Tutorial 4 (Functions)

1. Consider a function f with Domain $P = \{-2, -1, 0, 1, 2\}$ Codomain $Q = \{1,2,3,4,5\}$ and the rule $f(x) = 5 - x^2$. Draw a mapping diagram for f and state the range.

2. Let $P = \{1,2\}$ and $Q = \{a, b\}$. Give the mappings for each possible function $f: P \to Q$ and indicate whether each is injective, surjective, bijective, or not any of them.

3. Suppose that $X = \{n \in Z : n^2 < 9\}$ and $Y = \{n \in W: n^2 < 9\}$

   (a) List all of the elements of X, and all of the elements of Y
   (b) How many subsets does X have?
   (c) How many elements are there in the set X x Y ?
   (d) How many functions $f : X \to Y$ are there?
   (e) Are there more functions from X to Y or from Y to X?
   (f) How many bijective functions $f : X \to Y$ are there?

4. Determine if the following functions is injective, surjective, or bijective. If it is bijective, find the inverse.

   $f: Z \to Z$ defined by $f(x) = 3x - 2$

5. Show that the function $f: R \to R$ where $f(x) = 3 - 4x$ is one-to-one, surjective and therefore bijective

6. Evaluate the following function when $x = -1, 0, 1$

$$f(x) = x^2 + 1 \ \ when\ x < 0$$
$$f(x) = x - 1 \quad when\ x \geq 0$$

7. Which of the following is a one-to-one function? Proof using a Cartesian graph
   a) $y\ \{(2,3), (1,2), (5,2), (3,17)\}$
   b) $y = |x|$
   c) $y = 3$

8. Find the inverse function of *f* given by
$$f(x) = (x - 3)^2, \quad x \geq 3,$$
$$f(x) = (x + 1)/(x - 2)$$
$$f(x) = (x + 1)^2, \quad x \geq -1$$
$$f(x) = (x + 1)/(x - 1,)$$
$$f(x) = x^2 - 2x$$
$$f(x) = 2 / x$$

9. For each of the following functions, write down the composite function: $f \circ g,\ g \circ f,\ f \circ f,\ g \circ g$

   a) $f(x) = 2x + 3$ and $g(x) = -x^2 + 1$
   b) $f(x) = 2x + 1, g(x) = x - 3$
   c) $f(x) = 2x - 1,\ g(x) = x^2$
   d) $f(x) = x^2,\ g(x) = 2x - 1$
   e) $f(x) = x + 3,\ g(x) = x - 3$
   f) $f(x) = x/3 - 2, g(x) = 3x^2$
   g) $f(x) = 3x^2, g(x) = x/3 - 2$