Lecture 8 Permutation & Combination

8.1 What are they about?

Combination and permutation is about finding the number of ways to do a particular task

8.2 How are they different?

Here's an easy way to remember: **permutation sounds complicated**, doesn't it? And it is. With permutations, every little detail matters. Alice, Bob and Charlie are different from Charlie, Bob and Alice.

Combinations, on the other hand, are pretty easy going. The details don't matter. Alice, Bob and Charlie are the same as Charlie, Bob and Alice.

Key point:
Permutations are for lists (order matters)
Combinations are for groups (order doesn't matter)

8.3 Permutation

Let's start with permutations, or, all possible ways of listing something in order

Say we have 8 contestants who competed in a game show
– Alice, Bob, Charlie, David, Eve, Frank, George and Horatio

How many ways can we award a 1st, 2nd, 3rd place prizes among the 8 contestants?

Solution

Always ask these questions

a) ~~Who makes the list (n) – answer; the 8 contestants~~
b) ~~What is the list for (r) – answer; to award 1st, 2nd, 3rd place prizes~~
c) Initial list – 8 contestants without winners
d) Ordered list – 3 contestants winning 1st, 2nd and 3rd prizes

Apply the formula $\frac{n!}{(n-r)!} = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 8 * 7 * 6 = 336$

Another common problem is listing your 5 favourite books out of 10 books?

Solution

a) Initial list – 10 books
b) Ordered list – 5 favourite books

Apply formula $\frac{n!}{(n-r)!} = \frac{10!}{(10-5)!} = \frac{10!}{5!} = 10 * 9 * 8 * 7 * 6 = 30240$

8.4 Combination

Combinations are easy going. Order doesn't matter. You can mix it up and it looks the same. Let's say you want to give 3 cans of coke to 8 people and you are wondering

How many ways can I give 3 cans of coke to 8 people?

Well, in this case, the order you pick people doesn't matter. If you give a can to Alice, Bob and then Charlie, it's the same as giving to Charlie, Alice and then Bob. Either way, they're equally happy

So here, you don't need to be making a list and arrange the people in order. You can simply put them in groups. But what groups?

~~a) Who makes the group (n) – answer; the 8 people~~
~~b) What is the group for (r) – answer; to give 3 cans~~
c) Initial group – 8 people without coke
d) Updated group – 3 people get coke

Apply formula $\frac{n!}{(n-r)!r!} = \frac{8!}{(8-3)!3!} = \frac{8!}{5!3!} = \frac{8*7*6*5!}{5!3*2*1} = 56$

Another example is in finding the number of ways to choose 5 vowels out of a 20 letters word

a) Initial group – 20 letters word
b) Updated group – 5 vowels to take out

Apply formula

$$\frac{20!}{(20-5)!5!} = \frac{20!}{15!5!} = \frac{20*19*18*17*16*15!}{15!\ 5*4*3*2*1} = 19 * 3 * 17 * 16 = 15504$$

Try this:

1. Picking a team of 7 people from a group of 10
2. Picking a President, VP and Water-boy from a group of 10
3. Choosing 8 desserts from a menu of 10
4. Listing your 3 favourite desserts, in order, from a menu of 5

Answer

1. Picking a team of 7 people from a group of 10 (Combination problem)

   a) Initial group – 10 people
   b) Updated group – pick out 7 people

   Apply formula = $\frac{7!}{(7-3)!\,3!} = \frac{10!}{4!\,3!} = \frac{10*9*8*7*6*5*4!}{4!*3*2*1} = 25200$

2. Picking a President, VP and Water-boy from a group of 10 (Permutation problem)
   a) Initial list – 10 people without positions
   b) Ordered list – 3 people with positions (President, VP and water-boy)

   Apply formula $\frac{10!}{(10-3)!} = \frac{10!}{7!} = 10 * 9 * 8 = 720$

3. Choosing 8 desserts from a menu of 10 (Combination problem)
   a) Initial group – 10 desserts option
   b) Updated group – choosing 8 desserts

   Apply formula = $\frac{10!}{(10-8)!\,8!} = \frac{10!}{2!\,8!} = \frac{10*9*8!}{2*1*8!} = 45$

4. Listing your 3 favourite desserts, in order, from a menu of 5 (Permutation problem)
   a) Initial list – 5 desserts option
   b) Ordered list – 3 favourite desserts

   Apply formula $\frac{5!}{(5-3)!} = \frac{5!}{2!} = 5 * 4 * 3 = 60$