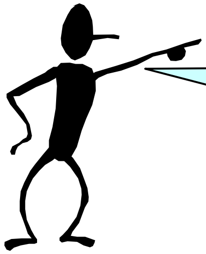
RELATIVE GAIN MEASURE OF INTERACTION

We have seen that interaction is important. It affects whether feedback control is possible, and if possible, its performance.

Do we have a quantitative measure of interaction?

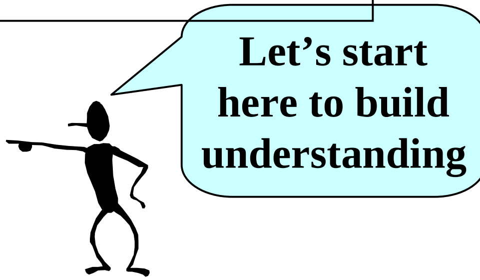
The answer is yes, we have several! Here, we will learn about the RELATIVE GAIN ARRAY.



Our main challenge is to understand the correct interpretations of the RGA.

RELATIVE GAIN MEASURE OF INTERACTION

OUTLINE OF THE PRESENTATION



1. DEFINITION OF THE RGA

2. EVALUATION OF THE RGA

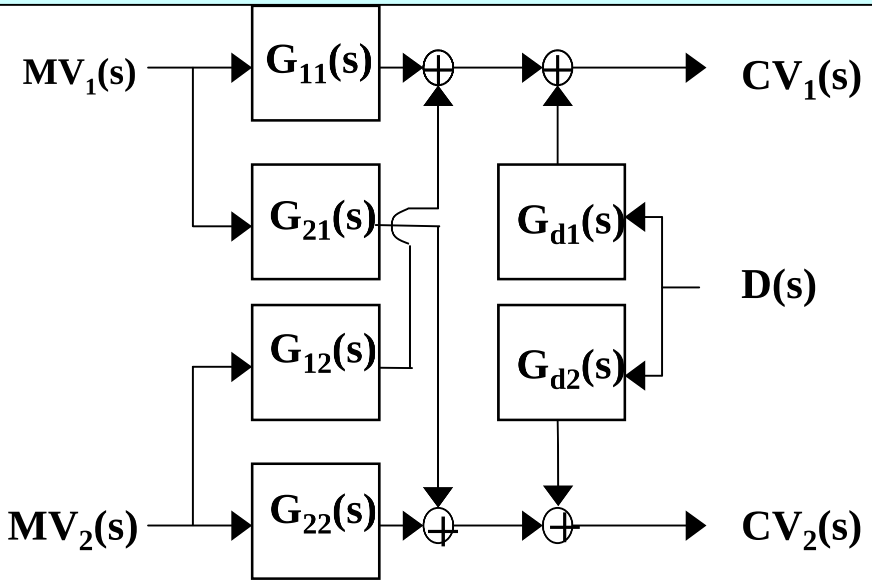
3. INTERPRETATION OF THE RGA

4. PRELIMINARY CONTROL DESIGN IMPLICATIONS OF RGA

RELATIVE GAIN MEASURE OF INTERACTION

The relative gain between MV_j and CV_i is λ_{ij} . It given in the following equation.

Explain in words.



$$\lambda_{ij} = \frac{\left(\frac{CV_i}{MV_j}\right)_{MV_k = \text{constant}}}{\left(\frac{CV_i}{MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\frac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\frac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

Assumes that loops have an integral mode

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RELATIVE GAIN MEASURE OF INTERACTION

1. The RGA can be calculated from open-loop values.

$$\lambda_{ij} = \frac{\left(\frac{CV_i}{MV_j}\right)_{MV_k = \text{constant}}}{\left(\frac{CV_i}{MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\frac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\frac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

$$[CV] = [K][MV] \qquad \mathbf{k_{ij}} = \left({\partial CV_i}/{\partial MV_j} \right)$$

$$[MV] = [K]^{-1}[CV] \qquad \mathbf{kI_{ij}} = \left({\partial MV_i}/{\partial CV_j} \right)$$

The relative gain array is the element-by-element product of K with K^{-1}

$$\Lambda = K \otimes \left(K^{-1}\right)^{\mathbf{T}} \qquad \lambda_{ij} = \left(k_{ij}\right)\left(kI_{ji}\right)$$

RELATIVE GAIN MEASURE OF INTERACTION

1. The RGA can be calculated from open-loop values.

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k = \textbf{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k = \textbf{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\textbf{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\textbf{other loops closed}}}$$

The relative gain array for a 2x2 system is given in the following equation.

$$\lambda_{11} = \frac{1}{1 - \dfrac{K_{12}K_{21}}{K_{11}K_{22}}}$$

What is true for the RGA to have 1's on diagonal?

RELATIVE GAIN MEASURE OF INTERACTION

2. The RGA elements are scale independent.

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

<u>Original units</u>

$$\begin{bmatrix} CV_1 \\ CV_2 \end{bmatrix} = \begin{bmatrix} 10 & 10 \\ 9 & 10 \end{bmatrix} \begin{bmatrix} MV_1 \\ MV_2 \end{bmatrix}$$

<u>Modified units</u>

$$\begin{bmatrix} CV_1 \\ CV^*_2 \end{bmatrix} = \begin{bmatrix} 1 & 10 \\ .09 & 1 \end{bmatrix} \begin{bmatrix} MV^*_1 \\ MV_2 \end{bmatrix}$$

Changing the units of the CV or the capacity of the valve does not change λ_{ij}.

$$\begin{array}{ccc} & \begin{array}{c} MV_1 \textbf{ or} \\ 10MV_1 \end{array} & MV_2 \\ CV_1 & 10 & -9 \\ CV_2 \textbf{ or } CV_2/10 & -9 & 10 \end{array}$$

RELATIVE GAIN MEASURE OF INTERACTION

3. The rows and columns of the RGA sum to 1.0.

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

$$\begin{array}{ccc} & MV_1 & MV_2 \\ CV_1 & 10 & -9 \\ CV_2 & -9 & 10 \end{array}$$

For a 2x2 system, how many elements are independent?

RELATIVE GAIN MEASURE OF INTERACTION

4. In some cases, the RGA is very sensitive to small errors in the gains, K_{ij}.

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k=\text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k=\text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

$$\lambda_{11} = \frac{1}{1 - \dfrac{K_{12}K_{21}}{K_{11}K_{22}}}$$

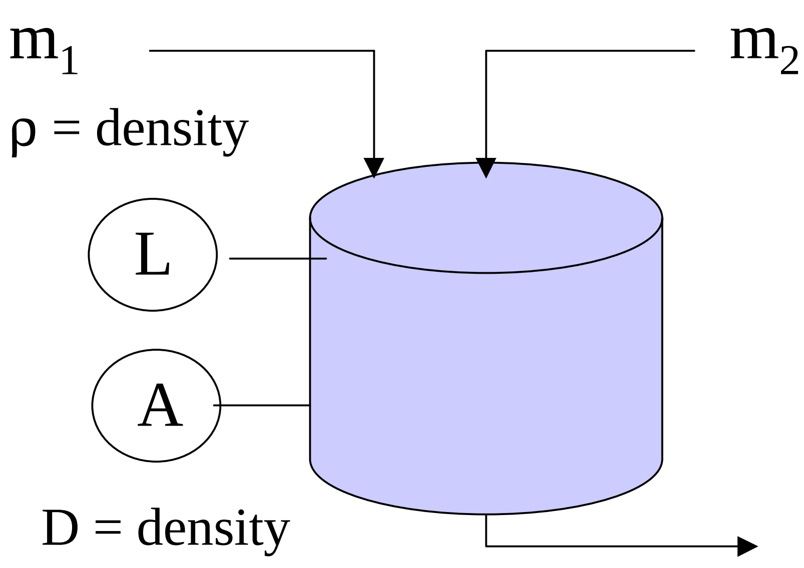
When is this equation very sensitive to errors in the individual gains?

RELATIVE GAIN MEASURE OF INTERACTION

5. We can evaluate the RGA of a system with integrating processes, such as levels.

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k = \textbf{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k = \textbf{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\textbf{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\textbf{other loops closed}}}$$

Redefine the output as the derivative of the level; then, calculate as normal.



$$A\frac{dL}{dt} = A\varepsilon = m_1 + m_2 - F_{out}$$

RELATIVE GAIN MEASURE OF INTERACTION

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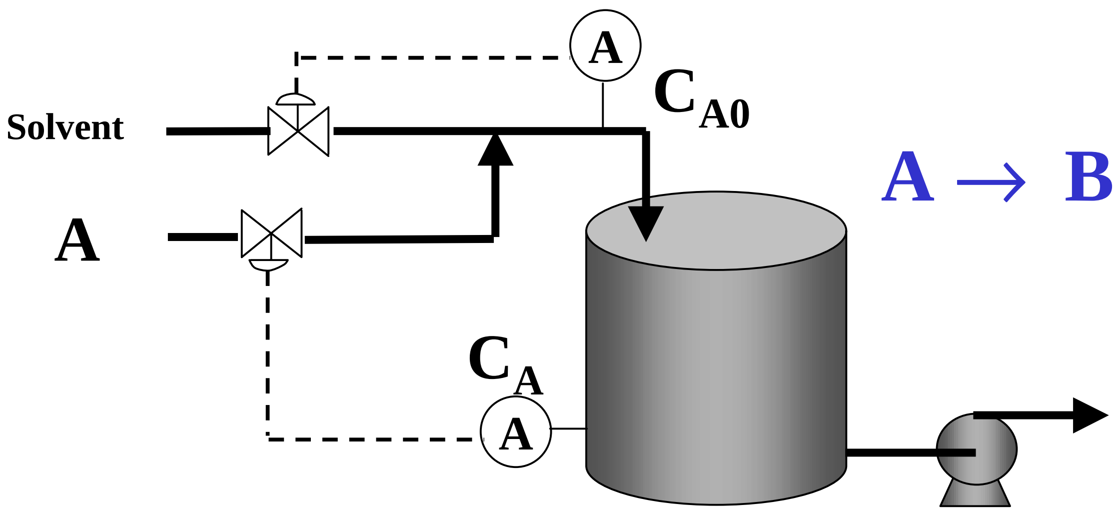
$$\mathbf{MV_j} \rightarrow \mathbf{CV_i}$$

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k=\text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k=\text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

$\lambda_{ij} < 0$ In this case, the steady-state gains have different signs depending on the status (auto/manual) of other loops



Discuss interaction in this system.

RELATIVE GAIN MEASURE OF INTERACTION

$$\mathbf{MV_j} \rightarrow \mathbf{CV_i}$$

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k=\text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k=\text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

$\lambda_{ij} < 0$ In this case, the steady-state gains have different signs depending on the status (auto/manual) of other loops

We can achieve stable multiloop feedback by using the sign of the controller gain that stabilizes the multiloop system.

Discuss what happens when the other interacting loop is placed in manual!

RELATIVE GAIN MEASURE OF INTERACTION

$$\mathbf{MV_j} \rightarrow \mathbf{CV_i}$$

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k=\text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k=\text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

$\lambda_{ij} < 0$ the steady-state gains have different signs

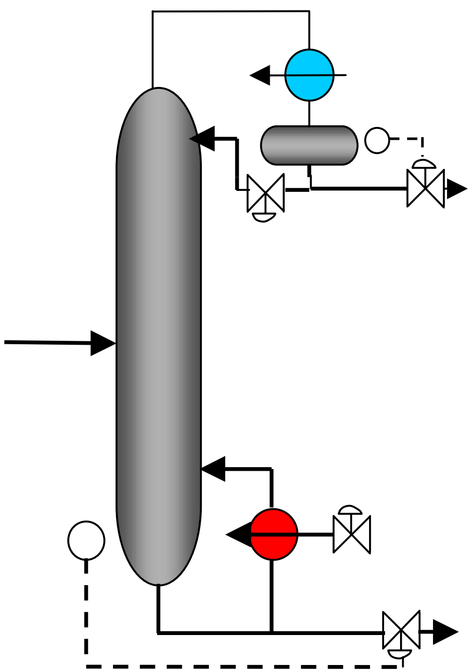
For $\lambda_{ij} < 0$, one of three BAD situations occurs

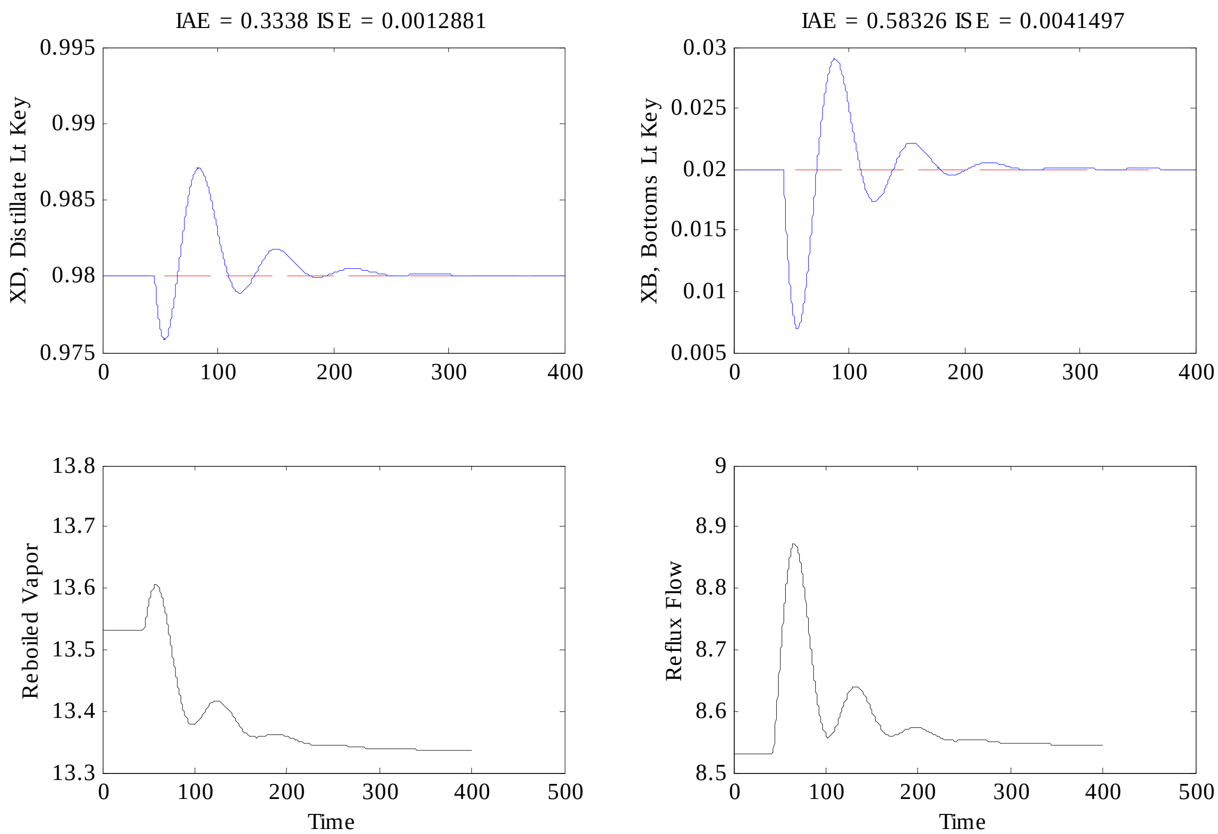
1. **Multiloop is unstable with all in automatic.**
2. **Single-loop ij is unstable when others are in manual.**
3. **Multiloop is unstable when loop ij is manual and other loops are in automatic**

Example of pairing on a negative RGA (-5.09). XB controller has a Kc with opposite sign from single-loop control! The system goes unstable when a constraint is encountered.

FR $\rightarrow$ XB

FRB $\rightarrow$ XD



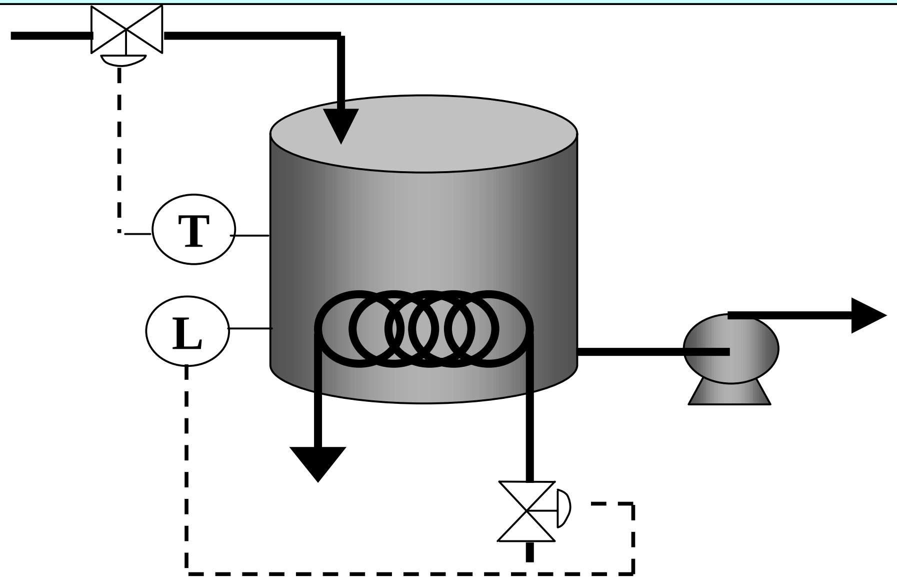


RELATIVE GAIN MEASURE OF INTERACTION

$$\mathbf{MV_j} \rightarrow \mathbf{CV_i}$$

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

$\lambda_{ij} = 0$ In this case, the steady-state gain is zero when all other loops are open, in manual.



Could this control system work?

What would happen if one controller were in manual?

RELATIVE GAIN MEASURE OF INTERACTION

$$\mathbf{MV_j} \rightarrow \mathbf{CV_i}$$

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

$0<\lambda_{ij}<1$ In this case, the steady-state (ML) gain is larger than the SL gain.

What would be the effect on tuning of opening/closing the other loop?

Discuss the case of a 2x2 system paired on $\lambda_{ij} = 0.1$

RELATIVE GAIN MEASURE OF INTERACTION

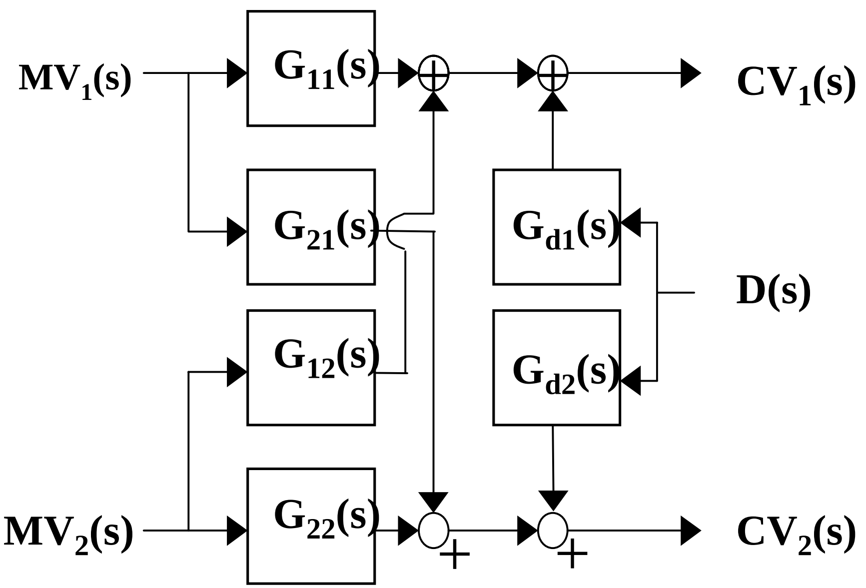
$$\mathbf{MV_j \rightarrow CV_i}$$

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k=\text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k=\text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

λ_{ij}= 1 In this case, the steady-state gains are identical in both the ML and the SL conditions.

What is generally true when λ_{ij}= 1 ?

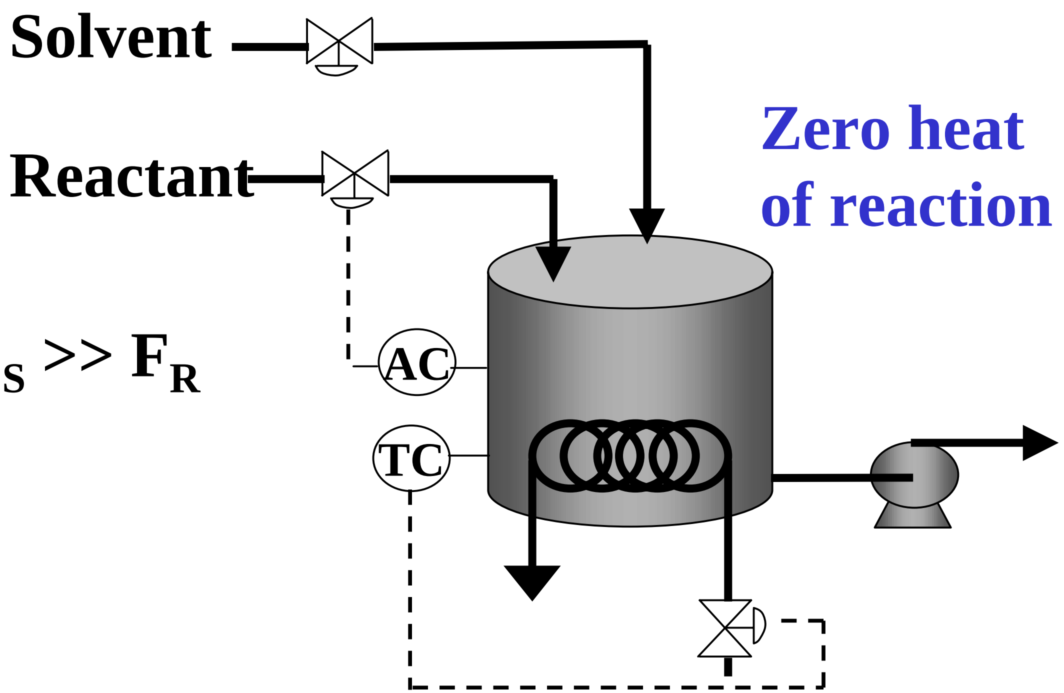
Does λ_{ij}= 1 indicate <u>no</u> interaction?



RELATIVE GAIN MEASURE OF INTERACTION

λ_{ij} = 1 In this case, the steady-state gains are identical in both the ML and the SL conditions.

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$



$F_S >> F_R$

Calculate the relative gain.

Discuss interaction in this system.

RELATIVE GAIN MEASURE OF INTERACTION

λ_{ij} = 1   In this case, the steady-state gains are identical in both the ML and the SL conditions.

$$\lambda_{ij} = \frac{\left(\frac{CV_i}{MV_j}\right)_{MV_k = \text{constant}}}{\left(\frac{CV_i}{MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\frac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\frac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

Diagonal gain matrix

$$\mathbf{K} = \begin{bmatrix} \mathbf{k}_{11} & & 0 & \\ & \mathbf{k}_{22} & & \\ & & .. & \\ 0 & & & .. \\ & & & & .. \end{bmatrix}$$

Lower diagonal gain matrix

$$\mathbf{K} = \begin{bmatrix} \mathbf{k}_{11} & & 0 & & \\ \mathbf{k}_{21} & \mathbf{k}_{22} & & & \\ .. & & .. & & \\ .. & & & .. & \\ \mathbf{k}_{n1} & .. & .. & .. & .. \end{bmatrix}$$

Both give an RGA that is diagonal!

$$\mathbf{RGA} = \begin{bmatrix} 1 & & & 0 & \\ & 1 & & & \\ & & 1 & & \\ 0 & & & 1 & \\ & & & & 1 \end{bmatrix} = \mathbf{I}$$

RELATIVE GAIN MEASURE OF INTERACTION

$$MV_j \rightarrow CV_i$$

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

$1 < \lambda_{ij}$ In this case, the steady-state (ML) gain is larger than the SL gain.

What would be the effect on tuning of opening/closing the other loop?

Discuss a the case of a 2x2 system paired on $\lambda_{ij} = 10$.

RELATIVE GAIN MEASURE OF INTERACTION

$$\mathbf{MV_j} \rightarrow \mathbf{CV_i}$$

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

$\lambda_{ij} = \infty$ **In this case, the gain in the ML situation is zero. We conclude that ML control is not possible.**

How can we improve the situation?

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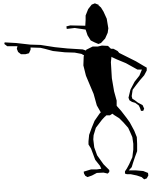
Let's evaluate some design guidelines based on RGA



RELATIVE GAIN MEASURE OF INTERACTION

Proposed Guideline #1

Select pairings that do not have any λ_{ij}<0

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

- **Review the interpretation, i.e., the effect on behavior.**
- **What would be the effect if the rule were violated?**
- **Do you agree with the Proposed Guideline?**

RELATIVE GAIN MEASURE OF INTERACTION

<u>Proposed Guideline #2</u>

Select pairings that do not have any λ_{ij}=0

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

- **Review the interpretation, i.e., the effect on behavior.**
- **What would be the effect if the rule were violated?**
- **Do you agree with the Proposed Guideline?**

RELATIVE GAIN MEASURE OF INTERACTION

RGA and INTEGRITY

- We conclude that the RGA provides excellent insight into the INTEGRITY of a multiloop control system.
- INTEGRITY: A multiloop control system has good integrity when after one loop is turned off, the remainder of the control system remains stable.
- "Turning off" can occur when (1) a loop is placed in manual, (2) a valve saturates, or (3) a lower level cascade controller no lower changes the valve (in manual or reached set point limit).
- Pairings with negative or zero RGA's have poor integrity

RELATIVE GAIN MEASURE OF INTERACTION

Proposed Guideline #3

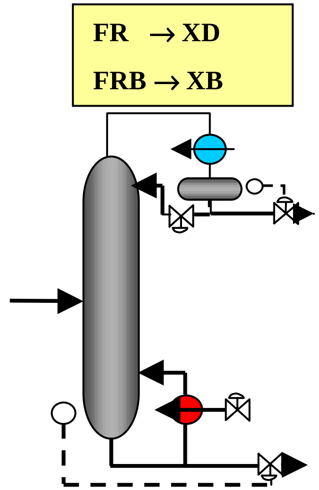
Select a pairing that has RGA elements as close as possible to λ_{ij}=1

$$\lambda_{ij} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{MV_k = \text{constant}}}{\left(\dfrac{CV_i}{MV_j}\right)_{CV_k = \text{constant}}} = \frac{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops open}}}{\left(\dfrac{CV_i}{MV_j}\right)_{\text{other loops closed}}}$$

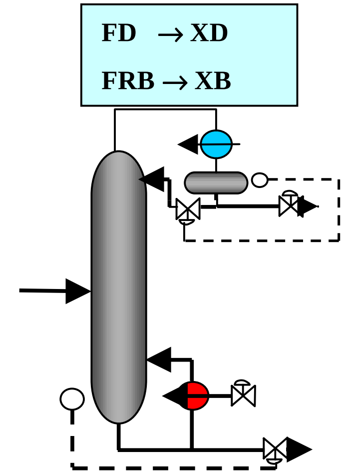
- **Review the interpretation, i.e., the effect on behavior.**
- **What would be the effect if the rule were violated?**
- **Do you agree with the Proposed Guideline?**

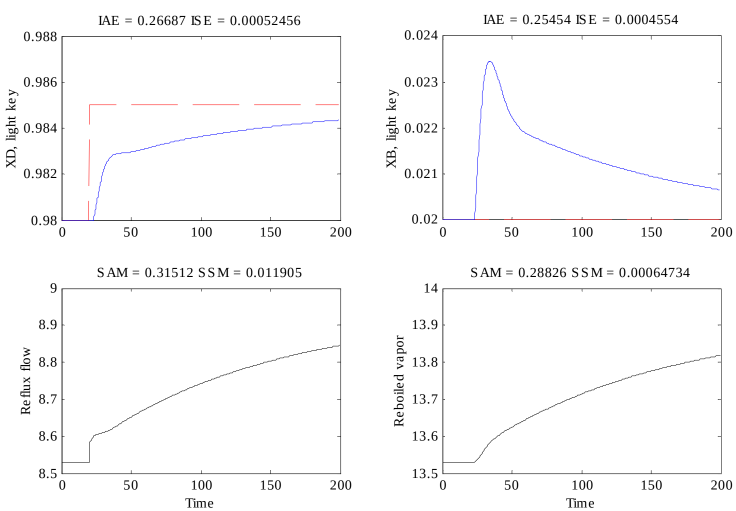
For set point response, RGA closer to 1.0 is better

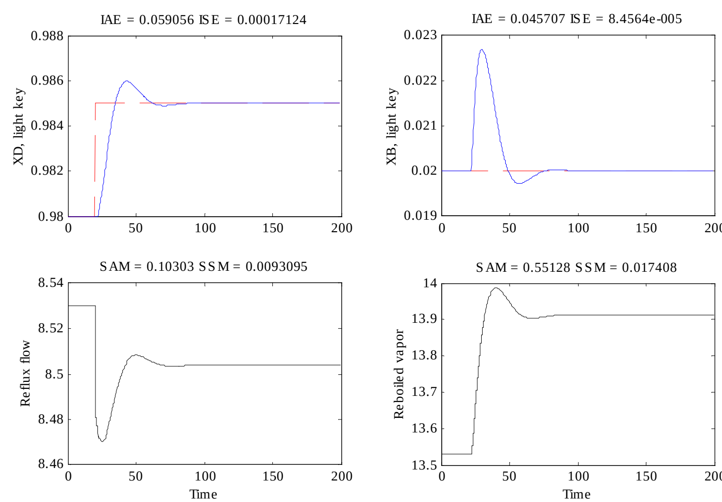
RGA = 6.09



RGA = 0.39

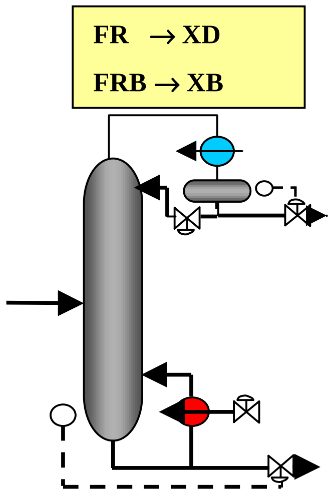






For set point response, RGA farther from 1.0 is better

RGA = 6.09



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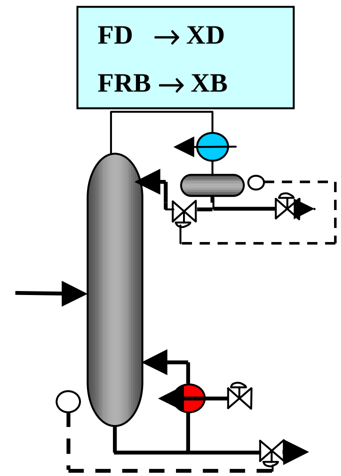

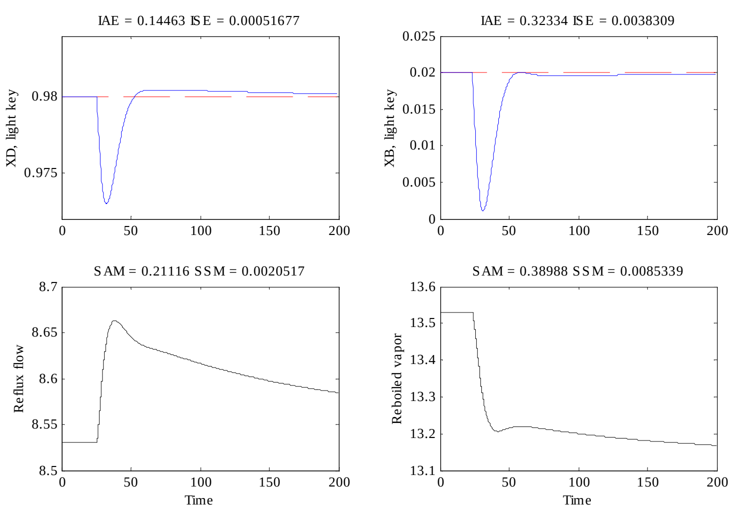

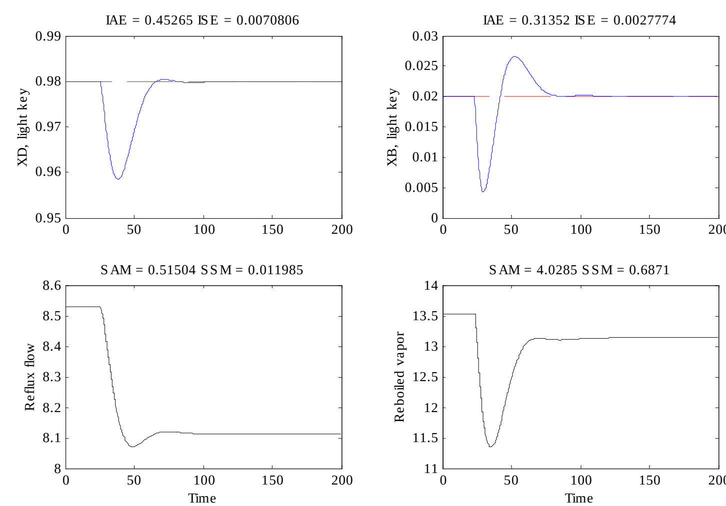

RELATIVE GAIN MEASURE OF INTERACTION

The RGA gives useful conclusions from S-S information

- **Tells us about the integrity of multiloop systems and something about the differences in tuning as well.**
- **Uses only gains from feedback process!**

Powerful results from limited information!

- **Does not use following information**
  - **Control objectives**
  - **Dynamics**
  - **Disturbances**

Can we design controls without this information?

- **Lower diagonal gain matrix can have strong interaction but gives RGAs = 1**

"Interaction?"