K}_{eq,b}^T\end{aligned} \quad , \tag{3}$$

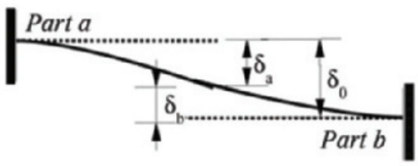


Figure 1; Equilibrium position of assembled parts as shown in [Johnson and Wichern, 2002]. The covariance of the part misalignment depends on the material stiffness and is therefore called the *material covariance*.

2.3. Stress and Springback Solution

The springback and residual stresses throughout each compliant part due to assembly are obtained from the mean and covariance of the part misalignment. An individual FE solution is performed for each compliant part, where the part has prescribed displacements at all mating edges. Two FE solutions provide maximum and minimum values for the springback and stress or, if a statistical solution is required, SFEM is used. Typically, the misalignment is small and the problem is treated as linear, elastic with small variations. If the variations are too large to be treated in this way, the resultant springback and residual stress would likely render the assembly unfit for use.

After the residual stresses and springback are found for each part, the performance of the assembly can be evaluated. If the stresses and springback violate design constraints, the critical tolerances are iterated until the performance is within design limits. To identify the critical tolerances, the critical closure points are identified with an FE sensitivity analysis and the critical tolerances are then identified with an STA sensitivity analysis. The design iterations are efficient, as they do not require repeated simulations or remeshing.

3. GEOMETRIC COVARIANCE

As explained above, when part data is not available, *i.e.* the parts are not in production, the geometric covariance is modeled because STA does not provide any covariant information. [Bihlmaier, 1999] shows that to accurately model the geometric covariance, the surface variation of the mating parts must be accounted for. It is useful to divide the surface variation into three frequency domains;

1. Warping – Wavelengths longer than the part-length

2. Waviness – From one to five wavelengths over the part-length

3. Roughness – More than five wavelengths over the part-length

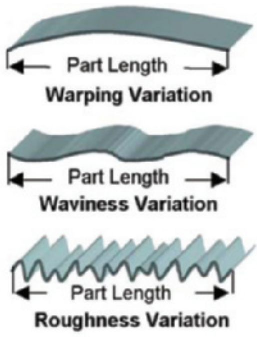


Figure 2: Examples of types of surface variation

where Figure 2 shows examples of each type of variation. The surface variation on any surface is composed of a combination of variation from the three domains and therefore an effective geometric covariance model should be capable of quantifying variation from all three domains. Often short wavelength variation also has small amplitude and need not be accounted for, but the amplitude that can be neglected varies with application and should be decided by the engineer.

Several covariance models have been proposed, each with different modeling capabilities. Promising models are presented in [Bihlmaier, 1999] and [Tonks and Chase, 2004]. These models require two sets of inputs, the variance of the misalignment at each closure point and descriptors that define typical surface variation caused by the manufacturing process. The methods are summarized below.

3.1. Frequency Spectrum Model

The geometric covariance model of [Bihlmaier, 1999] uses the frequency spectrum to model the surface variation. In the frequency spectrum model, the average autospectrum of the mating part surfaces, found by multiplying the frequency spectrum by its complex conjugate, describes the surface variation. To find the geometric covariance from the average autospectrum, **a,** the autocorrelation **c** is first found using the inverse discrete Fourier transform

$$c_i = \frac{1}{N}\sum_{0}^{N-1} a_j e^{\frac{2\pi ji}{N}} . \tag{4}$$

The autocorrelation function describes the correlation of two points on the surface and the center value corresponds to the normalized variance. To construct the geometric covariance matrix, **c** is placed along each row of the covariance matrix so that the peak value falls along the diagonal. Each row is then scaled such that the diagonal values equal the input variances at each node of the mating surfaces. Figure 3 depicts this shifting of the autocorrelation function.

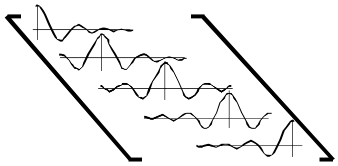


Figure 3; Finding the covariance matrix from the autocorrelation, [Bihlmaier, 1999]

The frequency spectrum model accurately predicts the covariance from waviness and roughness variation, but cannot provide information about warping variation. This is a serious shortcoming because warping is often the dominant type of surface variation in thin, compliant parts.

3.2. Orthogonal Polynomial Model

[Soman, 1999] shows that the warping variation can be separated from the waviness and roughness variation by fitting and subtracting polynomials from the variation data. [Tonks, 2002] finds that a series of orthogonal polynomials are an effective means of modeling the warping variation. He develops a model that uses a series of Legendre polynomials to model the covariance. Typical surface variation resulting from a manufacturing process is defined by the average polynomial coefficient vector **a**.

In the orthogonal polynomial model, an uncorrelated covariance matrix $\mathbf{\Sigma}_0$ is created by placing the closure point variances down the diagonal of a diagonal matrix. A correlation matrix is created according to

$$S_{ij} = \sum_{l=o}^{M-1} a_l \sqrt{\frac{2l+1}{N}} P_l(x_i) P_l(x_j) \tag{5}$$

where $P_l(x_i)$ is the lth order Legendre polynomial at point x_i. The geometric covariance, $\mathbf{\Sigma}_\delta$, is found using $\mathbf{\Sigma}_0$ and **S** according to

$$\Sigma_\delta = \mathbf{S}\Sigma_0\mathbf{S} \tag{6}$$

Orthogonal polynomials were found to accurately model warping variation, but the accuracy quickly decreased for variation with more than one wavelength over the part-length (see Table 1). Above four wavelengths, the polynomials could not accurately model the variation.

3.3. Hybrid Covariance Model

The frequency spectrum model accurately predicts the geometric covariance of waviness and roughness variation and the orthogonal polynomial model accurately predicts the geometric covariance of warping variation, but neither method models the entire range of variation. [Tonks and Chase, 2004] show that when the surface variation is dominated by warping variation the orthogonal polynomial method accurately models

Table 1; Number of polynomials needed to accurately model surface variation

Wavelengths over part-length:	**1**	**2**	**3**	**4**
Number of polynomials:	7	11	15	*inaccurate*

the geometric covariance, but when this is not the case, neither model is sufficient. [Soman, 1999] showed that after subtracting the warping variation with a polynomial fit, the remaining higher frequency variation could be modeled using the frequency spectrum method. Following this approach, a hybrid model has been developed that combines the warping variation covariance found from the orthogonal polynomial model with the waviness and roughness variation covariance found from the frequency spectrum model to obtain the full surface variation covariance.

Given a vector **a** that defines the variation of a surface, a linear combination of a vectors representing the normalized warping variation, $\mathbf{a}_w$, and the normalized waviness and roughness variation, $\mathbf{a}_{wr}$, can be obtained such that

$$\mathbf{a} = c_w \mathbf{a}_w + c_{wr} \mathbf{a}_{wr} \tag{7}$$

where c_{w} and c_{wr} scale the normalized vectors. Assuming any dependence between $\mathbf{a}_w$ and $\mathbf{a}_{wr}$ is negligible, the covariance of **a** is found from the covariances of $\mathbf{a}_w$ and $\mathbf{a}_{wr}$,

$$\Sigma_{\mathbf{a}} = c_w^2 \Sigma_w + c_{wr}^2 \Sigma_{wr} \tag{8}$$

where Σ_w is found from Eq. (6) and Σ_{wr} is found using the frequency spectrum method. The hybrid covariance model uses three variation descriptors to accurately model the geometric covariance for a general surface. The descriptors are the coefficients c_{w} and c_{wr}, the frequency spectrum of the waviness and/or roughness variation, and the polynomial coefficient matrix of the warping variation.

The hybrid method is carried out according to the following steps:

1. Model the geometric covariance of the waviness and roughness variation using the frequency spectrum method. Inputs: frequency spectrum
2. Model the geometric covariance of the warping variation using the orthogonal polynomial method. Inputs: polynomial coefficient vector
3. Combine the two covariances according to Eq. (8) to model the total covariance. Inputs: c_{w} and c_{wr}

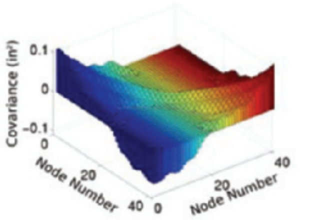
To develop the descriptors for the hybrid model, typical variation from a manufacturing process is required. The descriptors are obtained by first obtaining the warping portion of the variation data by fitting and subtracting successive polynomials; the remaining variation is the waviness and roughness portion. The warping variation and the waviness and roughness variation are then normalized such that their average values have a maximum of one. Finally, the average polynomial coefficient matrix is found from the normalized waviness variation and the average autospectrum is found from the normalized waviness and roughness variation.

4. HYBRID COVARIANCE MODEL EVALUATION

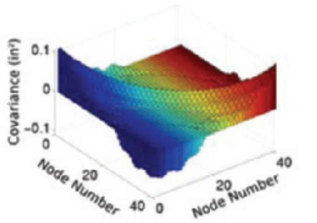
To evaluate the modeling capability of the hybrid covariance model, two comparisons are shown; first, with simulated surface variation data and second, with measured surface data.

4.1. Simulated Data

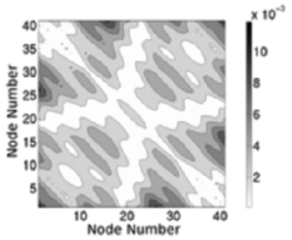
A set of surfaces were simulated as if being manufactured using a process that gives surface variation of all three types and for which descriptors of typical variation have been identified. 10,000 surfaces were generated and the covariance was calculated directly from the simulated data. The covariance was also calculated using the hybrid method according to the known variation descriptors. Figure 4 shows four plots comparing the simulated covariance to the covariance generated with the hybrid model, a 3D bar graph of the geometric covariance matrix generated by each method (Fig. 4 a. and 4 b.), a plot of the error in the hybrid covariance matrix (Fig. 4 c.), and a plot of the variances (the diagonal of the two covariance matrices) (Fig. 4 d.). The hybrid model accurately reproduced the covariance calculated from the simulated data



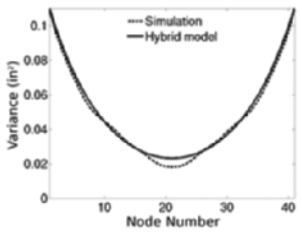
a). Simulated data covariance



b). Hybrid model covariance



c). Hybrid error plots

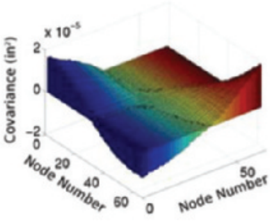


d). Variance plots

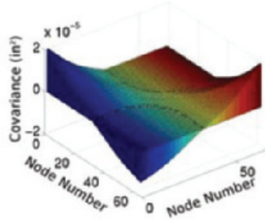
Figure 4; Comparison to simulated data

4.2. Measured Part Data

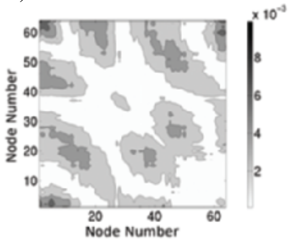
The hybrid model was also used to model the geometric covariance from a set of 6 sheet-metal parts for which the surface variation was obtained using a coordinate measure machine (CMM) and characterized using the three variation descriptors. The covariance was calculated directly from the CMM data and was also modeled using the hybrid method. Similar plots to those in Fig. 4 are shown in Fig. 5 and demonstrate that the hybrid model effectively modeled the geometric covariance of the part data.



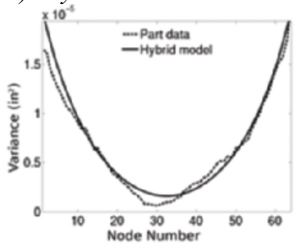
a). Part data covariance



b). Hybrid model covariance



c). Hybrid error plot



d). Variance plots

Figure 5; Comparison to measured part data

5. CONCLUSIONS

The hybrid covariance model provides a robust means of modeling the geometric covariance of surface variation in assemblies with compliant parts. Three descriptors of typical manufacturing process variation are required, but databases of these descriptors can be gathered for different processes to make this information readily available for design. The hybrid model uses the orthogonal polynomial model, requiring the polynomial coefficient matrix, for the warping variation and the frequency spectrum model, requiring the average autospectrum, for waviness and roughness variation. The two covariances are combined using the coefficients c_w and c_{wr}. The hybrid model has been shown to efficiently and accurately predict the geometric covariance from simulated and real data.

Combining STA with FEA provides a tool for performing variation analysis on compliant assemblies. The CSTA method, using the hybrid covariance model, can efficiently estimate the residual stresses and springback in each compliant part in an assembly due to the dimensional and surface variation of the parts. The analysis may be repeated with different tolerances without requiring remeshing, providing an efficient means of experimenting with tolerances to maintain the springback and residual stress within design limits. Thus, this tool provides an effective means for analysis and design of robust assemblies using rigid and compliant parts and for the simulation of current assembly processes and evaluation of product quality.

REFERENCES

[Liu and Hu, 1997] Liu, S.; Hu, S.; "Variation Simulation for Deformable Sheet Metal Assemblies using Finite Element Methods"; *Manufacturing. Sci. and Eng*., Trans. of the ASME, 119(3), pp. 368-374; 1997.

[Chang and Gossard, 1997] Chang, M.; Gossard, D.C.; "Modeling the Assemblies of Compliant, Non-ideal Parts"; Computer Aided Design, 29(10), pp. 701-708; 1997

[Sellem and Riviere, 1998] Sellem, E.; Riviere, A.; "Tolerance Analysis of Deformable Assemblies"; Design Automation Conference*, ASME Design Eng. Tech. Conf*, Atlanta, GA, 1998; DETC98 – DAC4471.

[Camelio *et al.*, 2003] Camelio, J.; Hu, S. J.; Ceglarek, D.; "Modeling Variation Propagation of Multi-Station Assembly Systems with Compliant Parts," *Journal of Mechanical Design*, 125, pp 673-681; 2003.

[Camelio *et al.*, 2004] Camelio, J.; Hu, S. J.; Marin, S. P.; "Compliant Assembly Variation Analysis Using Component Geometric Covariance," *Journal of Manufacturing Science and Engineering*, 126, pp 355-360; 2004.

[Merkley *et al.,* 1996] Merkley, K., Chase; K.W., Perry, E.; "An Introduction to Tolerance Analysis of Flexible Systems"; *MSC World Users Conference; 1996*.

[Bihlmaier, 1999] Bihlmaier, B.; *Tolerance Analysis of Flexible Assemblies Using Finite Element and Spectral Analysis*; MS. Thesis. BYU, Provo, UT; 1999.

[Huang and Ceglarek, 2002] Huang, W.; Ceglarek, D.; "Mod-based Decomposition of Part Form Error by Discrete-Cosine-Transform with Implementation to Assembly and Stamping System with Compliant Parts"; *Annals of CIRP*, 51, pp. 21-26; 2002

[Soman, 1999] Soman, S., *Functional Surface Characterization for Tolerance Analysis of Flexible Assemblies*; MS. Thesis. BYU, Provo, UT; 1999.

[Stout, 2000] Stout, J. B; *Geometric Covariance in Compliant Assembly Tolerance Analysis*; MS Thesis. BYU, Provo, UT; 2000.

[Tonks and Chase, 2004] Tonks, M. R.; Chase, K. W.; "Covariance Modeling Method for Use in Compliant Assembly Tolerance Analysis"; In*: Proceedings of DETC'04*; DETC2004-57066, SLC, UT; 2004.

[Chase and Parkinson, 1991] Chase, K.W.; and Parkinson, A.; "Survey of Research in the Application of Tolerance Analysis to the Design of Mechanical Assemblies"; *Research in Engineering*, 3, pp. 23-27.

[Shen *et al*., 2004] Shen, Z.; Ameta, G.; Shah, J. J.; Davidson, J. K.; "A Comparative Study of Tolerance Analysis Methods"; In*: Proceedings of DETC'04*; DETC 2004-57699, SLC, UT; 2004.

[Mortensen, 2002] Mortensen, A. J.; *An Integrated Methodology for Statistical Tolerance Analysis of Flexible Assemblies*; MS. Thesis, BYU, Provo, UT; 2002.

[Johnson and Wichern, 2002] Johnson, R.A.; Wichern, D.W.; *Applied Multivariate Statistical Analysis*; Prentice Hall, Upper Saddle River, N.J., p. 77; 2002;

[Tonks, 2002] Tonks, M. R.; *A Robust Geometric Covariance Method for Flexible Assembly Tolerance Analysis,* BYU, Provo, UT; 2002.