PROBLEMS

5.1 Given the current density $\mathbf{J} = -10^4(\sin 2x\, e^{-2y}\mathbf{a}_x + \cos 2x\, e^{-2y}\mathbf{a}_y)$ kA/m^2: (*a*) find the total current crossing the plane $y = 1$ in the $\mathbf{a}_y$ direction in the region $0 < x < 1$, $0 < z < 2$. Find the total current leaving the region $0 < x, y < 1$, $2 < z < 3$ by: (*b*) integrating $\mathbf{J} \cdot d\mathbf{S}$ over the surface of the cube; (*c*) employing the divergence theorem.

5.2 Let the current density be $\mathbf{J} = 2\rho \cos^2 \phi\, \mathbf{a}_\rho - \rho \sin 2\phi\, \mathbf{a}_\phi$ A/m^2 within the region $2.1 < \rho < 2.5$, $0 < \phi < 0.1$ rad, $6 < z < 6.1$. Find the total current I crossing the surface: (*a*) $\rho = 2.2$, $0 < \phi < 0.1$, $6 < z < 6.1$ in the $\mathbf{a}_\rho$ direction; (*b*) $\phi = 0.05$, $2.2 < \rho < 2.5$, $6 < z < 6.1$, in the $\mathbf{a}_\phi$ direction. (*c*) Evaluate $\nabla \cdot \mathbf{J}$ at $P(\rho = 2.4, \phi = 0.08, z = 6.05)$.

5.3 Let $\mathbf{J} = \dfrac{400 \sin \theta}{r^2 + 4}$ A/m^2. (*a*) Find the total current flowing through that portion of the spherical surface $r = 0.8$ bounded by $\theta = 0.1\pi$, $\theta = 0.3\pi$, $0 < \phi < 2\pi$. (*b*) Find the average value of J over the defined area.

5.4 The cathode of a planar vacuum tube is at $z = 0$. Let $\mathbf{E} = -4 \times 10^6 \mathbf{a}_z$ V/m for $z > 0$. An electron ($e = 1.602 \times 10^{-19}$ C, $m = 9.11 \times 10^{-31}$ kg) is emitted from the cathode with zero initial velocity at $t = 0$. (*a*) Find $v(t)$. (*b*) Find $z(t)$, the electron location as a function of time. (*c*) Determine $v(z)$. (*d*) Make the assumption that electrons are emitted continuously as a beam with a 0.25-mm radius and a total current of 60 μA. Find $\mathbf{J}(z)$ and $\rho_v(z)$.

5.5 Let $\mathbf{J} = \dfrac{25}{\rho}\mathbf{a}_\rho - \dfrac{20}{\rho^2 + 0.01}\mathbf{a}_z$ A/m^2, and: (*a*) find the total current crossing the plane $z = 0.2$ in the $\mathbf{a}_z$ direction for $\rho < 0.4$. (*b*) Calculate $\dfrac{\partial \rho_v}{\partial t}$. (*c*) Find the total outward current crossing the closed surface defined by $\rho = 0.01$, $\rho = 0.4$, $z = 0$, and $z = 0.2$. (*d*) Show that the divergence theorem is satisified for $\mathbf{J}$ and the surface specified.

5.6 Let $\epsilon = \epsilon_0$ and $V = 90z^{4/3}$ in the region $z = 0$. (*a*) Obtain expressions for $\mathbf{E}$, $\mathbf{D}$, and ρ_v as functions of z. (*b*) If the velocity of the charge density is given as $v_x = 5 \times 10^6 z^{2/3}$ m/s, find J_z at $z = 0$ and $z = 0.1$ m.

5.7 Assuming that there is no transformation of mass to energy or vice versa, it is possible to write a continuity equation for mass. (*a*) If we use the continuity equation for charge as our model, what quantities correspond to $\mathbf{J}$ and ρ_v? (*b*) Given a cube 1 cm on a side, experimental data show that the rates at which mass is leaving each of the six faces are 10.25, −9.85, 1.75, −2.00, −4.05, and 4.45 mg/s. If we assume that the cube is an incremental volume element, determine an approximate value for the time rate of change of density at its center.

5.8 The continuity equation for mass equates the divergence of the mass rate of flow (mass per second per square meter) to the negative of the density (mass per cubic meter). After setting up a cartesian coordinate system inside a star, Captain Kirk and his intrepid crew make measurements over the faces of a cube centered at the origin with edges 40 km long and parallel to the coordinate axes. They find the mass rate of flow of material outward across the six faces to be -1112, 1183, 201, -196, 1989, and $-1920\,\text{kg/km}^2 \cdot \text{s}$. (*a*) Estimate the divergence of the mass rate of flow at the origin. (*b*) Estimate the rate of change of the density at the origin.

5.9 (*a*) Using data tabulated in Appendix C, calculate the required diameter for a 2-m long nichrome wire that will dissipate an average power of 450 W when 120-V rms at 60 Hz is applied to it. (*b*) Calculate the rms current density in the wire.

5.10 A steel wire has a radius of 2 mm and a conductivity of 6×10^6 S/m. The steel wire has an aluminum ($\sigma = 3.8 \times 10^7$ S/m) coating of 2-mm thickness. Let the total current carried by this hybrid conductor be 80 A dc. Find: (*a*) J_{st}; (*b*) J_{Al}; (*c*) E_{st}; (*d*) E_{Al}; (*e*) the voltage between the ends of the conductor if it is 1 mi long.

5.11 Two perfectly conducting cylindrical surfaces are located at $\rho = 3$ and $\rho = 5$ cm. The total current passing radially outward through the medium between the cylinders is 3 A dc. (*a*) Find the voltage and resistance between the cylinders, and **E** in the region between the cylinders, if a conducting material having $\sigma = 0.05$ S/m is present for $3 < \rho < 5$ cm. (*b*) Show that integrating the power dissipated per unit volume over the volume gives the total dissipated power.

5.12 The spherical surfaces $r = 3$ and $r = 5$ cm are perfectly conducting, and the total current passing radially outward through the medium between the surfaces is 3 A dc. (*a*) Find the voltage and resistance between the spheres, and **E** in the region between them, if a conducting material having $\sigma = 0.05$ S/m is present for $3 < r < 5$ cm. (*b*) Repeat if $\sigma = 0.0005/r$ for $3 < r < 5$ cm. (*c*) Show that integrating the power dissipated per unit volume in part *b* over the volume gives the total dissipated power.

5.13 A hollow cylindrical tube with a rectangular cross section has external dimensions of 0.5 in by 1 in and a wall thickness of 0.05 in. Assume that the material is brass for which $\sigma = 1.5 \times 10^7$ S/m. A current of 200 A dc is flowing down the tube. (*a*) What voltage drop is present across a 1 m length of the tube? (*b*) Find the voltage drop if the interior of the tube is filled with a conducting material for which $\sigma = 1.5 \times 10^5$ S/m.

5.14 Find the magnitude of the electric field intensity in a conductor if: (*a*) the current density is $5\,\text{MA/m}^2$, the electron mobility is $3 \times 10^{-3}\,\text{m}^2/\text{V}\cdot\text{s}$, and the volume charge density is $-2.4 \times 10^{10}\,\text{C/m}^3$; (*b*) $J = 3\,\text{MA/m}^2$ and the resistivity is $3 \times 10^{-8}\,\Omega \cdot \text{m}$.

5.15 Let $V = 10(\rho + 1)z^2 \cos\phi$ V in free space. (*a*) Let the equipotential surface $V = 20$ V define a conductor surface. Find the equation of the conductor surface. (*b*) Find ρ and **E** at that point on the conductor surface where $\phi = 0.2\pi$ and $z = 1.5$. (*c*) Find $|\rho_S|$ at that point.

5.16 A potential field in free space is given as $V = (80 \cos\theta \sin\phi)/r^3$ V. Point $P(r = 2, \theta = \pi/3, \phi = \pi/2)$ lies on a conducting surface. (*a*) Write the equation of the conducting surface. (*b*) Find a unit normal directed outward to the surface, assuming the origin is inside the surface. (*c*) Find **E** at *P*.

5.17 Given the potential field $V = \dfrac{100xz}{x^2+4}$ V in free space: (*a*) find **D** at the surface $z = 0$. (*b*) Show that the $z = 0$ surface is an equipotential surface. (*c*) Assume that the $z = 0$ surface is a conductor and find the total charge on that portion of the conductor defined by $0 < x < 2$, $-3 < y < 0$.

5.18 Let us assume a field $\mathbf{E} = 3y^2z^3\mathbf{a}_x + 6xyz^3\mathbf{a}_y + 9xy^2z^2$ V/m in free space, and also assume that point $P(2, 1, 0)$ lies on a conducting surface. (*a*) Find ρ_v just adjacent to the surface at P. (*b*) Find ρ_S at P. (*c*) Show that $V = -3xy^2z^3$ V. (*d*) Determine V_{PQ}, given $Q(1, 1, 1)$.

5.19 Let $V = 20x^2yz - 10z^2$ V in free space. (*a*) Determine the equations of the equipotential surfaces on which $V = 0$ and 60 V. (*b*) Assume these are conducting surfaces and find the surface charge density at that point on the $V = 60$-V surface where $x = 2$ and $z = 1$. It is known that $0 \le V \le 60$ V is the field-containing region. (*c*) Give the unit vector at this point that is normal to the conducting surface and directed toward the $V = 0$ surface.

5.20 A conducting plane is located at $z = 0$ in free space, and a 20-nC point charge is present at $Q(2, 4, 6)$. (*a*) If $V = 0$ at $z = 0$, find V at $P(5, 3, 1)$. (*b*) Find **E** at P. (*c*) Find ρ_S at $A(5, 3, 0)$.

5.21 Let the surface $y = 0$ be a perfect conductor in free space. Two uniform infinite line charges of 30 nC/m each are located at $x = 0$, $y = 1$, and $x = 0$, $y = 2$. (*a*) Let $V = 0$ at the plane $y = 0$, and find V at $P(1, 2, 0)$. (*b*) Find **E** at P.

5.22 Let the plane $x = 0$ be a perfect conductor in free space. Locate a point charge of 4 nC at $P_1(7, 1, -2)$, and a point charge of -3 nC at $P_2(4, 2, 1)$. (*a*) Find **E** at $A(5, 0, 0)$. (*b*) Find $|\rho_S|$ at $B(3, 0, 0)$.

5.23 A dipole with $\mathbf{p} = 0.1\mathbf{a}_z\ \mu\text{C}\cdot\text{m}$ is located at $A(1, 0, 0)$ in free space, and the $x = 0$ plane is perfectly conducting. (*a*) Find V at $P(2, 0, 1)$. (*b*) Find the equation of the 200-V equipotential surface in cartesian coordinates.

5.24 The mobilities for intrinsic silicon at a certain temperature are $\mu_e = 0.14\ \text{m}^2/\text{V}\cdot\text{s}$ and $\mu_h = 0.035\ \text{m}^2/\text{V}\cdot\text{s}$. The concentration of both holes and electrons is $2.2 \times 10^{16}\ \text{m}^{-3}$. Determine both the conductivity and resistivity of this silicon sample.

5.25 Electron and hole concentrations increase with temperature. For pure silicon suitable expressions are $\rho_h = -\rho_e = 6200T^{1.5}e^{-7000/T}\ \text{C/m}^3$. The functional dependence of the mobilities on temperature is given by $\mu_h = 2.3 \times 10^5 T^{-2.7}\ \text{m}^2/\text{V}\cdot\text{s}$ and $\mu_e = 2.1 \times 10^5 T^{-2.5}\ \text{m}^2/\text{V}\cdot\text{s}$. Find σ at: (*a*) 0° C; (*b*) 40° C; (*c*) 80° C.

5.26 A little donor impurity, such as arsenic, is added to pure silicon so that the electron concentration is 2×10^{17} conduction electrons per cubic meter while the number of holes per cubic meter is only 1.1×10^{15}. If $\mu_e = 0.15\ \text{m}^2/\text{V}\cdot\text{s}$ for this sample, and $\mu_h = 0.045\ \text{m}^2/\text{V}\cdot\text{s}$, determine the conductivity and resistivity.

5.27 Atomic hydrogen contains 5.5×10^{25} atoms/m^3 at a certain temperature and pressure. When an electric field of 4 kV/m is applied, each dipole formed by the electron and the positive nucleus has an effective length of 7.1×10^{-19} m. Find: (*a*) P; (*b*) ϵ_R.

5.28 In a certain region where the relative permittivity is 2.4, $\mathbf{D} = 2\mathbf{a}_x - 4\mathbf{a}_y + 5\mathbf{a}_z$ nC/m^2. Find: (*a*) $\mathbf{E}$; (*b*) $\mathbf{P}$; (*c*) $|\nabla V|$.

5.29 A coaxial conductor has radii $a = 0.8$ mm and $b = 3$ mm and a polystyrene dielectric for which $\epsilon_R = 2.56$. If $\mathbf{P} = \dfrac{2}{\rho}\mathbf{a}_\rho$ nC/m^2 in the dielectric, find: (*a*) $\mathbf{D}$ and $\mathbf{E}$ as functions of ρ; (*b*) V_{ab} and χ_e. (*c*) If there are 4×10^{19} molecules per cubic meter in the dielectric, find $\mathbf{p}(\rho)$.

5.30 Given the potential field $V = 200 - 50x + 20y$ V in a dielectric material for which $\epsilon_R = 2.1$, find: (*a*) $\mathbf{E}$; (*b*) $\mathbf{D}$; (*c*) $\mathbf{P}$; (*d*) ρ_v; (*e*) ρ_b; (*f*) ρ_T.

5.31 The surface $x = 0$ separates two perfect dielectrics. For $x > 0$ let $\epsilon_R = \epsilon_{R1} = 3$, while $\epsilon_{R2} = 5$ where $x < 0$. If $\mathbf{E}_1 = 80\mathbf{a}_x - 60\mathbf{a}_y - 30\mathbf{a}_z$ V/m, find: (*a*) E_{N1}; (*b*) $\mathbf{E}_{t1}$; (*c*) E_{t1}; (*d*) E_1; (*e*) the angle θ_1 between $\mathbf{E}_1$ and a normal to the surface; (*f*) D_{N2}; (*g*) D_{t2}; (*h*) $\mathbf{D}_2$; (*i*) $\mathbf{P}_2$ (*j*) the angle θ_2 between $\mathbf{E}_2$ and a normal to the surface.

5.32 In Fig. 5.18 let $\mathbf{D}_1 = 3\mathbf{a}_x - 4\mathbf{a}_y + 5\mathbf{a}_z$ nC/m^2 and find: (*a*) $\mathbf{D}_2$; (*b*) $\mathbf{D}_{N2}$; (*c*) $\mathbf{D}_{t2}$; (*d*) the energy density in each region; (*e*) the angle that $\mathbf{D}_2$ makes with $\mathbf{a}_z$; (*f*) D_2/D_1; (*g*) P_2/P_1.

5.33 Two perfect dielectrics have relative permittivities $\epsilon_{R1} = 2$ and $\epsilon_{R2} = 8$. The planar interface between them is the surface $x - y + 2z = 5$. The origin lies in region 1. If $\mathbf{E}_1 = 100\mathbf{a}_x + 200\mathbf{a}_y - 50\mathbf{a}_z$ V/m, find $\mathbf{E}_2$.

5.34 Let the spherical surfaces $r = 4$ cm and $r = 9$ cm be separated by two perfect dielectric shells, $\epsilon_{R1} = 2$ for $4 < r < 6$ cm, and $\epsilon_{R2} = 5$ for $6 < r < 9$ cm. If $\mathbf{E}_1 = \dfrac{2000}{r^2}\mathbf{a}_r$ V/m, find: (*a*) $\mathbf{E}_2$; (*b*) the total electrostatic energy stored in each region.

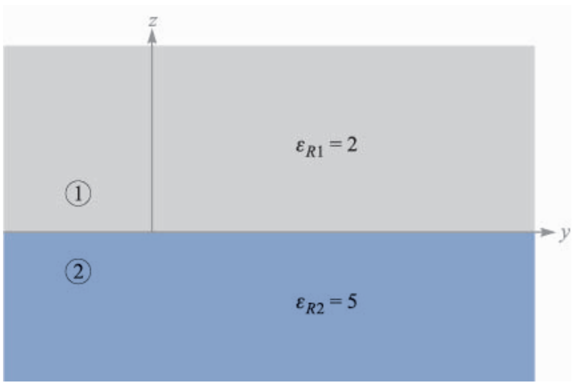

FIGURE 5.18
See Prob. 32.

5.35 Let the cylindrical surfaces $\rho = 4\,\text{cm}$ and $\rho = 9\,\text{cm}$ enclose two wedges of perfect dielectrics, $\epsilon_{R1} = 2$ for $0 < \phi < \pi/2$, and $\epsilon_{R2} = 5$ for $\pi/2 < \phi < 2\pi$. If $\mathbf{E}_1 = \dfrac{2000}{\rho}\mathbf{a}_\rho$ V/m, find: (*a*) $\mathbf{E}_2$; (*b*) the total electrostatic energy stored in a 1-m length of each region.

5.36 Let $S = 120\,\text{cm}^2$, $d = 4\,\text{mm}$, and $\epsilon_R = 12$ for a parallel-plate capacitor. (*a*) Calculate the capacitance. (*b*) After connecting a 40-V battery across the capacitor, calculate E, D, Q, and the total stored electrostatic energy. (*c*) The source is now removed and the dielectric carefully withdrawn from between the plates. Again calculate E, D, Q, and the energy. (*d*) What is the voltage between the plates?

5.37 Capacitors tend to be more expensive as their capacitance and maximum voltage $V_{\max}$ increase. The voltage $V_{\max}$ is limited by the field strength at which the dielectric breaks down, E_{BD}. Which of these dielectrics will give the largest $CV_{\max}$ product for equal plate areas: (*a*) air: $\epsilon_R = 1$, $E_{BD} = 3\,\text{MV/m}$; (*b*) barium titanate: $\epsilon_R = 1200$, $E_{BD} = 3\,\text{MV/m}$; (*c*) silicon dioxide: $\epsilon_R = 3.78$, $E_{BD} = 16\,\text{MV/m}$; (*d*) polyethylene: $\epsilon_R = 2.26$, $E_{BD} = 4.7\,\text{MV/m}$.

5.38 A dielectric circular cylinder used between the plates of a capacitor has a thickness of 0.2 mm and a radius of 1.4 cm. The dielectric properties are $\epsilon_R = 400$ and $\sigma = 10^{-5}\,\text{S/m}$. (*a*) Calculate C. (*b*) Find the quality factor $Q_{QF}(Q_{QF} = \omega RC)$ of the capacitor at $f = 10\,\text{kHz}$. (*c*) If the maximum field strength permitted in the dielectric is 2 MV/m, what is the maximum permissible voltage across the capacitor? (*d*) What energy is stored when this voltage is applied?

5.39 A parallel-plate capacitor is filled with a nonuniform dielectric characterized by $\epsilon_R = 2 + 2 \times 10^6 x^2$, where x is the distance from one plate. If $S = 0.02\,\text{m}^2$ and $d = 1\,\text{mm}$, find C.

5.40 (*a*) The width of the region containing ϵ_{R1} in Fig. 5.19 is 1.2 m. Find ϵ_{R1} if $\epsilon_{R2} = 2.5$ and the total capacitance is 60 nF. (*b*) Find the width of each region (containing ϵ_{R1} and ϵ_{R2}) if $C_{\text{total}} = 80\,\text{nF}$, $\epsilon_{R2} = 3\epsilon_{R1}$, and $C_1 = 2C_2$.

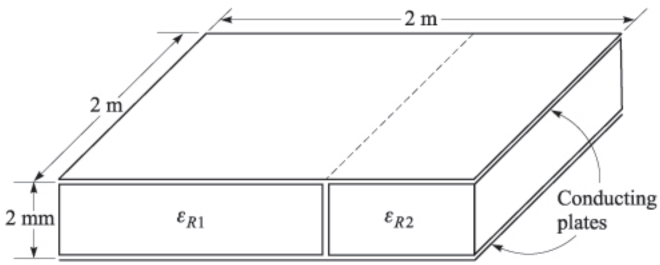

FIGURE 5.19
See Prob. 40.

5.41 Let $\epsilon_{R1} = 2.5$ for $0 < y < 1$ mm, $\epsilon_{R2} = 4$ for $1 < y < 3$ mm, and ϵ_{R3} for $3 < y < 5$ mm. Conducting surfaces are present at $y = 0$ and $y = 5$ mm. Calculate the capacitance per square meter of surface area if: (*a*) ϵ_{R3} is air; (*b*) $\epsilon_{R3} = \epsilon_{R1}$; (*c*) $\epsilon_{R3} = \epsilon_{R2}$; (*d*) ϵ_{R3} is silver.

5.42 Cylindrical conducting surfaces are located at $\rho = 0.8$ cm and 3.6 cm. The region 0.8 cm $< \rho < a$ contains a dielectric for which $\epsilon_R = 4$, while $\epsilon_R = 2$ for $a < \rho < 3.6$ cm. (*a*) Find a so that the voltage across each dielectric layer is the same. (*b*) Find the total capacitance per meter.

5.43 Two coaxial conducting cylinders of radius 2 cm and 4 cm have a length of 1 m. The region between the cylinders contains a layer of dielectric from $\rho = c$ to $\rho = d$ with $\epsilon_R = 4$. Find the capacitance if: (*a*) $c = 2$ cm, $d = 3$ cm; (*b*) $d = 4$ cm, and the volume of dielectric is the same as in part a.

5.44 Conducting cylinders lie at $\rho = 3$ and 12 mm; both extend from $z = 0$ to $z = 1$ m. Perfect dielectrics occupy the interior region: $\epsilon_R = 1$ for $3 < \rho < 6$ mm, $\epsilon_R = 4$ for $6 < \rho < 9$ mm, and $\epsilon_R = 8$ for $9 < \rho < 12$ mm. (*a*) Calculate C. (*b*) If the voltage between the cylinders is 100 V, plot $|E_\rho|$ versus ρ.

5.45 Two conducting spherical shells have radii $a = 3$ cm and $b = 6$ cm. The interior is a perfect dielectric for which $\epsilon_R = 8$. (*a*) Find C. (*b*) A portion of the dielectric is now removed so that $\epsilon_R = 1, 0 < \phi < \pi/2$, and $\epsilon_R = 8$, $\pi/2 < \phi < 2\pi$. Again find C.

5.46 Conducting cylinders lie at $\rho = 3$ and 12 mm; both extend from $z = 0$ to $z = 1$ m. Perfect dielectrics occupy the interior region: $\epsilon_R = 1$ for $3 < \rho < 6$ mm, $\epsilon_R = 4$ for $6 < \rho < 9$ mm, and $\epsilon_R = 8$ for $9 < \rho < 12$ mm. (*a*) Calculate C. (*b*) If the voltage between the cylinders is 100 V, plot $|E_\rho|$ versus ρ.

5.47 With reference to Fig. 5.17, let $b = 6$ m, $h = 15$ m, and the conductor potential be 250 V. Take $\epsilon = \epsilon_0$. Find values for K_1, ρ_L, a, and C.

5.48 A potential function in free space is given by $V = -20 + 10 \ln \dfrac{(5+y)^2 + x^2}{(5-y)^2 + x^2}$ V. Describe: (*a*) the 0-V equipotential surface; (*b*) the 10-V equipotential surface.

5.49 A 2-cm-diameter conductor is suspended in air with its axis 5 cm from a conducting plane. Let the potential of the cylinder be 100 V and that of the plane be 0 V. Find the surface charge density on the: (*a*) cylinder at a point nearest the plane; (*b*) plane at a point nearest the cylinder.