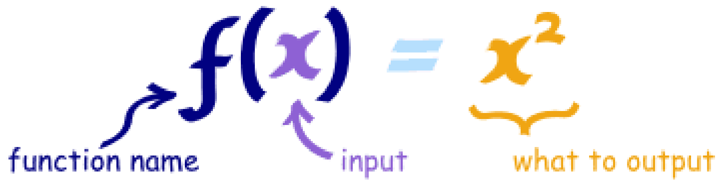
Lecture 4 Functions

4.1 What is a function?

Generally a function relates an input to an output
$f(x) = \ldots$ The classic way to write a function



4.2 Examples

$f(x) = x^2$	'f of x equals x squared'
$f(x) = x^3 + 1$	'f of x equals x cubed plus one'
$f(q) = 1 - q + q^2$	
$h(m) = m^3 + 2m + 1$	
$s(\alpha) = \alpha - 10$	
$f: x \to x^2$	same as above notation
$f: N \to N$	'f has a domain of Natural numbers, and range of N too'
$f: R \to W$	'f has a domain of Real numbers & range of Whole numbers'

Sometimes there is no function name:

$y = x^2$

$y = x^3 + 1$

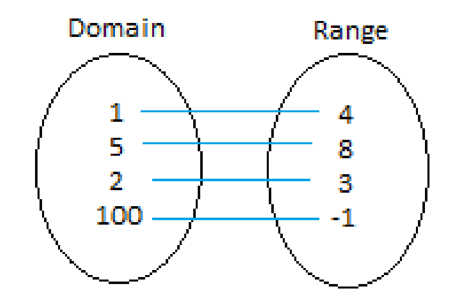
$\vdots$

4.3 What can functions do?

Function processes sets by taking elements of a set and gives back elements of a set
Which means a set can be an input and an output to a function

Function is always in the form of **ordered set** (coordinates)

Example: {(1,4), (5,8), (2,3), (100,-1)}
Where,
The input values = {1, 5, 2, 100} (domain)
The output values = {4, 8, 3, -1} (range)

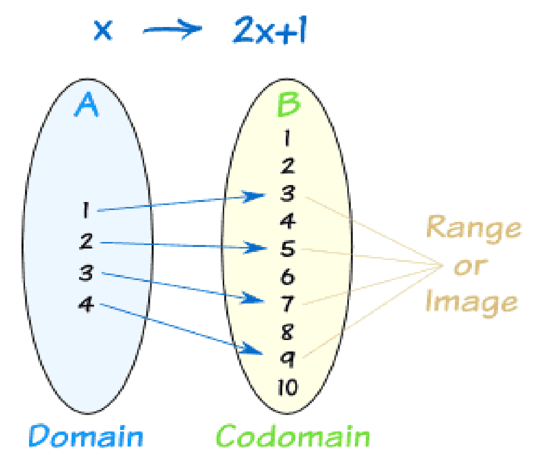


4.4 Characteristics of a function

From set A to B, a function must satisfy the following rules:

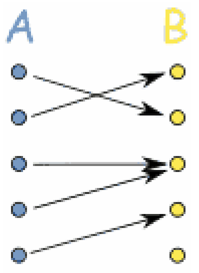
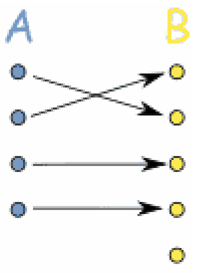
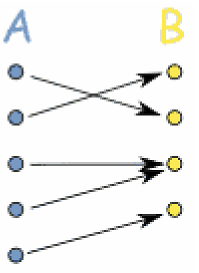
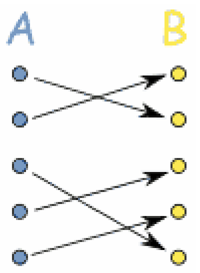
1. Each element in A must be matched with an element in B
2. Some elements in B may not be matched with any element in A
3. Two or more elements in A may be matched with the same element in B
4. An element in A (domain) cannot be matched with two different elements in B

4.5 Domain, Codomain and Range

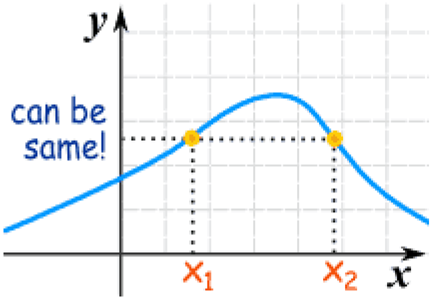
 	f(x) = 2x+1 Set A is the **domain** (input) Set B is the codomain (output) Elements in B that is selected as actual output is the **range**/image Domain = {1,2,3,4} Codomain = {1,2,3, …, 8,9,10} Range = {3,5,7,9}

4.6 Properties of Functions

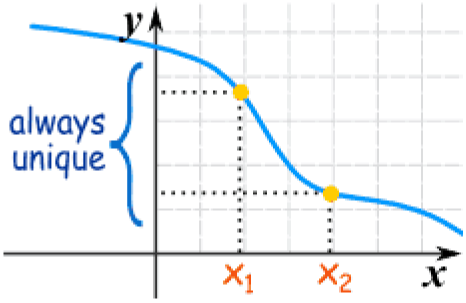
A function's behaviour is defined as **injective**, **surjective** or **bijective**

			
General Function	Injective Not surjective	Surjective Not injective	Bijective (injective and surjective)
More than one A maps to one B (**many to one**)	Only one A can map to one B (**one to one**)	All B has at least one matching A	Combination of both Injective and Surjective
B without a matching A is OK	B without a matching A is OK	B cannot be left out	Perfect one to one

4.7 On the graph



General Function



"Injective" (one-to-one)

- **Plot** the ordered pairs of a function on a Cartesian coordinate
- Do a Horizontal or a Vertical **line test**
- If the test line **intersects** more than once like in the General Function above, then it is a function just not an injective function

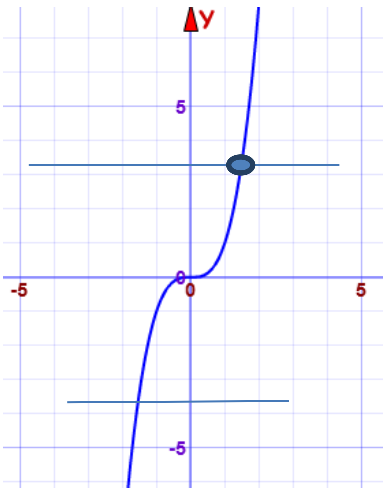
Example

State the following function:

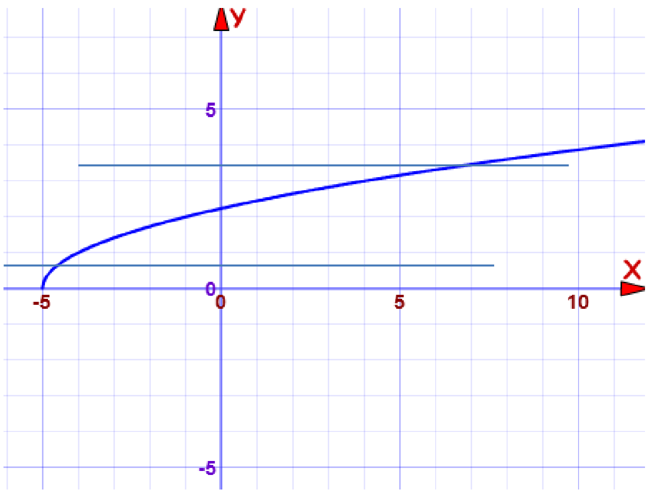
a) {(2,3), (4,5), (1,5), (3,4)}
b) {(2,3), (4,2), (1,5), (3,4)}
c) $f: R \to R$ where $y = x^3$
d) $f: R \to R$ where $f(x) = \sqrt{x+5}$
e) $f: Z \to Z$ where $y = 9 - x^2$

Solution

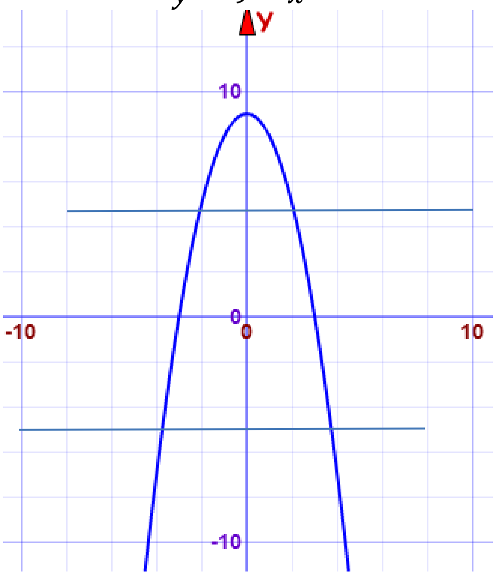
$y = x^3$



$f(x) = \sqrt{x+5}$

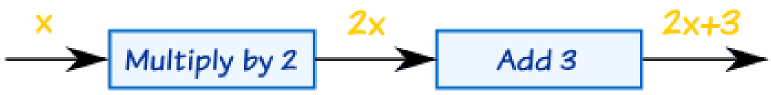


$y = 9 - x^2$

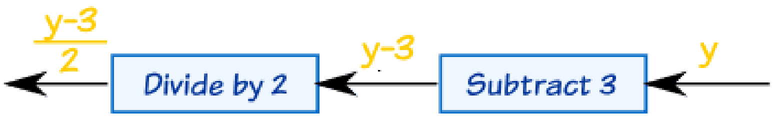


4.8 Inverse Functions

Let f be a function from set A to set B. The inverse function of f is noted as f^{-1}. Say we have $f(x) = 2x + 3$. In a flow diagram this can be represented as follow:



The inverse function of $f(x) = 2x + 3$ would be as follow:



We usually solve the inverse function using the algebra methods:

<u>Step</u>

1	The function	$f(x) = 2x + 3$
2	Put y for $f(x)$	$y = 2x + 3$
3	Solve x in the form of y	$y - 3 = 2x$ $\frac{y-3}{2} = x$
4	Swap sides	$x = \frac{y-3}{2}$
5	Put $f^{-1}(y)$ for x	$f^{-1}(y) = \frac{y-3}{2}$

4.9 Composition of Functions

Function composition is applying one function to the result of another

The result of $f()$ is sent through $g()$
You can write the composition above as $(g \circ f)(x)$ or $g(f(x))$

<u>Example</u>
Given $f(x) = 2x + 3$ and $g(x) = x^2$. Find the following composition:

a) $f \circ g = f\big(g(x)\big) = f(x^2) = 2x^2 + 3$
b) $g \circ f = g\big(f(x)\big) = g(2x + 3) = (2x + 3)^2$
c) $f \circ f = f\big(f(x)\big) = f(2x + 3) = 2(2x + 3) + 3 = 4x + 9$
d) $g \circ g = g\big(g(x)\big) = g(x^2) = (x^2)^2 = x^4$

4.10 Formula

Calculating number of functions in $f: X \to Y$

$|Y|^{|X|}$ where |Y|= number of elements in Y and
|X|= number of elements in X

Calculating number of bijective functions in $f: X \to Y$

$|X|!$ where |X| = number of elements in X

Calculating number of subsets in X

$2^{|X|}$ where |X| = number of elements in X