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Lecture 1

1.1 Number

The table below provides a brief summary on the types of number in theory and applied mathematics. Please pay attention to the symbol and definition used as they are going to appear in some other modules we have in the future.

Numbers	Symbol	Examples	Brief Description
Natural	N	1, 2, 3, 4 ,5, …	Most familiar number for counting
Whole	W	0, 1, 2, 3, 4, 5, …	Starts with 0, then continues with all Natural numbers
Integers	Z	…, -3, -2, -1, 0, 1, 2, 3, …	Has all Whole Numbers including their opposites i.e. negative numbers
Real	R	$-2, \frac{\sqrt{10}}{4}, \pi, e, \sin 20^0,$ $\log 2, etc$ …	Include all measuring numbers. May have decimal, positive and negatives values, it is either a Rational number or Irrational number
Rational	Q	$\frac{1}{2}, \frac{2}{3}, \frac{10}{100}, \frac{-97}{3}, \frac{-20}{-20}, \frac{8}{2},$ 1.75, etc…	Always in the form of simple fraction ($\frac{m}{n}$) with m always an integer, and n is non-zero. All Real numbers is a Rational numbers
Irrational		$e, \pi, \log_2 3\,, \sqrt{2}$ 0.7162162162 …, -7.162162162…,	Any Real numbers that cannot be expressed as a simple fraction. The decimal expansion never repeats or terminates
Algebraic		$-4, \frac{3}{2}, 6\sqrt{7}, \frac{-3-\sqrt[5]{4}}{17.2}$	Real numbers that can occur as roots of a polynomial equations that have integer coefficients. For example, all Rational numbers are algebraic
Imaginary	I	$(e)i, 3i, -9.3i, and\ (\pi)i$	Is based on the imaginary number *i* which is equal to $\sqrt{-1}$. Any Real numbers times *i* is an imaginary number
Complex/ Imaginary	C / I	$a + bi$, or, $a + ib$ $4 + 6i, 2 - 5i,$ $3.2 + 0i, 0 + 2i$	Extension of the Real numbers with greater level of abstraction

1.2 More on Rational Numbers

- They can be written as a *ratio* of two integers (i.e. a simple fraction)

  Example: 1.5 is rational, because it can be written as the ratio 3/2
  Example: 7 is rational, because it can be written as the ratio 7/1
  Example: 0.317 is rational, because it can be written as the ratio 317/1000

1.3 More on Irrational Numbers

- But some numbers cannot be written as a ratio of two integers - they are called Irrational Numbers. It is called irrational because it cannot be written as a ratio (or fraction), not because it is crazy!

  Example: π (Pi) is an irrational number.
  Pi (π) = 3.14159265358979323846264338327950 (and more...)
  You cannot write down a simple fraction that equals Pi
  The popular approximation of 22/7 = 3.1428571428571... is close but not accurate.

  Example: *e* the Euler number (Leonhard Euler, 1720s)
  It's value is approximately 2.718281828459045... and has been calculated to 869,894,101 decimal places by Sebastian Wedeniwski

  An effective way to calculate the value of *e* is to use the following infinite sum:
  e = 1/0! + 1/1! + 1/2! + 1/3! + 1/4! + …

  As an example, here is the computation of *e* to 22 decimal places:

```
1/0!  = 1/1       = 1.00000000000000000000000000
1/1!  = 1/1       = 1.00000000000000000000000000
1/2!  = 1/2       = 0.50000000000000000000000000
1/3!  = 1/6       = 0.16666666666666666666666667
1/4!  = 1/24      = 0.04166666666666666666666667
1/5!  = 1/120     = 0.00833333333333333333333333
1/6!  = 1/720     = 0.00138888888888888888888889
1/7!  = 1/5040    = 0.00019841269841269841269841
1/8!  = 1/40320   = 0.00002480158730158730158730
1/9!  = 1/362880  = 0.00000275573192239858906526
1/10! = 1/3628800 = 0.00000027557319223985890653
1/11!             = 0.00000002505210838544171877
1/12!             = 0.00000000208767569878680990
1/13!             = 0.00000000016059043836821615
1/14!             = 0.00000000001147074559772973
1/15!             = 0.00000000000076471637318198
1/16!             = 0.00000000000004779477332387
1/17!             = 0.00000000000000281145725434
1/18!             = 0.00000000000000015619206968
1/19!             = 0.00000000000000000822063524
1/20!             = 0.00000000000000000041103176
1/21!             = 0.00000000000000000001957294
1/22!             = 0.00000000000000000000088968
1/23!             = 0.00000000000000000000003868
1/24!             = 0.00000000000000000000000161
1/25!             = 0.00000000000000000000000006
                   -----------------------------
                    2.71828182845904523536028747
```

- Another clue is that the decimal in Irrational number goes on forever without repeating

1.4 More on Ratio

- When one calculates the ratio (or the fraction $\frac{a}{b}$), it means one is performing some *division* of numbers
- The result of the division is usually called the Quotient (from a Latin word Quotiens) (This is also the reason why the Rational number's symbol is Q, due to the relation between Quotient and Ratio)
- For **example**, the result of $\frac{6}{2}$ is 3. Here, 3 is the quotient, 6 is called the dividend and 2 is called the divisor. In other words, 6 divides 2 to equal 3 with 0 remainder
  If it is $\frac{7}{2}$ then the quotient is 3 with 1 as the remainder

  Notation: $\frac{a}{b}$ = a|b which pronounces as "a divides b"
  if a does not divides b, it is notated as "a ∤ b"

  Example:
  30 = 5 x 6, which means 5 divides 30 (so does 6)

  But if the statement is:
  30 = 4 x 7 + 2, this means that 4 does not divide 30 (neither does 7)

- **The division rules**
  0 can be divided by any integer.
  Proof: $\frac{0}{2} = 0, -\frac{0}{1234} = 0$

  Every integer can be divided by 1
  Proof: $\frac{6}{1} = 6, -\frac{90}{1} = -90$

  Every integer can be divided by itself
  Proof: $\frac{2}{2} = 1, -\frac{345}{345} = 1$

1.5 Greatest Common Divisor (GCD)

- Also known as the Greatest Common Factor (GCF) or Highest Common Factor (HCF)
- The GCD of two non-zero integers is the largest positive integers that divides the numbers without a remainder

- **Example**
  The number 54 can be expressed as a product of two other integers in several different ways:

  54 x 1 = 27 x 2 = 18 x 3 = 9 x 6

  Thus the **divisors of 54** are:
  1, 2, 3, 6, 9, 18, 27, 54

Similarly **the divisors of 24** are:
1, 2, 3, 4, 6, 8, 12, 24

The numbers that these two lists share in common are the **common divisors** of 54 and 24:
1, 2, 3, 6

The greatest of these is 6. That is the **greatest common divisor** of 54 and 24 and it is written as:

gcd (54, 24) = 6

- **Other Examples:**
  The gcd of 8 and 12 is 4, written as gcd (8, 12) = 4
  The gcd of 3 and 5 is 1, written as gcd (3, 5) = 1
  The gcd of 12 and 60 is 12, written as gcd (12, 60) = 12
  The gcd of 12 and 90 is 6, written as gcd (12, 90) = 6

- **Reducing fractions**
  The greatest common divisor is useful for reducing fractions to be in the lowest term. For example, gcd(42, 56) = 14, therefore,

  $$\frac{42}{56} = \frac{3\,.\,14}{4\,.\,14} = \frac{3}{4}$$

- Two or more positive integers that have greatest common divisor 1 are said to be relatively prime to one another, often simply just referred to as being "***relatively prime***" (or "***co-prime***") as in the case of 9 and 28.

1.5.1 The Euclidean Algorithm

This is an efficient algorithm to calculate gcd by searching for the difference between the two numbers. In Step 1, simply take the larger number (a) and divide it with the smaller number (b). Remember the remainder the calculation produces. Then in Step 2, repeat the division but this time taking (b) and divide it with the remainder of the previous step. Continue repeating the division until there is no longer remainder calculated in the division.

Let a = 2322, b = 654

2322 = 654·3 + 360	gcd(2322, 654) = gcd(654, 360)
654 = 360·1 + 294	gcd(654, 360) = gcd(360, 294)
360 = 294·1 + 66	gcd(360, 294) = gcd(294, 66)
294 = 66·4 + 30	gcd(294, 66) = gcd(66, 30)
66 = 30·2 + 6	gcd(66, 30) = gcd(30, 6)
30 = 6·5	gcd(30, 6) = 6

Therefore, gcd(2322,654) = 6.

1.6 The Least Common Multiple (LCM)

- Also called the Lowest Common Multiple or Smallest Common Multiple
- The LCM of two integers, usually denoted as LCM (a, b) is the smallest positive integer that is divisible by both a and b. If either a or b is 0, then the LCM (a, b) is also 0
- This arithmetic process should be familiar to you since it is used to determine the least common denominator before fractions can be added, subtracted or compared
- The LCM of more than two integers can also be searched because it is the smallest integer that is divisible by each of them respectively

- **Example**

  The LCM of 4 and 6 is
  a) List multiples of 4:
  4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 48, 52, 56, 60, …
  b) List multiples of 6:
  6, 12, 18, 24, 30, 36, 42, 48, 54, 60, …

  Common multiples of 4 and 6 are simply the numbers that are both in the lists:
  12, 24, 36, 48, 60, …

  So from the list, the smallest number is the LCM for 4 and 6, and that is 12

1.6.1 Using Formula

It is sometimes a hassle to enlist all the common multiples as shown previously. Thus, the use of a formula makes it easier to calculate the LCM. Using GCD, the steps are as follows:

a) Find the Greatest Common Divisor (GCD) of the numbers
b) Multiply the numbers together
c) Divide the product of the numbers by the GCD

- **Example**:
  Find the LCM of 15 and 12

  Step 1
  Determine the Greatest Common Divisor of 15 and 12 which is 3

  Step 2
  Either multiply the numbers and divide by the GCD (15 x 12 = 180, then, 180/3 = 60)
  OR
  Divide one of the numbers by the GCD and multiply the answer times the other number (15/3 = 5, then, 5 x 12 = 60)

  Therefore, the LCM (15, 12) = 60

- **Example**:
  Find LCM (6, 9, 21)

  Step 1:
  Find LCM (6, 9)
  GCD (6, 9) = 3
  LCM (6, 9) = (6 x 9) / 3 = 18

  Step 2:
  Fnd LCM (18, 21)
  GCD (18, 21) = 3
  LCM (18, 21) = (18 x 21) / 3 = 126.

- Hence we can derive the formula as follows:

$$\boxed{\text{LCM }(a,b) = a \times b \,/\, \text{GCD }(a,b)}$$

1.7 Binary numbers

- Binary is a number system used by digital devices like computers, cd players, etc. Binary is Base 2, unlike our counting system decimal which is Base 10 (denary). In other words, Binary has only 2 different numerals (0 and 1) to denote a value, unlike Decimal which has 10 numerals (0, 1, 2, 3, 4, 5, 6, 7, 8 and 9). Here is an example of a binary number; 10011100

- As we can see, it is simply a bunch of zeroes and ones, there are 8 numerals in all which make this an 8 bit binary number. Bit is short for Binary Digit, and each numeral is classed as a *bit*.

- The bit on the far right, in this case a 0, is known as the least significant bit (LSB). The bit on the far left, in this case a 1, is known as the most significant bit (MSB).

- Notations used in the digital systems:
  4 bits = Nibble
  8 bits = Byte
  16 bits = Word
  32 bits = Double Word
  64 bits = Quad Word (paragraph)

- When writing binary numbers you will need to signify that the number is binary (base 2), for example, let's take the value 101. As it is written, it would be hard to work out whether it is a binary or decimal (denary) value. To get around this problem it is common to denote the base to which the number belongs, by writing the base value with the number, for example:
  101_2 is a binary number and 101_{10} is a decimal (denary) value

- Once we know the base then it is easy to work out the value, for example:
  101_2 = (1 x 2^2) + (0 x 2^1) + (1 x 2^0) = 5 (five)
  101_{10} = (1 x 10^2) + (0 x 10^1) + (1 x 10^0) = 101 (one hundred and one)
- One other thing about binary numbers is that it is common to signify a negative binary value by placing a 1 (one) at the left hand side (most significant bit) of the value, called a sign bit.

- Note that the electronically binary numbers are stored/processed using off or on electrical pulses. So a digital system will interpret these off and on states as 0 and 1. In other words if the voltage is low then it would represent 0 (off state), and if the voltage is high then it would represent a 1 (on state).

1.7.1 Converting binary to decimal

- To convert binary into decimal is very simple. Say for example we want to convert the 8 bit value 10011101 into a decimal value; we can use a formula like that below:

$2^7 = 128$	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$
1	0	0	1	1	1	0	1
128	0	0	16	8	4	0	1

Total = 128 + 16 + 8 + 4 + 1 = 157

Alternately, we may also write:
$1*2^7 + 0*2^6 + 0*2^5 + 1*2^4 + 1*2^3 + 1*2^2 + 0*2^1 + 1*2^0 = 157$

1.7.2 Converting decimal to binary

- To convert decimal to binary is also very simple, we need to divide the decimal value by 2 and then write down the remainder. Repeat this process until we cannot divide by 2 anymore.

- Example: converting the decimal value 157 into binary

157 / 2 = 78 + 1 → 1
78 / 2 = 39 + 0 → 0
39 / 2 = 19 + 1 → 1
19 / 2 = 9 + 1 → 1
9 / 2 = 4 + 1 → 1
4 / 2 = 2 + 0 → 0
2 / 2 = 1 + 0 → 0
1 / 2 = 1 + 1 → 1 Write upward to get the binary representation

Answer: 10011101_2

1.8 Linear Equations

- A linear equation is an algebraic equation of the form $y = mx + c$
- Linear equation has only a constant and a first-order (linear) term, where m is the slope and c is the y-intercept
- Occasionally, the above is called a "linear equation of two variables," where y and x are the variables
- Examples:
  $x = 9$ (A linear equations of one variable(x))
  $x + 2y - 4z - 8 = 0$ (A linear equation with three variables (x, y *and* z))

1.8.1 Solving Linear Equations

- A linear equation with one variable consists of numbers or constants and multiplies of a variable
- The standard form of such an equation is $\boldsymbol{ax + b = 0}$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are constants and $\boldsymbol{x}$ is a variable
- The solution of the equation is found by operating on the equation to get it into the form similar to $\boldsymbol{x = -b/a}$
- In other words, you want the $\boldsymbol{x}$ alone on the left side and the other items on the right side of the equation. The rule is what you do on the left side; you do on the right side
- An example:

$4a = 3 - x$	Add *x* on both sides of the equation, we get
$4a + x = 3 - x + x$	Minus 4 on both sides of the equation, we get
$4a + x - 4a = 3 - 4a$	Simplify, we get
$x = 3 - 4a$	

1.8.2 Solving Systems of Linear Equations

There are 4 common methods used for solution as listed below:

1) Substitution Method
2) Addition/Elimination Method
3) Matrix Method – will be covered in later module
4) Graphing Method (Linear Programming) – will not be covered in this course!

- Examples using the **Substitution Method**

Example 1: Given 2 linear equations as follows

$y = 2x + 1$ ... (1)
$\mathbf{2y = 3x - 2}$ ... (2)

Step 1: Use Equation (1) to substitute for y in Equation (2)

$2(2x + 1) = 3x - 2$ ... (3)

Step 2: Solve for x

$4x + 2 = 3x - 2$ ... (4)
$x + 2 = -2$ ... (5)
$x = -4$

Step 3: Replace $x = -4$ into either equation to obtain the value of y
Say equation (1) is chosen:
$y = 2(-4) + 1 = -7$

∴The solution is $x = -4$, $y = -7$

Example 2**:** Given 2 linear equations as follows

$y = 5x - 1$ ... (1)
$2y = 3x + 12$ ... (2)

Step 1: Use Equation (1) to substitute for y in Equation (2)

$2(5x - 1) = 3x + 12$

Step 2: Solve for x
$10x - 2 = 3x + 12$
$7x = 14$
$x = 2$

Step 3: Replace $x = 2$ into either equation to obtain the value of y

$y = 5(2) - 1$
$y = 9$

∴The solution is $x = 2$, $y = 9$

- Examples using the **Addition/Elimination Method**

Example1: Given 2 linear equations as follows

$$2x + y = 9 \ldots (1)$$
$$3x - y = 16 \ldots (2)$$

Step 1: Add equation (1) with equation (2), we get

$$5x = 25$$
$$x = 5,$$

Step 2: Replace $x = 5$ in (1) gives

$$2\,(5) + y = 9$$
$$y = -1$$

∴The solution is $x = 5,\ y = -1$

Example 2: Given 2 linear equations as follows

$$2x - 3y = 6 \ldots (1)$$
$$x + y = -12 \ldots (2)$$

Step 0: (2) $t$imes 2 so x coefficient is similar to (1)

$$2x + 2y = -24 \ldots (3)$$

Step 1: Minus equation (1) with equation (3), we get

$(1) - (3)\ gives,$

$$2x - 3y - (2x + 2y) = 6 - (-24)$$
$$-3y - 2y = 30$$
$$-5y = 30$$
$$y = -6$$

Step 2: Replace $y = -6$ in (2) gives

$$x + (-6) = -12$$
$$x = -12 + 6$$
$$x = -6$$

1.9 Quadratic Equations

- **Quadratic Equations** are equations in the form of (*Standard form):*

  $ax^2 + bx + c = 0$ where $a \neq 0$

- There are 4 common methods used for solution as listed below:

  1) Factoring
  2) Completing the Squares
  3) Quadratic Formula

1.9.1 Examples using the Factoring Method

Example 1:

Solve	$x^2 - 3 = 2x$
Set to zero	$x^2 - 2x - 3 = 0$
Factor	$(x - 3)(x + 1) = 0$
Solve each factor	$x = 3 \text{ or } x = -1$

Example 2:

Solve	$x^2 + 3x = 10$
Set to zero	$x^2 + 3x - 10 = 0$
Factor	$(x + 5)(x - 2) = 0$
Solve each factor	$x = -5 \text{ or } x = 2$

Example 3:

Solve	$5x^2 = 20$
Set to zero	$5x^2 - 20 = 0$
	$x^2 - 4 = 0$
Factor	$(x - 2)(x + 2) = 0$
Solve each factor	$x = 2 \text{ or } x = -2$

1.9.2 Theory used in the Completing the Square Method

Step1: Make sure that the coefficient on the x^2 term is equal to 1
If the coefficient of the x^2 term is already 1, then proceed to **Step 2**

Step2: Isolate the x^2 and x terms
In other words, rewrite it so that the $x^2$and x terms are on one side and the constant is on the other side

Step3: Complete the square

- Note:
At this point we will be creating a perfect square trinomial (PST). A PST is a trinomial of the form $x^2 \pm 2xy + y^2$ and it factors in the form $(x \pm y)^2$. When it is in that form it will allow us to continue onto the next step and take square root of both sides and finds a solution

We need to find a number that we can add to the x^2 and x terms so that we have a PST

We can get that magic number by doing the following:

Say we have	$x^2 + bx$
We can complete the square by adding the constant	$\left(\frac{b}{2}\right)^2$

In other words, we complete the square by taking ½ of b (the coefficient of the x term) and then squaring it. Make sure you remember to add it to BOTH sides to keep the equation balanced

- Example 1:
Solve $x^2 - 10x = -9$ by completing the square

$$\begin{aligned} x^2 - 10x &= -9 \\ b &= \left(-\frac{10}{2}\right)^2 = 25 \\ x^2 - 10x + 25 &= -9 + 25 \\ (x-5)^2 &= 16 \\ x - 5 &= \sqrt{16} \\ x &= 5 \pm 4 \\ \therefore\ x &= 9 \text{ or } x = 1 \end{aligned}$$

There are two solutions to this quadratic equation which is $x = 9$ or $x = 1$

- Note:
You can solve ANY quadratic equation by completing the square. This comes in handy when a quadratic equation does not factor or is difficult to factor

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1.9.3 Theory used in the Quadratic Formula Method

When the Quadratic Equation is written in the Standard Form of

$$ax^2 + bx + c = 0$$

then the roots of the equation can be obtained from the following formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

- Note :
  You can solve ANY quadratic equation by using the quadratic formula. This comes in handy when a quadratic equation does not factor or is difficult to factor

  Example 1:

  Solve $2x^2 - 5x + 1$ by using the quadratic formula

$$2x^2 - 5x + 1 = 0$$
$$a = 2, b = -5, c = 1$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-5) \pm \sqrt{25 - 4(2)(1)}}{4}$$
$$x = \frac{5 \pm \sqrt{17}}{4}$$

  There are two solutions to this quadratic equation which is

$$x = \frac{5 + \sqrt{17}}{4} \; or \; x = \frac{5 - \sqrt{17}}{4}$$

- **Discriminants**
  When the quadratic equation is in standard form($ax2 + bx + c = 0$), the expression $b^2 - 4ac$ that is found under the square root part of the quadratic formula is called the discriminant
- The discriminant can tell you how many solutions there are going to be and if the solutions are real numbers or complex imaginary numbers

Discriminant $b^2 - 4ac$	**Solution for** $ax^2 + bx + c = 0$
$b^2 - 4ac > 0$	**Two distinct real solutions** Note that the value of the discriminant is found under the square root and there is a + or - in front of it. So, if that value is positive, then there would be two distinct real number answers
$b^2 - 4ac = 0$	**One real solution** Note that the value of the discriminant is found under the square root and there is a + or - in front of it. So, if that value is zero, + or - zero is the same number, so there would be only one real number solution.

$b^2 - 4ac < 0$	**Two distinct complex imaginary solution** Note that the value of the discriminant is found under the square root and there is a + or - in front of it. So, if that value is negative, then there would be two distinct complex imaginary number answers.

Example 2:

Find the discriminant for the equation $3x^2 + x + 10 = 0$. Based on the 2 discriminant, indicate how many and what type of solutions there would be

$$3x^2 + x + 10 = 0$$
$$a = 3, b = 1, c = 10$$

$$b2 - 4ac = 1^2 - 4(3)(10) = -199$$
$$\therefore b^2 - 4ac < 0$$

Therefore, there are two distinct complex imaginary solutions

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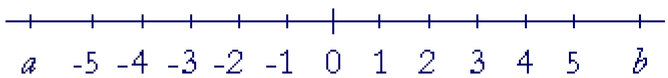
1.10 Inequalities

Sign	Meaning
$<$	Is smaller than
$>$	Is bigger than
$\leq$	Is smaller or equal to
$\geq$	Is bigger or equal to

- What do these mean:
  a) $2 < 5$
  b) $5 > 2$

- If < and > are the two signs to describe an inequality, what can you say about this number line? Gives some examples:



1.10.1 Basic Rules of Inequalities

- Inequalities usually have many solutions
- "Solving'' an inequality means finding all of its solutions. A "solution'' of an inequality is a number which when substituted for the variable makes the inequality a true statement.
- Here is an example: Consider the inequality
$$x - 2 > 5$$
- What can x be?

- Solve x for these inequalities:
a) $-2 < 10 - x$
b) $-x > 12$
c) $12 + x > 0$
d) $6 - x > -9$

- Are these the same?
a) $-2 < 10 - x$ and $2 < x - 10$
b) $-x > 12$ and $-x < -12$
c) $12 + x > 0$ and $12 - x < 0$
d) $6 - x > -9$ and $-9 + x < -6$

- <u>Note:</u>
Always make sure that if you add/subtract/multiply/divide anything to on the left hand side, you have to perform the same manipulations to the right hand side

- Only change the sign when there is a negation sign to x
<u>Examples:</u>
$-x < -10 \quad which\ means \quad x > 10\ \ or\ (10, \infty)$
$5 > -x \quad which\ means \quad x > -5 \quad or \quad (-5, \infty)$
$-2x > 4 \quad gives \quad -x > \frac{4}{2} \quad which\ means \quad x < -2 \quad or \quad (-\infty, -2)$

- Or when you multiply both sides with a negative number
<u>Examples:</u>
$-2 < 5$ (Multiply both sides with -4 gives) $8 > -20$

- Other than that, the sign pretty much stay the same
<u>Examples:</u>
$x + 2 \geq 6 \quad means \quad x \geq 6 - 2 \quad means \quad x \geq 4$
$2x - 1 \leq 0 \quad means \quad 2x \leq 1 \quad means \quad x \leq \frac{1}{2}$

- However,
$x^2 > x$ is not $x > 1$
Because we do not know at this point if x will be positive or negative
Proof: if x=-2, the left side and the right side is not the same

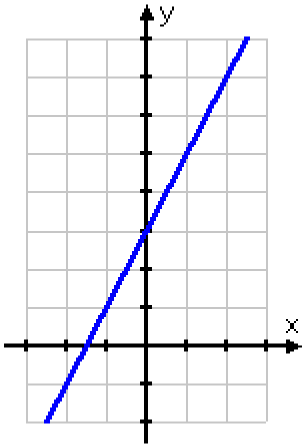
1.10.2 Graphing Inequalities

- Graph the solution to $y \leq 2x + 3$
  Step 1: Find the x and y axis

  When $x = 0, then\ y \leq 3$
  When $y = 0, then\ x \leq -\frac{3}{2}\ \ or\ \ x \geq -\frac{3}{2}$

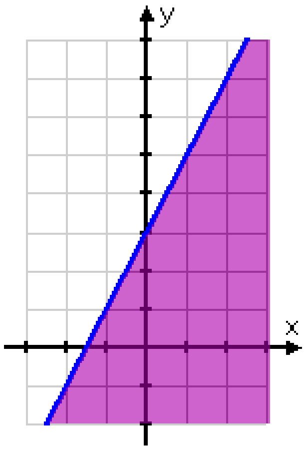
  Mark this information on the graph

  Join the two points on the graph and we have the following:



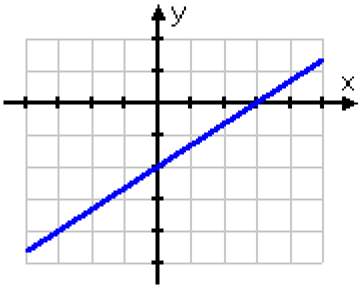
For equations which are y is **SMALLER THAN**, shade **BELOW** the line
For equations which are y **BIGGER THAN** shade **ABOVE** the line

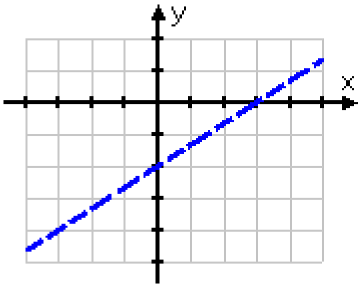
So for, $y \leq 2x + 3$, we get the following:



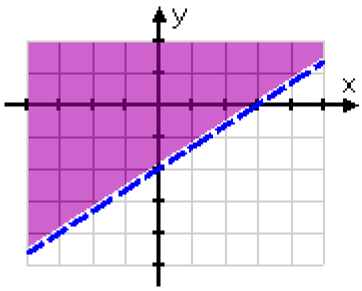
- Graph the solution to $2x - 3y < 6$

  When $x = 0, y > -2$
  When $y = 0, x < 3$





Note: The border is a dashed line, because this is a ‘strict’ inequality (sign is NOT ≥). Since this is a "*y* greater than" inequality, shade *above* the line, so the solution looks like this:



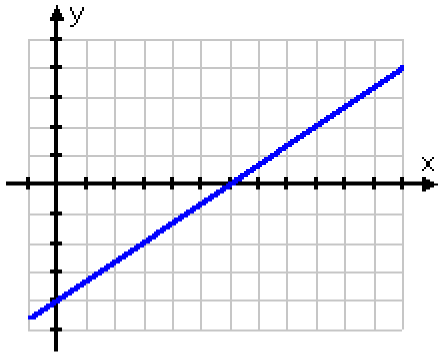
1.10.3 Solving Systems of Linear Inequalities

- Solving systems of linear inequalities means graphing each individual inequality, and then finding the overlaps of the various solutions
- <u>Example:</u>
  Given 2 inequalities
  $$2x - 3y \leq 12 \ldots (1)$$
  $$x + 5y \leq 20 \ldots (2)$$

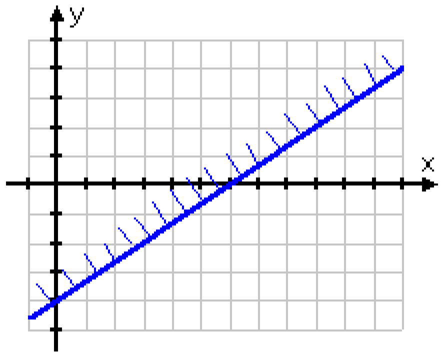
  So graph each inequality, and then find the overlapping portions of the solution regions

For (1): When $x = 0, y \geq -4$
When $y = 0, x \leq 6$

The line for the first inequality in the above system, $2x - 3y \leq 12$, looks like this:

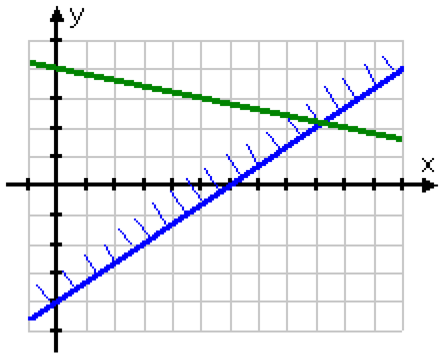


This inequality is a "bigger than" inequality, so shade above the line. However, since there will be more than one inequality on this graph, we just top the region by drawing a little "fringe" along the top side of the line, like this:

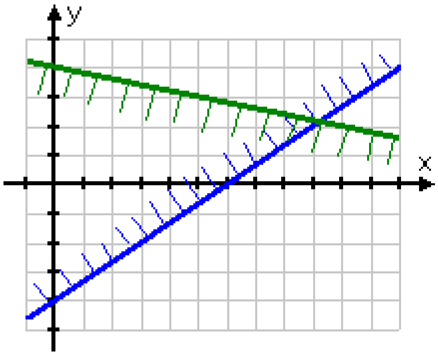


For (2): When $x = 0, y \leq 4$
When $y = 0, x \leq 20$

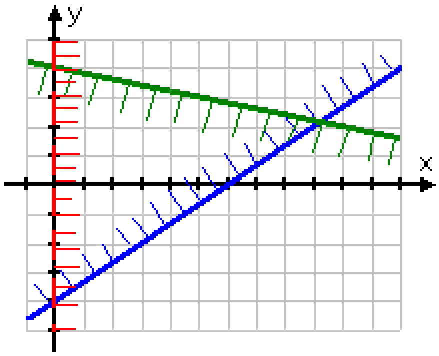
Now graph the line for the inequality (2) above is as follows:



...and, since this is a "less than" inequality, draw the fringe along the bottom of the line:



The last inequality is a common "real life" constraint: only allowing x to be positive (the line $x = 0$ on the y-axis)



The "solution" of the system is the region where all the inequalities are happy; that is, the solution is where all the inequalities work, the region where all three individual solution regions overlap. In this case, the solution is the shaded part in the middle: