Lecture 5 Logic / Proof Theory

5.1 Logic

Logic is the science of clear thinking and correct reasoning. It is used to determine the relations between statements, and to obtain conclusions from true statements

5.2 What is a statement?

A statement is a sentence that contains facts. The facts can be TRUE or FALSE, see example below:

The sky is blue	TRUE	p
Ferraris are expensive	TRUE	q
Harrison Ford is old	TRUE	r
The sky is not blue	FALSE	¬p
Ferraris are cheap	FALSE	¬q
Harrison Ford is a girl	FALSE	¬r

P, q and r is also called a proposition. It can only be TRUE or FALSE, not both at the same time.

5.3 How to proof a statement/ proposition

There are 2 ways to proof a statement's logic – using the truth table or the laws of logic

5.4 Truth Table

5.4.1 Rules to Remember!

TRUE (T)	1	ONE
FALSE (F)	0	ZERO
V	+	OR/DISJUNCTION
Λ	x	AND/CONJUNCTION
¬		NOT/NEGATES
→		IF … THEN/CONDITIONAL

V		Λ		→		¬	
0 + 0 = 0	F	0 x 0 = 0	F	0 → 0 = 1	T	¬ 0 = 1	T
0 + 1 = 1	T	0 x 1 = 0	F	0 → 1 = 1	T	¬ 1 = 0	F
1 + 0 = 1	T	1 x 0 = 0	F	1 → 0 = 0	F		
1 + 1 = 1	T	1 x 1 = 1	T	1 → 1 = 1	T		

5.4.2 Solving the truth table

Example 1: Proofing with one variable

$$p \vee \neg p = T$$

p	¬p	$p \vee \neg p$	
0	1	0+1 = 1	T
1	0	1+0 = 1	T

All T so statement it proven (TAUTOLOGY)

Example 2: Proofing with one variable

$$p \wedge \neg p = F$$

p	¬p	$p \wedge \neg p$	
0	1	0x1 = 0	F
1	0	1x0 = 0	F

All F so statement it proven (CONTRADICTION)

Example 3: Proofing with two variables

$$\neg(p \vee q) = \neg p \wedge \neg q$$

p	q	¬p	¬q	$p \vee q$	$\neg(p \vee q)$	$\neg p \wedge \neg q$
0	0	1	1			
0	1	1	0			
1	0	0	1			
1	1	0	0			

Example 4: Proofing with three variables

Construct the truth table for $\neg(p \wedge q) \wedge (\neg r)$

p	q	r	¬r	$p \wedge q$	$\neg(p \wedge q)$	$\neg(p \wedge q) \wedge (\neg r)$
0	0	0	1			
0	0	1	0			
0	1	0	1			
0	1	1	0			
1	0	0	1			
1	0	1	0			
1	1	0	1			
1	1	1	0			

5.5 Laws of Logic

5.5.1 The Laws

1	$\neg\neg p = p$	Double Negation
2	$p \wedge q = q \wedge p$	Commutative Law
3	$p \vee q = q \vee p$	Commutative Law
4	$(p \wedge q) \wedge r = p \wedge (q \wedge r)$	Associative Law
5	$(p \vee q) \vee r = p \vee (q \vee r)$	Associative Law
6	$p \wedge (q \vee r) = (p \wedge q) \vee (p \wedge r)$	Distributive Law
7	$p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$	Distributive Law
8	$p \wedge p = p$	Idempotent Law
9	$p = p$	Idempotent Law
10	$p \wedge T = p$	Properties of TRUE (T)
11	$p \vee T = T$	Properties of TRUE (T)
12	$p \vee F = p$	Properties of FALSE (F)
13	$p \wedge F = F$	Properties of FALSE (T)
14	$p \vee p = p$	Identity Law
15	$p \wedge p = p$	Identity Law
16	$p \vee \neg p = T$	Inverse Law
17	$p \wedge \neg p = F$	Inverse Law
18	$p \rightarrow q = \neg p \vee q$	Law of Implication
19	$p \wedge (p \vee q) = p$	Absorption Law
20	$p \vee (p \wedge q) = p$	Absorption Law
21	$\neg(p \vee q) = \neg p \wedge \neg q$	De Morgan's Law

5.5.2 Solving using the laws of logic

Some tips:

a) If ask to proof the statement is a TAUTOLOGY, the goal is to get a **T**
b) If ask to proof a CONTRADICTION, the goal is to get a **F**
c) If ask to proof a CONTINGENCY, you get a variable at the end
d) If ask to proof *something* = *something*, easier to solve the longer side
e) Always solve anything inside a bracket "( )" first
f) If there is an →, you usually starts with law of implication
g) Always try to arrange same variables together
h) Always think about using the Inverse and Identity Law when you have some similar variables together

Example 1: Proof $\neg\big(p \vee (\neg p \wedge q)\big) = \neg p \wedge \neg q$

$\neg\big(p \vee (\neg p \wedge q)\big)$	Distributive
$= \neg(p \vee \neg p \ \wedge \ p \vee q)$	Inverse
$= \neg(T \wedge p \vee q)$	De Morgan's
$= F \vee \neg p \wedge \neg q$	Properties of FALSE
$= \neg p \wedge \neg q$	Solved

Example 2: Show that $(p \wedge q) \rightarrow (p \vee q)$ is a tautology

Goal is to get T to proof tautology

$(p \wedge q) \rightarrow (p \vee q)$	Implication
$= \neg(p \wedge q) \vee (p \vee q)$	De Morgan's
$= \neg p \vee \neg q \vee p \vee q$	Commutative
$= \neg p \vee p \vee \neg q \vee q$	Properties of TRUE
$= T \vee T$	Properties of TRUE
$= T$	Solved