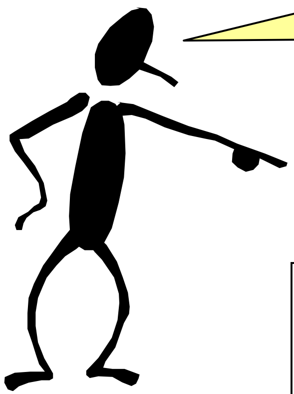
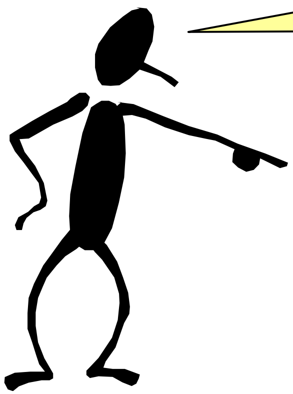
CHAPTER 10: STABILITY &TUNING



When I complete this chapter, I want to be able to do the following.

- **Determine the stability of a process without control**
- **Determine the stability of a closed-loop feedback control system**
- **Use these approaches to learn how dead time affects stability.**

CHAPTER 10: STABILITY &TUNING

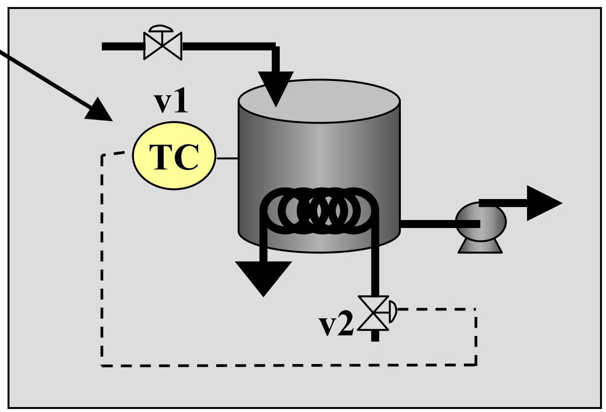


Outline of the lesson.

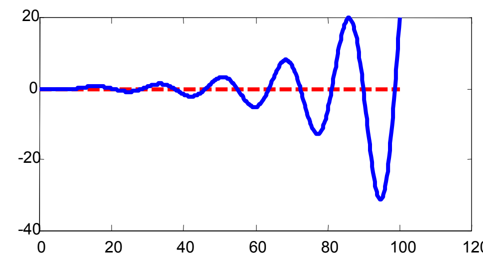
- **Define stability**
- **Review determining the roots of the characteristic equation**
- **Introduce the Bode stability method**
- **Apply to determine some general trends in feedback systems**

CHAPTER 10: STABILITY &TUNING

$$MV(t) = K_c \left[E(t) + \frac{1}{T_I} \int_0^{\infty} E(t')dt' - T_d \frac{d\,CV}{dt} \right] + I$$

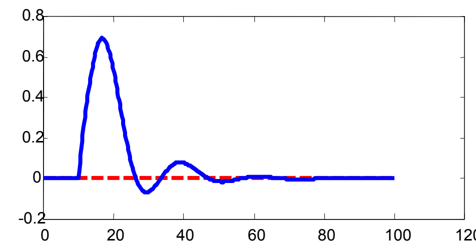


No!



or

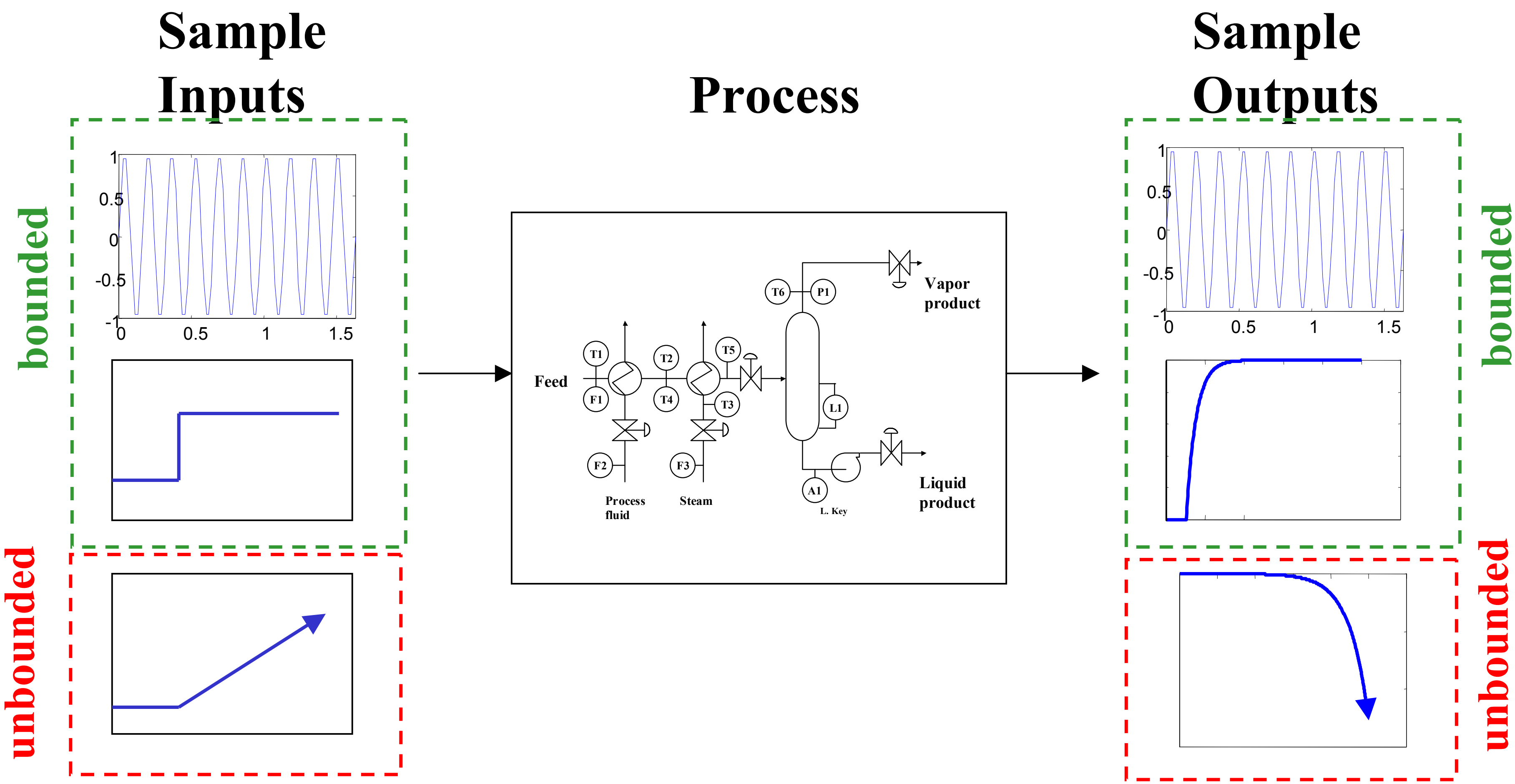
Yes!



We influence stability when we implement control. How do we achieve the influence we want?

CHAPTER 10: STABILITY &TUNING

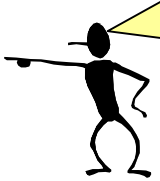
First, let's define stability: A system is stable if all bounded inputs to the system result in bounded outputs.



CHAPTER 10: STABILITY &TUNING

$$G(s) = Y(s)/X(s)$$

$$Y(s) = [N(s)/D(s)]\, X(s)$$



Let's review how we determine the stability of a model.

With α_i the solution to the denominator of the transfer function being zero, D(s) = 0 giving s = α_1, α_2 , $\alpha_{i\,...}$.

$$Y(t) = A_0 + A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} + \ldots + (B_0 + B_1 t + B_2 t^2 + ..)e^{\alpha_p t} + \ldots + [C_1 \cos(\omega t) + C_2 \sin(\omega t)]e^{\alpha_q t} + \ldots$$

Real, distinct α_i

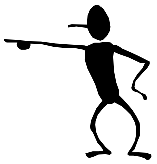
Complex α_i

Real, repeated α_i

If all α_i are ???, Y(t) is stable

If any one α_i is ???, Y(t) is unstable

Class exercise



CHAPTER 10: STABILITY &TUNING

With α_i the solutions to D(s) = 0, which is a polynomial.

$$Y(t) = A_0 + A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} + \ldots + (B_0 + B_1 t + B_2 t^2 + ..)e^{\alpha_p t} +$$

$$\ldots + [C_1 \cos(\omega t) + C_2 \sin(\omega t)]e^{\alpha_q t} + \ldots$$

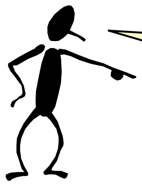
1. **If all real [α_i] are < 0, Y(t) is stable**

   If any one real [α_i] is $\geq$ 0, Y(t) is unstable

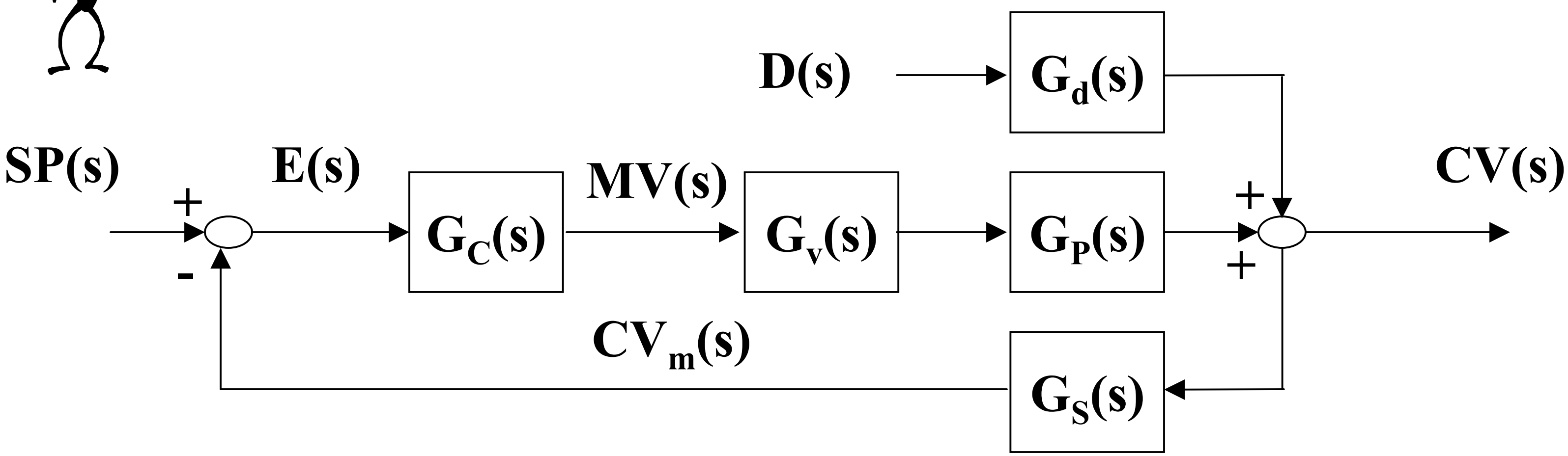
2. **If all α_i are real, Y(t) is overdamped (does not oscillate)**

   If one pair of α_i are complex, Y(t) is underdamped

CHAPTER 10: STABILITY &TUNING



Quick review of model for closed-loop feedback system.



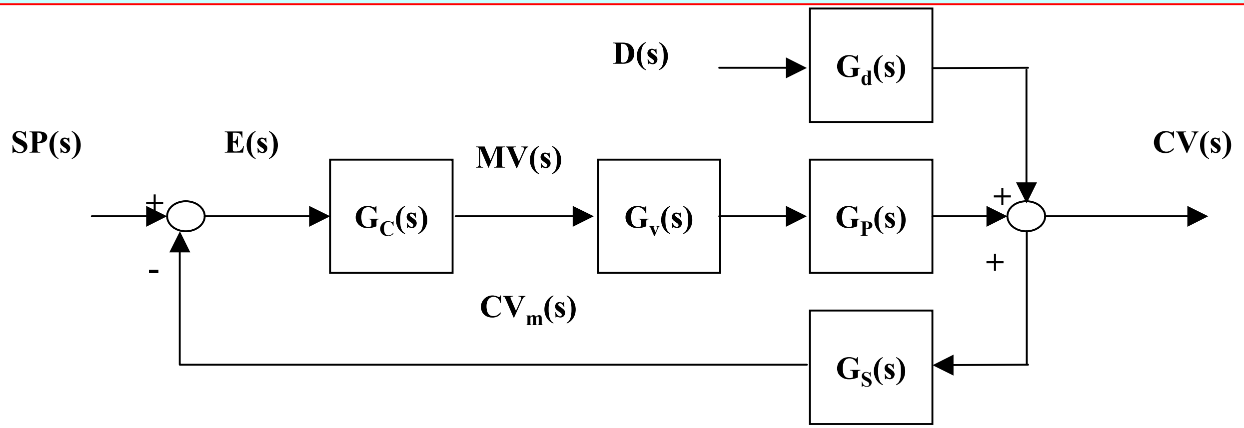
<u>Transfer functions</u>

$G_C(s)$ = controller
$G_v(s)$ = valve +
$G_P(s)$ = feedback process
$G_S(s)$ = sensor +
$G_d(s)$ = disturbance process

<u>Variables</u>

CV(s) = controlled variable
$CV_m(s)$ = measured value of CV(s)
D(s) = disturbance
E(s) = error
MV(s) = manipulated variable
SP(s) = set point

CHAPTER 10: STABILITY &TUNING

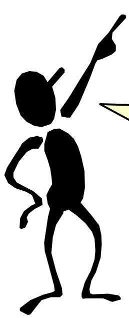


Set point response

$$\frac{CV(s)}{SP(s)} = \frac{G_p(s)G_v(s)G_c(s)}{1 + G_p(s)G_v(s)G_c(s)G_S(s)}$$

Disturbance Response

$$\frac{CV(s)}{D(s)} = \frac{G_d(s)}{1 + G_p(s)G_v(s)G_c(s)G_S(s)}$$



The denominator determines the stability of the closed-loop feedback system! We call it the characteristic equation.

CHAPTER 10: STABILITY &TUNING

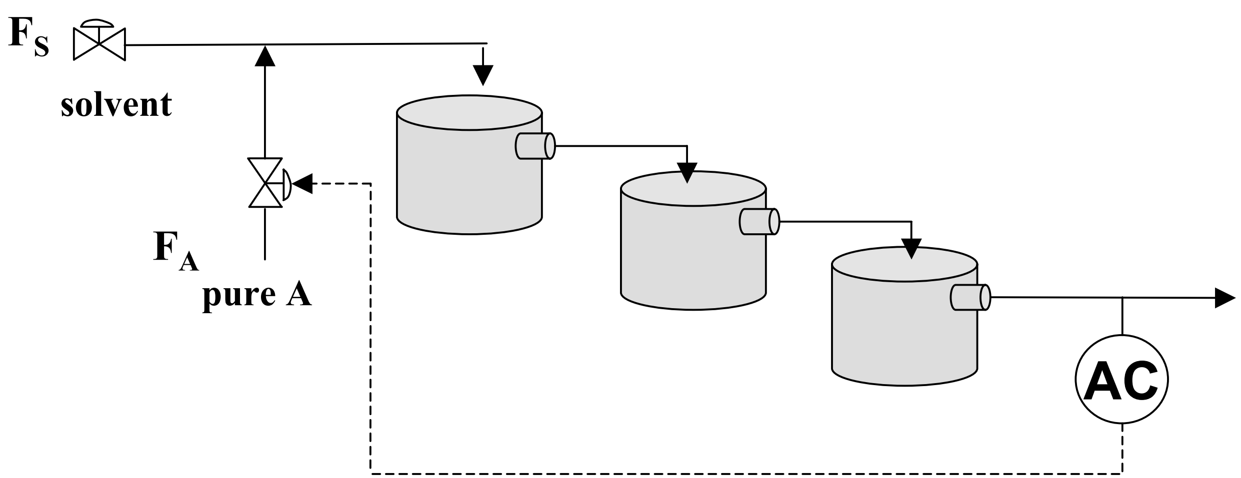
Direction Solution for the Roots to determine the stability

Controller is a P-only controller. Is the system stable? Let's evaluate the roots of the characteristic equation.

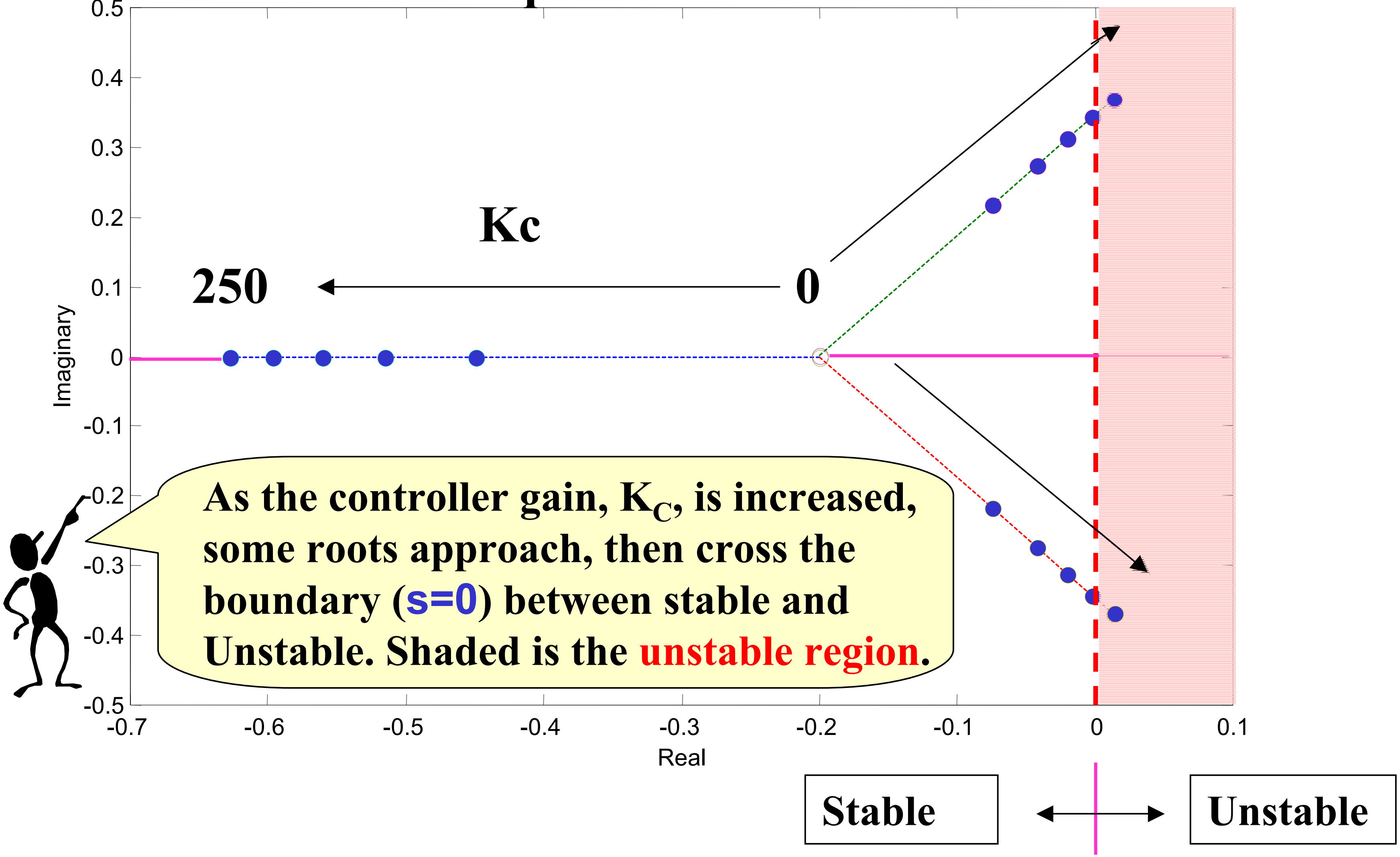
$$1 + G_p(s)G_v(s)G_c(s)G_S(s)$$

$$1 + \frac{K_C K_P}{(1+\tau s)^3} = 1 + \frac{K_C(0.039)}{(1+5s)^3} = 0$$

$$\boxed{125s^3 + 75s^2 + 15s + 1 + 0.039K_c = 0}$$

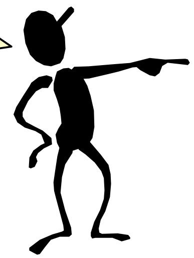


Plot of real and imaginary parts of the roots of the characteristic equation - three roots for cubic



CHAPTER 10: STABILITY &TUNING

The denominator determines the stability of the closed-loop feedback system!



Set point response

$$\frac{CV(s)}{SP(s)} = \frac{G_p(s)G_v(s)G_c(s)}{1 + G_p(s)G_v(s)G_c(s)G_S(s)}$$

$$\textbf{For the mixer:}\ 125s^3 + 75s^2 + 15s + 1 + 0.039K_c = 0$$

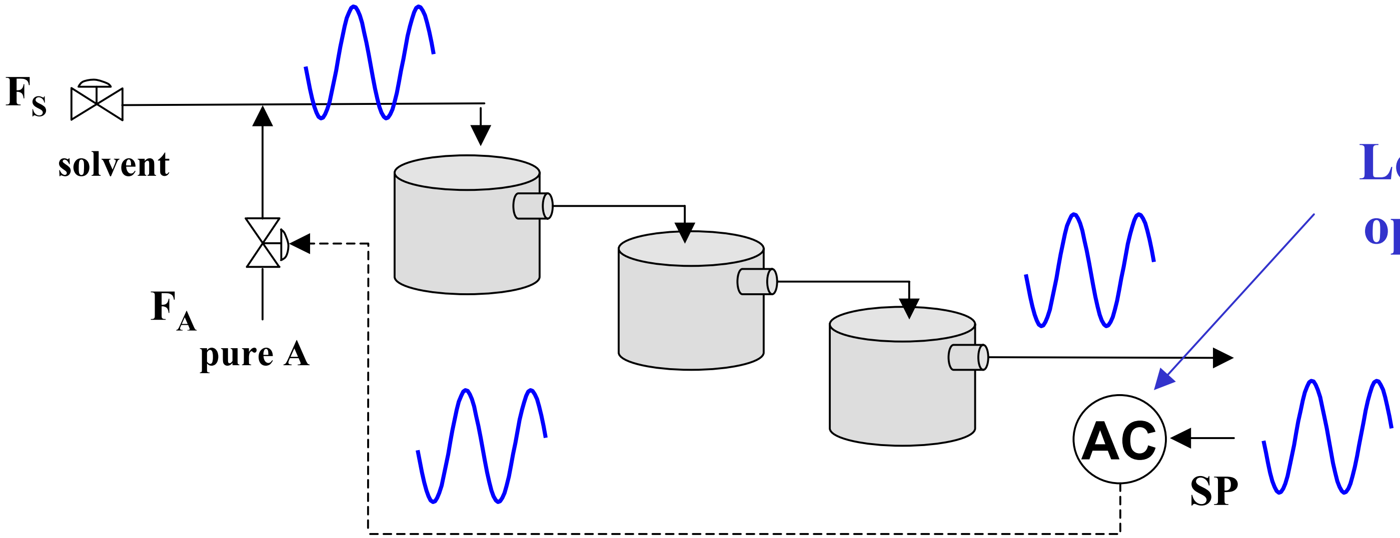
Bode Stability Method

Calculating the roots is easy with standard software. However, if the equation has a dead time, the term $e^{-\theta s}$ appears. Therefore, we need another method.

Th method we will use next is the Bode Stability Method.

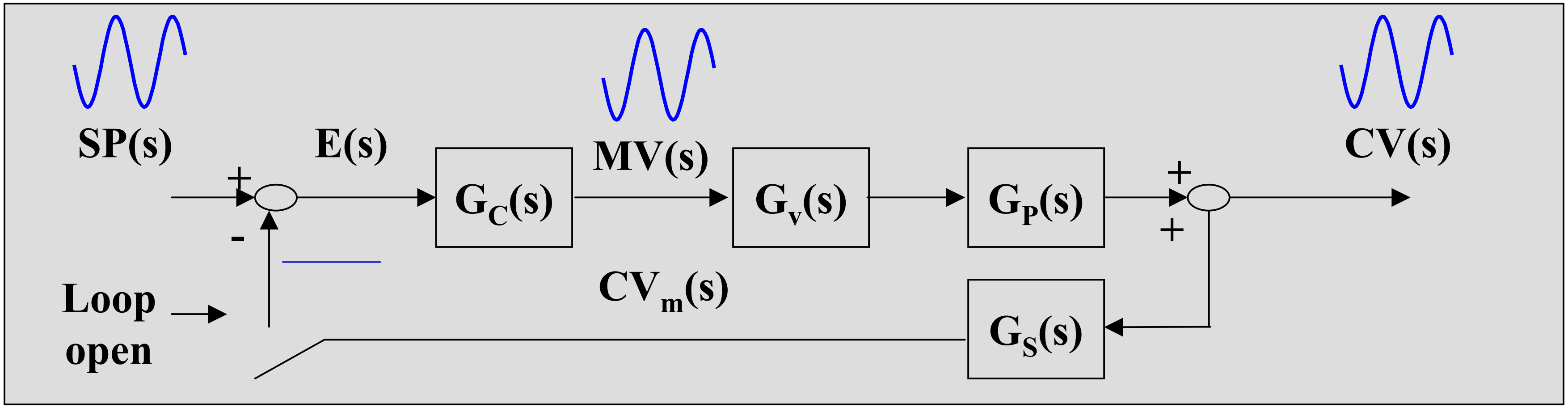
CHAPTER 10: STABILITY &TUNING

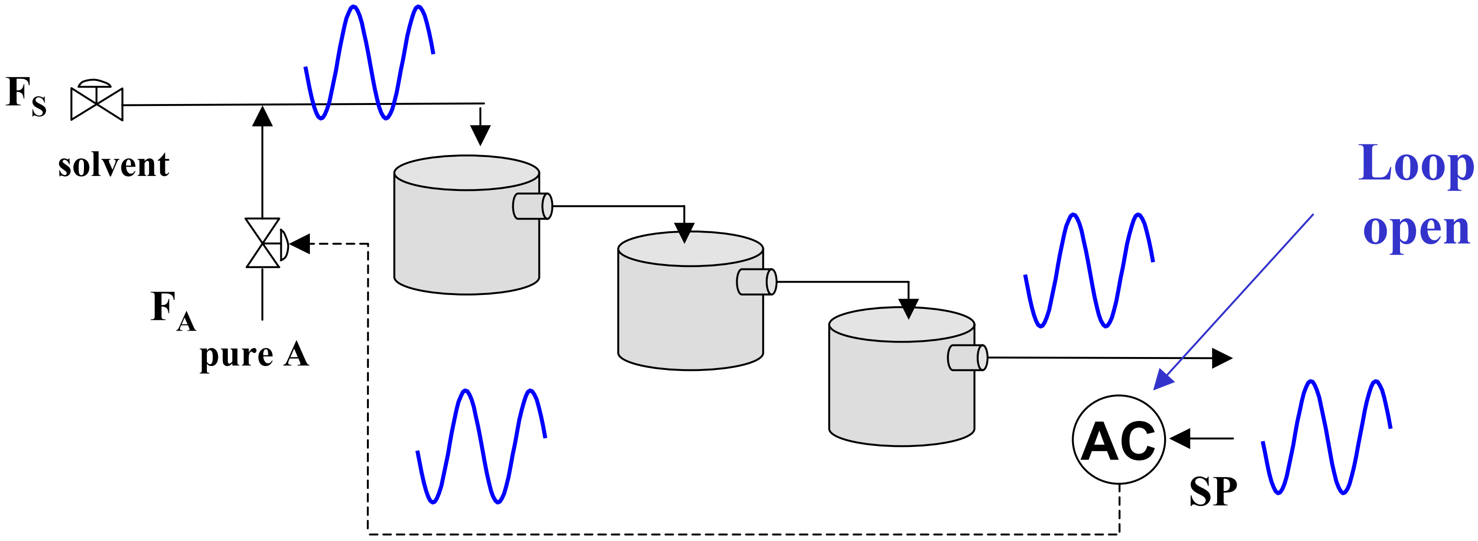
Bode Stability: To understand, let's do a thought experiment



CHAPTER 10: STABILITY &TUNING

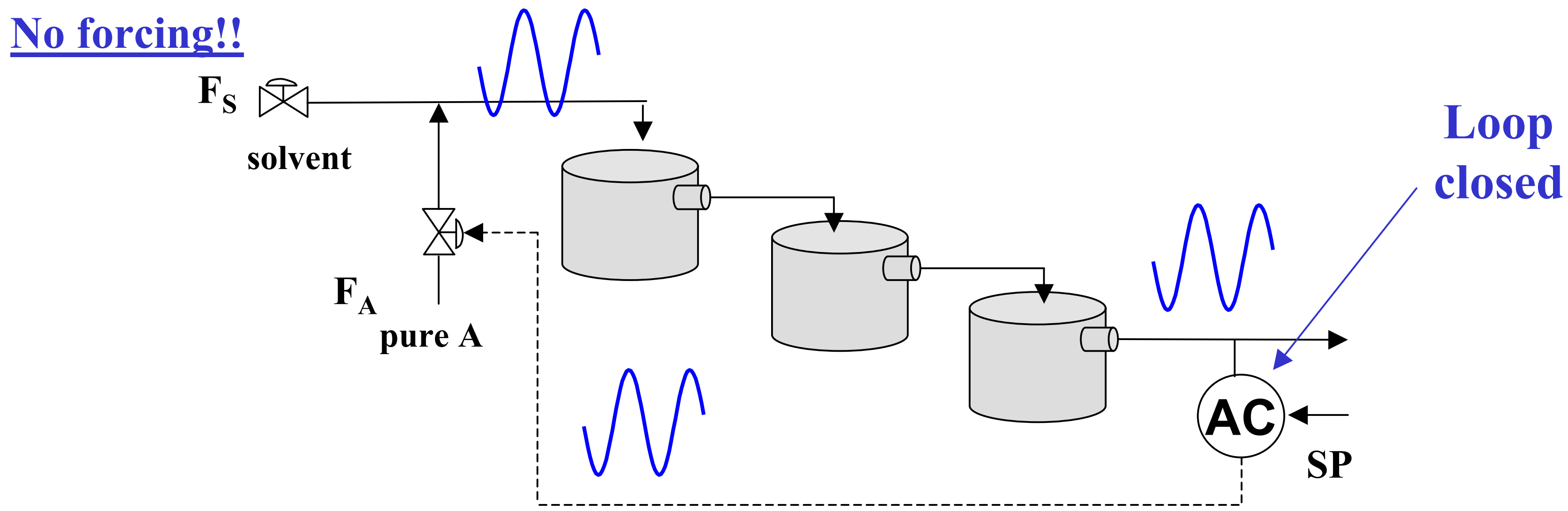
Bode Stability: To understand, let's do a thought experiment





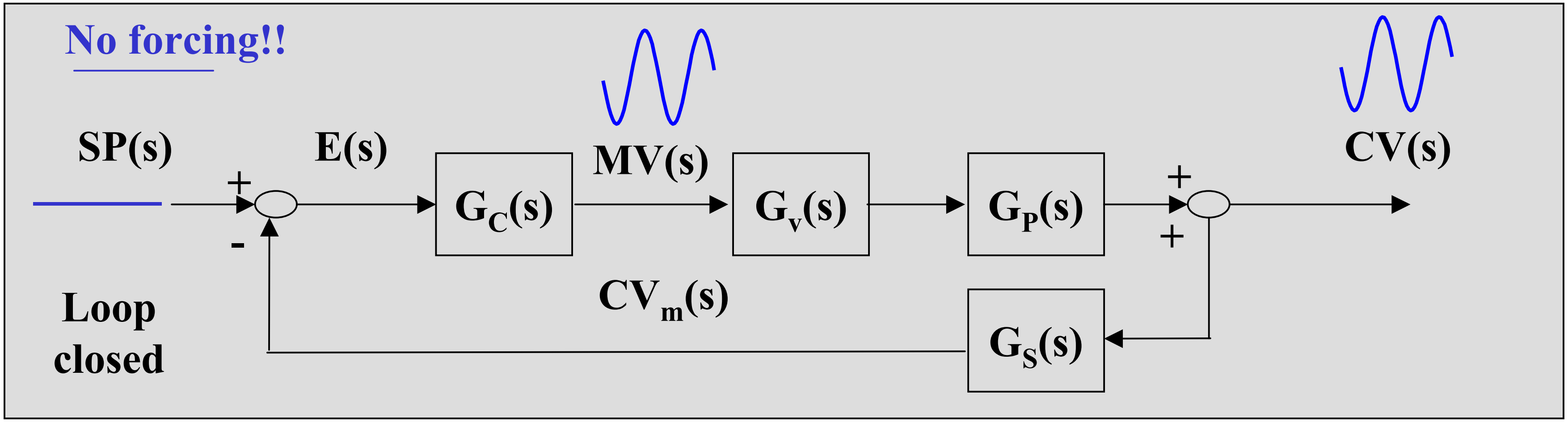
CHAPTER 10: STABILITY &TUNING

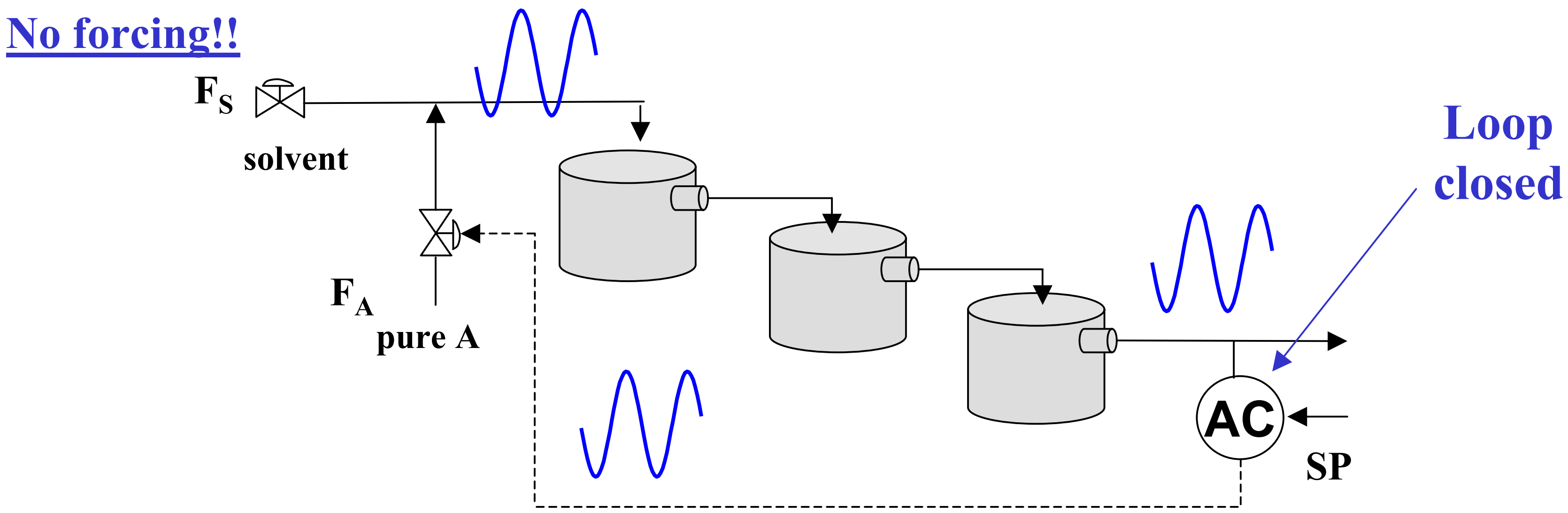
Bode Stability: To understand, let's do a thought experiment



CHAPTER 10: STABILITY &TUNING

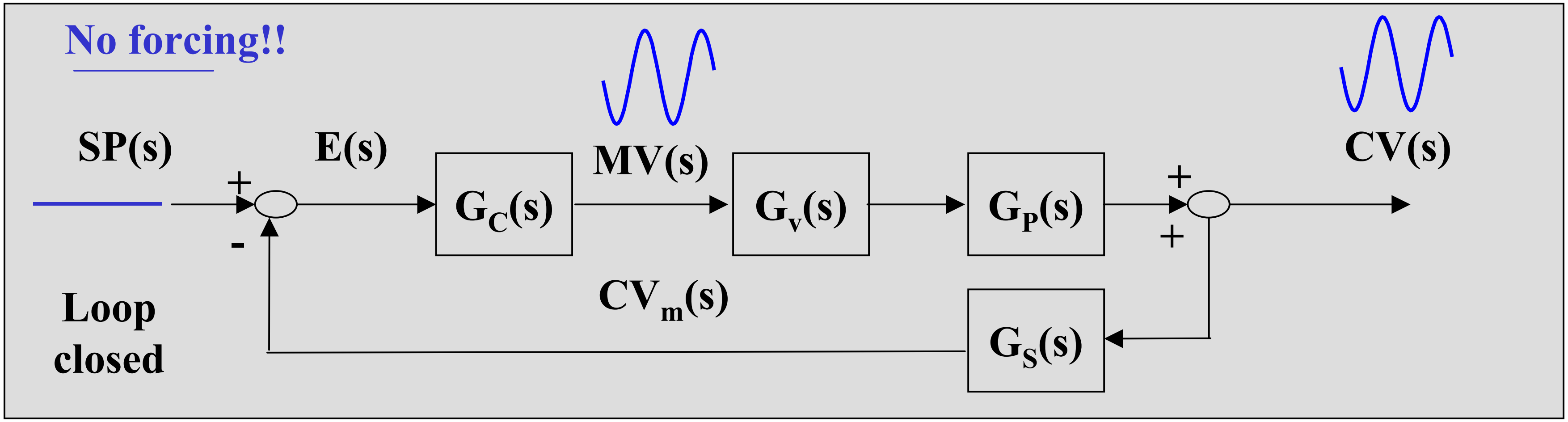
Bode Stability: To understand, let's do a thought experiment

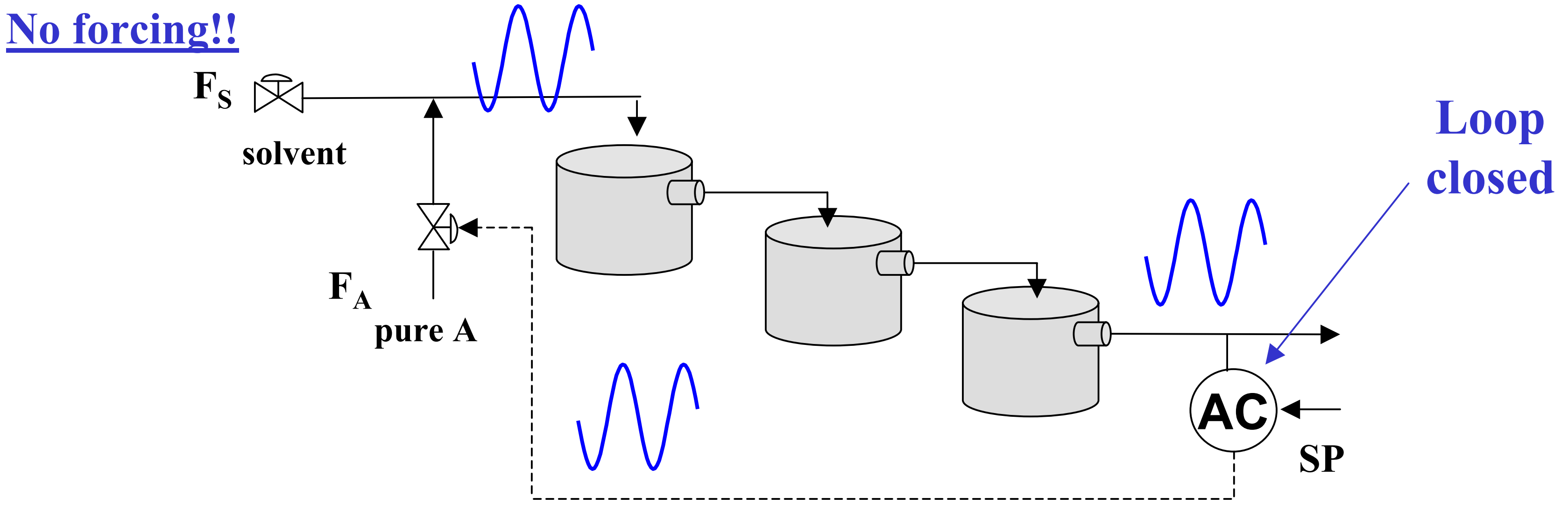




CHAPTER 10: STABILITY &TUNING

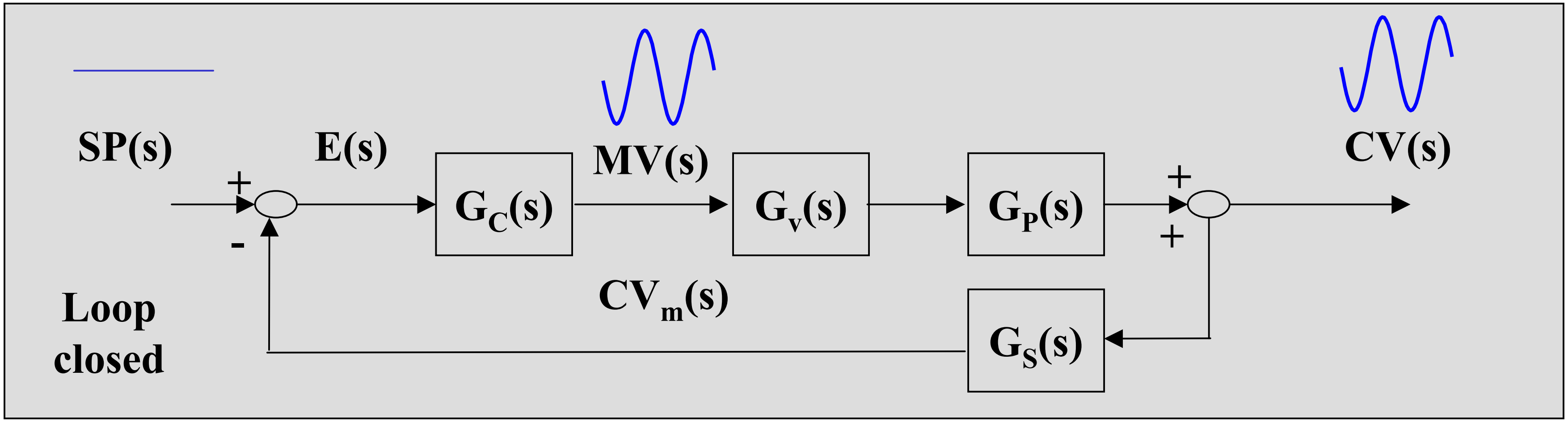
Bode Stability: To understand, let's do a thought experiment





CHAPTER 10: STABILITY &TUNING

Bode Stability: To understand, let's do a thought experiment

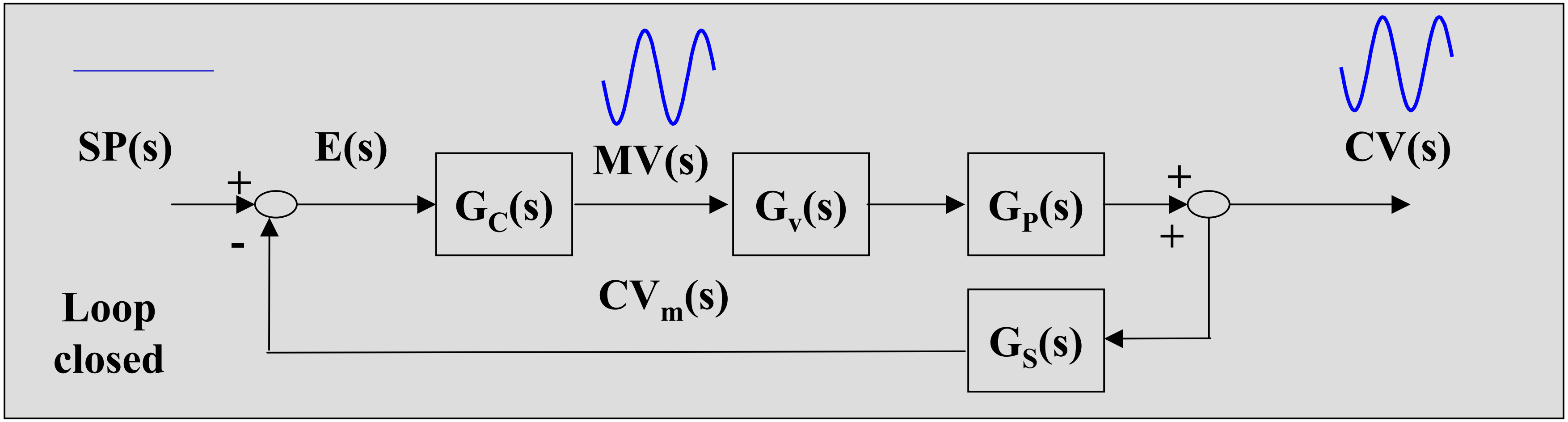


Under what conditions is the system stable (unstable)?

Hint: think about the sine wave as it travels around the loop once.

CHAPTER 10: STABILITY &TUNING

Bode Stability: To understand, let's do a thought experiment



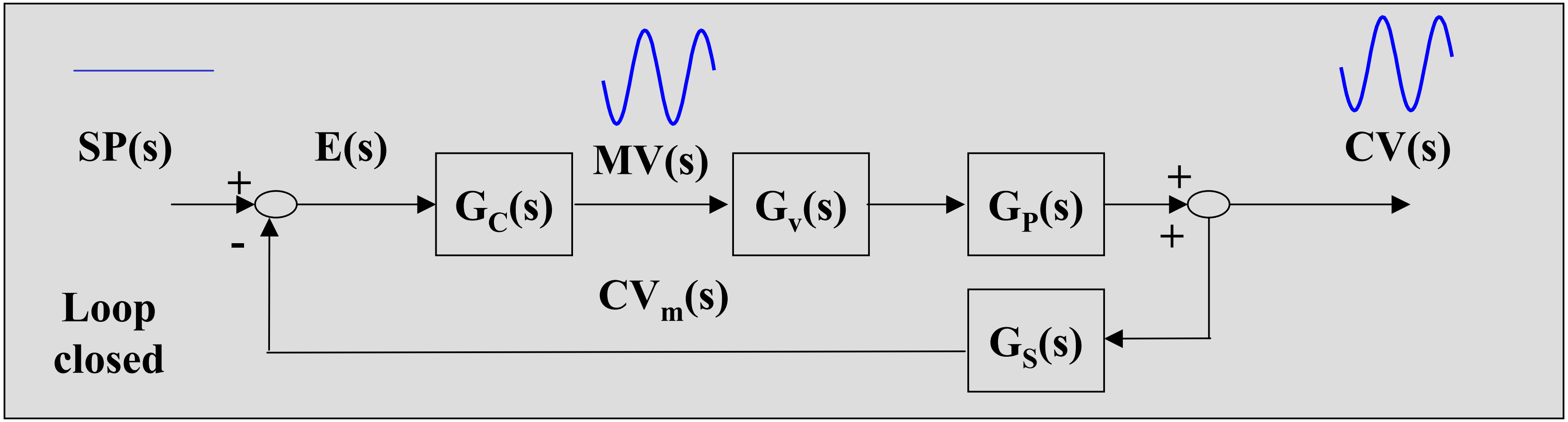
Under what conditions is the system stable (unstable)?

If the sine is larger in amplitude after one cycle; then it will increase each "time around" the loop. The system will be unstable.

Now: at what frequency does the sine most reinforce itself?

CHAPTER 10: STABILITY &TUNING

Bode Stability: To understand, let's do a thought experiment



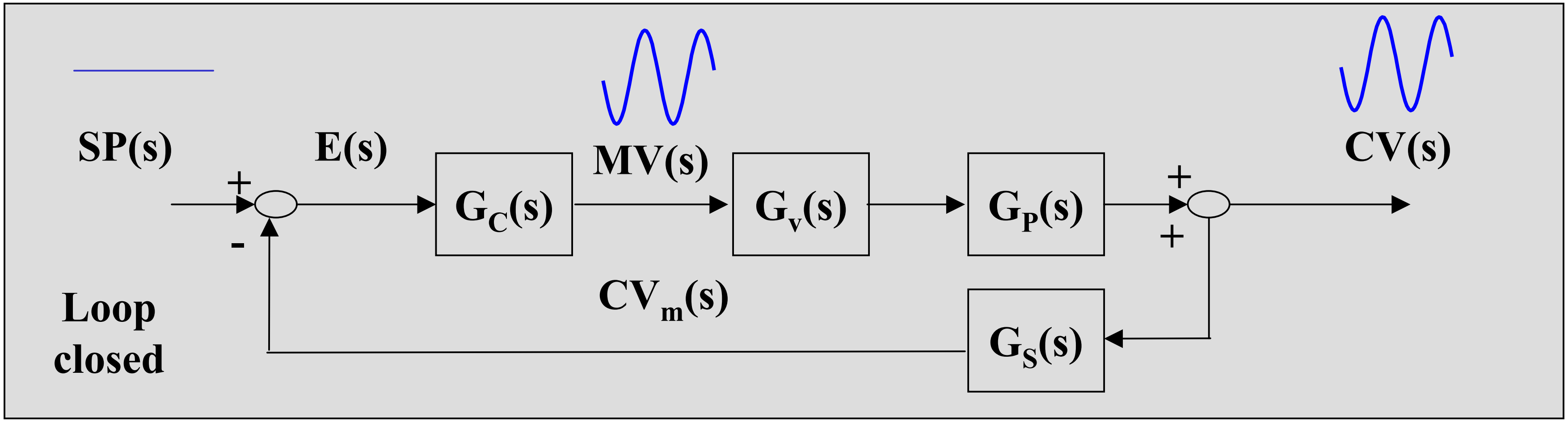
Now: at what frequency does the sine most reinforce itself?

When the sine has a lag of 180° due to element dynamics, the feedback will reinforce the oscillation (remember the - sign).

This is the **critical frequency**.

CHAPTER 10: STABILITY &TUNING

Bode Stability: To understand, let's do a thought experiment



Let's put the results together. $G_{OL}(s)$ includes all elements in the closed loop.

At the critical frequency: $\angle G_{OL}(\omega_c j) = -180°$

The amplitude ratio: $|G_{OL}(\omega_c j)| < 1$ for stability

$|G_{OL}(\omega_c j)| > 1$ for stability

See textbook for limitations

CHAPTER 10: STABILITY &TUNING

Bode Stability: **Let's do an example: three-tank mixer with 5 minutes dead time added**

$\angle G_{OL}(\omega_c j) = -180°$ $\qquad$ $|G_{OL}(\omega_c j)| < 1$ for stability

$$G_{OL}(s) = G_c(s)G_v(s)G_p(s)G_s(s) = \boxed{\frac{K_P e^{-\theta s}}{(1+\tau s)^3}} \boxed{\left(K_c\left[1+\frac{1}{T_I s}\right]\right)}$$

Process

$K_P = 0.039$ **% A/% open**

$\tau = 5$ **min (each tank)**

$\theta = 5$ **min**

Controller tuning w/o dead time

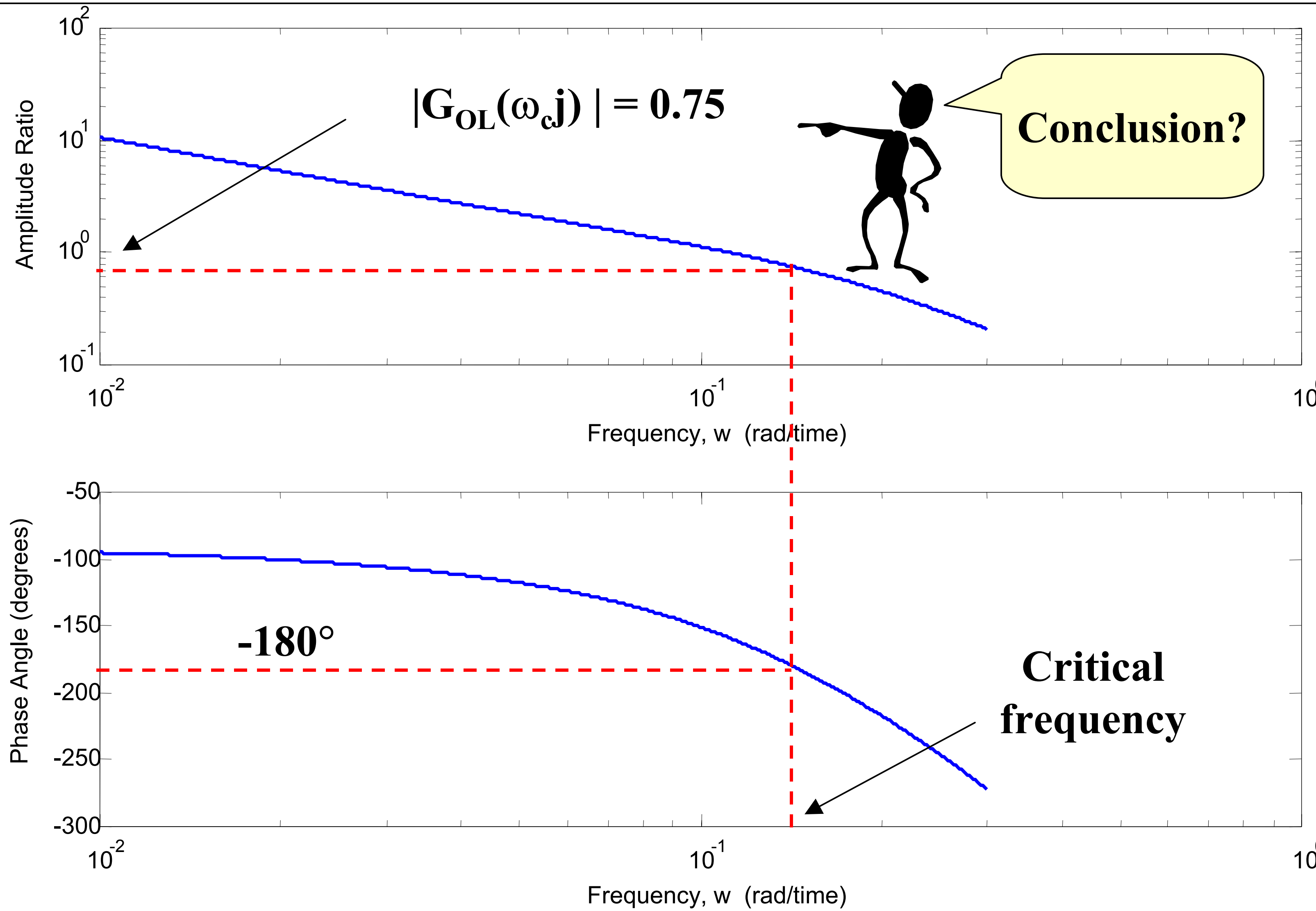
$K_c = 30$ **% open/% A**

$T_I = 11$ **min**

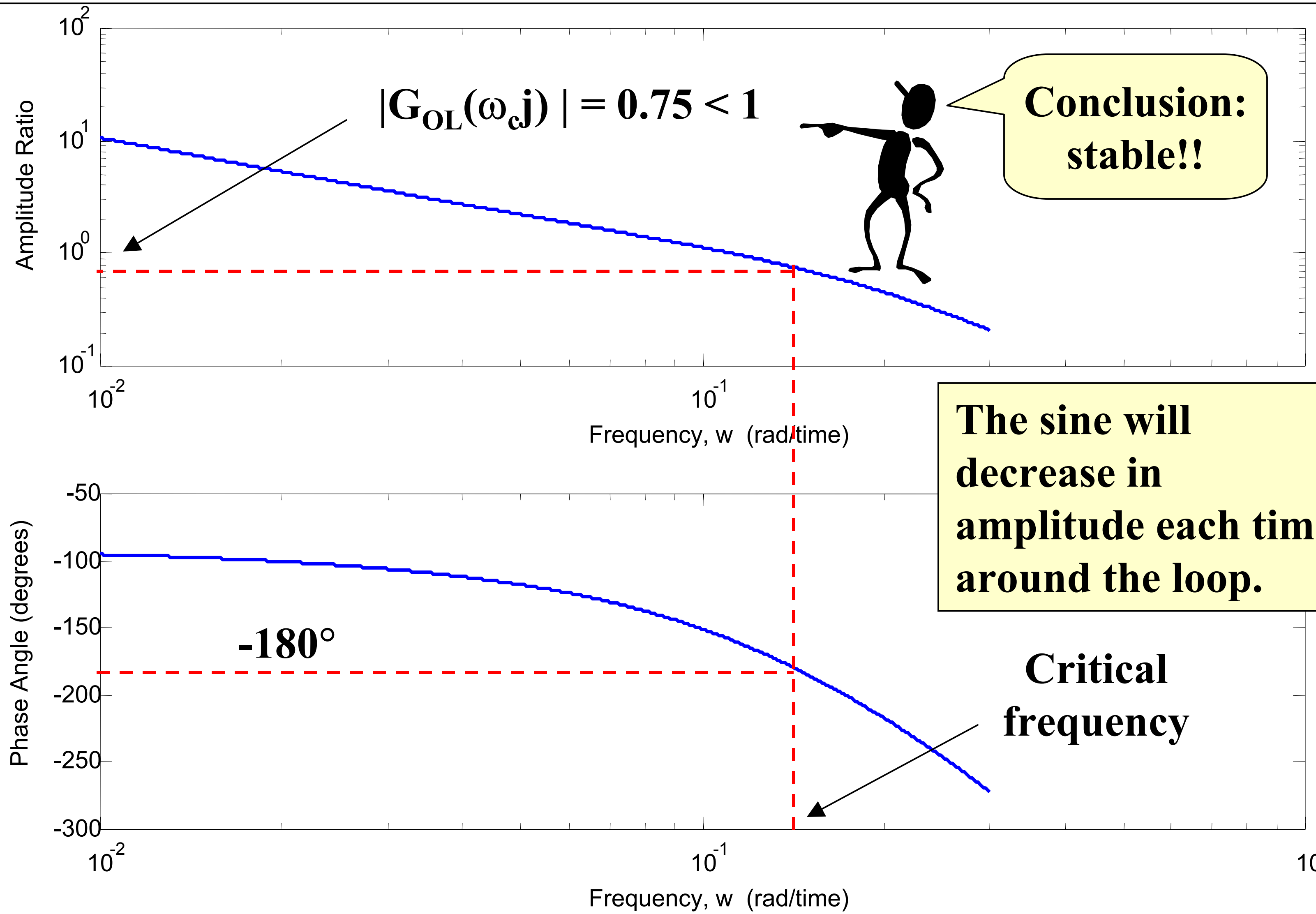
From Ciancone correlations

CHAPTER 10: STABILITY &TUNING

Bode Stability: $\angle G_{OL}(\omega_c j) = -180°$ $|G_{OL}(\omega_c j)| < 1$ for stability

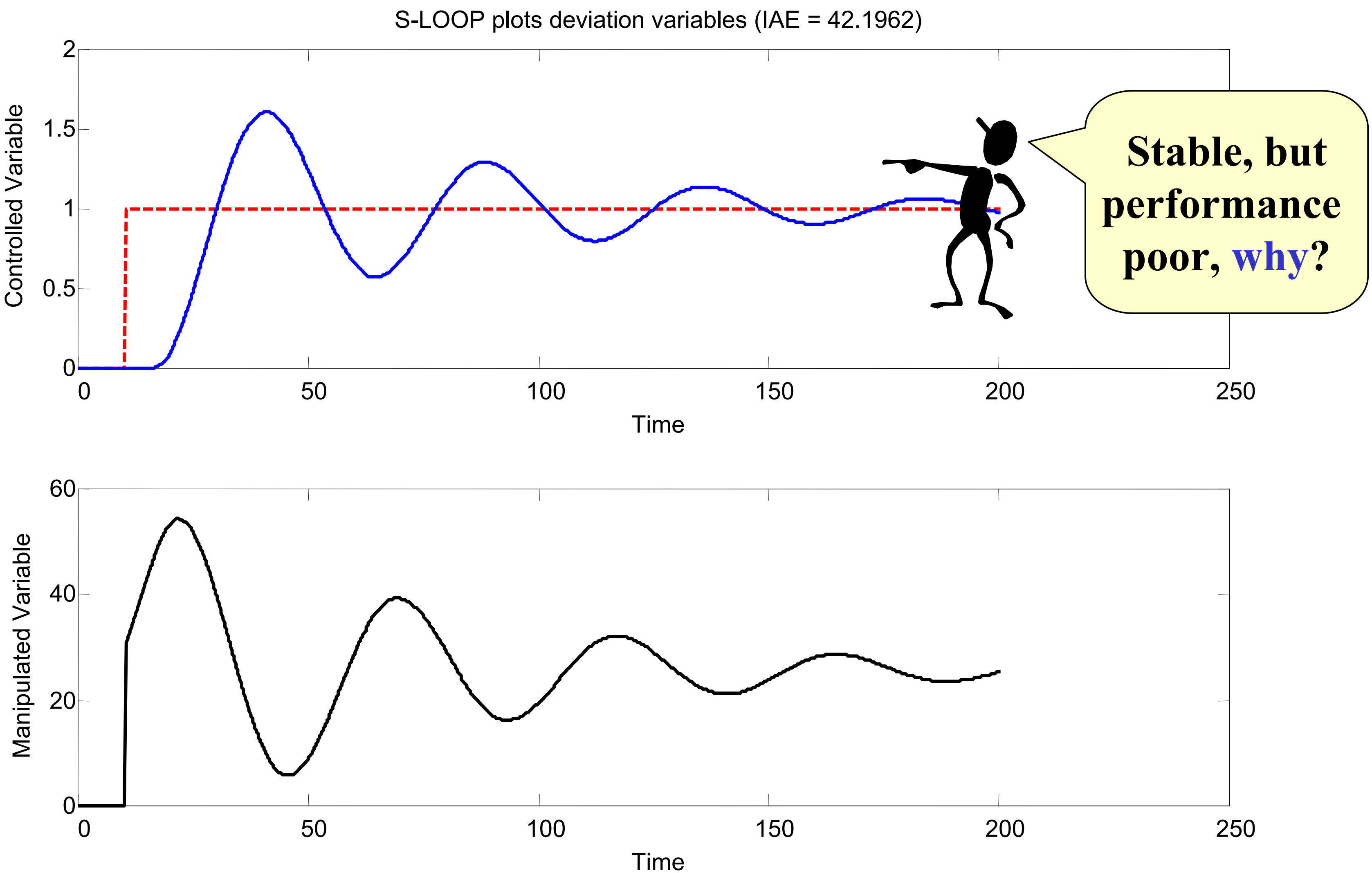


Bode Stability: $\angle G_{OL}(\omega_c j) = -180°$ $|G_{OL}(\omega_c j)| < 1$ for stability

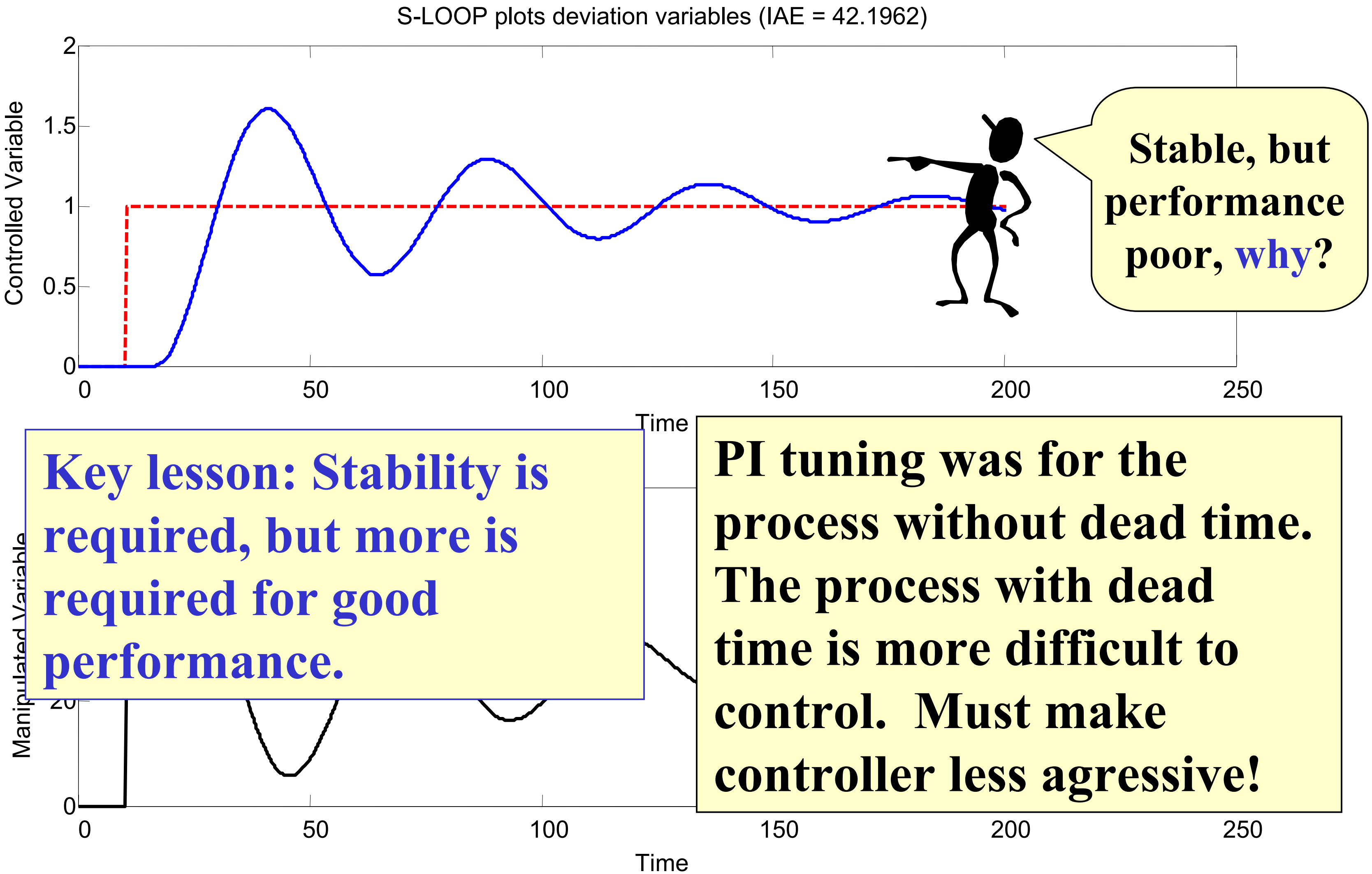


The sine will decrease in amplitude each time around the loop.

CHAPTER 10: STABILITY &TUNING



CHAPTER 10: STABILITY &TUNING



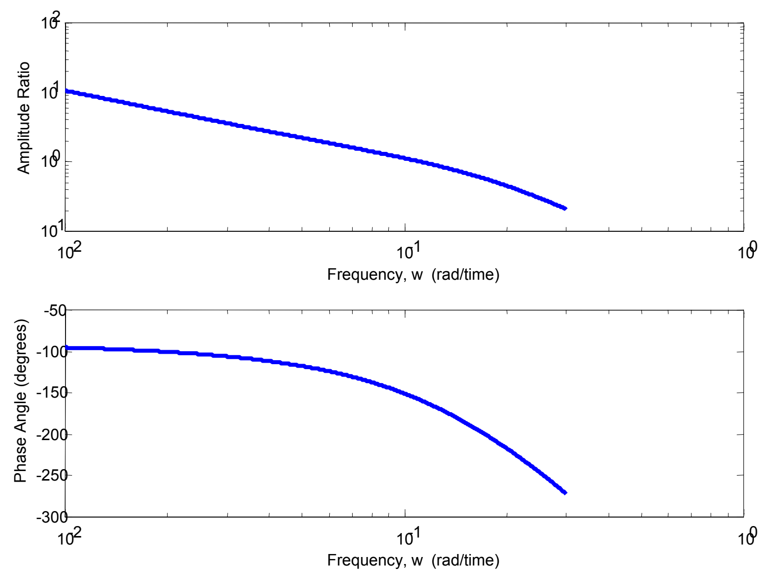
Key lesson: Stability is required, but more is required for good performance.

PI tuning was for the process without dead time. The process with dead time is more difficult to control. Must make controller less agressive!

Bode calculations can be done by hand, easier with S_LOOP

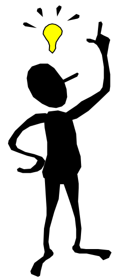
```
************************************************************
*      S_LOOP:  SINGLE LOOP CONTROL SYSTEM ANALYSIS        *
*            BODE PLOT OF GOL(s) = Gp(s)Gc(s)              *
*                                                          *
*          Characteristic Equation = 1 + GOL(s)            *
************************************************************
SELECT THE APPROPRIATE MENU ITEM
MODIFY...
                              PRESENT VALUES
1) Lowest Frequency         0.01
2) Highest Frequency        0.30
3) Create Bode plot
   and calculate the results at critical frequency
4) Return to main menu
Enter the desired selection:
```

```
*************************************************
      Critical frequency and amplitude ratio
              from Bode plot of GOL
 *************************************************

Caution:  1) cross check with plot because of possible
             MATLAB error in calculating the phase angle
          2) the program finds the first crossing of -180

The critical frequency is between          0.14263
                                       and 0.14287

The amplitude ratio at the critical frequency is 0.74797
```



Or, write your own program in MATLAB.

CHAPTER 10: STABILITY &TUNING



Let's review what we have accomplished so far.

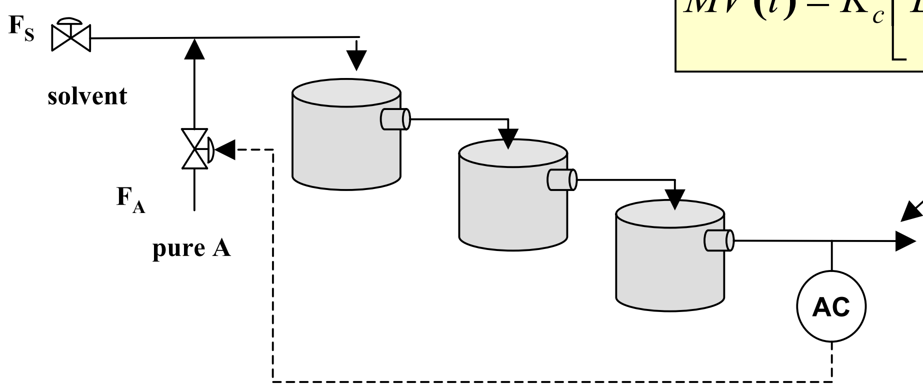
- **We can evaluate the stability of a process without control by evaluating the roots of char. equation**
- **We can evaluate the stability of a process under feedback by either**
  - evaluating the roots of char. equation
  - Bode method (required for process with dead time)
- **These are local tests, caution about non-linearity**
- **Stability does not guarantee good performance !!!!**
- **Unstable system performance always poor!!!**

CHAPTER 10: STABILITY &TUNING

1. What else can we do with this neat technology?

Tune controllers

$$MV(t) = K_c\left[E(t) + \frac{1}{T_I}\int_0^{\infty} E(t')dt' - T_d\frac{d\,CV}{dt}\right] + I$$



Ziegler-Nichols Tuning

We can tune controllers. The basic idea is to keep a "reasonable" margin from instability limit. This "reasonable" margin might give good performance.

CHAPTER 10: STABILITY &TUNING

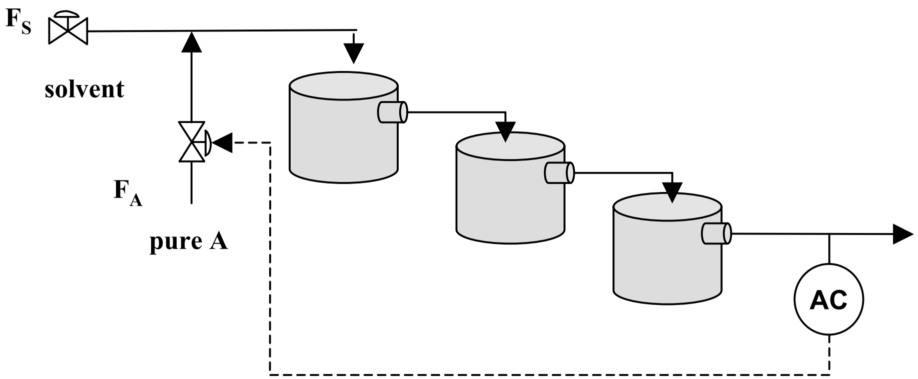
1. What else can we do with this neat technology?

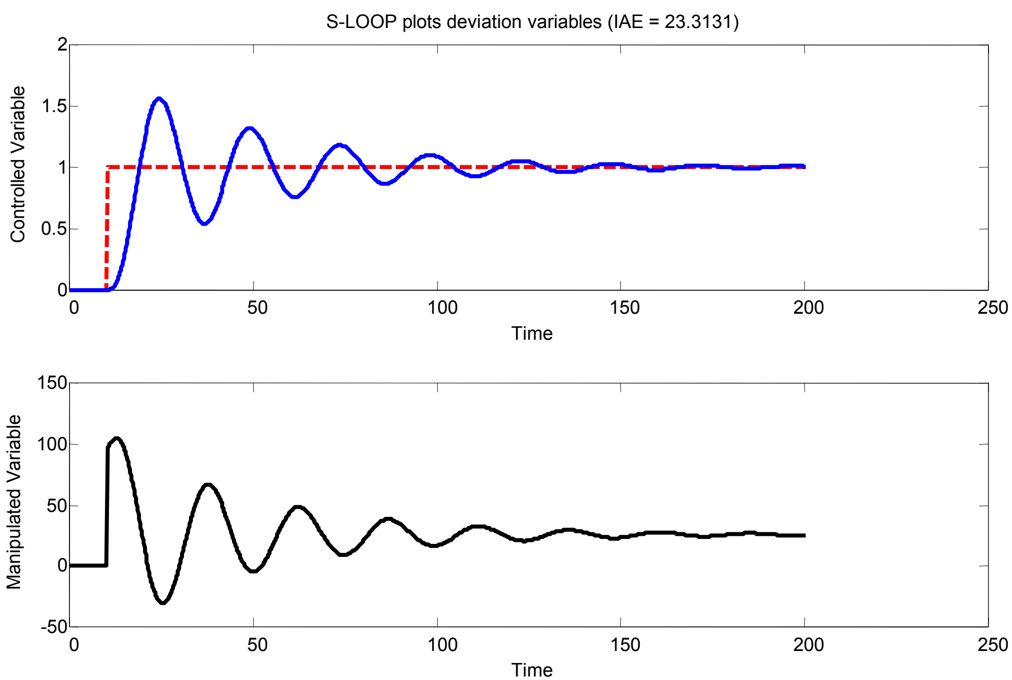
Tune controllers

Controller	Kc	TI	Td
P-only	Ku/2	---	---
PI	Ku/2.2	Pu/1.2	---
PID	Ku/1.7	Pu/2.0	Pu/8

- Gain margin is approximately 2
- Integral mode is required for zero s-s offset
- Derivative has stabilizing effect

CHAPTER 10: STABILITY &TUNING





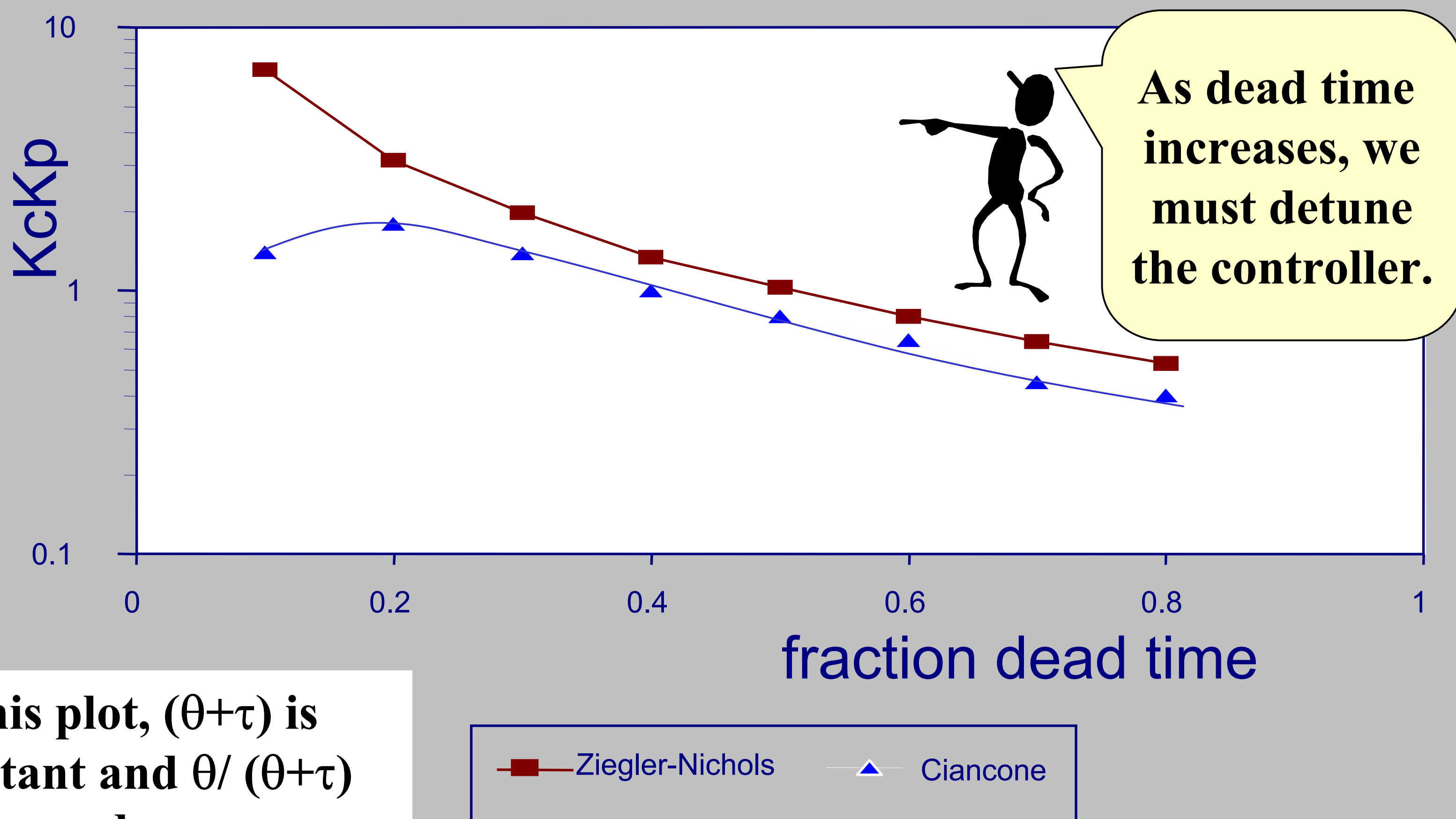
Ziegler-Nichols tuning

- Generally, Ziegler-Nichols tuning is not the best initial tuning method.
- However, these two guys were **real pioneers** in the field! Its taken 50 years to surpass their guidelines.

CHAPTER 10: STABILITY &TUNING

2. What else can we do with this neat technology?

Understand why detuning is required for tough processes.

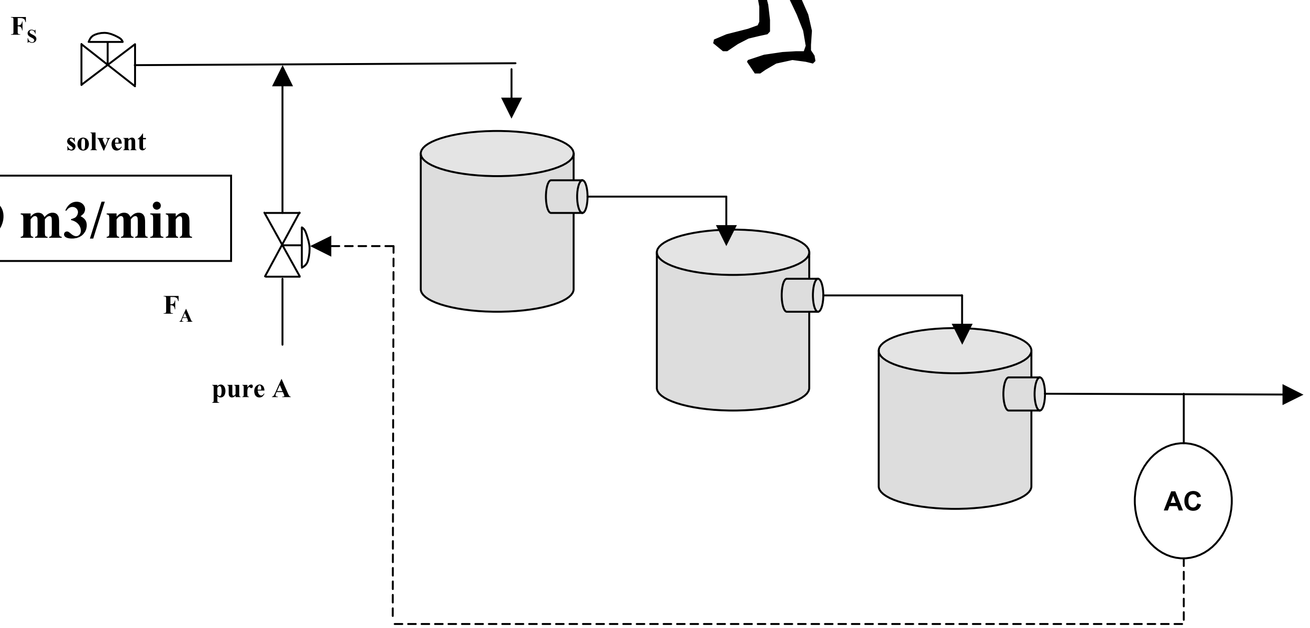


In this plot, $(\theta+\tau)$ is constant and $\theta/(\theta+\tau)$ is changed.

CHAPTER 10: STABILITY &TUNING

3. What else can we do with this neat technology?
Understand need for "robustness".

After we tune the controller, we change the flow of solvent. What happens?

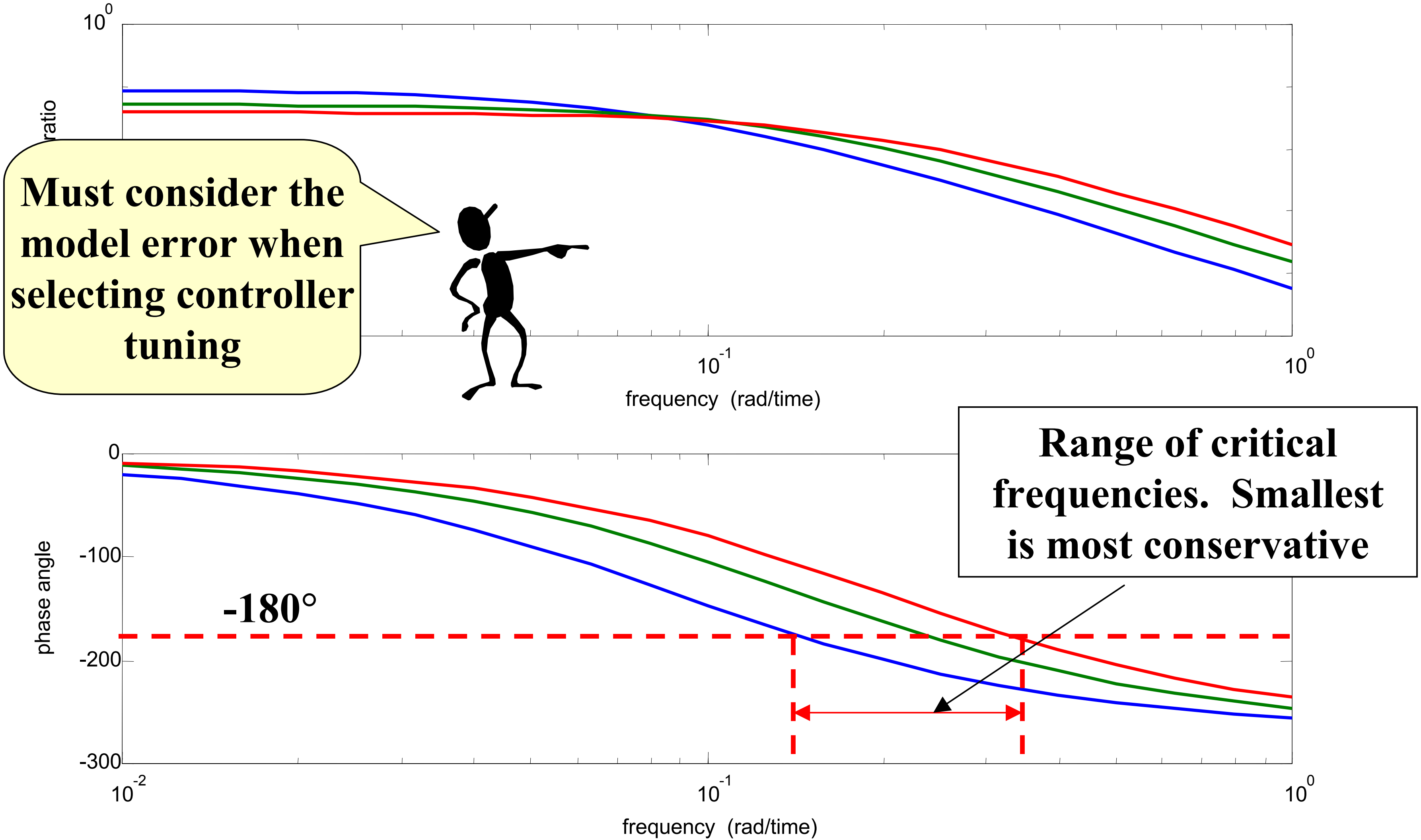


F_S = 3.0 to 6.9 m3/min

CHAPTER 10: STABILITY &TUNING

3. What else can we do with this neat technology?

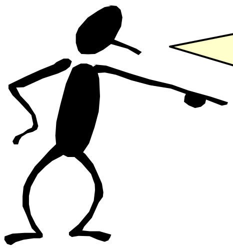
Understand need for “robustness”.



CHAPTER 10: STABILITY &TUNING

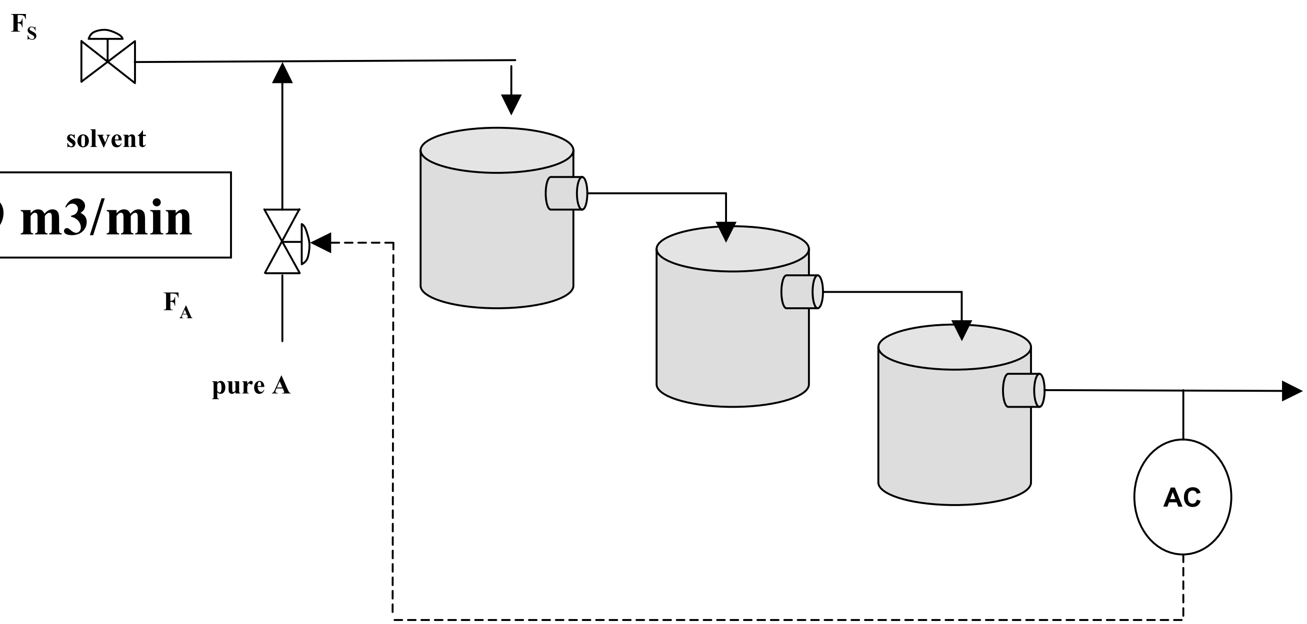
3. What else can we do with this neat technology?

Understand need for "robustness".



Tune for the process response that is slowest, has highest fraction dead time, and largest process gain.

This will give least aggressive controller.



F_S = 3.0 to 6.9 m3/min

CHAPTER 10: STABILITY &TUNING

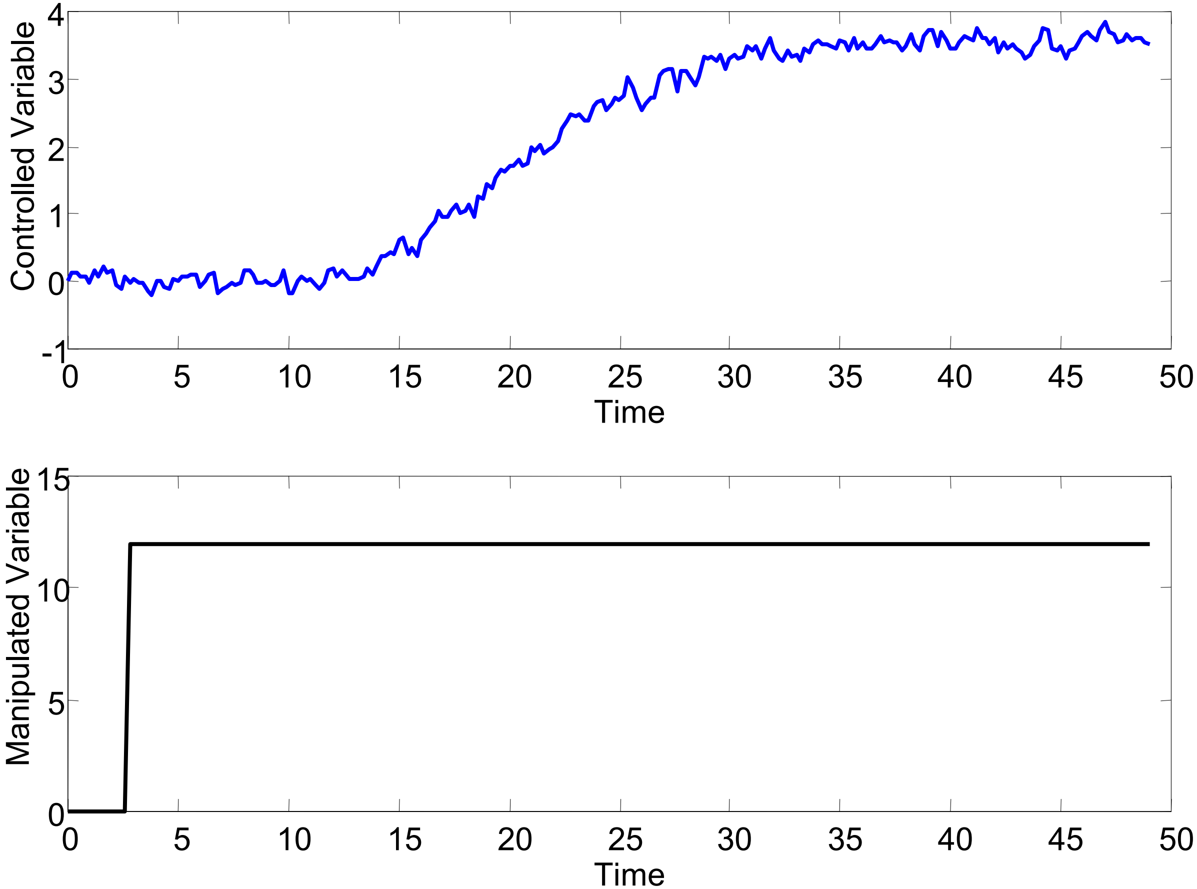
Match your select of tuning method to tuning goals!

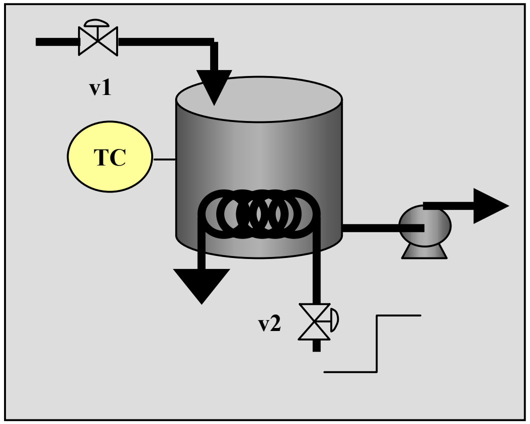
Summary of PID tuning methods

Tuning method	**Stability objective**	**Objective for CV(t)**	**Objective for MV(t)**	**Model error**	**Noise on CV(t)**	**Input** SP = set point D = disturbance
Ciancone (Chapter 9)	None explicit	Min IAE	Overshoot and variation with noise	±25%	Yes	SP and D individually
Fertik (1974)	None explicit	Min ITAE with limit on overshoot	None	None explicit	No	SP and D individually
Gain/phase margin (Section 10.8)	Gain margin or phase margin	None	None	Depends on margins	No	n/a
IMC tuning (Section 19.7)	For specified model error	ISE (robust performance)	None	Tune λ, see Morari and Zafiriou (1989)	No	SP and D (step and ramp) individually
Lopez et al. (1969)	None explicit	IAE, ISE, or ITAE	None	None	No	SP and D individually
Ziegler-Nichols closed-loop (Section 10.7)	Implicit margin for stability (GM $\approx$ 2)	4 : 1 decay ratio	None	None explicit	No	n/a
Ziegler-Nichols open-loop (Section 10.9)	Implicit margin for stability (GM $\approx$ 2)	4 : 1 decay ratio	None	None explicit	No	n/a

CHAPTER 10: TUNING & STABILITY WORKSHOP 1

The data below is a <u>process reaction curve</u> for a process, plotted in deviation variables. Determine the tuning for a PID controller using the Ziegler-Nichols method.





CHAPTER 10: TUNING & STABILITY WORKSHOP 2

Answer true or false to each of the following questions and explain each answer.

A. **A closed-loop system is stable only if the process and the controller are both stable.**

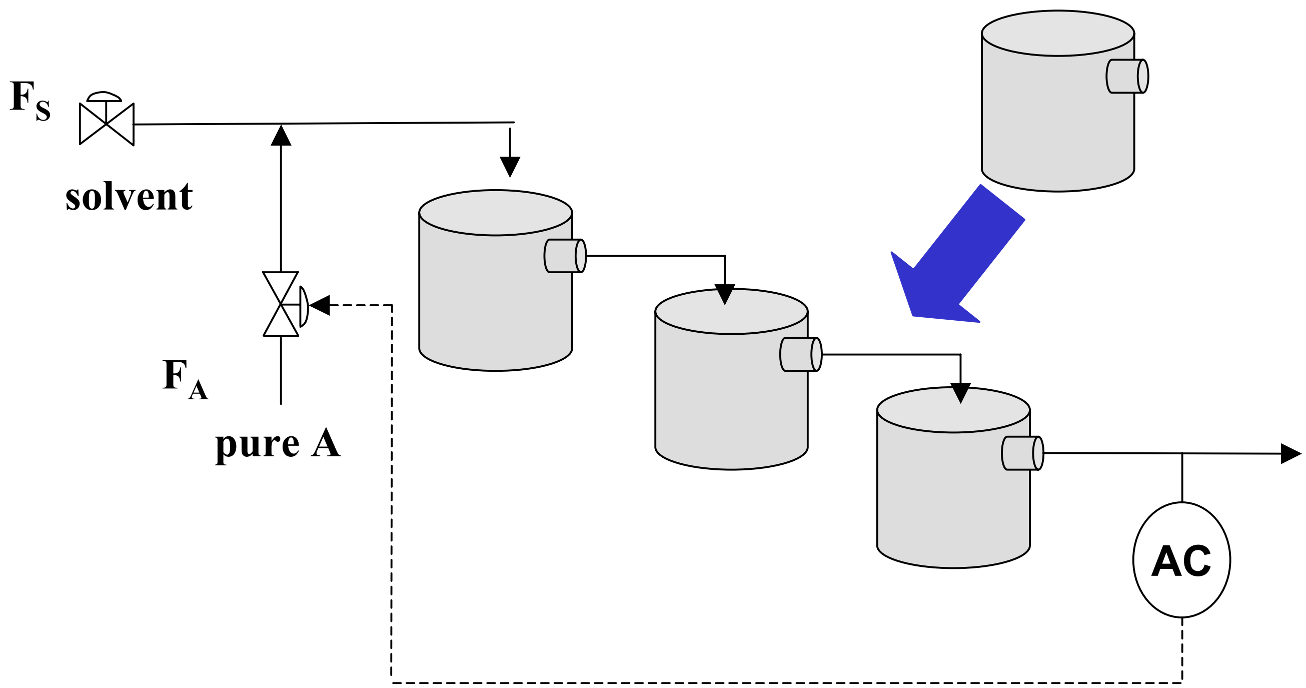
B. **The Bode stability method proves that the closed-loop system is stable for only sine inputs.**

C. **$G_{OL}(s)$ is the process model, $G_P(s)$, and sensor, final element, and signal transmission dynamics**

D. **A process would be stable if it had three poles with the following values: -1, -.2, and 0.**

CHAPTER 10: TUNING & STABILITY WORKSHOP 3

The PID controller has been tuned for a three-tank mixer. Later, we decide to include another mixing tank in the process. If we do not retune the controller, will the control system be stable with the four-tank mixer?

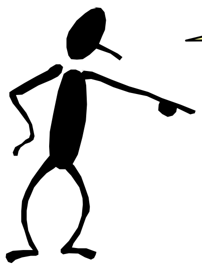


$K_c = 30$

$T_I = 11$

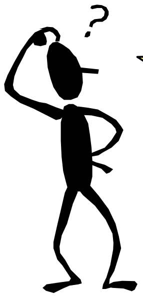
$T_d = 0.8$

CHAPTER 10: STABILITY &TUNING



When I complete this chapter, I want to be able to do the following.

- Determine the stability of a process without control
- Determine the stability of a closed-loop feedback control system
- Use these approaches to learn how dead time affects stability.



Lot's of improvement, but we need some more study!

- Read the textbook
- Review the notes, especially learning goals and workshop
- Try out the self-study suggestions
- Naturally, we'll have an assignment!

CHAPTER 10: LEARNING RESOURCES

- **<u>SITE PC-EDUCATION WEB</u>**
  - Instrumentation Notes
  - Interactive Learning Module (Chapter 10)
  - Tutorials (Chapter 10)

- **S_LOOP**
  - You can perform the stability and frequency response calculations uses menu-driven features. Then, you can simulate in the time domain to confirm your conclusions.

CHAPTER 10: SUGGESTIONS FOR SELF-STUDY

1. **Determine the stability for the example in textbook Table 9.2 (recommended tuning). Use the nominal process parameters. How much would K_C have to be increased until the system became unstable?**

2. **Determine the Ziegler-Nichols tuning for the three-tank mixer process. Simulate the dynamic response using S_LOOP.**

3. **Discuss applying the Bode stability method to a process without control.**

CHAPTER 10: SUGGESTIONS FOR SELF-STUDY

4. **We do not want to operate a closed-loop system "too close" to the stability limit. Discuss measures of the closeness to the limit and how they could be used in calculating tuning constant values.**