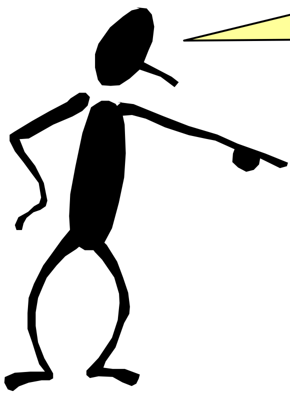
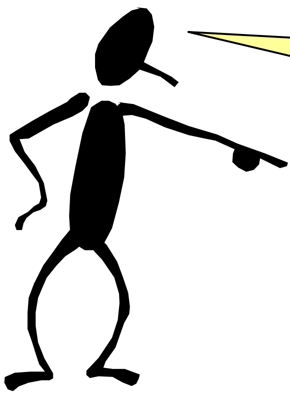
CHAPTER 8: THE PID CONTROLLER



When I complete this chapter, I want to be able to do the following.

- **Understand the strengths and weaknesses of the three modes of the PID**
- **Determine the model of a feedback system using block diagram algebra**
- **Establish general properties of PID feedback from the closed-loop model**

CHAPTER 8: THE PID CONTROLLER



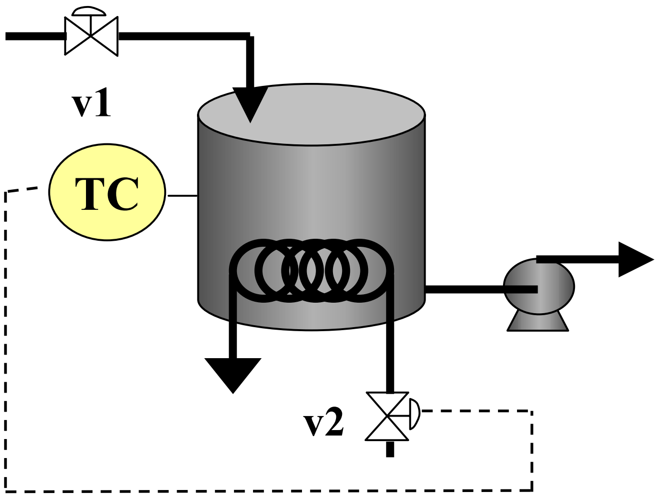
Outline of the lesson.

- General Features and history of PID
- Model of the Process and controller - the Block Diagram
- The Three Modes with features
  - Proportional
  - Integral
  - Derivative
- Typical dynamic behavior

CHAPTER 8: THE PID CONTROLLER

PROPERTIES THAT WE SEEK IN A CONTROLLER

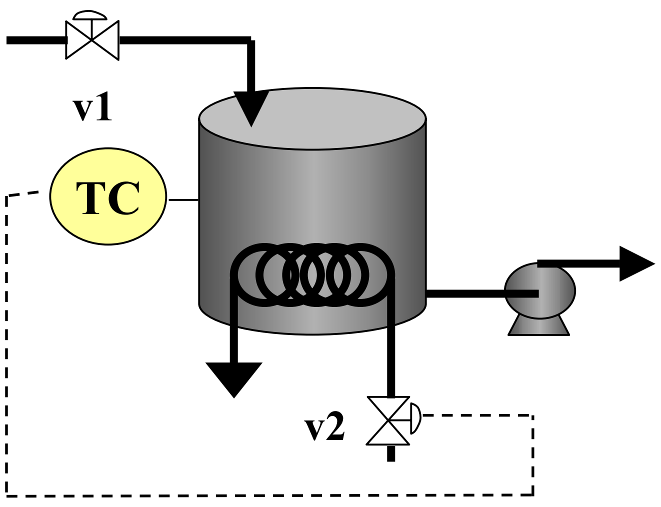
- **Good Performance** - feedback measures from Chapter 7
- **Wide applicability** - adjustable parameters
- **Timely calculations** - avoid convergence loops
- **Switch to/from manual** - bumplessly
- **Extensible** - enhanced easily



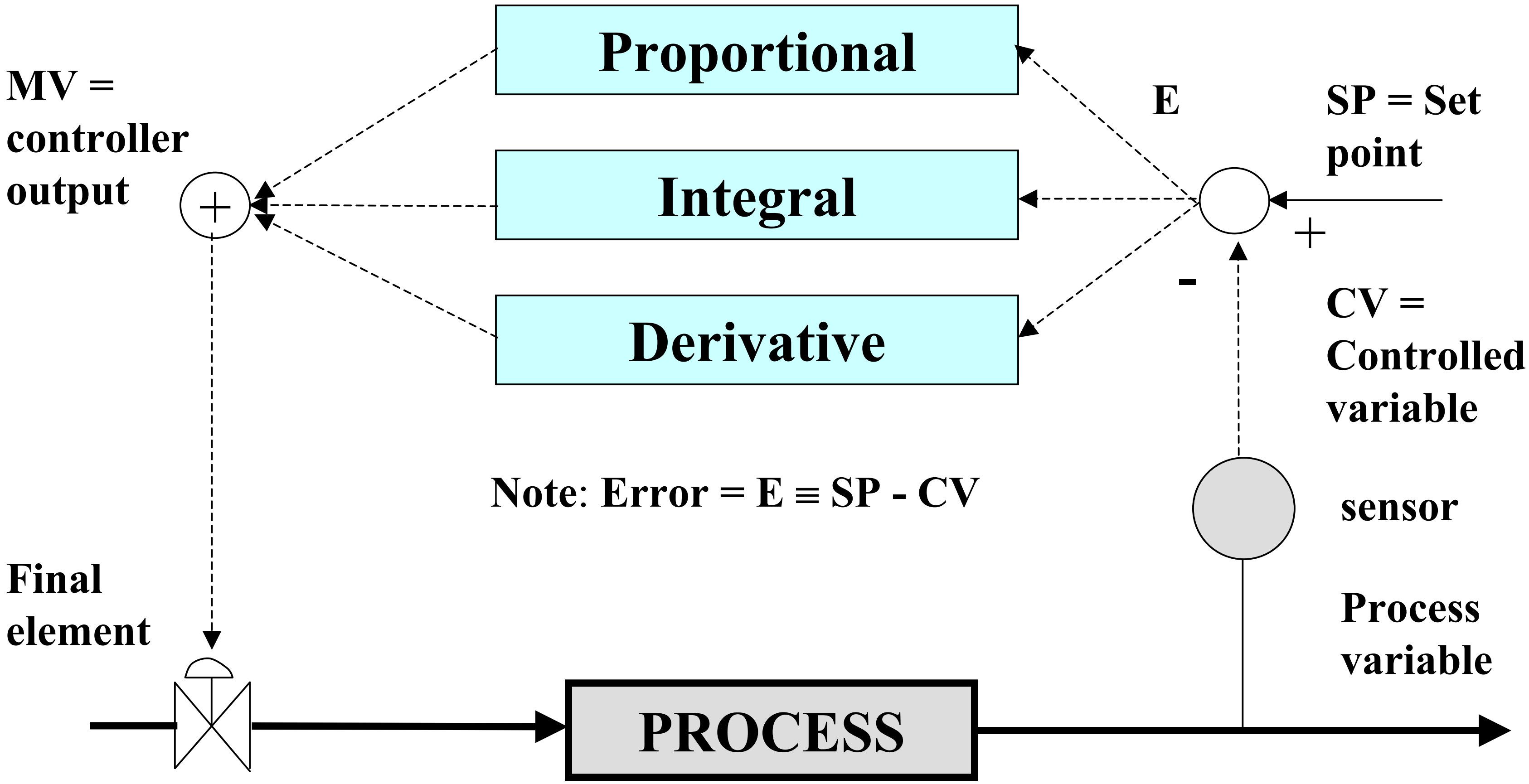
CHAPTER 8: THE PID CONTROLLER

SOME BACKGROUND IN THE CONTROLLER

- Developed in the 1940's, remains workhorse of practice
- Not "optimal", based on good properties of each mode
- Programmed in digital control equipment
- ONE controlled variable (CV) and ONE manipulated variable (MV). Many PID's used in a plant.



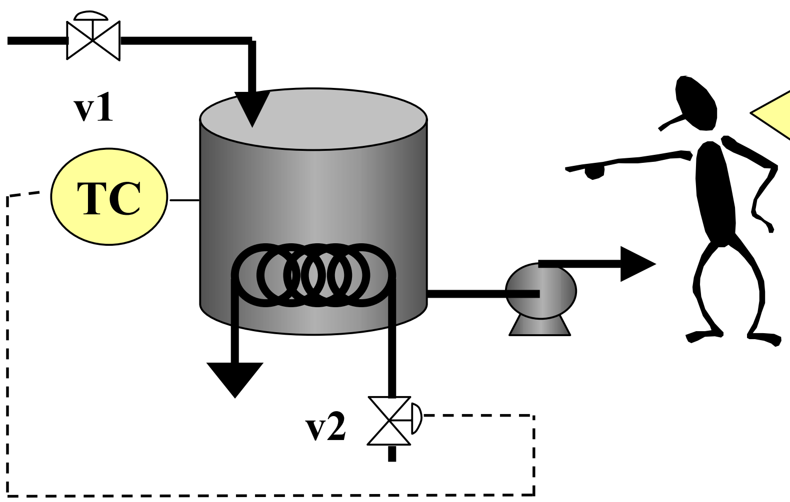
CHAPTER 8: THE PID CONTROLLER



Three "modes": Three ways of using the time-varying behavior of the measured variable

CHAPTER 8: THE PID CONTROLLER

Closed-Loop Model: **Before we learn about each calculation, we need to develop a general dynamic model for a closed-loop system - that is the process and the controller working as an integrated system.**

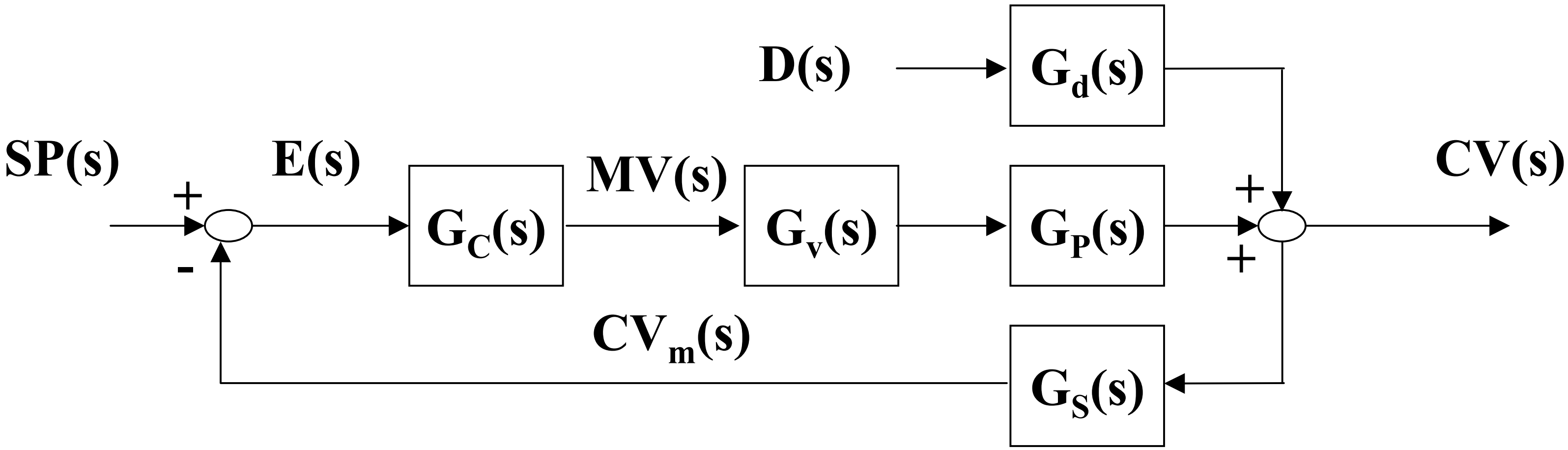


This is an example; how can we generalize?

- **What if we measured pressure, or flow, or ...?**
- **What if the process were different?**
- **What if the valve were different?**

CHAPTER 8: THE PID CONTROLLER

GENERAL CLOSED-LOOP MODEL BASED ON BLOCK DIAGRAM



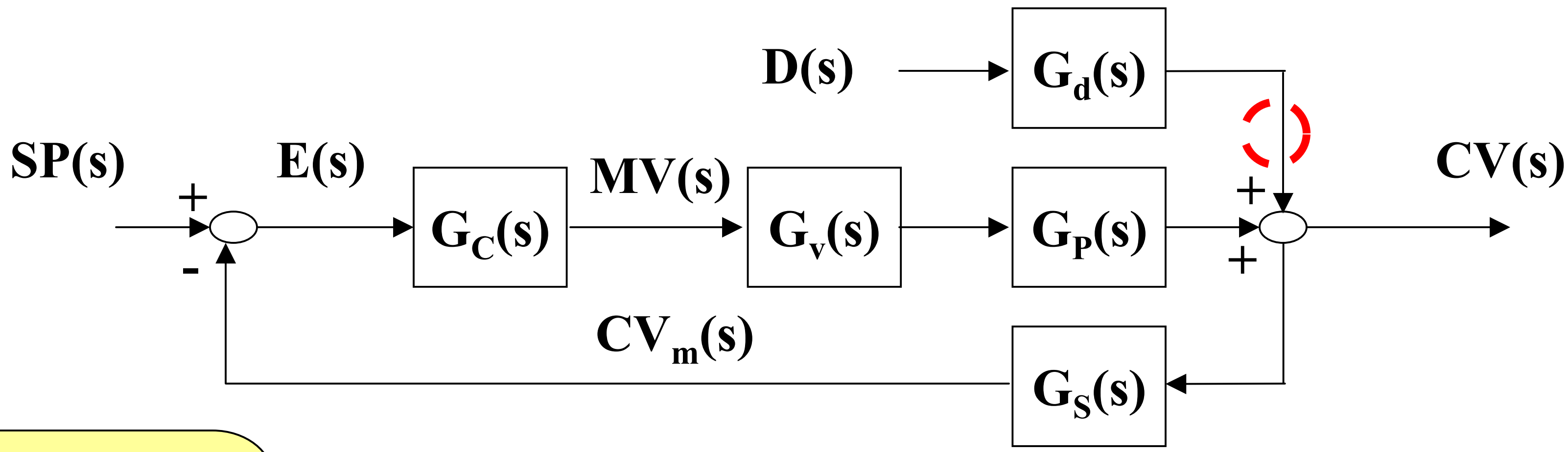
Transfer functions

$G_C(s)$ = controller
$G_v(s)$ = valve +
$G_P(s)$ = feedback process
$G_S(s)$ = sensor
$G_d(s)$ = disturbance process

Variables

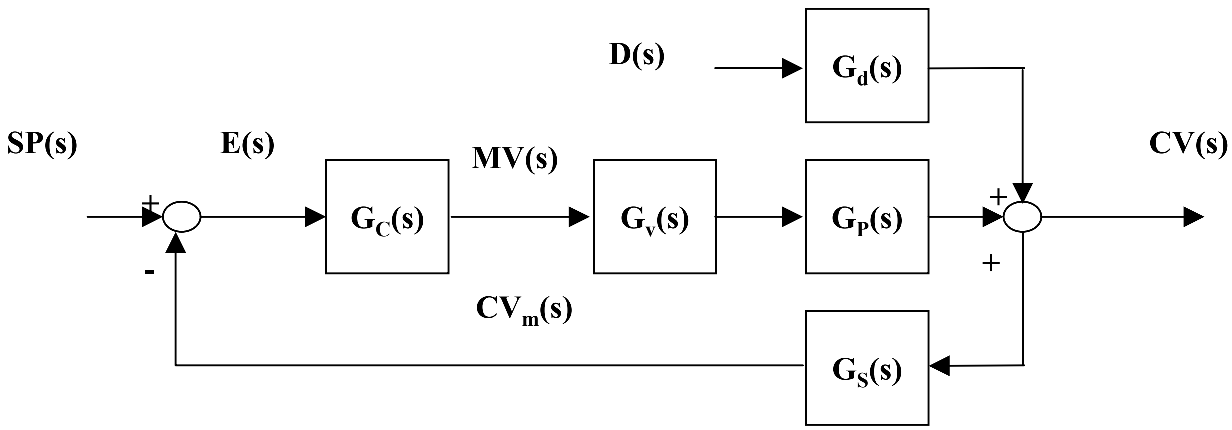
CV(s) = controlled variable
$CV_m(s)$ = measured value of CV(s)
D(s) = disturbance
E(s) = error
MV(s) = manipulated variable
SP(s) = set point

CHAPTER 8: THE PID CONTROLLER



- Where are the models for the transmission, and signal conversion?
- What is the difference between CV(s) and $CV_m(s)$?
- What is the difference between $G_P(s)$ and $G_d(s)$?
- How do we measure the variable whose line is indicated by the red circle?
- Which variables are determined by a person, which by computer?

CHAPTER 8: THE PID CONTROLLER

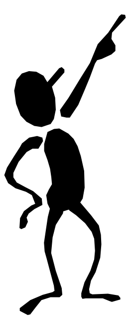


Set point response

$$\frac{CV(s)}{SP(s)} = \frac{G_p(s)G_v(s)G_c(s)}{1 + G_p(s)G_v(s)G_c(s)G_S(s)}$$

Disturbance Response

$$\frac{CV(s)}{D(s)} = \frac{G_d(s)}{1 + G_p(s)G_v(s)G_c(s)G_S(s)}$$

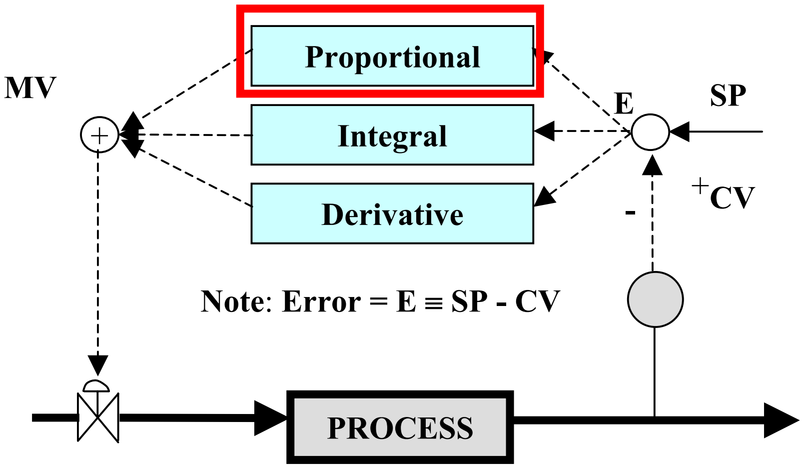


- **Which elements in the control system affect system stability?**
- **Which elements affect dynamic response?**

CHAPTER 8:

THE PID CONTROLLER,

The Proportional Mode

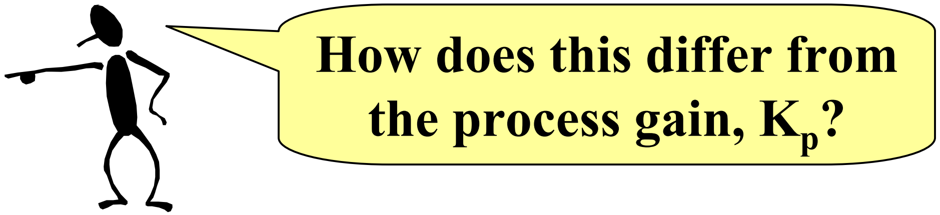


“correction proportional to error.”

$$\textbf{Time domain :}\quad MV(t) = K_c E(t) + I_p$$

$$\textbf{Transfer function :}\quad G_C(s) = \frac{MV(s)}{E(s)} = K_C$$

K_C = controller gain

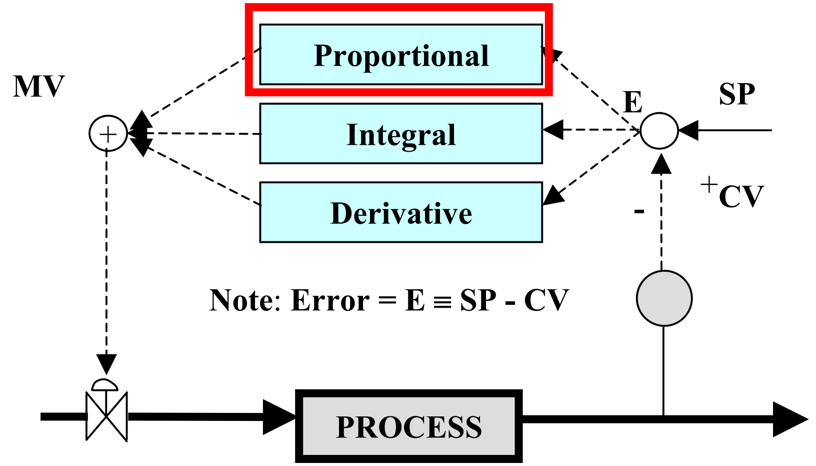


CHAPTER 8:

THE PID CONTROLLER,

The Proportional Mode

"correction proportional to error."



Time domain : $MV(t) = K_c E(t) + I_p$

K_C = controller gain

K_p depends upon the process (e.g., reactor volume, flows, temperatures, etc.)

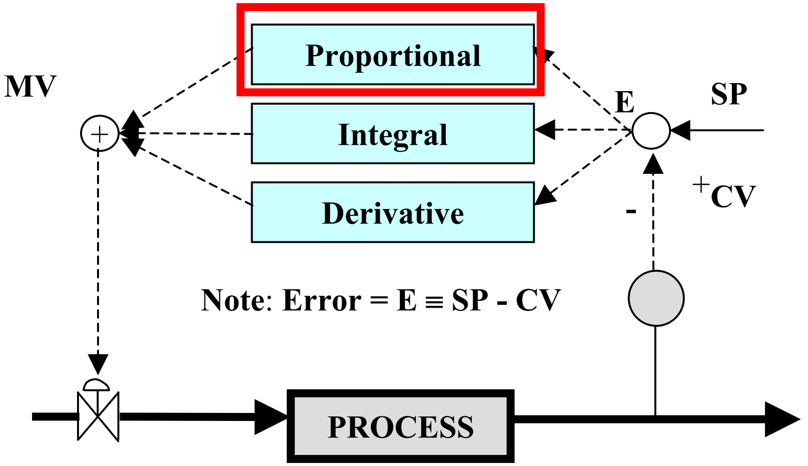
K_C is a number <u>we select</u>; it is used in the computer each time the controller equation is calculated

How does this differ from the process gain, K_p?

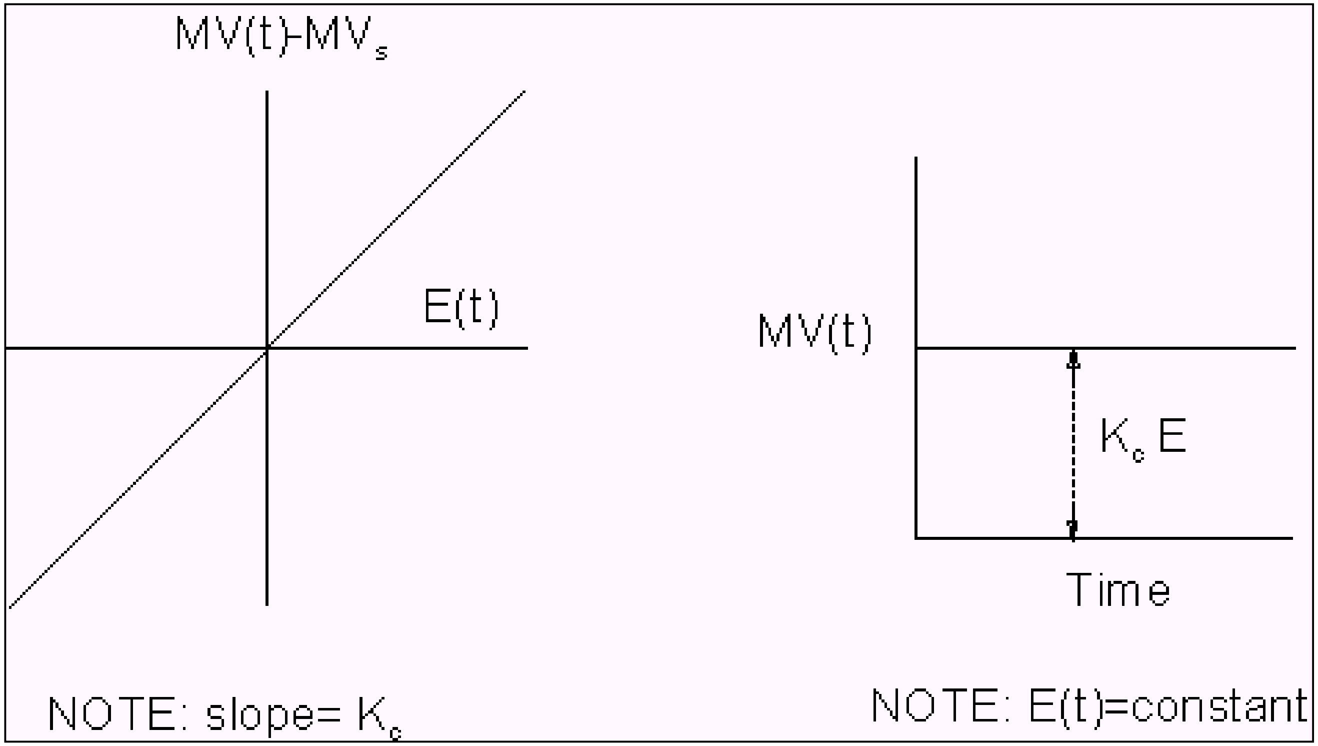
CHAPTER 8:

THE PID CONTROLLER,

The Proportional Mode



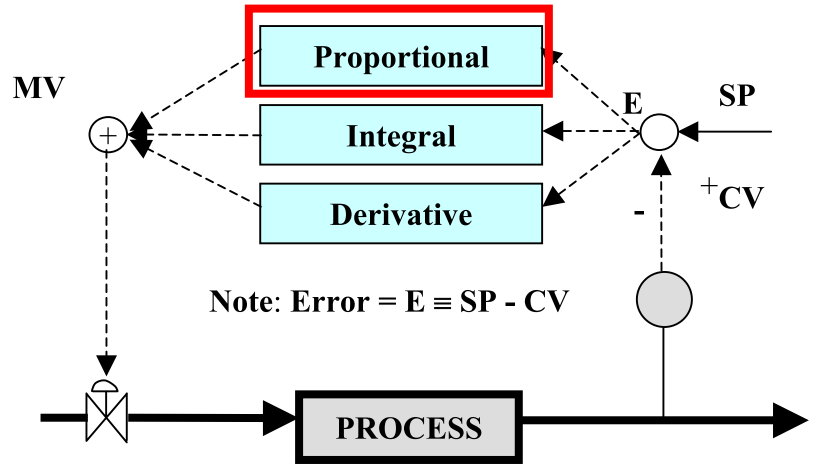
$$\textbf{Time domain :}\ MV(t) = K_c E(t) + I_p$$



CHAPTER 8:

THE PID CONTROLLER,

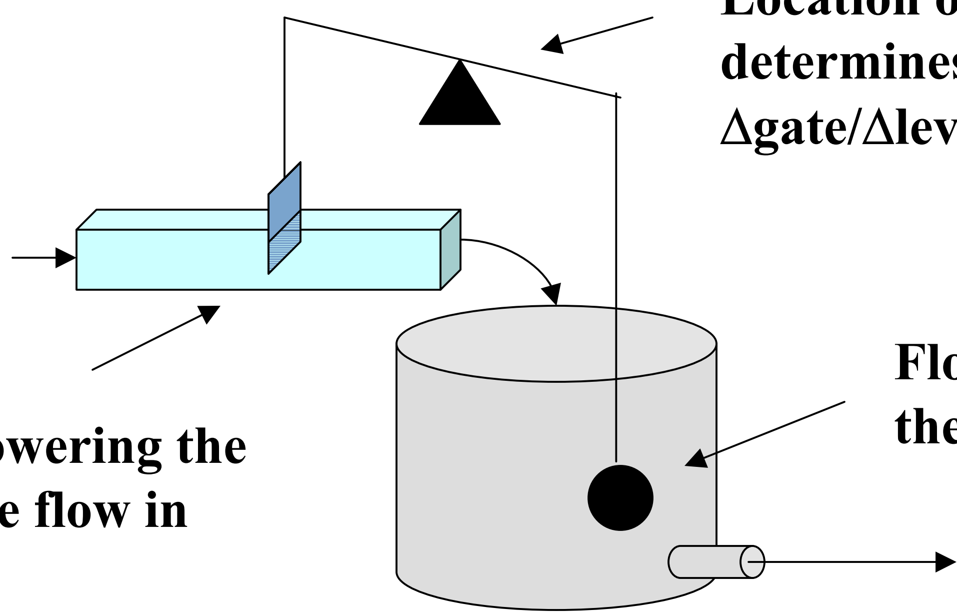
The Proportional Mode



$$\textbf{Time domain :}\ MV(t) = K_c E(t) + I_p$$

Physical Device:

Location of the fulcrum determines the Δgate/Δlevel



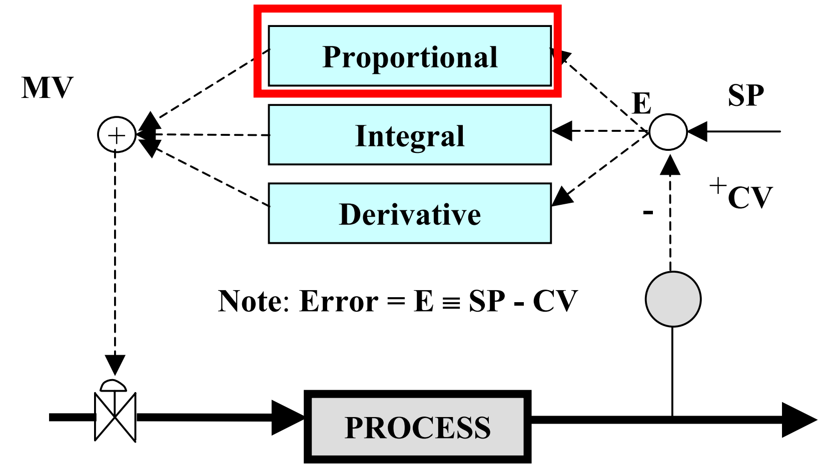
Raising and lowering the gate affects the flow in

Float measures the liquid level

CHAPTER 8:

THE PID CONTROLLER,

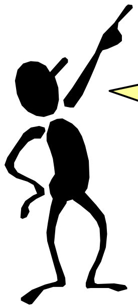
The Proportional Mode



Key feature of closed-loop performance with P-only

Final value after disturbance:

$$CV(t)\Big|_{t\to\infty} = \lim_{s\to 0} s\frac{\Delta D}{s}\frac{K_d}{1+K_cK_p} = \frac{\Delta D \; K_d}{1+K_cK_p} \neq 0$$

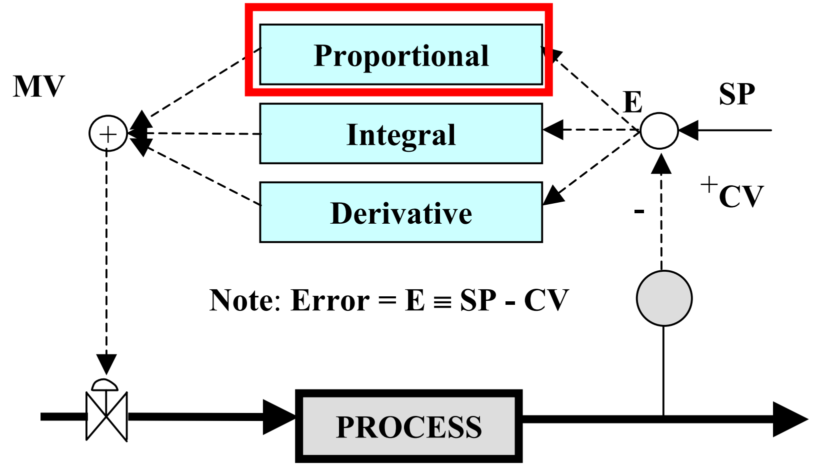


- **We do not achieve zero offset; <u>don't return to set point</u>!**
- **How can we get very close by changing a controller parameter?**
- **Any possible problems with suggestion?**

CHAPTER 8:

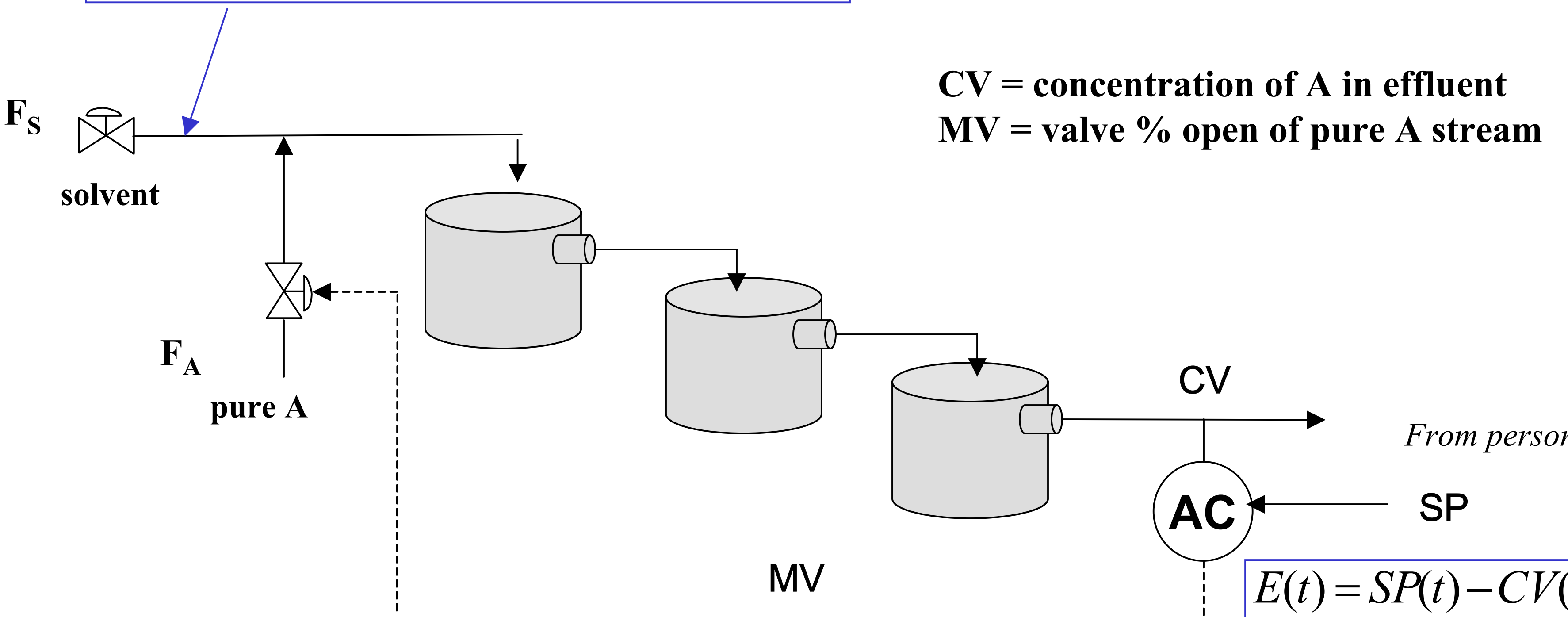
THE PID CONTROLLER,

The Proportional Mode



Disturbance in concentration of A in the solvent

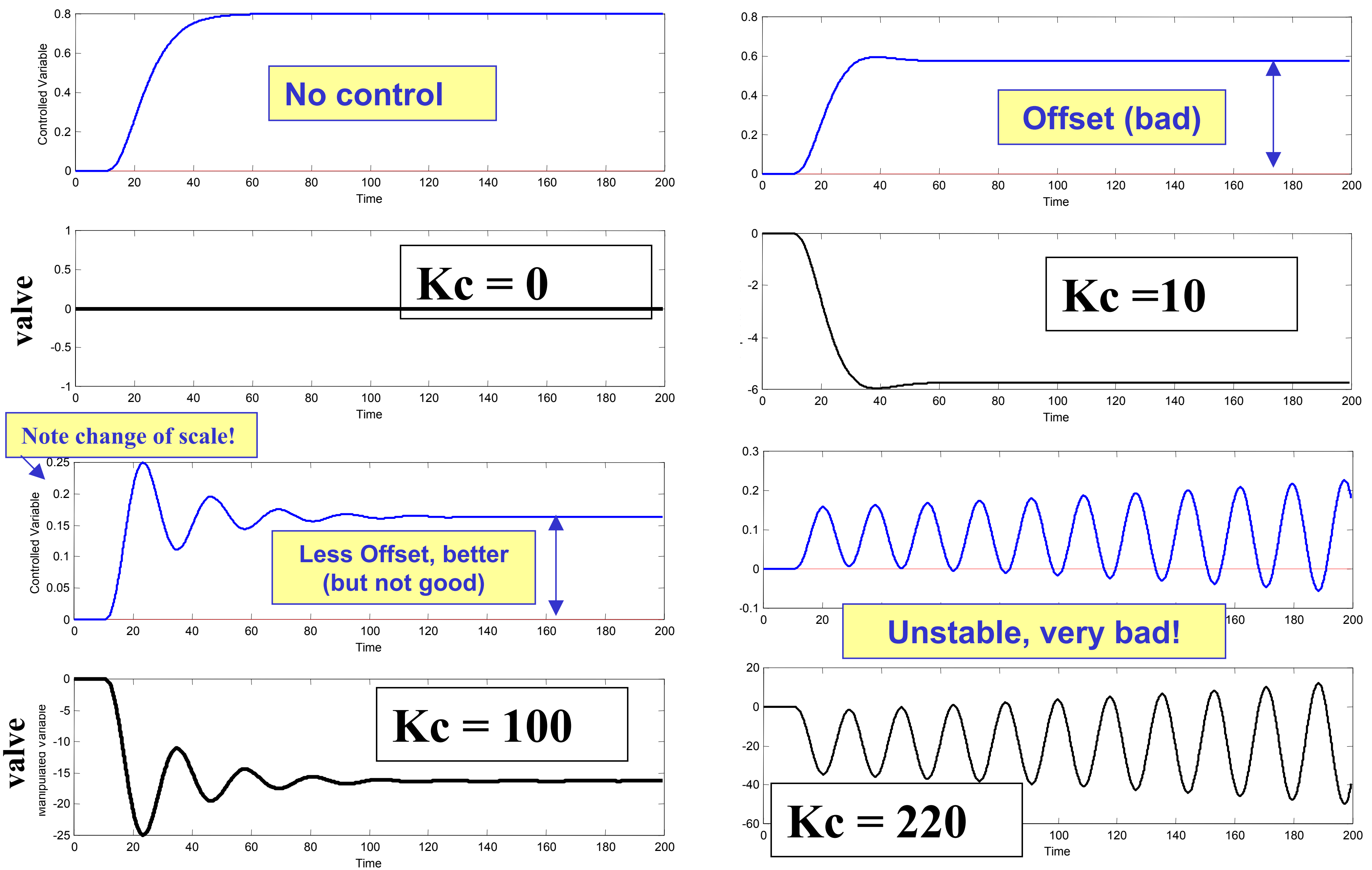
CV = concentration of A in effluent
MV = valve % open of pure A stream



$$E(t) = SP(t) - CV(t)$$

$$MV(t) = K_c\left[E(t)\right] + I$$

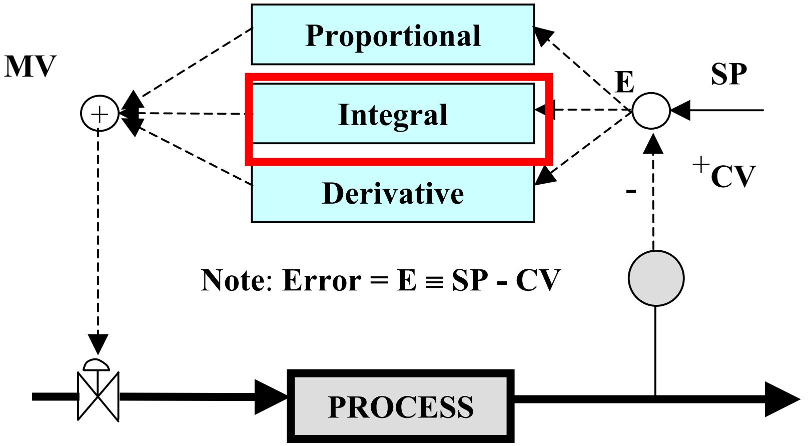
THE PID CONTROLLER,The Proportional Mode



CHAPTER 8:

THE PID CONTROLLER,

The Integral Mode



"The persistent mode"

$$\text{Time domain}: \quad MV(t) = \frac{K_c}{T_I}\int_0^t E(t')dt' + I_I$$

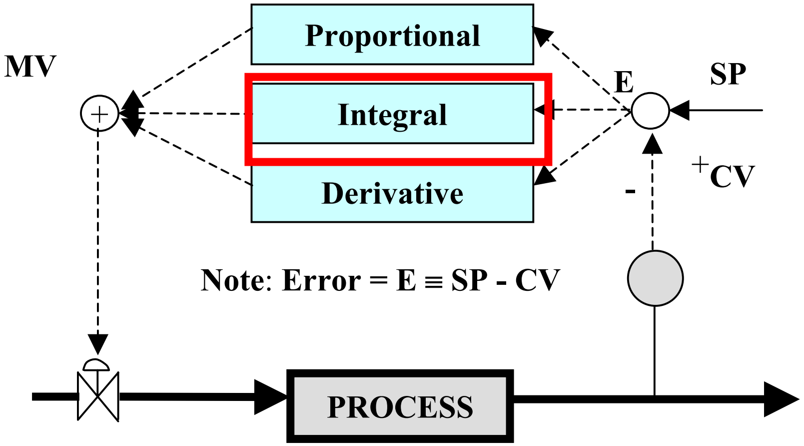
$$\textbf{Transfer function}: \quad G_C(s) = \frac{MV(s)}{E(s)} = \frac{K_C}{T_I}\frac{1}{s}$$

T_I = controller integral time (in denominator)

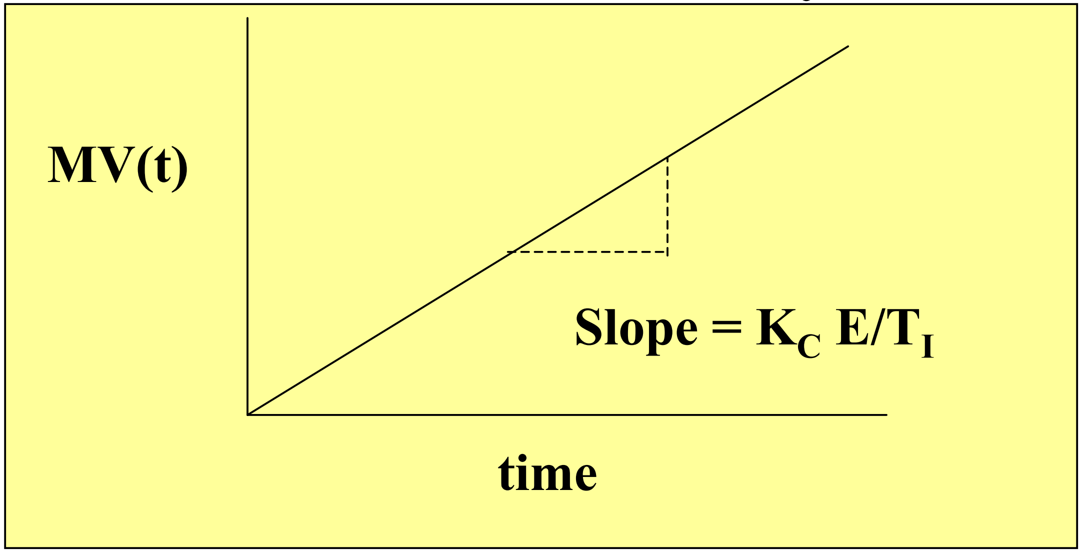
CHAPTER 8:

THE PID CONTROLLER,

The Integral Mode



$$\text{Time domain}: \quad MV(t) = \frac{K_c}{T_I}\int_0^t E(t')dt' + I_I$$

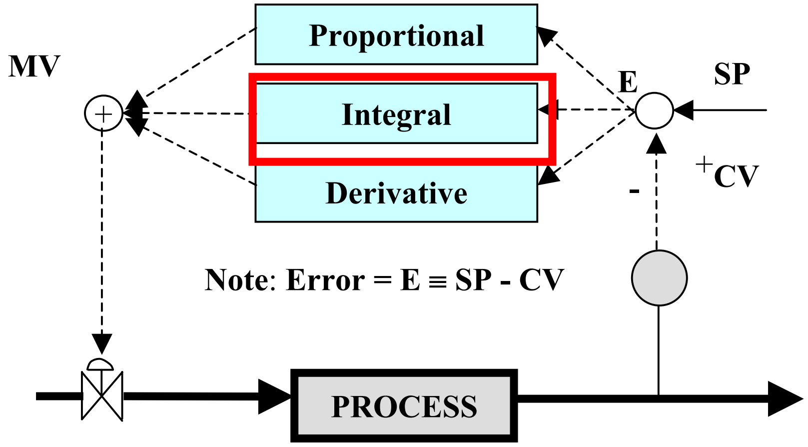


Behavior when E(t) = constant

CHAPTER 8:

THE PID CONTROLLER,

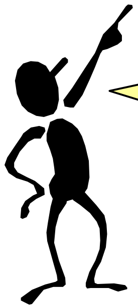
The Integral Mode



Key feature of closed-loop performance with I mode

Final value after disturbance:

$$CV(t)\Big|_{t\to\infty} = \lim_{s\to 0} s\,\frac{\Delta D}{s}\,\frac{K_d}{1+\dfrac{K_c K_p}{sT_I}} = 0$$

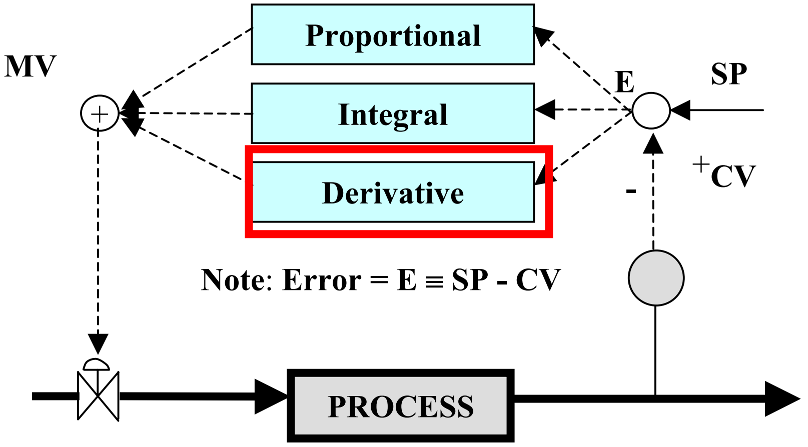


- **We achieve zero offset for a step disturbance; return to set point!**
- **Are there other scenarios where we do not?**

CHAPTER 8:

THE PID CONTROLLER,

The Derivative Mode



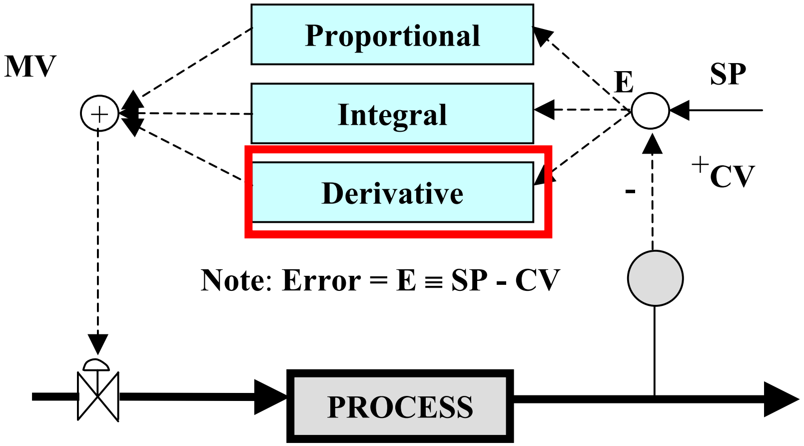
"The predictive mode"

Time domain : $$MV(t) = K_c T_D \frac{dE(t)}{dt} + I_D$$

Transfer function : $$G_C(s) = \frac{MV(s)}{E(s)} = K_c T_d s$$

T_D = controller derivative time

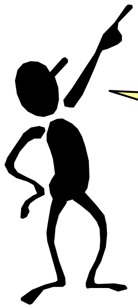
CHAPTER 8: THE PID CONTROLLER, The Derivative Mode



Key features using closed-loop dynamic model

Final value after disturbance:

$$CV(t)\Big|_{t\to\infty} = \lim_{s\to 0} s\,\frac{\Delta D}{s}\,\frac{K_d}{1+K_c T_d s} = \Delta D\, K_d$$

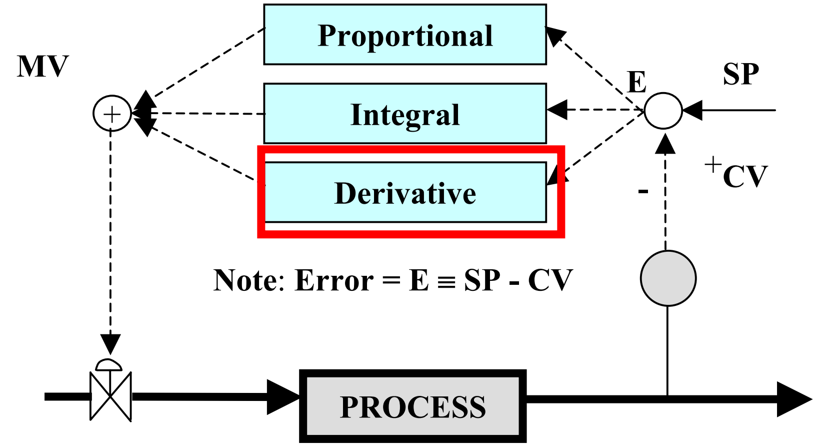


We do not achieve zero offset; do not return to set point!

CHAPTER 8:

THE PID CONTROLLER,

The Derivative Mode



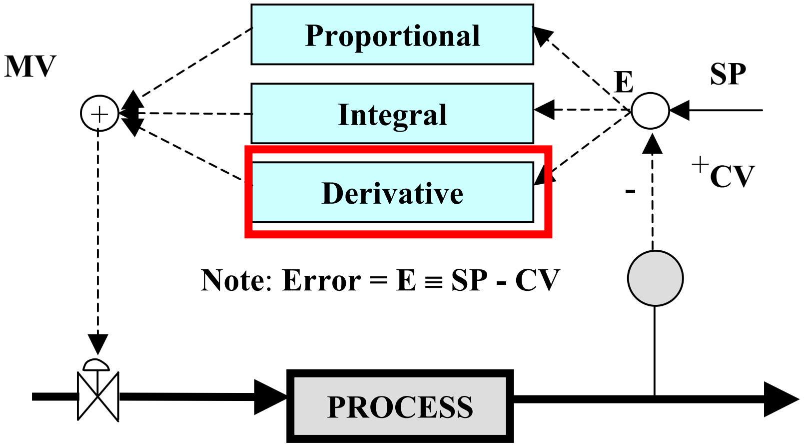
$$\textbf{Time domain :}\quad MV(t) = K_c T_D \frac{dE(t)}{dt} + I_D$$

- **What would be the behavior of the manipulated variable when we enter a step change to the set point?**
- **How can we modify the algorithm to improve the performance?**

CHAPTER 8:

THE PID CONTROLLER,

The Derivative Mode



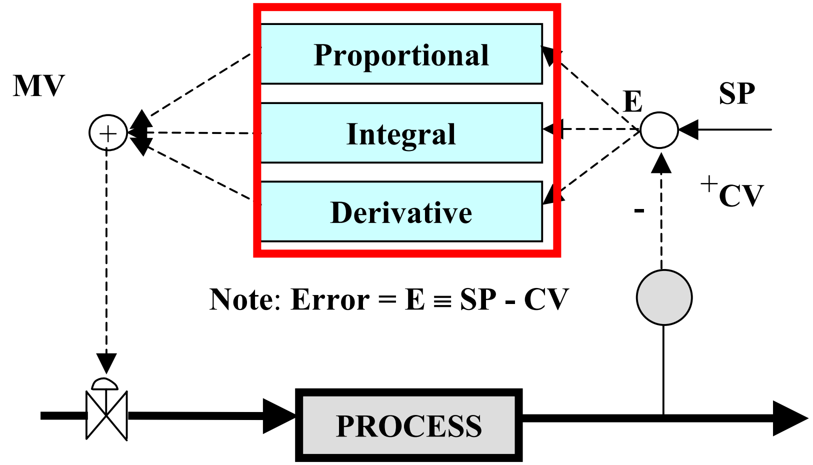
Time domain : $$MV(t) = K_c T_D \frac{dE(t)}{dt} + I_D$$

We do not want to take the derivative of the set point; therefore, we use only the CV when calculating the derivative mode.

Time domain : $$MV(t) = -K_c T_D \frac{d\,CV(t)}{dt} + I_D$$

CHAPTER 8:

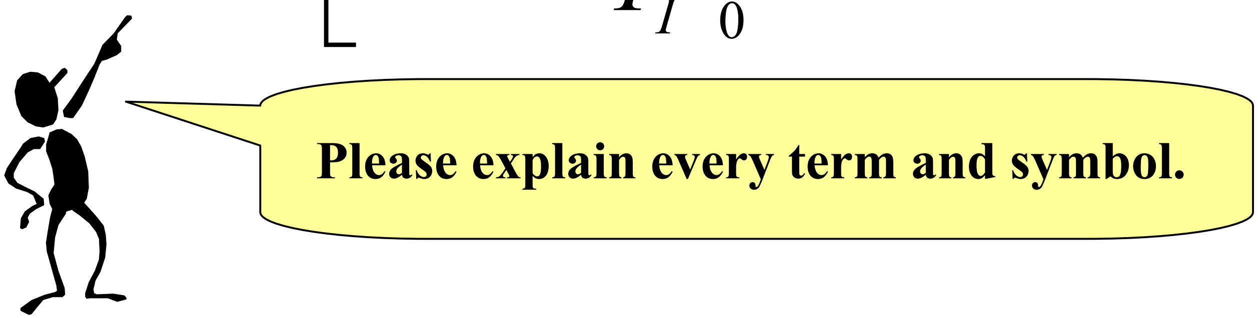
THE PID CONTROLLER



Let's combine the modes to formulate the PID Controller!

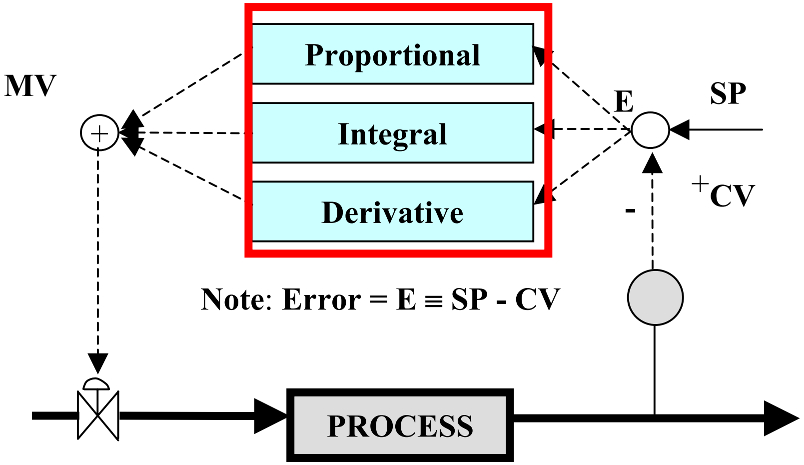
$$E(t) = SP(t) - CV(t)$$

$$MV(t) = K_c\left[E(t) + \frac{1}{T_I}\int_0^t E(t')dt' - T_d\frac{d\,CV}{dt}\right] + I$$



CHAPTER 8:

THE PID CONTROLLER



Let's combine the modes to formulate the PID Controller!

$$E(t) = SP(t) - CV(t) \quad \Big\} \; \boxed{\text{Error from set point}}$$

$$MV(t) = K_c\left[\underbrace{E(t)}_{\text{proportional}} + \underbrace{\frac{1}{T_I}\int_0^t E(t')\,dt'}_{\text{integral}} - \underbrace{T_d\,\frac{d\,CV}{dt}}_{\text{derivative}}\right] + I$$

Constant (bias) for bumpless transfer

CHAPTER 8: THE PID CONTROLLER

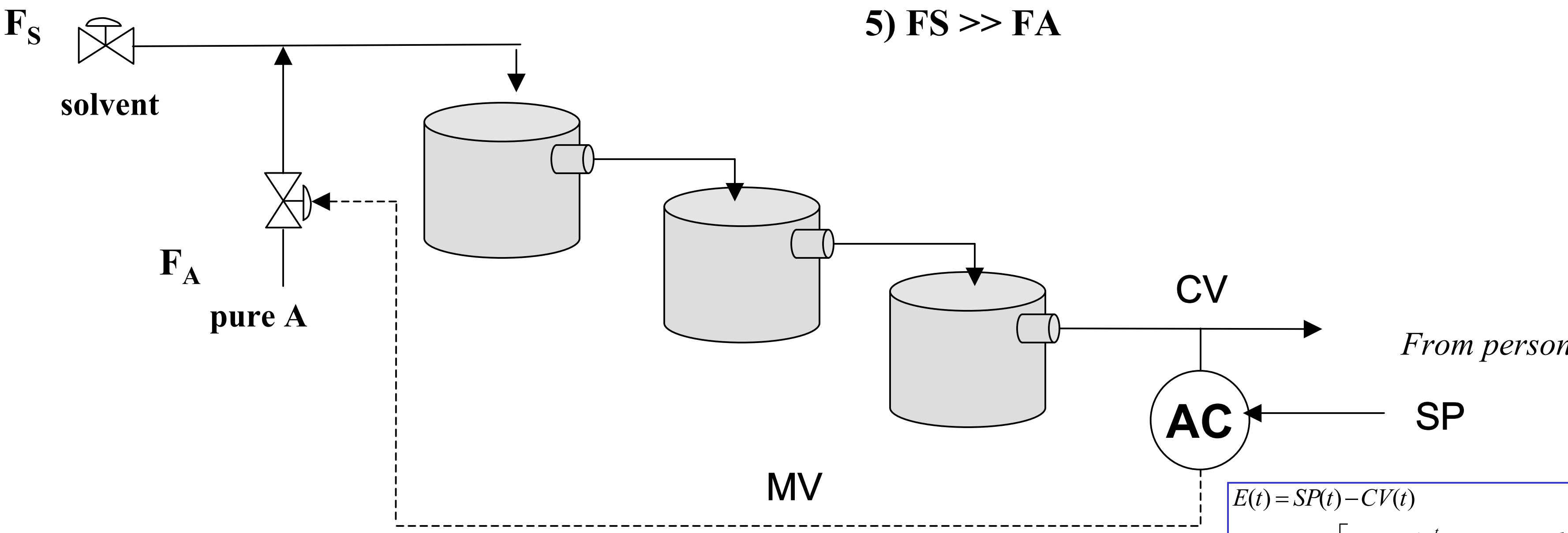
Let's apply the controller to the three-tank mixer (no rxn).

CV = concentration of A in effluent

MV = valve % open of pure A stream

Notes:

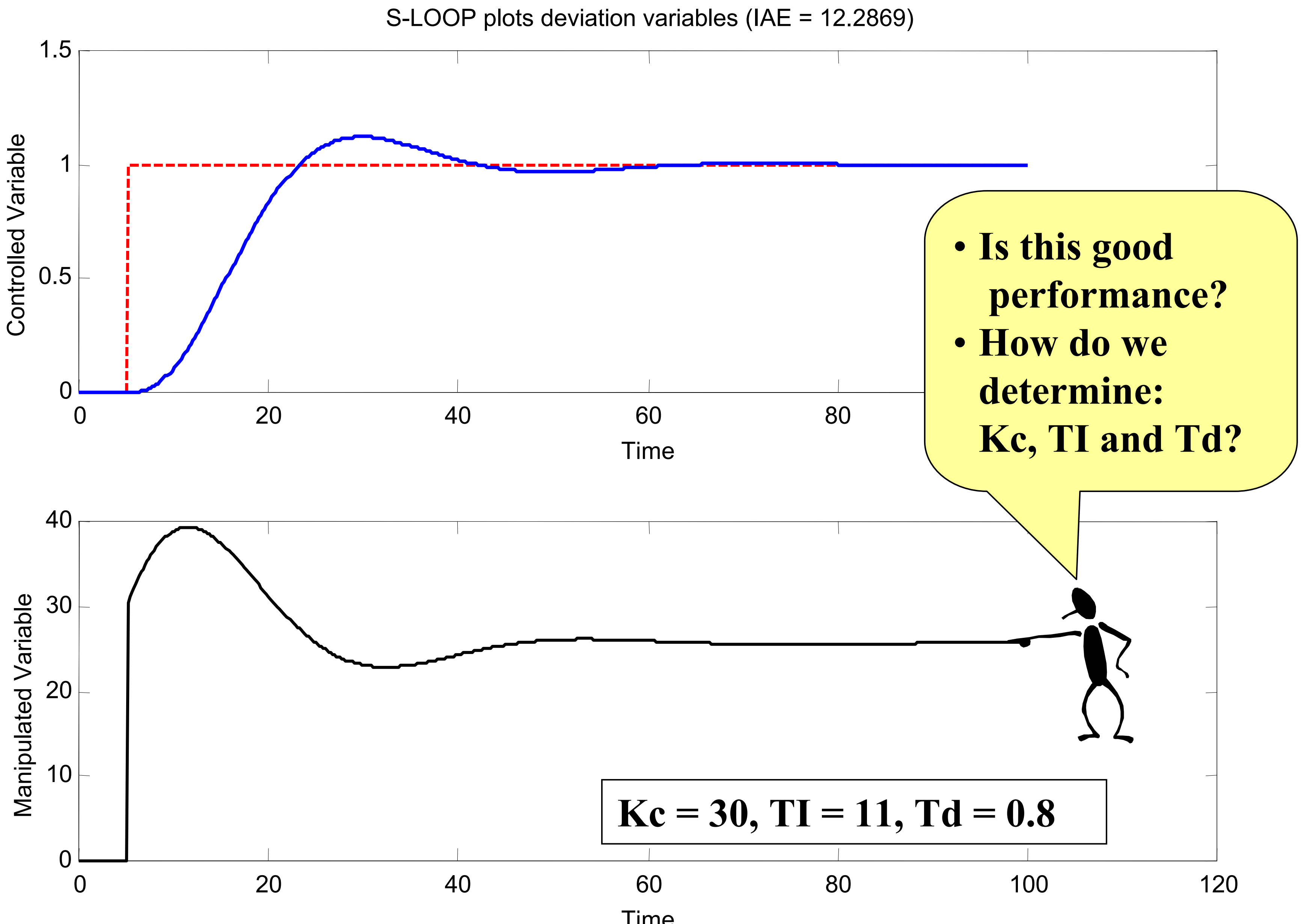
1) tanks are well mixed
2) liquid volumes are constant
3) sensor and valve dynamics are negligible
4) $F_A = K_v$ (v), with v = % opening
5) FS >> FA



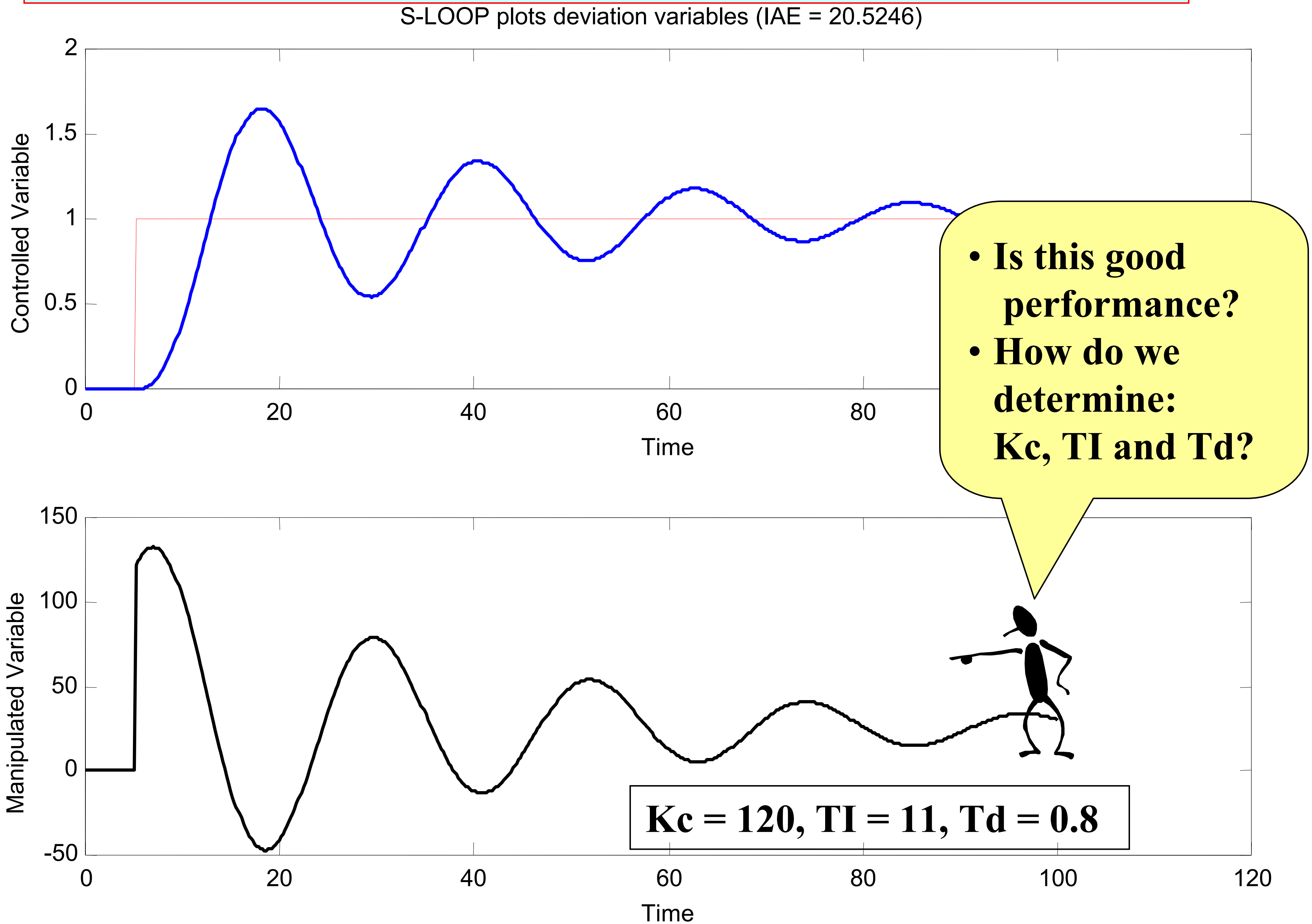
$$E(t) = SP(t) - CV(t)$$

$$MV(t) = K_c\left[E(t) + \frac{1}{T_I}\int_0^t E(t')dt' - T_d\frac{d\,CV}{dt}\right] + I$$

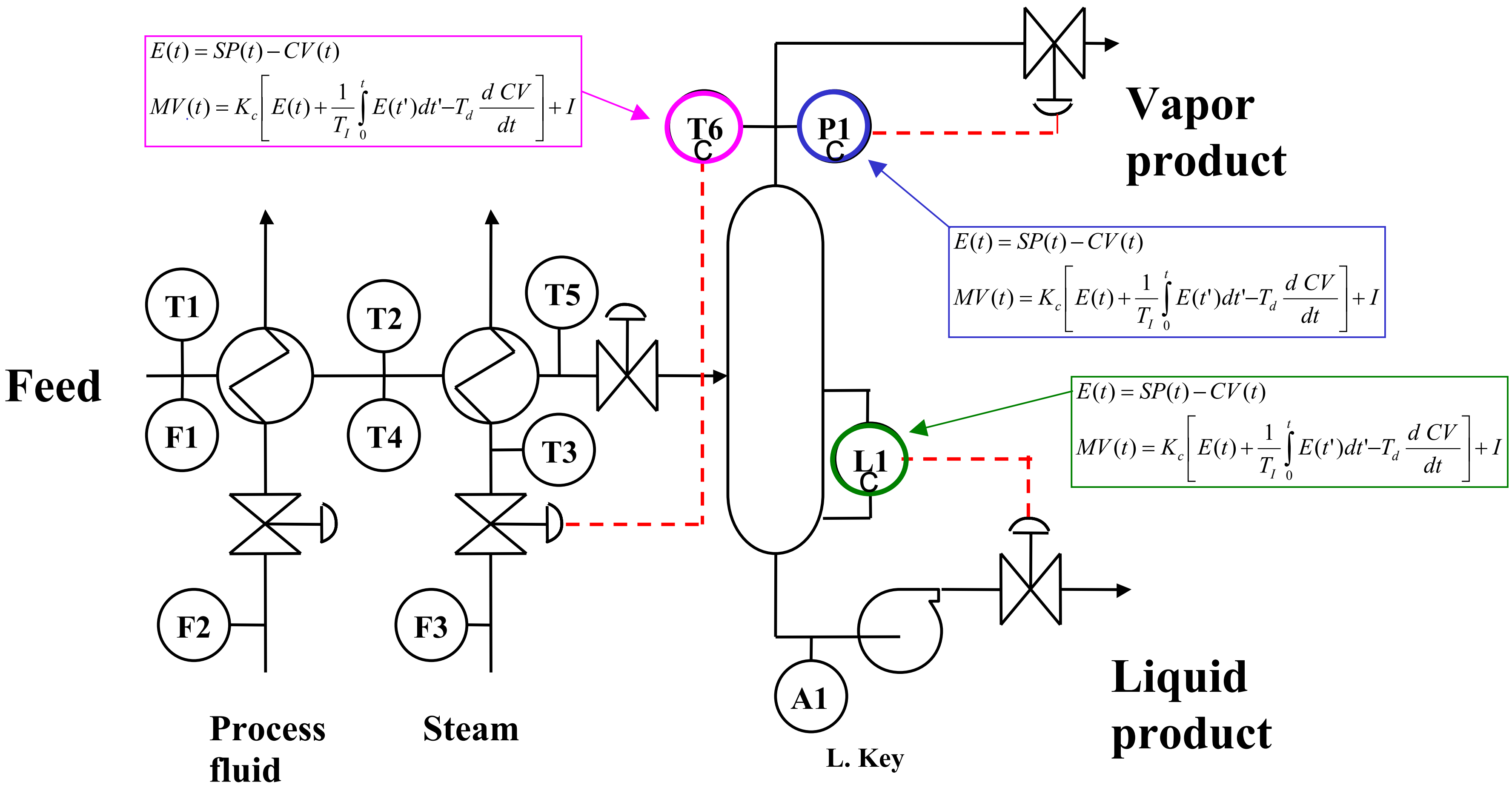
CHAPTER 8: THE PID CONTROLLER



CHAPTER 8: THE PID CONTROLLER



Lookahead: We can apply many PID controllers when we have many variables to be controlled!



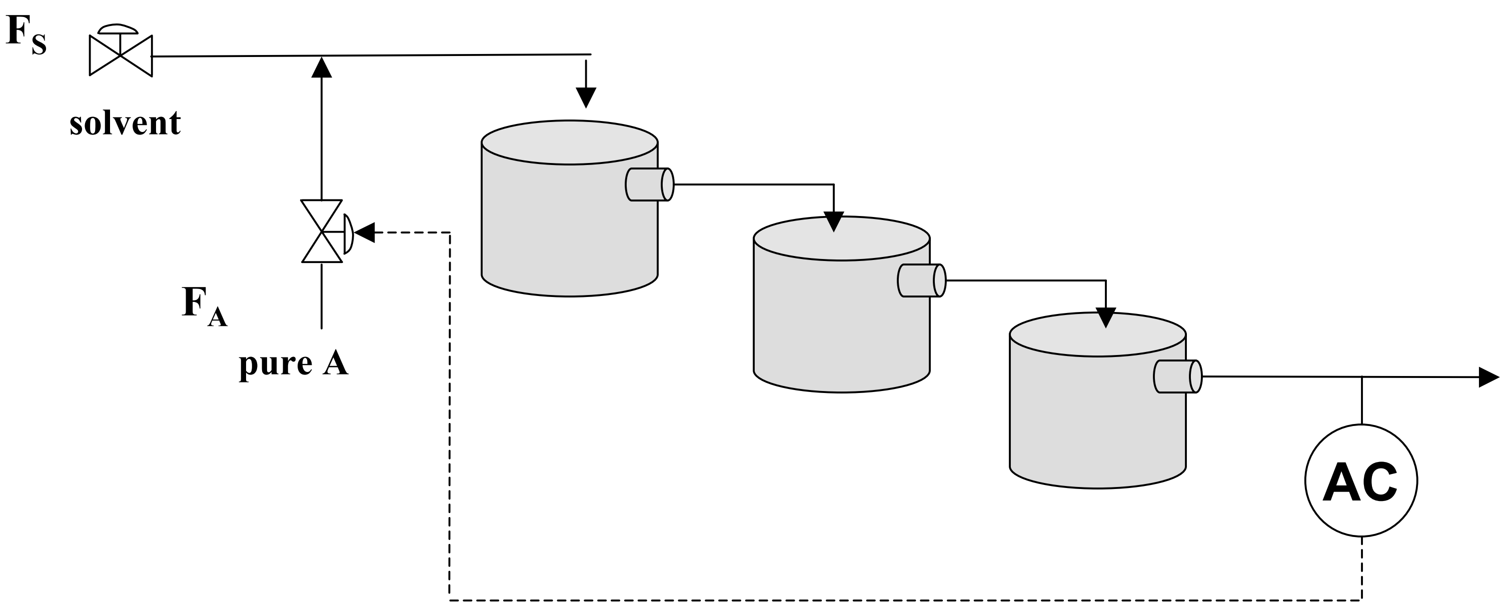
HOW DO WE EVALUATE THE DYNAMIC RESPONSE OF THE CLOSED-LOOP SYSTEM?

- **In a few cases, we can do this analytically (See Example 8.5)**
- **In most cases, we must solve the equations numerically. At each time step, we integrate**
  - **The differential equations for the process**
  - **The differential equation for the controller**
  - **Any associated algebraic equations**
- **Many numerical methods are available**
- **"S_LOOP" does this from menu-driven input**

CHAPTER 8: THE PID, WORKSHOP 1

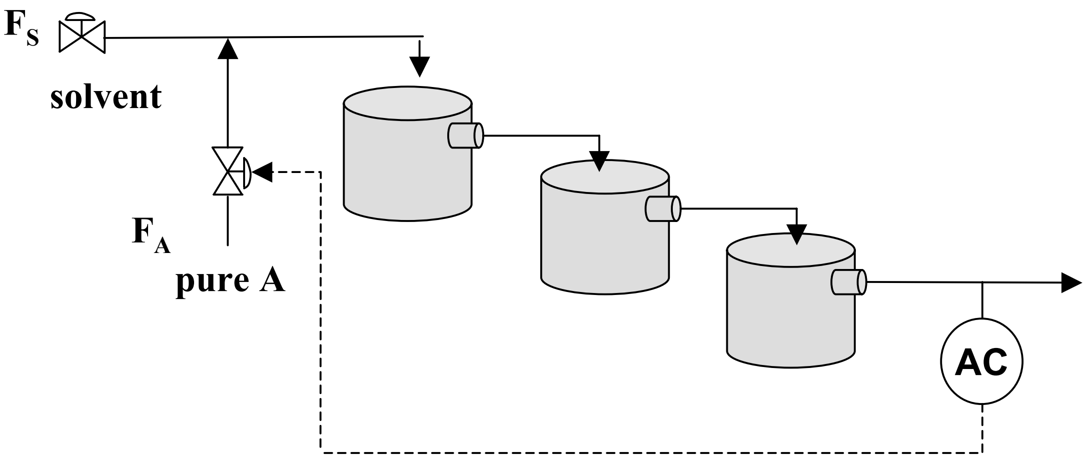
- **Model formulation**: Develop the equations that describe the dynamic behavior of the three-tank mixer and PID controller.

- **Numerical solution**: Develop the equations that are solved at each time step for the simulated control system (process and controller).



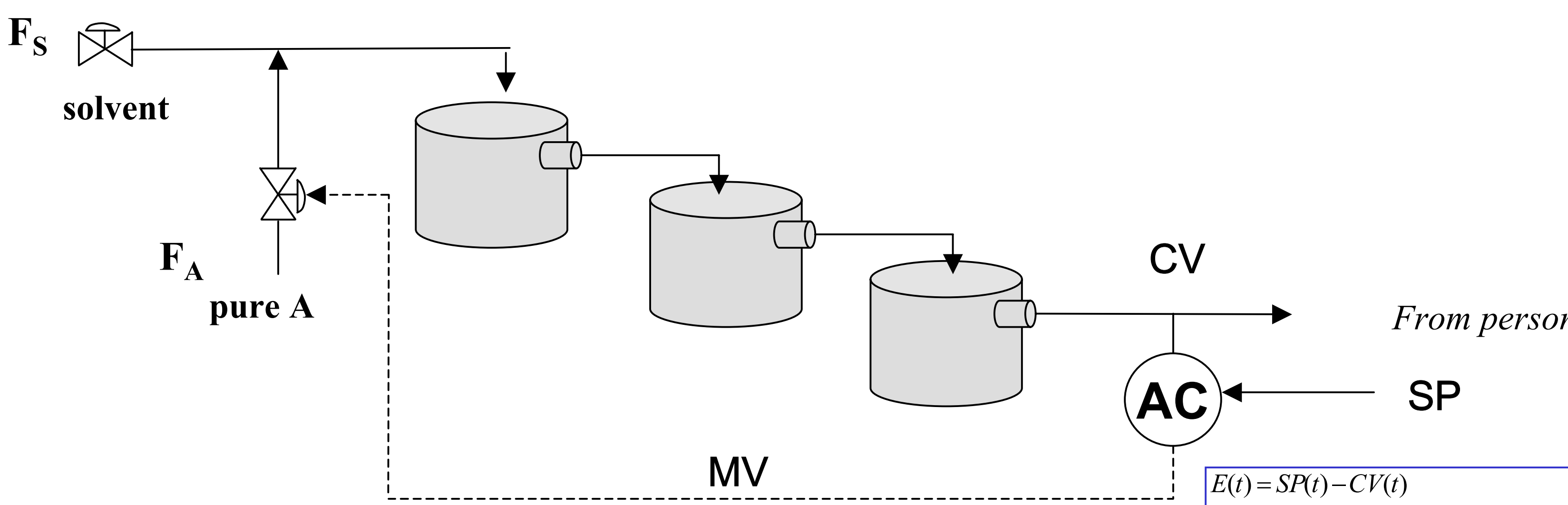
CHAPTER 8: THE PID, WORKSHOP 2

- The PID controller is applied to the three-tank mixer. Prove that the PID controller with provide zero steady-state offset when the set point is changed in a step, ΔSP.
- The three-tank process is stable. If we add a controller, could the closed-loop system become unstable?



CHAPTER 8: THE PID, WORKSHOP 3

- **Determine the engineering units for the controller tuning parameters in the system below.**
- **Explain how the initialization constant (I) is calculated. This is sometimes called the “bias”.**



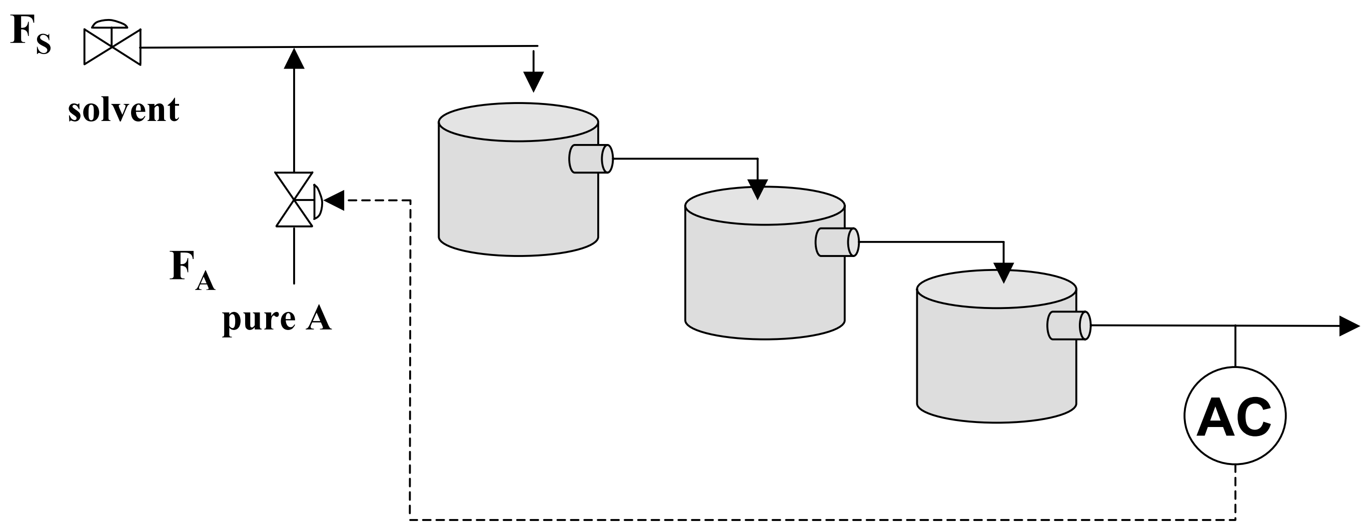
$$E(t) = SP(t) - CV(t)$$

$$MV(t) = K_c\left[E(t) + \frac{1}{T_I}\int_0^t E(t')dt' - T_d\frac{d\,CV}{dt}\right] + I$$

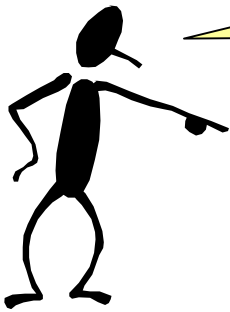
CHAPTER 8: THE PID, WORKSHOP 4

The PID controller must be displayed on a computer console for the plant operator. Design a console display and define values that

- The operator needs to see to monitor the plant
- The operator can change to “run” the plant
- The engineer can change

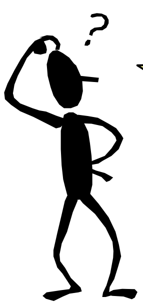


CHAPTER 8: THE PID CONTROLLER



When I complete this chapter, I want to be able to do the following.

- **Understand the strengths and weaknesses of the three modes of the PID**
- **Determine the model of a feedback system using block diagram algebra**
- **Establish general properties of PID feedback from the closed-loop model**



Lot's of improvement, but we need some more study!

- **Read the textbook**
- **Review the notes, especially learning goals and workshop**
- **Try out the self-study suggestions**
- **Naturally, we'll have an assignment!**

CHAPTER 8: LEARNING RESOURCES

- **SITE PC-EDUCATION WEB**
  - Instrumentation Notes
  - Interactive Learning Module (Chapter 8)
  - Tutorials (Chapter 8)

CHAPTER 8: SUGGESTIONS FOR SELF-STUDY

1. In your own words, explain each of the PID modes. Give at least one advantage and disadvantage for each.

2. Repeat the simulations for the three-tank mixer with PID control that are reported in these notes. You may use the MATLAB program "S_LOOP".

3. Select one of the processes modelled in Chapters 3 or 4. Add a PID controller to the numerical solution of the dynamic response in the MATLAB m-file.

4. Derive the transfer function for the PID controller.