Assignment 5

BCS 247: Topic in Computational Neuroscience

To goal of this exercise is to implement a generative model, and perform both exact and approximate probabilistic inference in it. Assume the Bayesian network defined in class with the following conditional probabilities:

p(sky=sunny)=0.5

sky	p(rain=on|sky)
Cloudy	0.7
Sunny	0.01

sky	p(sprinkler=on|sky)
Cloudy	0.5
Sunny	0.9

rain sprinkler	p(grass=wet|rain, sprinkler)
on on	0.99
off on	0.9
on off	0.9
off off	0.01

a) Enumerate all possible combinations (2^4 in total) over all 4 variables and compute the associated probabilities. Confirm that the probabilities sum to 1

b) Exactly (based on the table in (b)) compute the joint posterior over all unobserved variables conditioned on the grass being wet, and the grass being dry. This should yield 2 tables with 2^3 probabilities each. Confirm that the probabilities in each table sum to 1.

c) Write down the Gibbs sampling equations for each unobserved variable assuming the state of the grass is being observed. Generate 1000 full samples (i.e. 1000 for each unobserved variable) and compare the implied posterior probabilities with the exact ones obtained in (c).

d) Compute the pairwise correlations between the samples and discuss their relationship to the conditional probabilities that you defined in (a).

e) OPTIONAL: Change your conditional probabilities to make them more deterministic (i.e. closer to $1-0$ rather than $0.5-0.5$) and explain why the approximation error obtained in (d) increases.

Note: Briefly summarize/interpret each plot.