

Tutorial 1 (Numbers)

1(a) Given:

$$\text{LCM} = 45 \text{ HCF} \dots\dots\dots(1)$$

$$\text{LCM} + \text{HCF} = 1150 \dots\dots\dots(2)$$

Sub (1) into (2)

$$45 \text{ HCF} + \text{HCF} = 1150$$

$$46 \text{ HCF} = 1150$$

$$\text{HCF} = 1150/46$$

$$\text{HCF} = 25 \dots\dots\dots(3)$$

Sub (3) into (1)

$$\text{LCM} = 45 \text{ HCF}$$

$$\text{LCM} = 45 (25)$$

$$\text{LCM} = 1125$$

The LCM formula stated that:

$$\text{LCM} (a, b) = \frac{a \times b}{\text{HCF} (a, b)}$$

$$\text{This means: } 1125 = \frac{a \times b}{25}$$

Since one of the numbers is given as 125, we are able to find the other number as follows:

$$1125 = \frac{125 \times b}{25}$$

$$b = 225$$

5marks

1(b) Given

$$\text{LCM} (a, b) = 48 \dots\dots\dots(1)$$

Where a and b is in 2:3 ratio.

LCM is about multipliers hence we can write as follows

a : b (the 2 numbers)

2 : 3 (the ratio given)

2x : 3x (the multipliers)

4x : 6x

6x : 9x

8x : 12x

10x : 15x

: :

6x is then the LCM for a and b

Sub 6x in (1):

$$\text{LCM} (a, b) = 48$$

$$\therefore 6x = 48$$

$$x = 48/6$$

$$x = 8$$

So now we can find what each number is:

$$2x = 2(8) = 16$$

$$3x = 3(8) = 24$$

5marks

Remember that the question asks the **sum** of the two numbers, so the answer is $16 + 24 = 40$

1(c) What is the GCD (105, 1001, 2436)?

The strategy here is to find the GCD of 2 numbers (choose the two numbers example 1001 and 2436) then check if their GCD can also divide the third number (in this case, 105)

$$\text{Step1: } \begin{array}{r} 1001 \overline{)2436} \\ 2002 \rightarrow 434 \end{array}$$

$$\text{Step2: } \begin{array}{r} 434 \overline{)1001} \\ 868 \rightarrow 133 \end{array}$$

$$\text{Step3: } \begin{array}{r} 133 \overline{)434} \\ 399 \rightarrow 35 \end{array}$$

$$\text{Step4: } \begin{array}{r} 35 \overline{)133} \\ 105 \rightarrow 28 \end{array}$$

$$\text{Step5: } \begin{array}{r} 28 \overline{)35} \\ 28 \rightarrow 7 \end{array}$$

$$\text{Step6: } \begin{array}{r} 7 \overline{)28} \\ 28 \rightarrow 0 \end{array}$$

$$\text{So, GCD} (1001, 2436) = 7$$

Can 7 divide 105?

$$\text{Step 7: } \begin{array}{r} 7 \overline{)105} \\ 105 \rightarrow 0 \end{array}$$

Yes, 7 can divide 105.

So, 7 is the GCD for 105, 1001 and 2346

4marks

2. Convert Binary \rightarrow Decimal form

- (a) $10 = (1 \times 2^1) + (0 \times 2^0) = 2$
- (b) $101 = (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0) = 5$
- (c) $111 = (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = 7$
- (d) $110 = (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) = 6$
- (e) $1011 = (1 \times 2^3) + (1 \times 2^1) + (1 \times 2^0) = 11$
- (f) $1111 = (1 \times 2^3) + (1 \times 2^2) + (1 \times 2^1) + (1 \times 2^0) = 15$
- (g) $1001 = (1 \times 2^3) + (1 \times 2^0) = 9$
- (h) $1010 = (1 \times 2^3) + (1 \times 2^1) = 10$

8marks

3. Convert Decimal \rightarrow Binary form

(a) 10

- $10/2 = 5 \rightarrow 0$
 - $5/2 = 2 \rightarrow 1$
 - $2/2 = 1 \rightarrow 0$
- \therefore Answer is 1010_2

1marks

(b) 101

- $101/2 = 50 \rightarrow 1$
 - $50/2 = 25 \rightarrow 0$
 - $25/2 = 12 \rightarrow 1$
 - $12/2 = 6 \rightarrow 0$
 - $6/2 = 3 \rightarrow 0$
 - $3/2 = 1 \rightarrow 1$
- \therefore Answer is 1100101_2

1marks

(c) 111

- $111/2 = 55 \rightarrow 1$
 - $55/2 = 27 \rightarrow 1$
 - $27/2 = 13 \rightarrow 1$
 - $13/2 = 6 \rightarrow 1$
 - $6/2 = 3 \rightarrow 0$
 - $3/2 = 1 \rightarrow 1$
- \therefore Answer is 1101111_2

1marks

(d) 110

- $110/2 = 55 \rightarrow 0$
 - $55/2 = 27 \rightarrow 1$
 - $27/2 = 13 \rightarrow 1$
 - $13/2 = 6 \rightarrow 1$
 - $6/2 = 3 \rightarrow 0$
 - $3/2 = 1 \rightarrow 1$
- \therefore Answer is 1101110_2

1marks

(e) 101101

- $101101/2 = 50550 \rightarrow 1$
 - $50550/2 = 25275 \rightarrow 0$
 - $25275/2 = 12637 \rightarrow 1$
 - $12637/2 = 6318 \rightarrow 1$
 - \vdots
- \therefore Answer is $(11000101011101101)_2$

1marks

(f) 111110

- $111110/2 = 55555 \rightarrow 0$
 - $55555/2 = 27777 \rightarrow 1$
 - $27777/2 = 13888 \rightarrow 1$
 - $13888/2 = 6944 \rightarrow 0$
 - \vdots
- \therefore Answer is $(11011001000000110)_2$

1marks

(g) 101101

- $101101/2 = 50550 \rightarrow 1$
 - $50550/2 = 25275 \rightarrow 0$
 - $25275/2 = 12637 \rightarrow 1$
 - $12637/2 = 6318 \rightarrow 1$
 - \vdots
- \therefore Answer is $(11000101011101101)_2$

1marks

(h) 101110

- $101110/2 = 50555 \rightarrow 0$
 - $50555/2 = 25277 \rightarrow 1$
 - $25277/2 = 12638 \rightarrow 1$
 - $12638/2 = 6319 \rightarrow 0$
 - \vdots
- \therefore Answer is $(11000101011110110)_2$

1marks

For question 3(e) – 3(h), just show the first 4 steps of the calculation. You may use an online converter to complete it.

I used:

<http://www.binaryhexconverter.com/decimal-to-binary-converter>

4(a) Given:

$$\begin{aligned}x + y &= 8 \dots (1) \\2x - y &= 7 \dots (2) \\ \text{From (1), you get } y &= 8 - x \dots (3) \\ \text{Sub (3) into (2):} \\2x - (8 - x) &= 7 \\2x - 8 + x &= 7 \\2x + x &= 7 + 8 \\3x &= 15 \\x &= 5 \\ \text{So,} \\y &= 8 - 5 = 3\end{aligned}$$

4marks

$$\therefore x = 5, y = 3$$

(b) Given:

$$\begin{aligned}7x + 8y &= 11 \dots (1) \\5x + 6y &= 7 \dots (2) \\ \text{From (1) you get } x &= \frac{11-8y}{7} \dots (3)\end{aligned}$$

$$\begin{aligned}\text{Sub (3) into (2):} \\5\left(\frac{11-8y}{7}\right) + 6y &= 7 \\55 - 40y + 42y &= 49 \\-40y + 42y &= 49 - 55 \\2y &= -6 \\y &= -3 \\ \text{So,} \\x &= \frac{11-8(-3)}{7} = 5\end{aligned}$$

4marks

$$\therefore x = 5, y = -3$$

(c) Given:

$$\begin{aligned}x + 4y &= 3 \dots (1) \\2x + 8y &= 11 \dots (2) \\ \text{From (1) you get } x &= 3 - 4y \dots (3) \\ \text{Sub (3) into (2):} \\2(3 - 4y) + 8y &= 11 \\6 - 8y + 8y &= 11 \\6 &= 11\end{aligned}$$

4marks

\therefore Inconsistent equations thus cannot be solved

(d) Given:

$$\begin{aligned}\frac{1}{4}x + y &= 2 \dots (1) \\x &= 8 - 4y \dots (2) \\ \text{From (1) you get } x + 4y &= 8 \dots (3)\end{aligned}$$

Sub (3) into (2):

$$8 - 4y + 4y = 8$$

4marks

\therefore Inconsistent equations thus cannot be solved

5(a) $x^2 = 16$

$$\begin{aligned}x^2 - 16 &= 0 \\(x + 4)(x - 4) &= 0 \\ \therefore x &= \pm 4\end{aligned}$$

2marks

(b) $x^2 - 5x = 0$

$$\begin{aligned}x(x - 5) &= 0 \\ \therefore x &= 0 \text{ or } x = 5\end{aligned}$$

2marks

(c) $x^2 - 4 = 0$

$$\begin{aligned}(x - 2)(x + 2) &= 0 \\ \therefore x &= \pm 2\end{aligned}$$

2marks

(d) $3x^2 + x - 2 = 0$

$$\begin{aligned}(3x - 2)(x + 1) &= 0 \\ \therefore x &= \frac{2}{3} \text{ or } x = -1\end{aligned}$$

2marks

6(a) $x^2 - 2x - 1 = 0$

$$\begin{aligned}x^2 - 2x + \left(\frac{-2}{2}\right)^2 - \left(\frac{-2}{2}\right)^2 - 1 &= 0 \\(x^2 - 2x + 1) - 1 - 1 &= 0 \\(x - 1)^2 - 2 &= 0 \\(x - 1)^2 &= 2 \\x - 1 &= \sqrt{2} \\ \therefore x &= 1 \pm \sqrt{2}\end{aligned}$$

3marks

(b) $x^2 - 8x - 24 = 0$

$$\begin{aligned}x^2 - 8x + \left(\frac{-8}{2}\right)^2 - \left(\frac{-8}{2}\right)^2 - 24 &= 0 \\(x^2 - 8x + 16) - 16 + 24 &= 0 \\(x - 4)^2 &= -24 + 16 \\(x - 4)^2 &= -8 \\x - 4 &= \sqrt{8i} \\ \therefore x &= 4 \pm \sqrt{8i}\end{aligned}$$

3marks

6(c) $5x^2 - 6x = 8$

$$5\left(x^2 - \frac{6}{5}x - \frac{8}{5}\right) = 0$$

$$\left(x - \frac{6}{10}\right)^2 - \frac{8}{5} - \left(\frac{6}{-10}\right)^2 = 0$$

$$\left(x - \frac{6}{10}\right) = \pm \sqrt{\frac{49}{25}}$$

$$x = \frac{6}{10} \pm \frac{7}{5}$$

$$\therefore x = 2 \text{ or } x = -\frac{4}{5}$$

3marks

7. The quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(a) $x^2 + 4x + 0 = 0$

$$x = \frac{-(4) \pm \sqrt{4^2 - 4(1)(0)}}{2(1)}$$

$$x = \frac{-4 \pm 4}{2}$$

$$\therefore x = 0 \text{ or } x = -4$$

3marks

(b) $x^2 + 2x - 3 = 0$

$$x = \frac{-(2) \pm \sqrt{2^2 - 4(1)(-3)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4+12}}{2}$$

$$\therefore x = 1 \text{ or } x = -3$$

3marks

(c) $4x^2 + 8x + 7 = 4$

$$4x^2 + 8x + 3 = 0$$

$$x = \frac{-(8) \pm \sqrt{8^2 - 4(4)(3)}}{2(4)}$$

$$x = \frac{-8 \pm 4}{8}$$

$$\therefore x = -\frac{1}{2} \text{ or } x = -\frac{3}{2}$$

3marks

(d) $2x^2 - 7x - 13 = -10$

$$2x^2 - 7x - 3 = 0$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(-3)}}{2(2)}$$

$$x = \frac{7 \pm \sqrt{73}}{4}$$

$$x = \frac{7 + \sqrt{73}}{4} \text{ or } x = \frac{7 - \sqrt{73}}{4}$$

3marks

8. Solving the following inequalities:

(a) $x + 2 < 4$

$$x < 4 - 2$$

$$\therefore x < 2$$

2marks

(b) $\frac{x}{2} > 4$

$$x > 4(2)$$

$$\therefore x > 8$$

2marks

(c) $2x \leq 4$

$$x \leq \frac{4}{2}$$

$$\therefore x \leq 2$$

2marks

(d) $3 - 2x \geq 15$

$$-2x \geq 15 - 3$$

$$\therefore x \leq -6$$

2marks

(e) $1 - x > 0$

$$1 > x$$

$$\therefore x < 1$$

2marks

9. Solving system of inequalities

(a) $x - 2y > -12 \dots (1)$

When $x = 0, y < 6$

When $y = 0, x > -12$

y	0	6
x	-12	0

$4y + 8 \geq 2x - 4 \dots (2)$

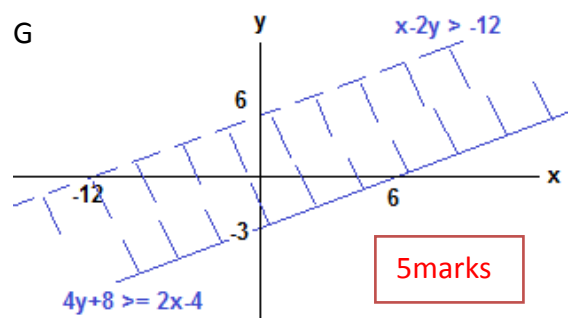
$$4y \geq 2x - 12$$

When $x = 0, y \geq -3$

When $y = 0, x \leq 6$

y	0	-3
x	6	0

Graph:



b) $y > -3x + 4 \dots (1)$

When $x = 0, y > 4$

When $y = 0, x < \frac{4}{3}$

y	4	0
x	0	$\frac{4}{3}$

$2x + 3y \geq 12 \dots (2)$

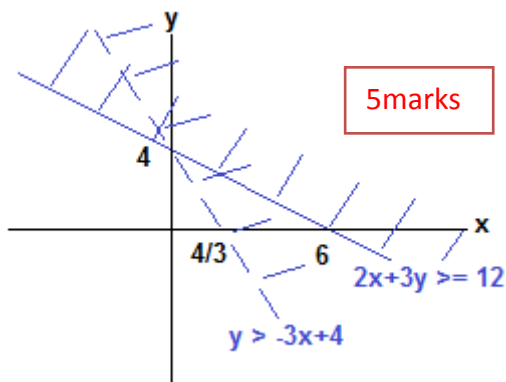
$3y \geq 12 - 2x$

When $x = 0, y \geq 4$

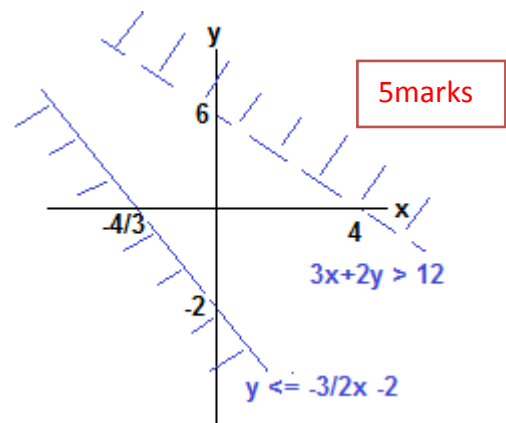
When $y = 0, x \leq 6$

y	0	4
x	6	0

Graph:



Graph:



c) $3x + 2y > 12 \dots (1)$

When $x = 0, y > 6$

When $y = 0, x > 4$

y	0	6
x	4	0

$y \leq -\frac{3}{2}x - 2 \dots (2)$

$3y \geq 12 - 2x$

When $x = 0, y \leq -2$

When $y = 0, x \leq -\frac{4}{3}$

y	0	-2
x	$-\frac{4}{3}$	0