# Assignment 3

*BCS 547: Computational Neuroscience*

## Computer problem 1

Simulate a population code over orientation, $\phi$, using von-Mises function-shaped tuning curves with offset $f_0$.

$$f_i(c, \phi) = f_{0,i} + c f_{\max} \exp\{\kappa[\cos(2(\phi - \phi_i)) - 1]\}$$

where $0 \leqslant c \leqslant 1$ is the contrast. Investigate the ability of two linear decoders to perform fine discrimination ($-\Delta\phi$ vs $+\Delta\phi$ , at $c = 1$) based on the population response. For the two decoders use the optimal linear weights, $w$, ignoring and including response correlations, respectively. The optimal linear weights are given by the Fisher Linear Discriminant: $\mathbf{w}^{\text{opt}} = \mathbf{C}^{-1}\mathbf{f}'$ where $\mathbf{f}'$ is the derivative, which can be approximated by $\mathbf{f}_{s2} - \mathbf{f}_{s1}$, of the population response vector with respect to the task-relevant variable and $\mathbf{C}$ is the covariance matrix.

a) Use $n = 100$ equispaced neurons (i.e. their preferred orientation tiles the $[0..\pi)$ space uniformly), $\kappa = 1$, $f_{\max} = 20$, $f_0 = 5$. Assume independent Poisson noise. Investigate the dependence of percent correct (or $d'$) on $n$ and $\kappa$.

- Write a function that returns the population response to a given stimulus orientation, $\phi$, and contrast, $c$. Plot example responses for $c = 1$ and $c = 0.1$, and for two pairs of to-be-distinguished orientations. Can you tell the population responses apart?
- Compute the covariance matrix which for this part only consists of the variances on the diagonal. Since we assume Poisson noise, the variance of each neuron is simply given by its mean. You can either compute it at the decision boundary ($\phi = 0$) or as the average of the two population responses, $C_{ii} = [f_i(c, \phi_1) + f_i(c, \phi_2)]/2$.
- Compute the read-out weights $\mathbf{w} = \mathbf{C}^{-1}[f_i(c, \phi_1) - f_i(c, \phi_2)]$ for $c = 1$ and some $\Delta\phi$ and plot them. How does $\mathbf{w}$ depend on $\Delta\phi$?
- Compute the decision variable $d = \mathbf{w}^\top \mathbf{r}$.
- Compute and plot the distributions over the decision variables for both stimuli, $\phi_1$ and $\phi_2$, and for $c = 1$, by simulating 1000 trials. How often would the decision-maker make the right decision based on $d$? ($d > 0$ corresponds to the decision that stimulus 1 was shown.) Compute the $d'$ (d-prime) of the decision-maker as the difference in the mean decision variable associated with each stimulus divided by the square root of the average variance: $d' = (\langle d_1 \rangle - \langle d_2 \rangle)/\sqrt{\text{var}d_1 + \text{var}d_2}$.
- Make a plot that shows the dependency of $d'$ on the number of neurons $n$ by varying $n$ from 2 to 1000.
- Make a plot that shows the dependency of $d'$ on the width of the tuning curves $\kappa$ by varying $\kappa$ from 0.5 to 10 while keeping $n$ constant at 100.

b) Now repeat the analysis including limited-range noise correlations such that two neurons are correlated depending on the difference in their preferred orientations: $c_{ij} = c_{\max} \exp(-|\phi_i - \phi_j|/\tau)$ where $c_{\max} = 0.3$ and $\tau = 0.5$ and repeat the above experiments. Keep $\kappa = 1$ fixed and vary $c_{\max}$ and $n$. Approximate Poisson variability by Gaussian variability where the variance of each neuron is proportional to its mean.

- Compute the covariance matrix from the correlation matrix defined above by multiplying the correlations with the variances. Assuming Poisson-like variability means that the variance equals the mean such that $C_{ij} = c_{ij}\sqrt{f_{0,i} f_{0,j}}$.
- Compute the optimal read-out weights taking the correlation into account by: $w^{\text{opt}} = \mathbf{C}^{-1}[f_i(c, \phi_1) - f_i(c, \phi_2)]$ for $c = 0.1$.
- Generating correlated Poisson spike counts is hard. However, it can be approximated by sampling responses from a correlated Gaussian distribution (implemented in Matlab).
- Plot the $d'$ for both decoders ($\mathbf{w}$ and $\mathbf{w}^{\text{opt}}$) as a function of $n$ as above, for at least 4 values of $c_{\max}$, e.g. 0, 0.1, 0.2 and 0.3.

c) Now repeat the analysis while drawing all parameters from 'reasonable' prior distributions, e.g. $\kappa$ from a Gamma-distribution with a mean of 1.5, and $f_{\text{max}}$ from a Gamma distribution with a mean of 20. For a fixed $n = 1000$, how does percent correct (or $d'$) depend on the variances of these prior distributions?

- Matlab hint: gamrnd($\mu/f$,$f$) generates a random sample from a Gamma distribution with mean $\mu$ and Fano Factor $f$.

d) Describe your general insights based on these numerical results.