EE 2000 SIGNALS AND SYSTEMS

Ch. 1 Continuous-Time Signals

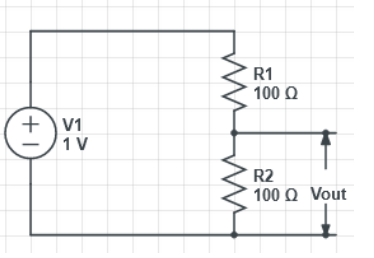
(These slides are taken from Dr. Jingxian Wu, University of Arkansas, 2020.)

OUTLINE

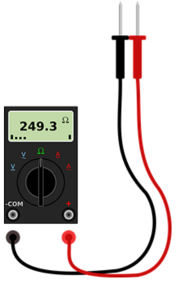
- **Introduction: what are signals and systems?**
- **Signals**
- **Classifications**
- **Basic Signal Operations**
- **Elementary Signals**

INTRODUCTION

- **Examples of signals and systems (Electrical Systems)**
  - Voltage divider
    - Input signal: x = 5V
    - Output signal: y = Vout
    - The system output is a fraction of the input ($y = \frac{R_2}{R_1+R_2}x$)
  - Multimeter
    - Input: the voltage across the battery
    - Output: the voltage reading on the LCD display
    - The system measures the voltage across two points
  - Radio or cell phone
    - Input: electromagnetic signals
    - Output: audio signals
    - The system receives electromagnetic signals and convert them to audio signal



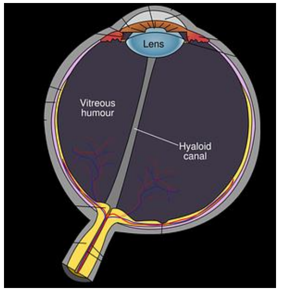
Voltage divider



multimeter

INTRODUCTION

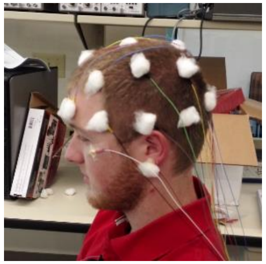
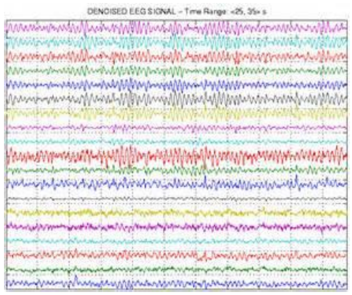
- **Examples of signals and systems (Biomedical Systems)**
  - Central nervous system (CNS)
    - Input signal: a nerve at the finger tip senses the high temperature, and sends a neural signal to the CNS
    - Output signal: the CNS generates several output signals to various muscles in the hand
    - The system processes input neural signals, and generate output neural signals based on the input
  - Retina
    - Input signal: light
    - Output signal: neural signals
    - Photosensitive cells called rods and cones in the **retina** convert incident light energy into signals that are carried to the brain by the optic nerve.



Retina

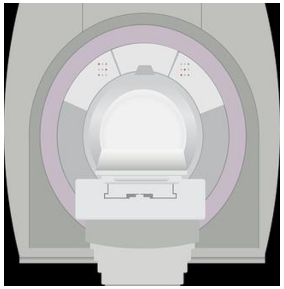
INTRODUCTION

- **Examples of signals and systems (Biomedical Instrument)**
  - EEG (Electroencephalography) Sensors
    - Input: brain signals
    - Output: electrical signals
    - Converts brain signal into electrical signals

EEG signal collection

  - Magnetic Resonance Imaging (MRI)
    - Input: when apply an oscillating magnetic field at a certain frequency, the hydrogen atoms in the body will emit radio frequency signal, which will be captured by the MRI machine
    - Output: images of a certain part of the body
    - Use strong magnetic fields and radio waves to form images of the body.




MRI

INTRODUCTION

- **Signals and Systems**
  - Even though the various signals and systems could be quite different, they share some common properties.
  - In this course, we will study:
    - How to represent signal and system?
    - What are the properties of signals?
    - What are the properties of systems?
    - How to process signals with system?
  - The theories can be applied to any general signals and systems, be it electrical, biomedical, mechanical, or economical, etc.

OUTLINE

- **Introduction: what are signals and systems?**
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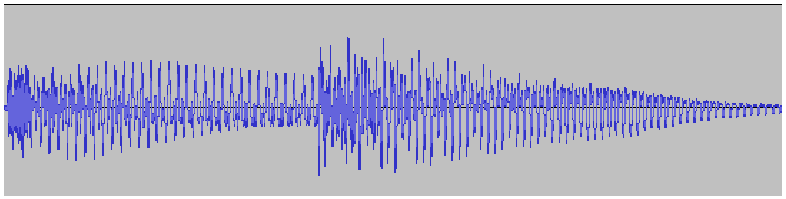
- **Basic Signal Operations**
- **Elementary Signals**

SIGNALS AND CLASSIFICATIONS

- **What is signal?**
  - Physical quantities that carry information and changes with respect to time.
  - E.g. voice, television picture, telegraph.
- **Electrical signal**
  - Carry information with electrical parameters (e.g. voltage, current)
  - All signals can be converted to electrical signals
    - Speech → Microphone → Electrical Signal → Speaker → Speech



audio signal

  - Signals changes with respect to time

SIGNALS AND CLASSIFICATIONS

- **Mathematical representation of signal:**
  - Signals can be represented as a function of time t

$$s(t), \qquad t_1 \leq t \leq t_2$$

  - Support of signal: $t_1 \leq t \leq t_2$
  - E.g. $s_1(t) = \sin(2t) \qquad -\infty \leq t \leq +\infty$
  - E.g. $s_2(t) = \sin(2t) \qquad 0 \leq t \leq \pi$
    - $s_1(t)$ and $s_2(t)$ are two different signals!

  - The mathematical representation of signal contains two components:
    - The expression: $s(t)$
    - The support: $t_1 \leq t \leq t_2$
      - The support can be skipped if $-\infty \leq t \leq +\infty$
      - E.g. $s_1(t) = \sin(2t)$

SIGNALS AND CLASSIFICATIONS

- **Classification of signals: signals can be classified as**
  - Continuous-time signal v.s. discrete-time signal
  - Analog signal v.s. digital signal
  - Finite support v.s. infinite support
  - Even signal v.s. odd signal
  - Periodic signal v.s. Aperiodic signal
  - Power signal v.s. Energy signal
  - ……

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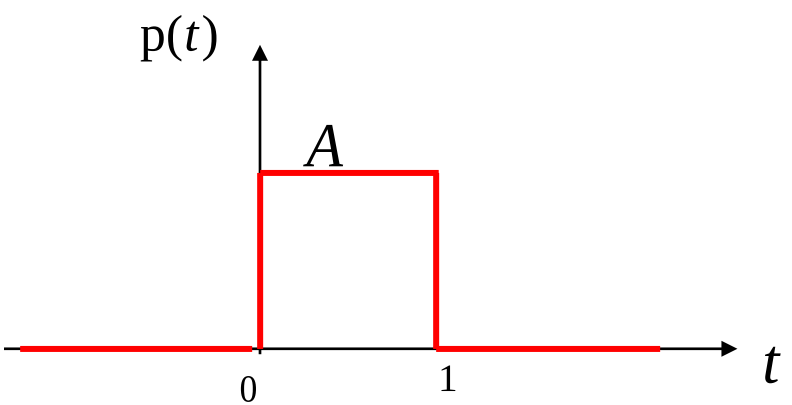
SIGNALS: CONTINUOUS-TIME V.S. DISCRETE-TIME

- **Continuous-time signal**
  - If the signal is defined over continuous-time, then the signal is a continuous-time signal
    - E.g. sinusoidal signal $s(t) = \sin(4t)$
    - E.g. voice signal
    - E.g. Rectangular pulse function

$$\mathrm{p}(t) = \begin{cases} A, & 0 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$



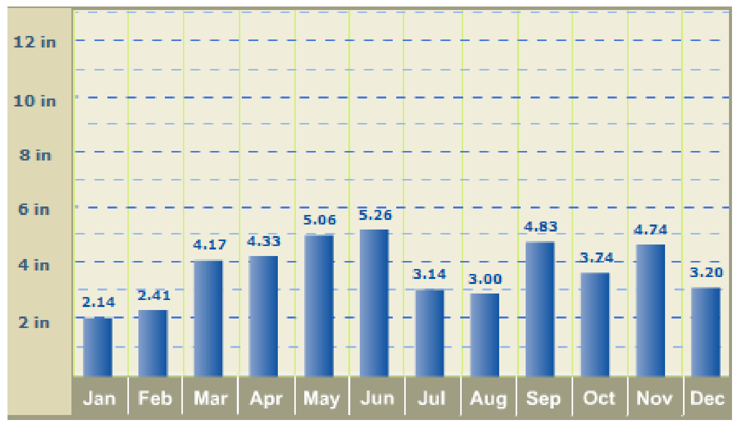
Rectangular pulse function

SIGNALS: CONTINUOUS-TIME V.S. DISCRETE-TIME

- **Discrete-time signal**
  - If the time t can only take discrete values, such as,

$$t = kT_s \qquad\qquad k = 0, \pm 1, \pm 2, \cdots$$

    then the signal $s(t) = s(kT_s)$ is a discrete-time signal
  - E.g. the monthly average precipitation at Fayetteville, AR (weather.com)



Monthly average precipitation

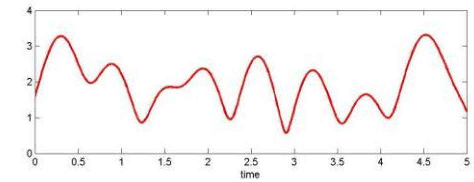
$$T_s = 1 \text{ month}$$

$$k = 1, 2, \cdots, 12$$

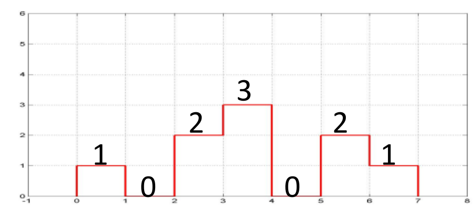
  - What is the value of $s(t)$ at $(k-1)T_s < t < kT_s$?
    - Discrete-time signals are undefined at $t \neq kT_s$!!!
    - Usually represented as $s(k)$

SIGNALS: ANALOG V.S. DIGITAL

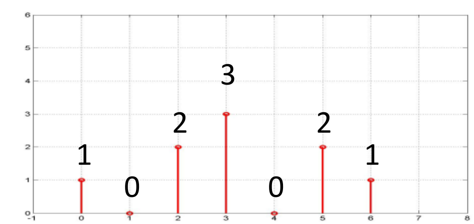
- **Analog v.s. digital**
  - Continuous-time signal
    - continuous-time, continuous amplitude→ analog signal
      - Example: speech signal



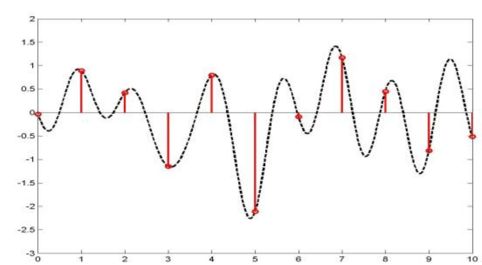
    - Continuous-time, discrete amplitude
      - Example: traffic light



  - Discrete-time signal
    - Discrete-time, discrete-amplitude → digital signal
      - Example: Telegraph, text, roll a dice



    - Discrete-time, continuous-amplitude
      - Example: samples of analog signal, average monthly temperature



Different types of signals

SIGNALS: EVEN V.S. ODD

- **Even v.s. odd**
  - x(t) is an even signal if: $x(t) = x(-t)$
    - E.g. $x(t) = \cos(2t)$
  - x(t) is an odd signal if: $x(-t) = -x(t)$
    - E.g. $x(t) = \sin(2t)$
  - Some signals are neither even, nor odd
    - E.g. $x(t) = e^t$ $x(t) = \cos(2t), t > 0$
  - Any signal can be decomposed as the sum of an even signal and odd signal

$$y(t) = \underbrace{y_e(t)}_{\text{even}} + \underbrace{y_o(t)}_{\text{odd}}$$

$$\begin{aligned} y_e(t) &= 0.5\,[y(t) + y(-t)] \\ y_o(t) &= 0.5\,[y(t) - y(-t)] \end{aligned}$$

    - proof

SIGNALS: EVEN V.S. ODD

- **Example**
  - Find the even and odd decomposition of the following signal

$$x(t) = e^{t}$$

SIGNALS: EVEN V.S. ODD

- **Example**
  - Find the even and odd decomposition of the following signal

$$x(t) = \begin{cases} 2\sin(4t), & t > 0 \\ 0 & \text{otherwise} \end{cases}$$

SIGNALS: PERIODIC V.S. APERIODIC

- **Periodic signal v.s. aperiodic signal**
  - An analog signal is periodic if
    - There is a positive real value T such that $s(t) = s(t + nT)$
    - It is defined for all possible values of t, $-\infty \leq t \leq \infty$ (why?)
  - Fundamental period T_0: the smallest positive integer T_0 that satisfies

$$s(t) = s(t + nT_0)$$

- E.g. $T_1 = 2T_0$

$$s(t + nT_1) = s(t + 2nT_0) = s(t)$$

  - So T_1 is a period of s(t), but it is not the fundamental period of s(t)

SIGNALS: PERIODIC V.S. APERIODIC

- **Example**
  - Find the period of $s(t) = A\cos(\Omega_0 t + \theta)$ $-\infty \le t \le \infty$
  - Amplitude: A
  - Angular frequency: Ω_0
  - Initial phase: θ
  - Period: $T_0 =$
  - Linear frequency: $f_0 =$

SIGNALS: PERIODIC V.S. APERIODIC

- **Complex exponential signal**
  - Euler formula: $e^{jx} = \cos(x) + j\sin(x)$
  - Complex exponential signal

$$e^{j\Omega_0 t} = \cos(\Omega_0 t) + j\sin(\Omega_0 t)$$

  - Complex exponential signal is periodic with period $T_0 = \dfrac{2\pi}{\Omega_0}$
    - Proof:

Complex exponential signal has same period as sinusoidal signal!

SIGNALS: PERIODIC V.S. APERIODIC

- **The sum of two periodic signals**
  - $x(t)$ has a period T_1
  - $y(t)$ has a period T_2
  - Define $z(t) = a\,x(t) + b\,y(t)$
  - Is $z(t)$ periodic?

$$z(t+T) = ax(t+T) + by(t+T)$$

- In order to have $x(t)=x(t+T)$, T must satisfy $T = kT_1$
- In order to have $y(t)=y(t+T)$, T must satisfy $T = lT_2$
- Therefore, if $T = kT_1 = lT_2$

$$z(t+T) = ax(t+kT_1) + by(t+lT_2) = ax(t) + by(t) = z(t)$$

- The sum of two periodic signals is periodic if and only if the ratio of the two periods can be expressed as a rational number.

$$\frac{T_1}{T_2} = \frac{l}{k}$$

  - The period of the sum signal is $T = kT_1 = lT_2$

SIGNALS: PERIODIC V.S. APERIODIC

- **Example**

$$x(t) = \sin(\frac{\pi}{3}t) \qquad y(t) = \exp(j\frac{2\pi}{9}t) \qquad z(t) = \exp(j\frac{2}{9}t)$$

  - Find the period of $x(t), y(t), z(t)$
  - Is $2x(t) - 3y(t)$ periodic? If periodic, what is the period?
  - Is $x(t) + z(t)$ periodic? If periodic, what is the period?
  - Is $y(t)z(t)$ periodic? If periodic, what is the period?

- Aperiodic signal: any signal that is not periodic.

SINGALS: ENERGY V.S. POWER

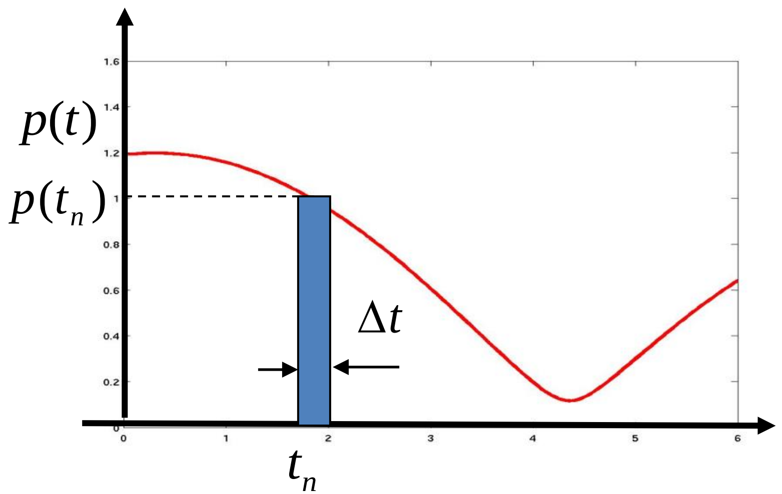
- **Signal energy**
  - Assume $x(t)$ represents voltage across a resistor with resistance R.
  - Current (Ohm's law): $i(t) = x(t)/R$
  - Instantaneous power: $p(t) = x^2(t)/R$
  - Signal power: the power of signal measured at R = 1 Ohm: $p(t) = x^2(t)$
  - Signal energy at: $[t_n, t_n + \Delta t]$

$$E_n \approx p(t_n)\Delta t$$

  - Total energy

$$E = \lim_{\Delta t \to 0} \sum_n p(t_n)\Delta t = \int_{-\infty}^{+\infty} p(t)dt$$

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 dt$$



Instantaneous power

- Review: integration over a signal gives the area under the signal.

SINGALS: ENERGY V.S. POWER

- **Energy of signal *x*(*t*) over** $t \in [-\infty, +\infty]$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

  – If $0 < E < \infty$, then $x(t)$ is called an energy signal.

- **Average power of signal *x*(*t*)**

$$P = \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

  – If $0 < P < \infty$, then $x(t)$ is called a power signal.

- A signal can be an energy signal, or a power signal, or neither, but not both.

SINGALS: ENERGY V.S. POWER

- **Example 1:** $x(t) = A\exp(-t) \qquad t > 0$

- **Example 2:** $x(t) = A\sin(\Omega_0 t + \theta)$

- **Example 3:** $x(t) = (1 + j)e^{j\pi t} \qquad 0 \le t \le 10$

- **All periodic signals are power signal with average power:**

$$P = \frac{1}{T}\int_0^T |x(t)|^2 dt$$

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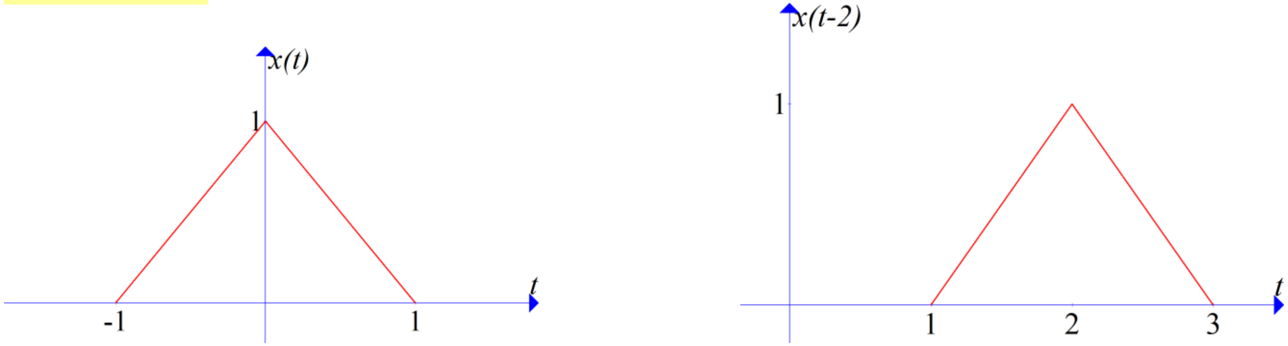
OPERATIONS: SHIFTING

- **Shifting operation**
  - $x(t-t_0)$: shift the signal $x(t)$ to the right by t_0



Shifting to the right by two units

  - Why right?

$$x(0) = A \qquad y(t) = x(t-t_0) \qquad y(t_0) = x(t_0 - t_0) = x(0) = A$$

$$x(0) = y(t_0)$$

OPERATIONS: SHIFTING

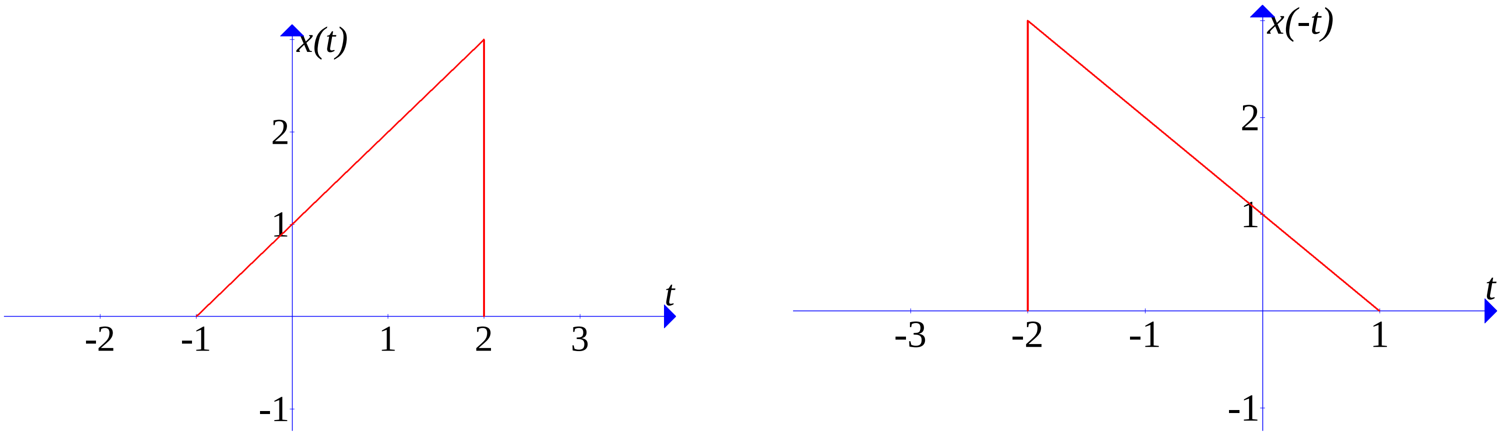
- **Example**

$$x(t) = \begin{cases} t+1 & -1 \le t \le 0 \\ 1 & 0 < t \le 2 \\ -t+3 & 2 < t \le 3 \\ 0 & \text{o.w.} \end{cases}$$

  - Find $x(t+3)$

OPERATIONS: REFLECTION

- **Reflection operation**
  - $x(-t)$ is obtained by reflecting $x(t)$ w.r.t. the y-axis ($t = 0$)



Reflection

OPERATIONS: REFLECTION

- **Example:**

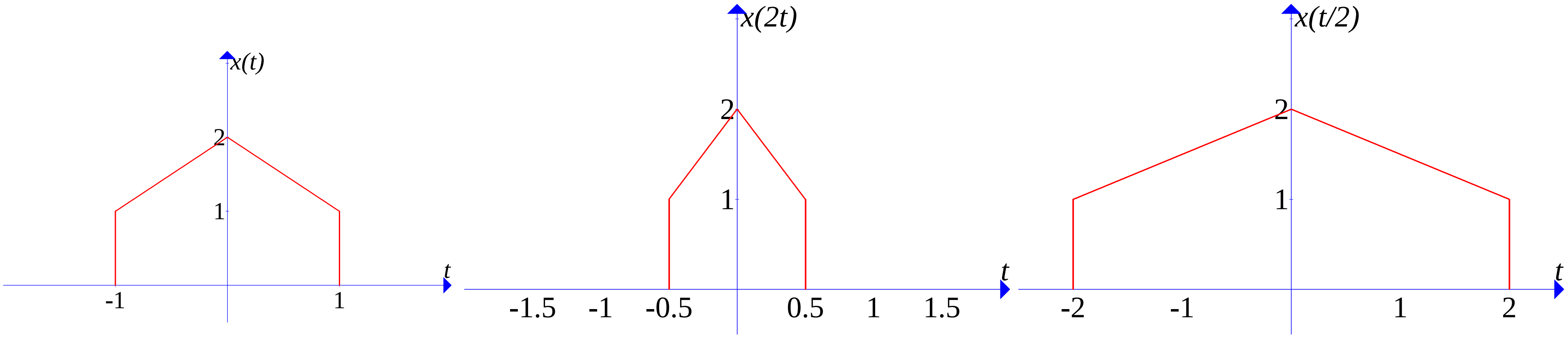
$$x(t)=\begin{cases} t+1 & -1\le t\le 0 \\ 1 & 0<t\le 2 \\ 0 & \text{o.w.} \end{cases}$$

  – Find $x(3-t)$

- The operations are always performed w.r.t. the time variable t directly!

OPERATIONS: TIME-SCALING

- **Time-scaling operation**
  - $x(at)$ is obtained by scaling the signal $x(t)$ in time.
    - $|a| > 1$, signal shrinks in time domain
    - $|a| < 1$, signal expands in time domain



Time scaling

OPERATIONS: TIME-SCALING

- **Example:**

$$x(t)=\begin{cases} t+1 & -1\le t\le 0 \\ 1 & 0<t\le 2 \\ -t+3 & 2<t\le 3 \\ 0 & \text{o.w.} \end{cases}$$

  - Find $x(3t-6)$

$x(at+b)$

1. scale the signal by a: $y(t) = x(at)$
2. left shift the signal by b/a: z(t) = y(t+b/a) = x(a(t+b/a))=x(at+b)

- The operations are always performed w.r.t. the time variable *t* directly (be careful about –*t* or *at*)!

OUTLINE

- Signals
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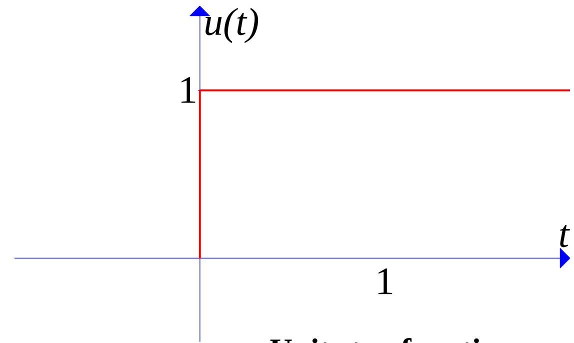
- Basic Signal Operations
- **Elementary Signals**

ELEMENTARY SIGNALS: UNIT STEP FUNCTION

- **Unit step function**

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

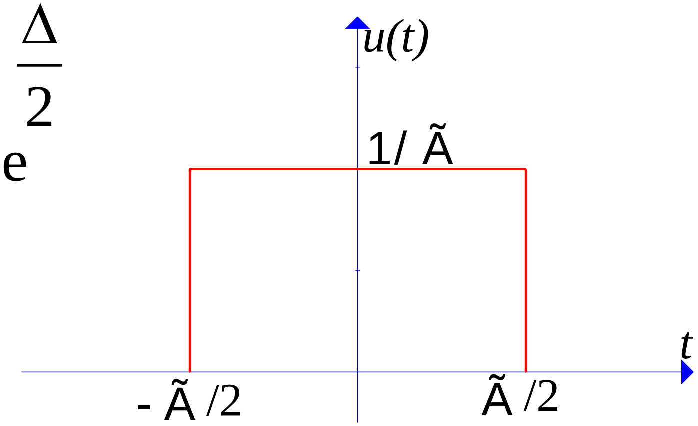


Unit step function

- **Example: rectangular pulse**

$$p_\Delta(t) = \begin{cases} \dfrac{1}{\Delta}, & -\dfrac{\Delta}{2} \le t \le \dfrac{\Delta}{2} \\ 0, & \text{otherwise} \end{cases}$$

Express $p_\Delta(t)$ as a function of $u(t)$

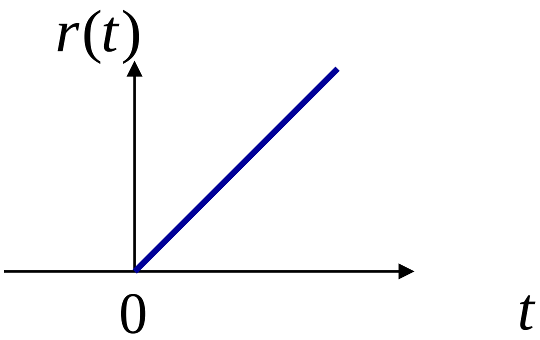


Rectangular pulse

ELEMENTARY SIGNALS: RAMP FUNCTION

- **The Ramp function**

$$r(t) = t \cdot u(t)$$



Unit ramp function

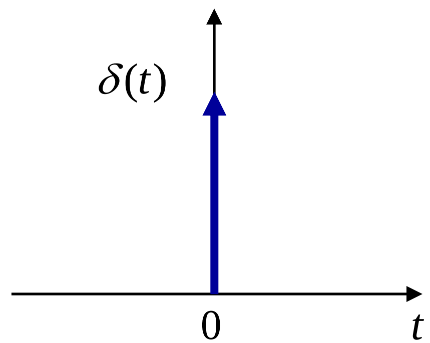
– The Ramp function is obtained by integrating the unit step function $u(t)$

$$\int_{-\infty}^{t} u(t)dt =$$

ELEMENTARY SIGNALS: UNIT IMPULSE FUNCTION

- **Unit impulse function (Dirac delta function)**

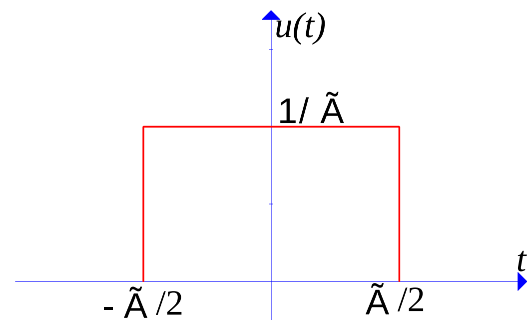
$$\delta(0) = \infty$$
$$\delta(t) = 0, t \neq 0$$
$$\int_{-\infty}^{t} \delta(t)dt = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



Unit impulse function

  – delta function can be viewed as the limit of the rectangular pulse

$$\delta(t) = \lim_{\Delta \to 0} p_{\Delta}(t)$$



Rectangular pulse

  – Relationship between $\delta(t)$ and $u(t)$

$$\int_{-\infty}^{t} \delta(t)dt = u(t)$$

$$\delta(t) = \frac{du(t)}{dt}$$

ELEMENTARY SIGNALS: UNIT IMPULSE FUNCTION

- **Sampling property**

$$x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$$

- **Shifting property**

$$\int_{-\infty}^{+\infty} x(t)\delta(t-t_0)dt = x(t_0)$$

  – Proof:

ELEMENTARY SIGNALS: UNIT IMPULSE FUNCTION

- **Scaling property**

$$\delta(at+b) = \frac{1}{|a|}\delta\left(t+\frac{b}{a}\right)$$

  – Proof:

ELEMENTARY SIGNALS: UNIT IMPULSE FUNCTION

- **Examples**

$$\int_{-2}^{4} (t + t^2)\delta(t-3)dt =$$

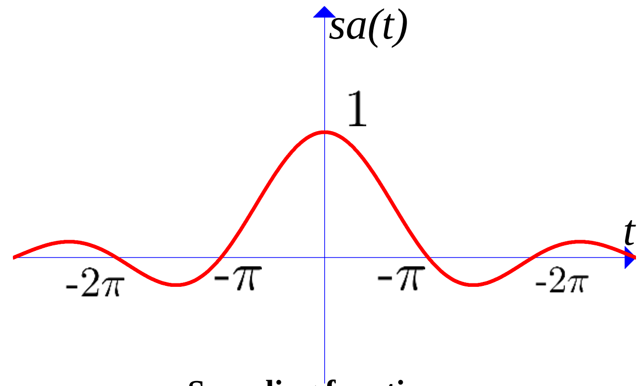
$$\int_{-2}^{1} (t + t^2)\delta(t-3)dt =$$

$$\int_{-2}^{3} \exp(t-1)\delta(2t-4)dt =$$

ELEMENTARY SIGNALS: SAMPLING FUNCTION

- **Sampling function**

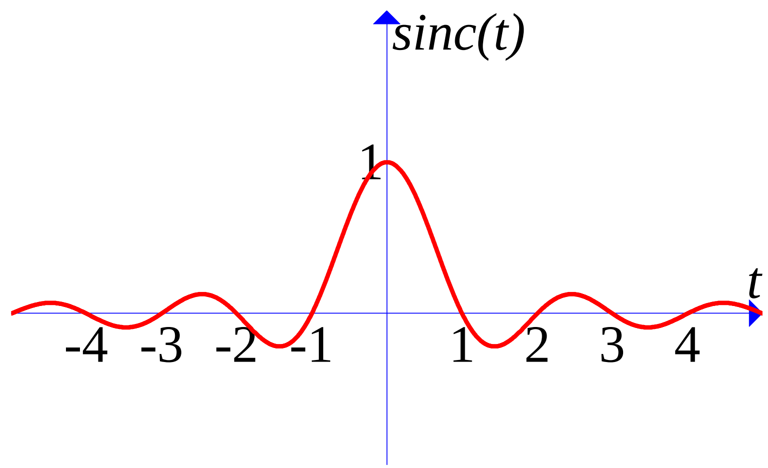
$$Sa(x) = \frac{\sin x}{x}$$



Sampling function

  - Sampling function can be viewed as scaled version of sinc(x)

$$\text{Sinc}\,(x) = \frac{\sin \pi x}{\pi x} = sa(\pi x)$$



Sinc function

ELEMENTARY SIGNALS: COMPLEX EXPONENTIAL

- **Complex exponential**

$$x(t) = e^{(r + j\Omega_0)t}$$

  - Is it periodic?

- **Example:**
  - Use Matlab to plot the real part of $x(t) = e^{(-1 + j2\pi)t}[u(t+2) - u(t-4)]$

SUMMARY

- **Signals and Classifications**
  - Mathematical representation $s(t), \qquad t_1 \le t \le t_2$
  - Continuous-time v.s. discrete-time
  - Analog v.s. digital
  - Odd v.s. even
  - Periodic v.s. aperiodic
  - Power v.s. energy
- **Basic Signal Operations**
  - Time shifting
  - reflection
  - Time scaling
- **Elementary Signals**
  - Unit step, unit impulse, ramp, sampling function, complex exponential