PROBLEMS

8.1 (*a*) Find $\mathbf{H}$ in cartesian components at $P(2, 3, 4)$ if there is a current filament on the z axis carrying 8 mA in the $\mathbf{a}_z$ direction. (*b*) Repeat if the filament is located at $x = -1$, $y = 2$. (*c*) Find $\mathbf{H}$ if both filaments are present.

8.2 A current filament of $3\mathbf{a}_x$ A lies along the x axis. Find $\mathbf{H}$ in cartesian components at $P(-1, 3, 2)$.

8.3 Two semi-infinite filaments on the z axis lie in the regions $-\infty < z < -a$ and $a < z < \infty$. Each carries a current I in the $\mathbf{a}_z$ direction. (*a*) Calculate $\mathbf{H}$ as a function of ρ and ϕ at $z = 0$. (*b*) What value of a will cause the magnitude of $\mathbf{H}$ at $\rho = 1$, $z = 0$, to be half the value obtained for an infinite filament?

8.4 (*a*) A filament is formed into a circle of radius a, centered at the origin in the plane $z = 0$. It carries a current I in the $\mathbf{a}_\phi$ direction. Find $\mathbf{H}$ at the origin. (*b*) A filament of the same length is shaped into a square in the $z = 0$ plane. The sides are parallel to the coordinate axes and a current I flows in the general $\mathbf{a}_\phi$ direction. Again find $\mathbf{H}$ at the origin.

8.5 The parallel filamentary conductors shown in Fig. 8.21 lie in free space. Plot $|\mathbf{H}|$ versus y, $-4 < y < 4$, along the line $x = 0$, $z = 2$.

8.6 (*a*) A current filament I is formed into a circle, $\rho = a$, in the $z = z'$ plane. Find H_z at $P(0, 0, z)$ if I flows in the $\mathbf{a}_\phi$ direction. (*b*) Find H_z at P caused by a uniform surface current density $\mathbf{K} = K_0\mathbf{a}_\phi$, flowing on the cylindrical surface, $\rho = a$, $0 < z < h$. The results of part (*a*) should help.

8.7 Given points $C(5, -2, 3)$ and $P(4, -1, 2)$, a current element $Id\mathbf{L} = 10^{-4}(4, -3, 1)\,\mathrm{A \cdot m}$ at C produces a field $d\mathbf{H}$ at P. (*a*) Specify the direction of $d\mathbf{H}$ by a unit vector $\mathbf{a}_H$. (*b*) Find $|d\mathbf{H}|$. (*c*) What direction $\mathbf{a}_1$ should $Id\mathbf{L}$ have at C so that $d\mathbf{H} = 0$?

8.8 For the finite-length current element on the z axis, as shown in Fig. 8.5, use the Biot-Savart law to derive Eq. (9) of Sec. 8.1.

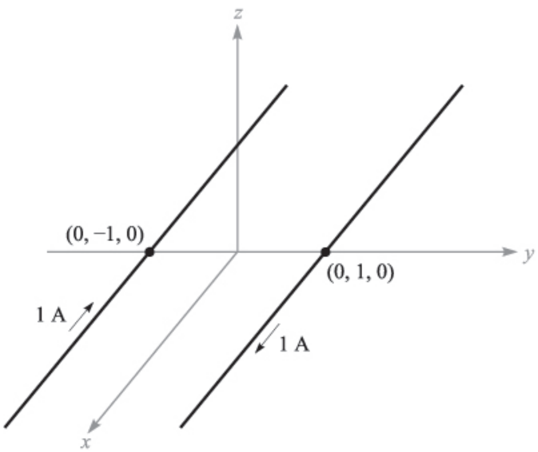

FIGURE 8.21
See Prob. 8.5.

8.9 A current sheet $\mathbf{K} = 8\mathbf{a}_x$ A/m flows in the region $-2 < y < 2$ m in the plane $z = 0$. Calculate **H** at $P(0, 0, 3)$.

8.10 Let a filamentary current of 5 mA be directed from infinity to the origin on the positive z axis and then back out to infinity on the positive x axis. Find **H** at $P(0, 1, 0)$.

8.11 An infinite filament on the z axis carries 20π mA in the $\mathbf{a}_z$ direction. Three uniform cylindrical current sheets are also present: 400 mA/m at $\rho = 1$ cm, -250 mA/m at $\rho = 2$ cm, and -300 mA/m at $\rho = 3$ cm. Calculate H_ϕ at $\rho = 0.5, 1.5, 2.5$, and 3.5 cm.

8.12 In Fig. 8.22, let the regions $0 < z < 0.3$ m and $0.7 < z < 1.0$ m be conducting slabs carrying uniform current densities of 10 A/m^2 in opposite directions as shown. Find **H** at $z =$: (*a*) -0.2; (*b*) 0.2 (*c*) 0.4; (*d*) 0.75; (*e*) 1.2 m.

8.13 A hollow cylindrical shell of radius a is centered on the z axis and carries a uniform surface current density of $K_a\mathbf{a}_\phi$. (*a*) Show that H is not a function of ϕ or z. (*b*) Show that H_ϕ and H_ρ are everywhere zero. (*c*) Show that $H_z = 0$ for $\rho > a$. (*d*) Show that $H_z = K_a$ for $\rho < a$. (*e*) A second shell, $\rho = b$, carries a current $K_b\mathbf{a}_\phi$. Find **H** everywhere.

8.14 A toroid having a cross section of rectangular shape is defined by the following surfaces: the cylinders $\rho = 2$ cm and $\rho = 3$ cm, and the planes $z = 1$ cm and $z = 2.5$ cm. The toroid carries a surface current density of $-50\mathbf{a}_z$ A/m on the surface $\rho = 3$ cm. Find **H** at the point $P(\rho, \phi, z)$: (*a*) $P_A(1.5\text{ cm}, 0, 2\text{ cm})$; (*b*) $P_B(2.1\text{ cm}, 0, 2\text{ cm})$; (*c*) $P_C(2.7\text{ cm}, \pi/2, 2\text{ cm})$; (*d*) $3.5\text{ cm}, \pi/2, 2\text{ cm})$.

8.15 Assume that there is a region with cylindrical symmetry in which the conductivity is given by $\sigma = 1.5e^{-150\rho}$kS/m. An electric field of $30\mathbf{a}_z$ V/m is present. (*a*) Find **J**. (*b*) Find the total current crossing the surface $\rho < \rho_0$, $z = 0$, all ϕ. (*c*) Make use of Ampere's circuital law to find **H**.

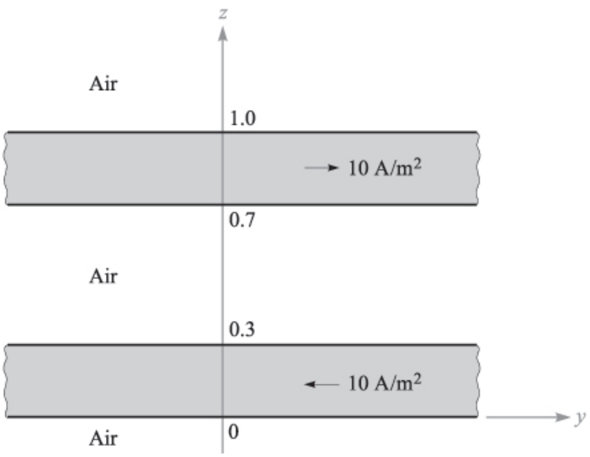

FIGURE 8.22
See Prob. 8.12.

8.16 The cylindrical shell, $2\,\text{mm} < \rho < 3\,\text{mm}$, carries a uniformly distributed total current of 8 A in the $-\mathbf{a}_z$ direction, and a filament on the z axis carries 8 A in the $\mathbf{a}_z$ direction. Find $\mathbf{H}$ everywhere.

8.17 A current filament on the z axis carries a current of 7 mA in the $\mathbf{a}_z$ direction, and current sheets of $0.5\mathbf{a}_z$ A/m and $-0.2\mathbf{a}_z$ A/m are located at $\rho = 1$ cm and $\rho = 0.5$ cm, respectively. Calculate $\mathbf{H}$ at $\rho =$: (*a*) 0.5 cm; (*b*) 1.5 cm; (*c*) 4 cm; (*d*) What current sheet should be located at $\rho = 4$ cm so that $\mathbf{H} = 0$ for all $\rho > 4$ cm?

8.18 Current density is distributed as follows: $\mathbf{J} = 0$ for $|y| > 2$ m, $\mathbf{J} = 8y\mathbf{a}_z$ A/m^2 for $|y| < 1$ m, $\mathbf{J} = 8(2 - y)\mathbf{a}_z$ A/m^2 for $1 < y < 2$ m, $\mathbf{J} = -8(2 + y)\mathbf{a}_z$ A/m^2 for $-2 < y < -1$ m. Use symmetry and Ampere's law to find $\mathbf{H}$ everywhere.

8.19 Calculate $\nabla \times [\nabla(\nabla \cdot \mathbf{G})]$ if $\mathbf{G} = 2x^2yz\mathbf{a}_x - 20y\mathbf{a}_y + (x^2 - z^2)\mathbf{a}_z$.

8.20 The magnetic field intensity is given in the square region $x = 0$, $0.5 < y < 1$, $1 < z < 1.5$ by $\mathbf{H} = z^2\mathbf{a}_x + x^3\mathbf{a}_y + y^4\mathbf{a}_z$ A/m. (*a*) Evaluate $\oint \mathbf{H} \cdot d\mathbf{L}$ about the perimeter of the square region. (*b*) Find $\nabla \times \mathbf{H}$. (*c*) Calculate $(\nabla \times \mathbf{H})_x$ at the center of the region. (*d*) Does $(\nabla \times \mathbf{H})_x = [\oint \mathbf{H} \cdot d\mathbf{L}]/\text{Area enclosed}$?

8.21 Points A, B, C, D, E, and F are each 2 mm from the origin on the coordinate axis indicated in Fig. 8.23. The value of $\mathbf{H}$ at each point is given. Calculate an approximate value for $\nabla \times \mathbf{H}$ at the origin.

8.22 In the cylindrical region $\rho \le 0.6$ mm, $H_\phi = \dfrac{2}{\rho} + \dfrac{\rho}{2}$ A/m, while $H_\phi = \dfrac{3}{\rho}$ A/m for $\rho > 0.6$ mm. (*a*) Determine $\mathbf{J}$ for $\rho < 0.6$ mm. (*b*) Determine $\mathbf{J}$ for $\rho > 0.6$ mm. (*c*) Is there a filamentary current at $\rho = 0$? If so, what is its value? (*d*) What is $\mathbf{J}$ at $\rho = 0$?

8.23 Given the field $\mathbf{H} = 20\rho^2\mathbf{a}_\phi$ A/m: (*a*) determine the current density $\mathbf{J}$; (*b*) integrate $\mathbf{J}$ over the circular surface $\rho = 1$, $0 < \phi < 2\pi$, $z = 0$, to determine the total current passing through that surface in the $\mathbf{a}_z$ direction; (*c*) find the total current once more, this time by a line integral around the circular path $\rho = 1$, $0 < \phi < 2\pi$, $z = 0$.

Point	H (A/m)		
A	$11.34\mathbf{a}_x$	$-13.78\mathbf{a}_y$	$+14.21\mathbf{a}_z$
B	$10.68\mathbf{a}_x$	$-12.19\mathbf{a}_y$	$+15.82\mathbf{a}_z$
C	$10.49\mathbf{a}_x$	$-12.19\mathbf{a}_y$	$+15.69\mathbf{a}_z$
D	$11.49\mathbf{a}_x$	$-13.78\mathbf{a}_y$	$+14.35\mathbf{a}_z$
E	$11.11\mathbf{a}_x$	$-13.88\mathbf{a}_y$	$+15.10\mathbf{a}_z$
F	$10.88\mathbf{a}_x$	$-13.10\mathbf{a}_y$	$+14.90\mathbf{a}_z$

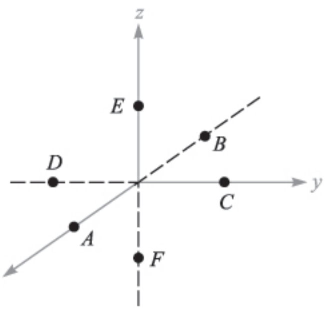

FIGURE 8.23
See Prob. 8.21.

8.24 Evaluate both sides of Stokes' theorem for the field $\mathbf{G} = 10 \sin\theta \mathbf{a}_\phi$ and the surface $r = 3$, $0 \le \theta \le 90^\circ$, $0 \le \phi \le 90^\circ$. Let the surface have the $\mathbf{a}_r$ direction.

8.25 Given the field $\mathbf{H} = \frac{1}{2}\cos\frac{\phi}{2}\mathbf{a}_\rho - \sin\frac{\phi}{2}\mathbf{a}_\phi$ A/m, evaluate both sides of Stokes' theorem for the path formed by the intersection of the cylinder $\rho = 3$ and the plane $z = 2$, and for the surface defined by $\rho = 3$, $0 \le z \le 2$, and $z = 0$, $0 \le \rho \le 3$.

8.26 Let $\mathbf{G} = 15r\mathbf{a}_\phi$. (*a*) Determine $\oint \mathbf{G} \cdot d\mathbf{L}$ for the circular path $r = 5$, $\theta = 25^\circ$, $0 \le \phi \le 2\pi$. Evaluate $\int_S (\nabla \times \mathbf{G}) \cdot d\mathbf{S}$ over the spherical cap $r = 5$, $0 \le \theta \le 25^\circ$, $0 \le \phi \le 2\pi$.

8.27 The magnetic field intensity is given in a certain region of space as $\mathbf{H} = \frac{x+2y}{z^2}\mathbf{a}_y + \frac{2}{z}\mathbf{a}_z$ A/m. (*a*) Find $\nabla \times \mathbf{H}$. (*b*) Find $\mathbf{J}$. (*c*) Use $\mathbf{J}$ to find the total current passing through the surface $z = 4$, $1 \le x \le 2$, $3 \le z \le 5$, in the $\mathbf{a}_z$ direction. (*d*) Show that the same result is obtained using the other side of Stokes' theorem.

8.28 Given $\mathbf{H} = (3r^2/\sin\theta)\mathbf{a}_\theta + 54r\cos\theta\mathbf{a}_\phi$ A/m in free space: (*a*) find the total current in the $\mathbf{A}_\theta$ direction through the conical surface $\theta = 20^\circ$, $0 \le \phi \le 2\pi$, $0 \le r \le 5$, by whichever side of Stokes' theorem you like the best. (*b*) Check the result by using the other side of Stokes' theorem.

8.29 A long straight nonmagnetic conductor of 0.2-mm radius carries a uniformly distributed current of 2 A dc. (*a*) Find $\mathbf{J}$ within the conductor. (*b*) Use Ampère's circuital law to find $\mathbf{H}$ and $\mathbf{B}$ within the conductor. (*c*) Show that $\nabla \times \mathbf{H} = \mathbf{J}$ within the conductor. (*d*) Find $\mathbf{H}$ and $\mathbf{B}$ within the conductor. (*e*) Show that $\nabla \times \mathbf{H} = \mathbf{J}$ outside the conductor.

8.30 A solid nonmagnetic conductor of circular cross section has a radius of 2 mm. The conductor is inhomogeneous, with $\sigma = 10^6(1 + 10^6\rho^2)$ S/m. If the conductor is 1 m in length and has a voltage of 1 mV between its ends, find: (*a*) H; (*b*) the total magnetic flux inside the conductor.

8.31 The cylindrical shell defined by $1\text{ cm} < \rho < 1.4\text{ cm}$ consists of a nonmagnetic conducting material and carries a total current of 50 A in the $\mathbf{a}_z$ direction. Find the total magnetic flux crossing the plane $\phi = 0$, $0 < z < 1$: (*a*) $0 < \rho < 1.2\text{ cm}$; (*b*) $1.4\text{ cm} < \rho < 1.4\text{ cm}$; (*c*) $1.4\text{ cm} < \rho < 20\text{ cm}$.

8.32 The free-space region defined by $1 < z < 4\text{ cm}$ and $2 < \rho < 3\text{ cm}$ is a toroid of rectangular cross section. Let the surface at $\rho = 3$ cm carry a surface current $\mathbf{K} = 2\mathbf{a}_z$ kA/m. (*a*) Specify the currents on the surfaces at $\rho = 2$ cm, $z = 1$ cm, and $z = 4$ cm. (*b*) Find $\mathbf{H}$ everywhere. (*c*) Calculate the total flux within the toroid.

8.33 Use an expansion in cartesian coordinates to show that the curl of the gradient of any scalar field G is identically equal to zero.

8.34 A filamentary conductor on the z axis carries a current of 16 A in the $\mathbf{a}_z$ direction, a conducting shell at $\rho = 6$ carries a total current of 12 A in the $-\mathbf{a}_z$ direction, and another shell at $\rho = 10$ carries a total current of 4 A in the $-\mathbf{a}_z$ direction. (*a*) Find $\mathbf{H}$ for $0 < \rho < 12$. (*b*) Plot H_ϕ versus ρ. (*c*) Find the total flux Φ crossing the surface $1 < \rho < 7$, $0 < z < 1$.

8.35 A current sheet, $\mathbf{K} = 20\mathbf{a}_z$ A/m, is located at $\rho = 2$, and a second sheet, $\mathbf{K} = -10\mathbf{a}_z$ A/m, is located at $\rho = 4$. (*a*) Let $V_m = 0$ at $P(\rho = 3, \phi = 0, z = 5)$ and place a barrier at $\phi = \pi$. Find $V_m(\rho, \phi, z)$ for $-\pi < \phi < \pi$. (*b*) Let $\mathbf{A} = 0$ at P and find $\mathbf{A}(\rho, \phi, z)$ for $2 < \rho < 4$.

8.36 Let $\mathbf{A} = (3y - z)\mathbf{a}_x + 2xz\mathbf{a}_y$ Wb/m in a certain region of free space. (*a*) Show that $\nabla \cdot \mathbf{A} = 0$. (*b*) At $P(2, -1, 3)$, find $\mathbf{A}$, $\mathbf{B}$, $\mathbf{H}$, and $\mathbf{J}$.

8.37 Let $N = 1000$, $I = 0.8$ A, $\rho_0 = 2$ cm, and $a = 0.8$ cm for the toroid shown in Fig. 8.12*b*. Find V_m in the interior of the toroid if $V_m = 0$ at $\rho = 2.5$ cm, $\phi = 0.3\pi$. Keep ϕ within the range $0 < \phi < 2\pi$.

8.38 The solenoid shown in Fig. 8.11*b* contains 400 turns, carries a current $I = 5$ A, has a length of 8 cm, and a radius $a = 1.2$ cm. (*a*) Find $\mathbf{H}$ within the solenoid. (*b*) If $V_m = 0$ at the origin, specify $V_m(\rho, \phi, z)$ inside the solenoid. (*c*) Let $\mathbf{A} = 0$ at the origin, and specify $\mathbf{A}(\rho, \phi, z)$ inside the solenoid if the medium is free space.

8.39 Planar current sheets of $\mathbf{K} = 30\mathbf{a}_z$ A/m and $-30\mathbf{a}_z$ A/m are located in free space at $x = 0.2$ and $x = -0.2$, respectively. For the region $-0.2 < x < 0.2$: (*a*) find $\mathbf{H}$; (*b*) obtain an expression for V_m if $V_m = 0$ at $P(0.1, 0.2, 0.3)$; (*c*) find $\mathbf{B}$; (*d*) obtain an expression for $\mathbf{A}$ if $\mathbf{A} = 0$ at P.

8.40 Let $\mathbf{A} = (3y^2 - 2z)\mathbf{a}_x - 2x^2z\mathbf{a}_y + (x + 2y)\mathbf{a}_z$ Wb/m in free space. Find $\nabla \times \nabla \times \mathbf{A}$ at $P(-2, 3, -1)$.

8.41 Assume that $\mathbf{A} = 50\rho^2\mathbf{a}_z$ Wb/m in a certain region of free space. (*a*) Find $\mathbf{H}$ and $\mathbf{B}$. (*b*) Find $\mathbf{J}$. (*c*) Use $\mathbf{J}$ to find the total current crossing the surface $0 \le \rho \le 1$, $0 \le \phi < 2\pi$, $z = 0$. (*d*) Use the value of H_ϕ at $\rho = 1$ to calculate $\oint \mathbf{H} \cdot d\mathbf{L}$ for $\rho = 1$, $z = 0$.

8.42 Show that $\nabla_2(1/R_{12}) = -\nabla_1(1/R_{12}) = \mathbf{R}_{21}/R_{12}^3$.

8.43 Compute the vector magnetic potential within the outer conductor for the coaxial line whose vector magnetic potential is shown in Fig. 8.20 if the outer radius of the outer conductor is $7a$. Select the proper zero reference and sketch the results on the figure.

8.44 By expanding Eq. (58), Sec. 8.7, in cartesian coordinates, show that (59) is correct.