Extra credit assignment: network of LIF neurons

Random network

1. Simulate a network of n leaky integrate and fire neurons with parameters as in the first assignment. Draw the connection strength between neuron i and neuron j, $W_{i\to j}$, randomly from a uniform distribution on $[-1,1]\times w$ obeying Dale's law. Give each neuron a constant input current I. Vary w and I and for every combination of values, compute mean firing rate and coefficient of variation (CV) for every neuron (for the CV calculation, ignore neurons that generate less than 10 spike over the course of your simulation). Can you find values such that average firing rate is between 10 and 100 spikes/s and CV is close to 1? Make 2d plots (imagesc) showing the mean firing rate and the mean CV across the network as a function of I and w for two network sizes, $n = 10$ and $n = 100$. Use 1ms time bins for the integration and simulate the network for at least 10s, ideally 100s. For all your simulations (including those below) initialize the membrane potential of each neuron by drawing it from a uniform distribution between reset and threshold potential.

Sensory network

1. Divide your network into two pools of neurons. Neurons within each pool all mutually excite each other with the same connection weight, i.e. $W_{i\to j} = w_E > 0$, neurons in different pools all inhibit each other, i.e. $W_{i\to j} = w_I < 0$. The external input to the first pool is I_1, the input to the second pool is I_2. Define a linear read-out variable, $d = \sum_{i=1}^{n} v_i r_i$, where v_i is the read-out weight of neuron i whose response is r_i, to infer whether $I_2 > I_1$. Compute and plot the psychometric curve implied by your read-out where on the x-axis you plot $I_2 - I_1$ and on the y-axis you plot the probability that $d > 0$ over 100 trials for each $I_2 - I_1$ value. Keep $I_1 + I_2$ constant for each curve. Assume a trial-length of 1s. Proceed in the following way:
   - Set: $w_E = w_I = 0$, $I_1 + I_2 = 0$, $v_i = 1$ for all neurons receiving I_2 as input and $v_i = -1$ for all neurons receiving I_1 as input.
   - Increase $I_1 + I_2$ until the average firing rate (=number of spike over the course of the 1s-long trial) across all neurons is 10 spikes/s in the $I_1 = I_2$ condition.
   - Simulate your system for a range of $I_1 - I_2$ (=signal strength in a discrimination task) and plot the resulting psychometric curve.
   - Plot multiple psychometric curves for decreasing w_I (make it increasingly negative). At what value is the psychometric function the steepest? Fix w_I at that value.
2. For a value of $I_2 - I_1$ for which your read-out is about 80% correct, plot the average firing rate of the neurons receiving input I_1 and the average firing rate of the neurons receiving input I_2 as a function of time within a trial (from 0 to 1s). Average over 10 trials. To compute the average firing rate as a function of trial, simply bin the spikes into 100ms bins (10 bins per trial) and plot the average firing rate in each bin over time.

Decision-making (attractor) network

1. Now restrict the read-out to the spikes in the last 100ms of the trial and replot the psychometric curve with the current values for all the parameters. Next increase w_E and plot multiple psychometric curves showing how they change as you increase w_E. Fix w_E at the value at which the curve is the steepest (i.e. performance the best).
2. For a value of $I_2 - I_1$ for which your read-out is about 80% correct, plot the average firing rate of the neurons receiving input I_1 and the average firing rate of the neurons receiving input I_2 as a function of time within a trial (from 0 to 1s). Average over 10 trials. To compute the average firing rate as a function of trial, simply bin the spikes into 100ms bins (10 bins per trial) and plot the average firing rate in each bin over time.

Remember to clearly label your plots and to add titles and legends where helpful. Also add a few sentences to each plot explaining what you expected to see, how you interpret what you see.