Selecting the Right Algorithm

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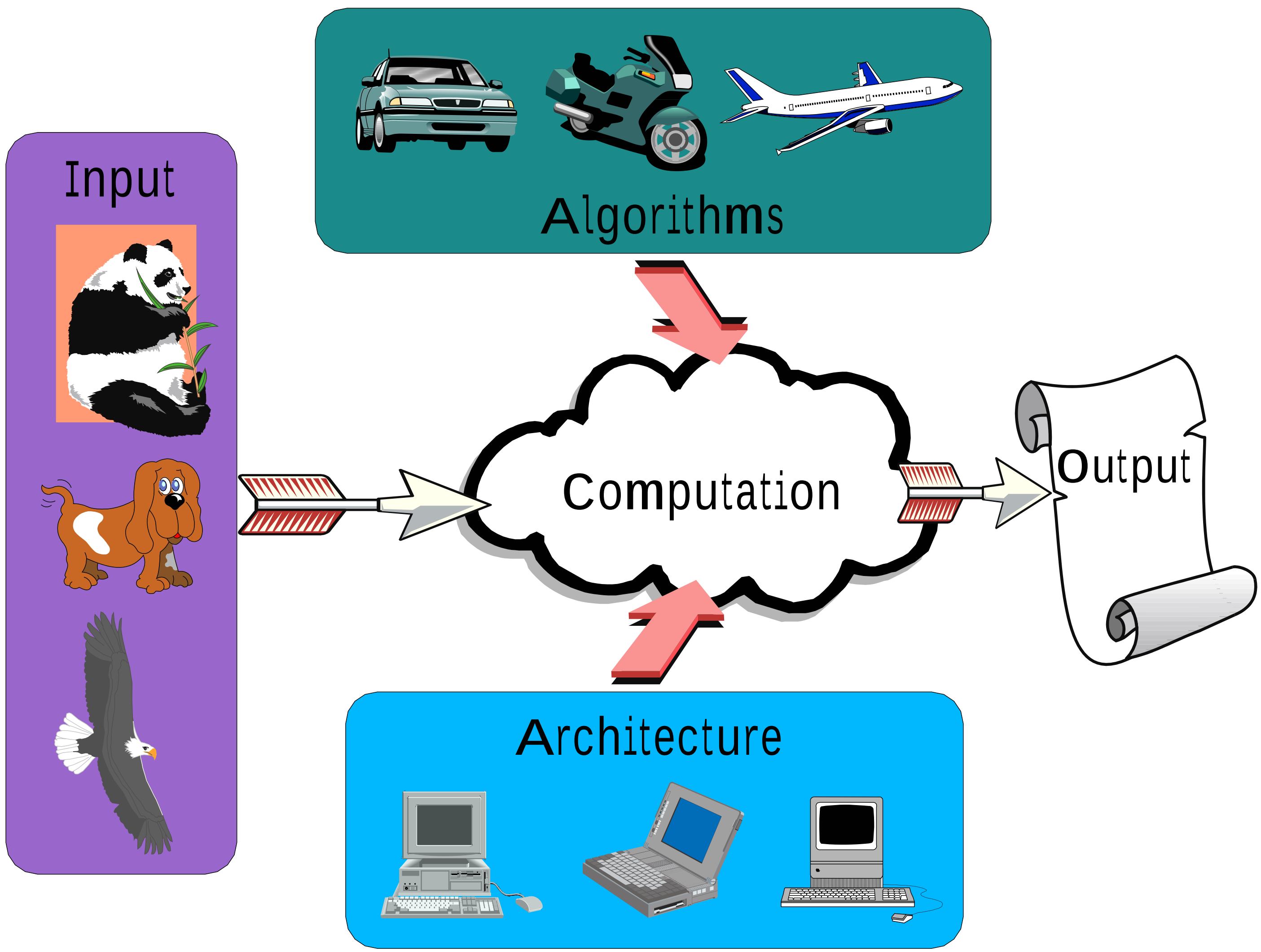
Uncertainty in Computation



A Little Quiz ...

Problem

Add some numbers

Question

Which method would you choose

?

- Paper and pencil
- Calculator
- MatLab program
- Pay somebody

The Answer ...

Answer

It depends!

- How many numbers?
- How many digits per number?
- What base?
- What format?

Now, it's clear!

- 142 + 304
- 34A4C324D6 + 7C37B5E349 + 1A8582234C + 326BDFFFF4 + ...

Sorting ...

Problem

Sort some numbers

Question

Which algorithm would you choose

?

- Bubble Sort
- Insertion Sort
- Quick Sort
- Merge Sort
- Shell Sort
- Distribution Sort
- Radix Sort
- Heap Sort

Outline

- *(Recursive) Algorithm Selection*
- *Markov Decision Processes (MDPs)*
- *Algorithm Selection as an MDP*
- *Case Study: Sorting*
- *Case Study: Results*
- *Discussion*
- *Related Work*
- *Conclusion*

Algorithm Selection

Given

- A *set of algorithms* for a given problem.
- A description (*set of features*) of the current instance.

Goal

- Dynamically *select the "**right**" (fastest) algorithm* for any given instance based on its features.

Recursive Algorithm Selection

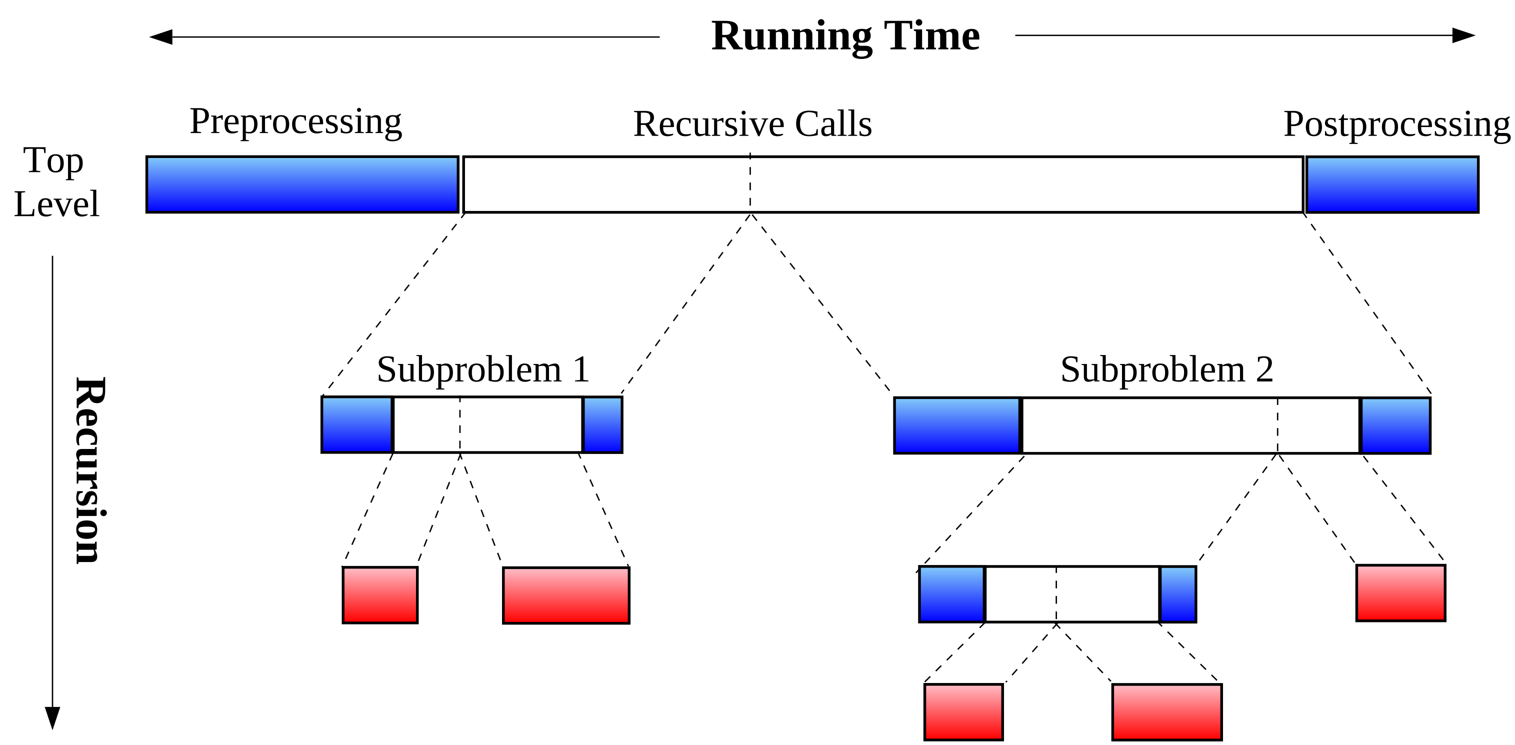
Previous Work

- Algorithms are treated as black-boxes
- *Cannot* do better than the best of the individual algorithms

Our work

- **Recursive** algorithms $\Longrightarrow$ **Multiple** algorithm selection problems
- *Any* algorithm can be selected at *each* recursive call
- Algorithm selection is viewed as a ***sequential decision task***
- Yields a hybrid algorithm!
- *Can* potentially do better than the best of the individual algorithms!

Recursive Computation



Markov Decision Processes (MDPs)

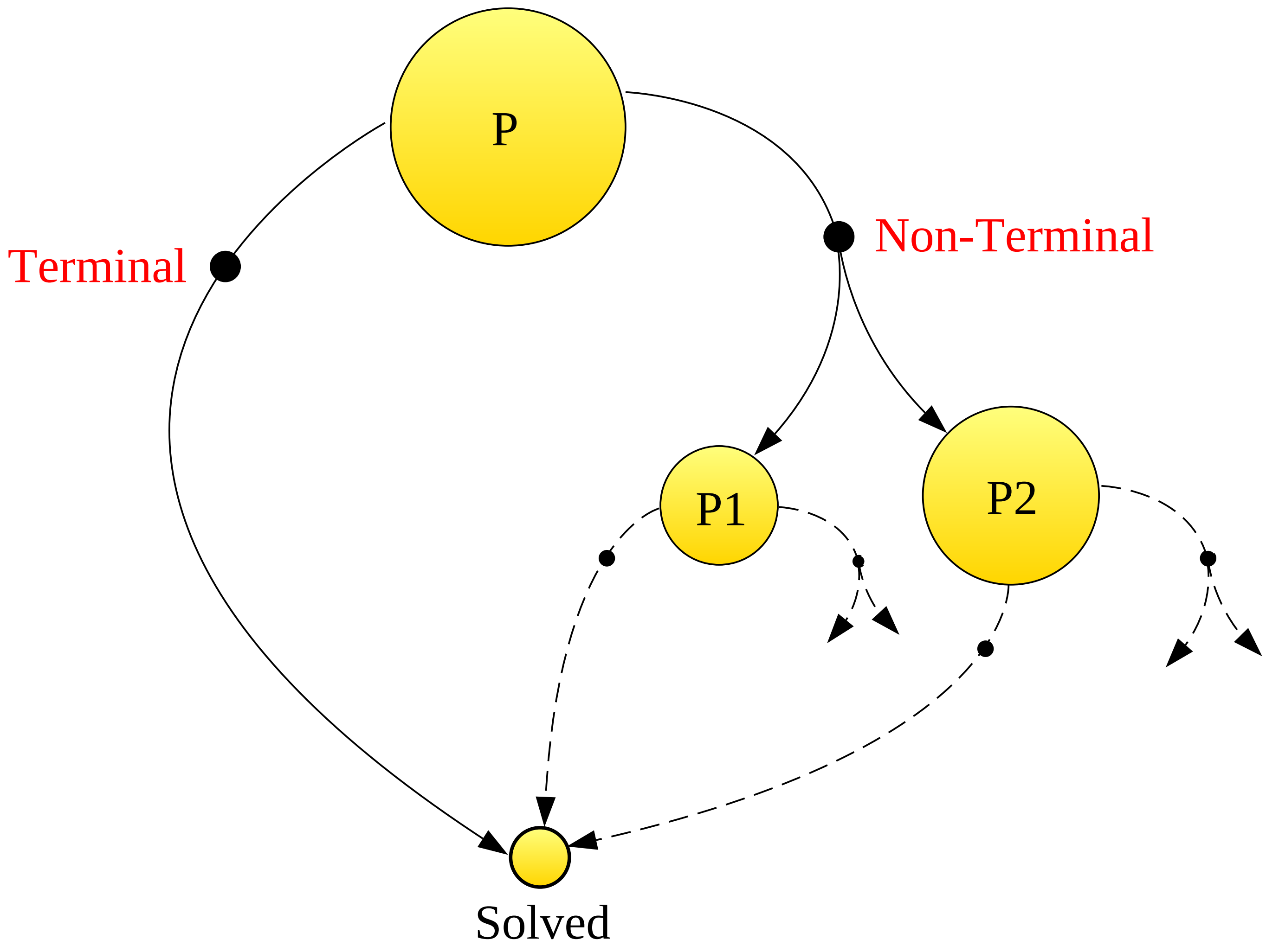
- **States** : $\mathcal{S}, \quad |\mathcal{S}| = n$
- **Actions** : $\mathcal{A}, \quad |\mathcal{A}| = m$
- **Transition Model** : $\mathcal{P}(s, a, s') = P(s'|s, a)$
- **Cost Function** : $\mathcal{R}(s, a)$
- **Episodes** : $s_1 \xrightarrow[r_1]{a_1} s_2 \xrightarrow[r_2]{a_2} s_3 \xrightarrow[r_3]{a_3} s_4 \quad \ldots \xrightarrow[r_{N-1}]{a_{N-1}} s_N$
- **Policy** : $\pi : \mathcal{S} \mapsto \mathcal{A}$
- ***Optimal policy*** : π^* (minimizes the expected total cost):

$$\pi^* = \arg\min_{\pi} E_{\pi} \left(\sum_t r_t \right)$$

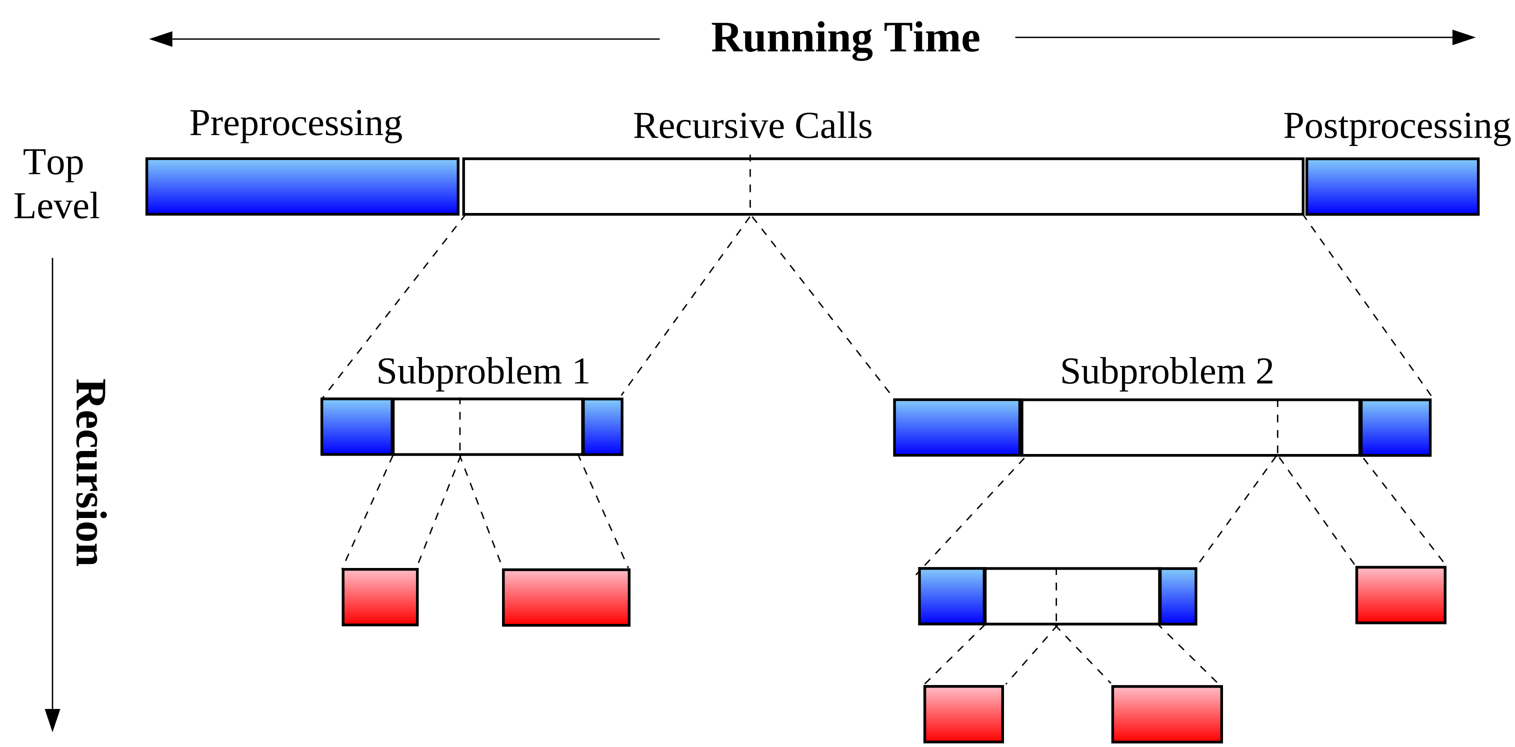
Algorithm Selection as an MDP

- ***Task***: Solve the current instance (episodic task)
- ***State***: Current instantiation of instance features
- ***Actions***: The available algorithms (terminal and non-terminal)
- ***State Transitions***: One-to-Many for non-terminal algorithms!
- ***Transition Cost***: Real time taken (excluding time in recursive calls)
- ***Total Cost***: The total execution time
- ***Objective***: A policy that minimizes the total cost

State Transitions



Time Cost


Running Time
Preprocessing
Recursive Calls
Postprocessing
Top
Level
Recursion
Subproblem 1
Subproblem 2

Case Study: Sorting

Problem

- Rearrange an array of n (unordered) numbers in ascending order.

Algorithms

- **InsertionSort** (terminal, worst-case $O(n^2)$)
- **Randomized QuickSort** (recursive, worst-case $O(n^2)$, average case $O(n \log n)$)
- **MergeSort** (recursive, worst-case $O(n \log n)$)

Sorting as an MDP

- States: Size of the array s
- Actions: The algorithms I, M, and Q.
- Transition Model? Cost Function?

Transition Model

InsertionSort (I)

$$\mathcal{P}(s, I, 1) = 1$$

MergeSort (M)

$$\mathcal{P}(s, M, \{\lfloor s/2 \rfloor, \lceil s/2 \rceil\}) = 1$$

QuickSort (Q)

$$\mathcal{P}(s, Q, \{1, s-1\}) = 2/s \quad \text{and} \quad \mathcal{P}(s, Q, \{p, s-p\}) = 1/s \ , \ p \in [2, s-1]$$

All transitions to states of smaller size!

Cost Function

The only unknown component.

Idea: *Estimate the cost function experimentally!*

- Run each algorithm on many inputs of different sizes.
- Measure the real execution time consumed for each transition.
- Average the measurements and store in a table.
- Execute on the target architecture.

Cost functions

$$I(s) = \mathcal{R}(s, I) \qquad M(s) = \mathcal{R}(s, M) \qquad Q(s) = \mathcal{R}(s, Q)$$

Dynamic Programming

- $Opt(s)$: the minimum expected cost for sorting an input of size s
- $Opt(1) = 0$
- If $Opt(.)$ is known up to size $s - 1$, then

$$Opt(s) = \min\{optQ(s), optI(s), optM(s)\}$$

- $optX(s)$: the total expected cost of choosing action $X \in \{I, Q, M\}$ in state s and following the optimal policy thereafter.
- Optimal choice :

$$\pi^*(s) = \arg\min\{optQ(s), optI(s), optM(s)\}$$

Dynamic Programming (cnt'd)

InsertionSort

$$optI(s) = I(s)$$

MergeSort

$$optM(s) = M(s) + Opt(\lceil s/2 \rceil) + Opt(\lfloor s/2 \rfloor)$$

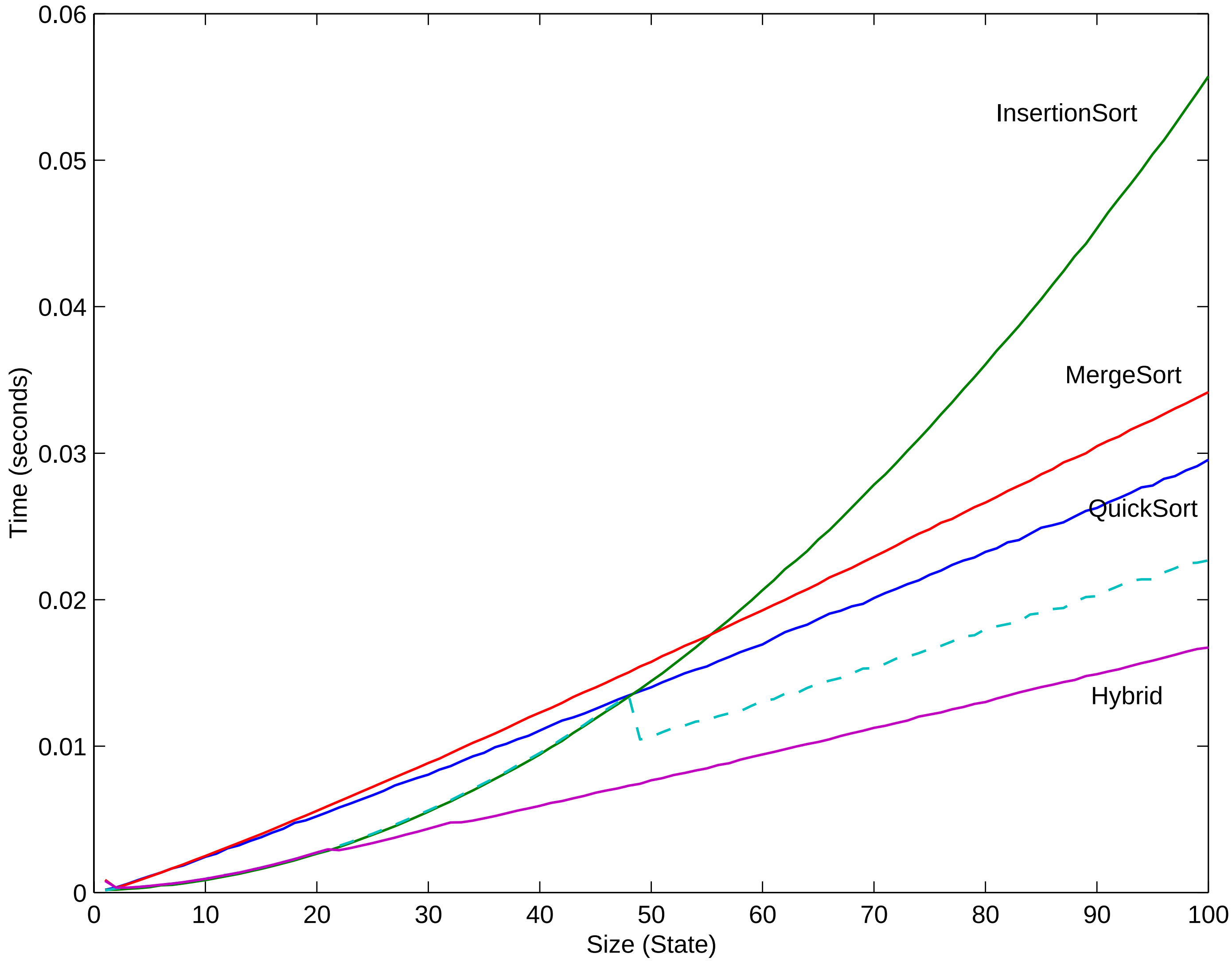
QuickSort

$$\begin{aligned} optQ(s) = Q(s) \quad &+ \quad \frac{1}{s}\sum_{p=2}^{s-1}\Big(Opt(p) + Opt(s-p)\Big) \\ &+ \quad \frac{2}{s}\Big(Opt(1) + Opt(s-1)\Big) \end{aligned}$$

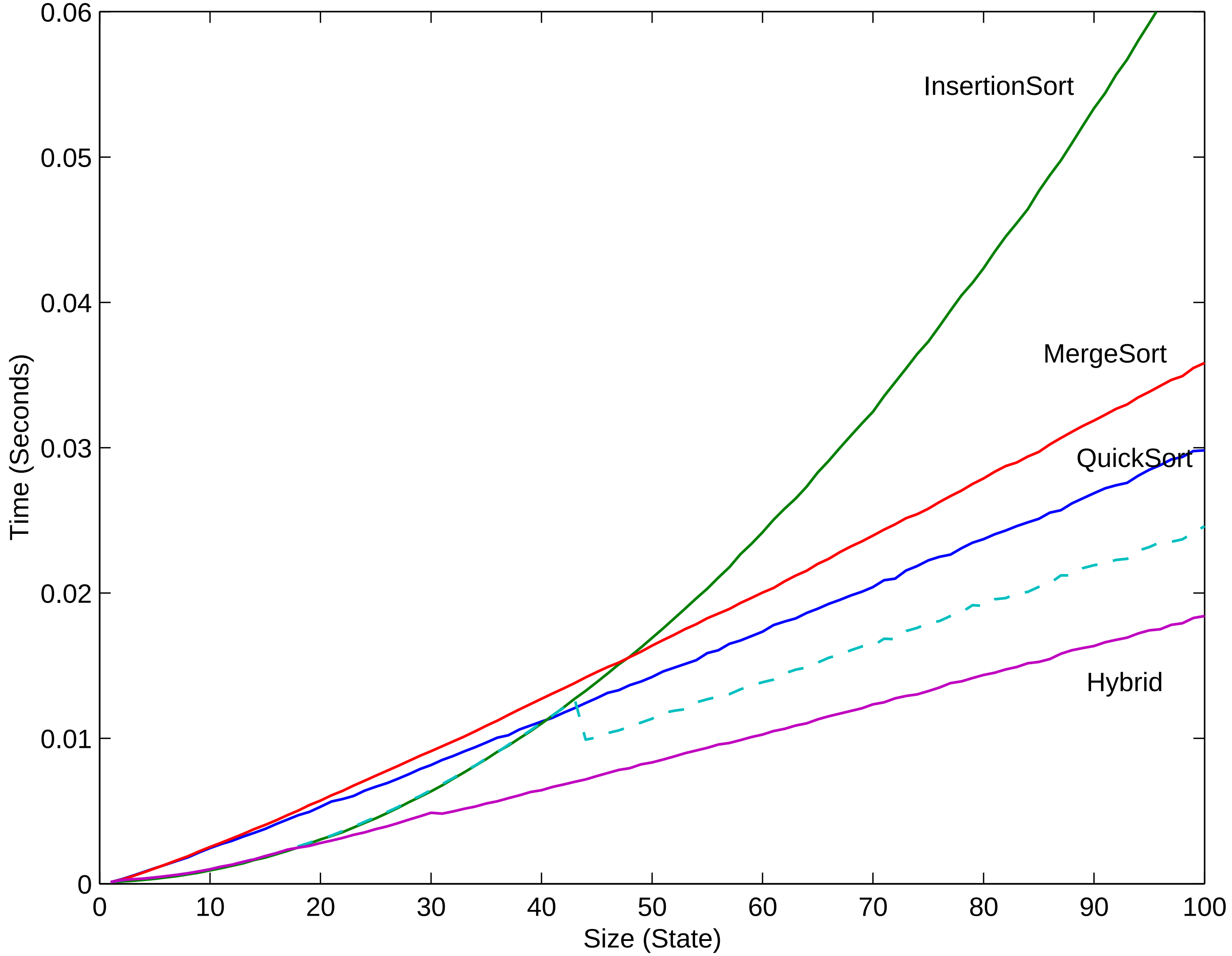
Optimal Hybrid Policies

Sparc	(Solaris)	**Pentium**	(Linux)
Size	Algorithm	Size	Algorithm
2 - 21	InsertionSort	2 - 17	InsertionSort
22 - 32	MergeSort	18 - 30	MergeSort
33 - ...	QuickSort	31 - ...	QuickSort

Performance on a Sparc/Solaris Architecture



Performance on a Pentium/Linux Architecture



Issues

Limitations

- State Description
- Hidden State
- Model Derivation

Adaptation

- Machine Learning
- Function Approximation
- Learning while Computing

Domains

- Hard Combinatorial Problems

Some Related Work

- FFTW (Fast Fourier Transform) [Frigo and Johnson, 1998]
- PYTHIA (Scientific Software) [Houstis et al., 1991-2000]
- LAPACK (Linear Algebra) [Anderson et al., 1987-2001]
- STAGE (Local Search) [Boyan, 1998]
- RLSAT (Satisfiability) [Lagoudakis and Littman, 2001]
- Bayesian Modelling [Horvitz at al., 2001]
- Branch-and-Bound Search [Lobjois and Lemaitre, 1998]
- Problem Solving [Fink, 1998]
- ...

Conclusion

The Past

- There is no uncertainty in computation.
- The best algorithm is the one with the best worst-case guarantees.
- Static Software (fixed algorithms)

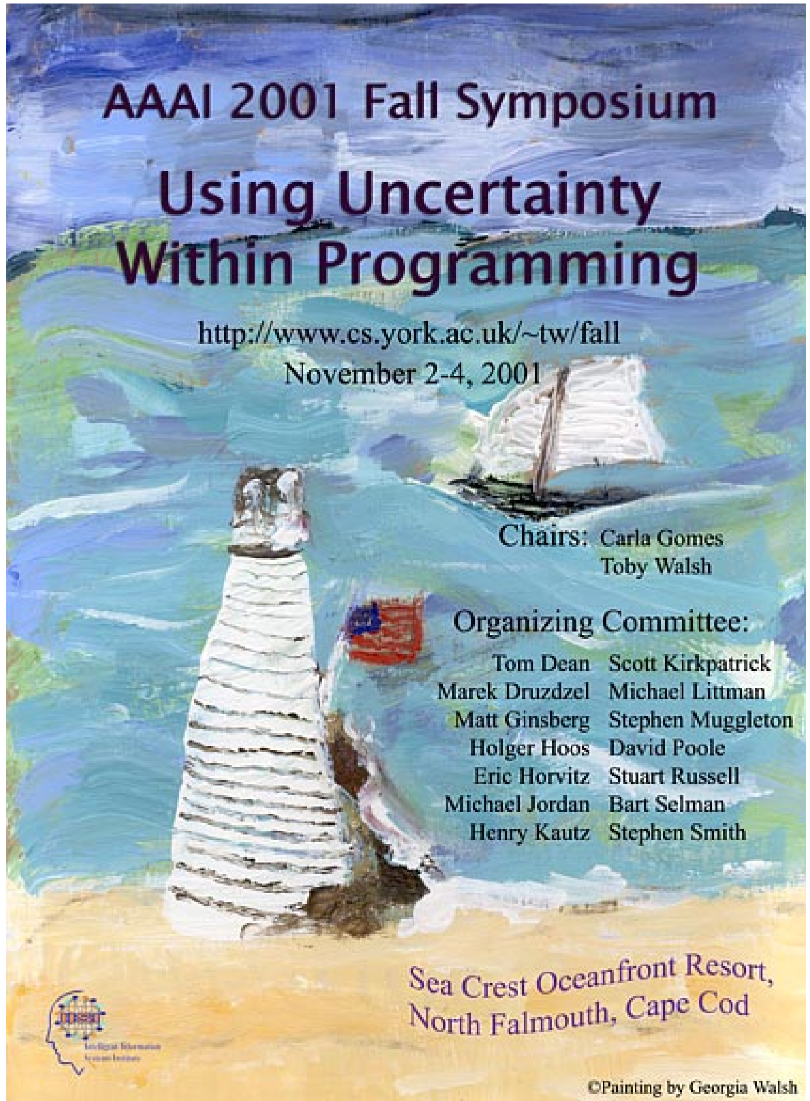
The Present

- There *is* uncertainty in computation.
- Optimization, reasoning, and learning methods can cope with uncertainty.
- Optimized Software (hybrid algorithms)

The Future

- Adaptive systems that encapsulate many algorithms.
- Systems that "learn" from past experience and improve their performance.
- Intelligent Software (AI?)

Thank You!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