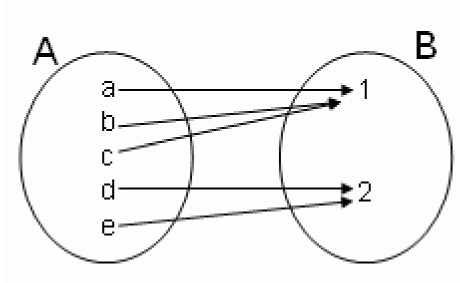
Tutorial 3

1. Name the following relations

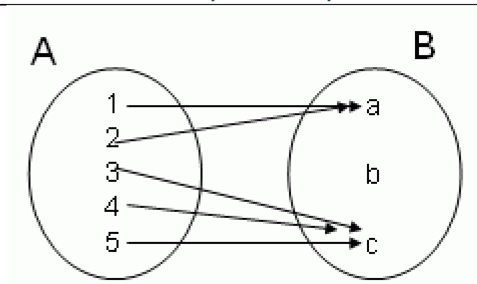
1marks each

a)



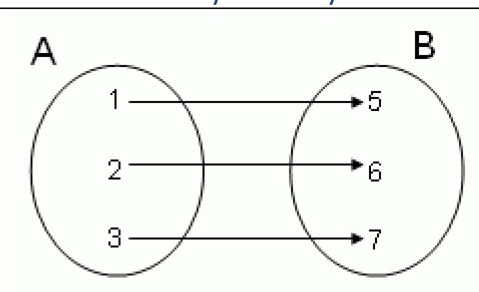
Many to Many

b)



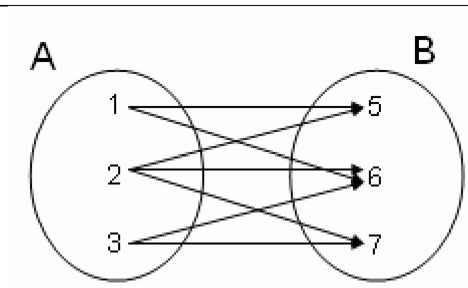
Many to Many

c)



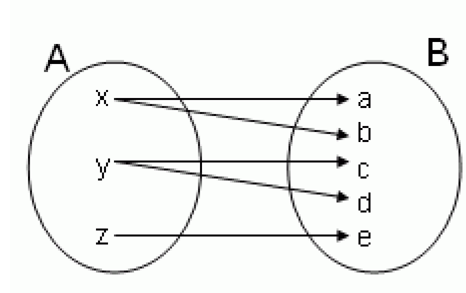
One to one

d)



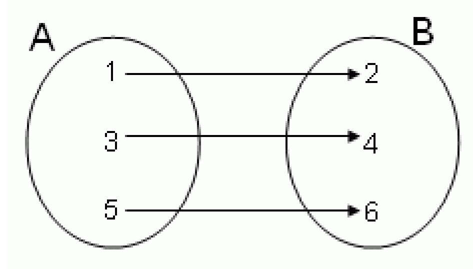
Many to many

e)



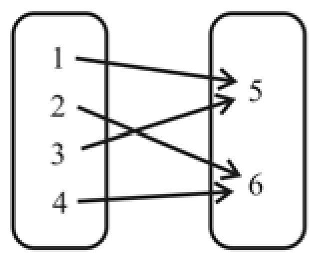
One to many

f)



One to one

2. Is the following a relation? Why?



Yes it is a relation because elements in the set on the left (domain) are mapped to elements on the set on the right (range) and it's a one-way mapping. This is called the Many to One relations

4marks

3. Determine which of the reflexive, symmetric, and transitive properties are satisfied by the given relation R defined on a set S. If the property is not satisfied give the reason why.

To tackle this question, you need to know what the rules for each property are. The rules say:

R is Reflexive if
$a \in S$ implies $(a, a) \in R$

R is Symmetric if
$(a, b) \in S$ implies $(b, a) \in R$

R is transitive if
$(a, b), (b, c) \in S$ implies $(a, c) \in R$

a) Given:
S = {1, 2, 9}
R = {(1, 1), (1, 2), (2, 1), (2, 2), (9,9)}

To solve Reflexive, look at R:
Since 1 and 2 is in S, R must have (1, 1) and (2, 2) to be reflexive. This is true, so R is reflexive

3marks

To solve Symmetric, look at R:
(1,2) and (2,1) is in R – so R is symmetric

3marks

To solve Transitive, look at R:
(1,1), (1,2) and (1,2) – true
(1,2), (2,1) and (1,1) – true
(1,2), (2,2) and (1,2) – true
(2,1), (1,1) and (2,1) – true
(2,1), (1,2) and (2,2) – true
So R is transitive

3marks

This means R is an equivalence relation because it is reflexive, symmetric and transitive on S

1marks

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b) Given
S = {1, 2, 3}
R = {(1,1), (1,3), (2,2), (2,3), (3,1), (3,2), (3,3)}

To solve Reflexive, look at R:
Since S has 1, 2, 3 in it, R must have (1,1), (2,2) and (3,3) to be reflexive.
This is true so R is reflexive

3marks

To solve Symmetric, look at R:
(1,3) and (3,1) is in R – true
(2,3) and (3,2) is in R – true
(3,2) and (2,3) is in R – true
So R is symmetric

3marks

To solve Transitive, look at R:
(1,3), (3,1) and (1,1) – true
(1,3), (3,2) and (1,2) – not true … (i)
(2,3), (3,1) and (2,1) – not true … (ii)
(3,1), (1,3) and (3,3) – true
So R is not transitive, check (i) and (ii)

3marks

This means R is reflexive, symmetric but not transitive – not an equivalence relation

1marks

4. Let A = {1, 2, 3, 4} and B = {a, b, c} and define the following two relations:

   a) r : { (a,a), (b,b), (a,b), (b,a) } (on B)

   b) s : 1 ~ 1, 2 ~ 2, 3 ~ 3, 4 ~ 4, 1 ~ 4, 4 ~ 1, 2 ~ 4, 4 ~ 2 (on A)

Given
A = {1, 2, 3, 4}
B = {a, b, c}

a) The question ask what is the relation of r on B. Rewrite as follows:
   B = {a, b, c}
   r = {(a,a), (b,b), (a,b), (b,a)}

   To check Reflexive, look at r:
   Since B has a, b and c in it, r must have (a,a), (b,b) and (c,c) in it. This is not true because (c,c) is missing in r. So r is not reflexive

   3marks

   To check Symmetric, look at r:
   (a,b) and (b,a) is in r – true
   (b,a) and (a,b) is in r – true
   So r is symmetric

   3marks

   To check Transitive, look at r:
   (a,b), (b,a) and (a,a) – true
   (b,a), (a,b) and (b,b) – true
   So r is transitive

   3marks

   This means, r is symmetric and transitive but not reflexive – not an equivalence relation ;)

   1marks

b) The question ask what is the relation of s on A. Rewrite as follows:
   A = {1, 2, 3, 4}
   s = {(1,1), (2,2), (3,3), (4,4), (1,4), (4,1), (2,4), (4,2)}

   To check Reflexive, look at s:
   Since A has 1,2,3,4 in it, s must have (1,1), (2,2), (3,3), (4,4). This is true so s is reflexive.

   3marks

   To check Symmetric, look at s:
   (1,4) and (4,1) – true
   (2,4) and (4,2) – true
   So s is symmetric

   3marks

   To check for Transitive, look at s:
   (1,4), (4,1) and (1,1) – true
   (1,4), (4,2) and (1,2) – not true … (i)
   (2,4), (4,1) and (2,1) – not true … (ii)
   (2,4), (4,2) and (2,2) – true
   s is not transitive because of the statement (i) and (ii)

   3marks

   This means s is not an equivalence relation because it is reflexive and symmetric but not transitive

   1marks