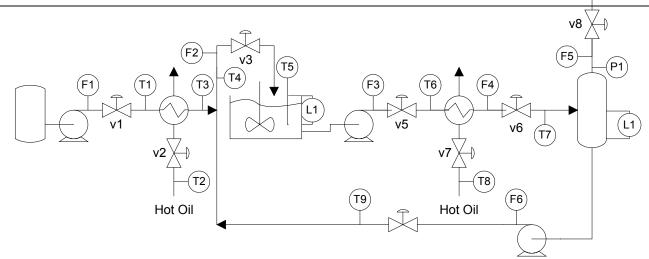
Process plants (or complex experiments) have many variables that must be controlled. The engineer must

- 1. Provide the needed sensors
- 2. Provide adequate manipulated variables
- 3. Decide how the CVs and MVs are paired (linked via the control design)

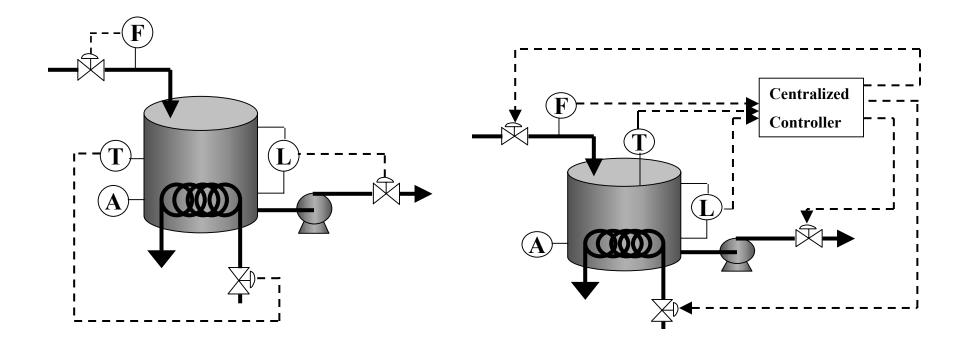
Fortunately, most of what we learned about single-loop systems applies, but we need to <a href="learn-more">learn more</a>! \*\*



## Two control approaches are possible

**Multiloop: Many independent PID controllers** 

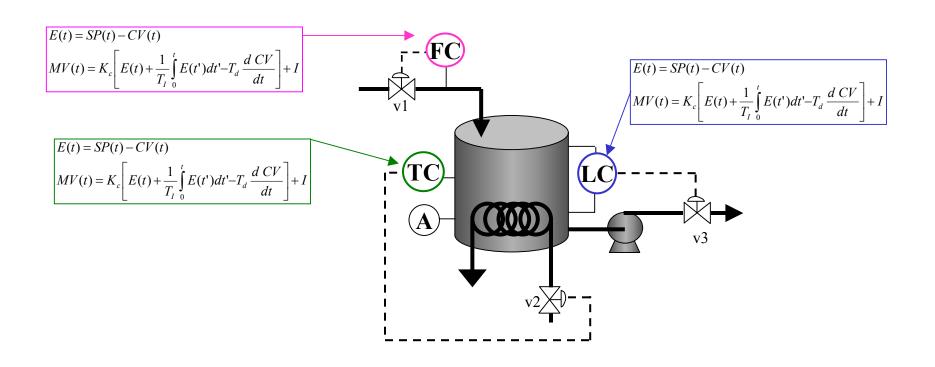
**Centralized:** Method covered in Chapter 23



## Two control approaches are possible

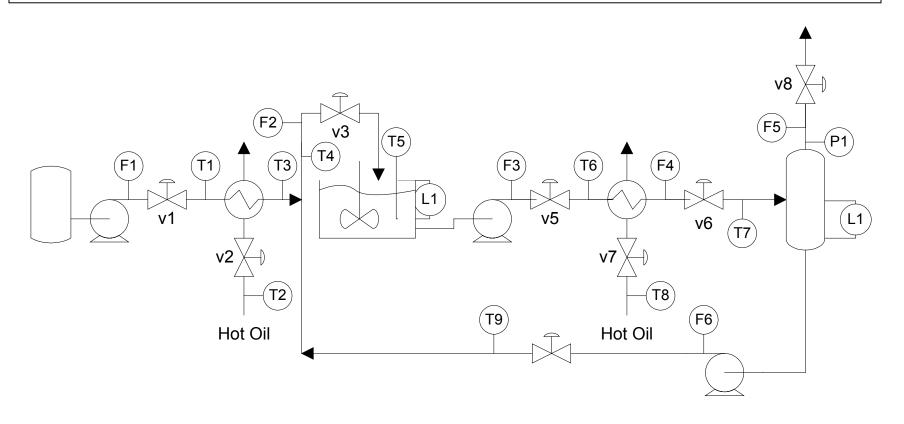
## We will concentrate on MULTILOOP

## **Multiloop:** Many independent PID controllers



Let's assume (for now) that we have the sensors and valves.

How do we ask the right questions in the best order to have a systematic method for pairing multiloop control?



# Some key questions whose answers help us design a multiloop control system.

## 1. <u>IS INTERACTION PRESENT?</u>

- If no interaction  $\Rightarrow$  All single-loop problems

Let's start here to build understanding

## 2. <u>IS CONTROL POSSIBLE</u>?

- Can we control the specified CVs with the available MVs?

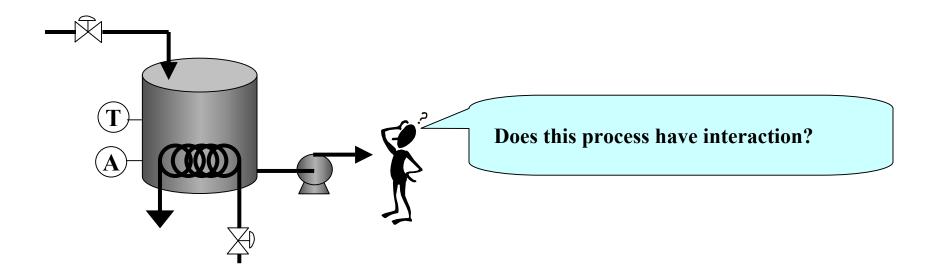
## 3. WHAT IS S-S AND DYNAMIC BEHAVIOR?

- Over what range can control keep CVs near the set points?

What is different when we have multiple MVs and CVs?

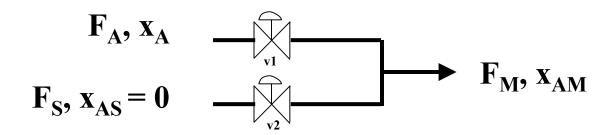
## **INTERACTION!!**

<u>Definition</u>: A multivariable process has interaction when input (manipulated) variables affect more than one output (controlled) variable.



#### How can we determine how much interaction exists?

## One way - Fundamental modelling



#### Fundamental (n-l)

$$F_{A} + F_{S} = F_{M}$$

$$F_{A}x_{A} + F_{S}x_{AS} = F_{M}x_{AM}$$

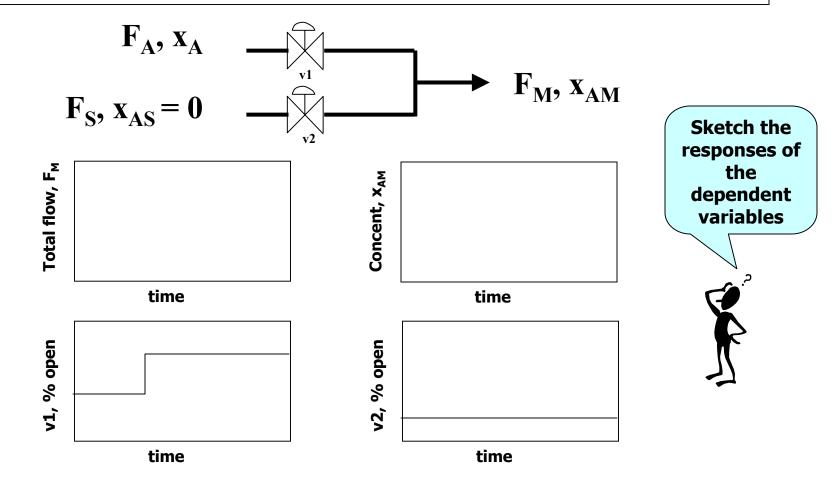
#### **Fundamental linearized**

$$\mathbf{F'_M} = \mathbf{F'_A} + \mathbf{F'_S}$$

$$\mathbf{x'_{AM}} = \left[ \frac{\mathbf{F_S}}{(\mathbf{F_S} + \mathbf{F_A})^2} \right]_{ss} \mathbf{F'_A} + \left[ \frac{-\mathbf{F_A}}{(\mathbf{F_S} + \mathbf{F_A})^2} \right]_{ss} \mathbf{F'_S}$$

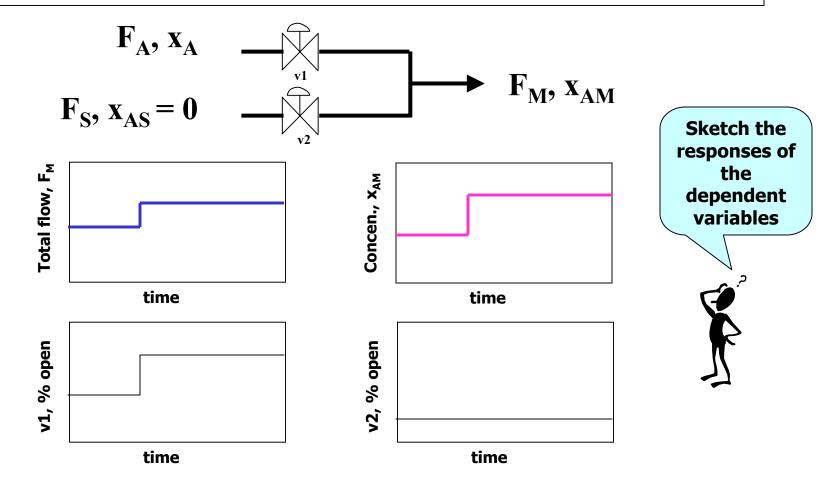
#### How can we determine how much interaction exists?

## One way - Fundamental modelling



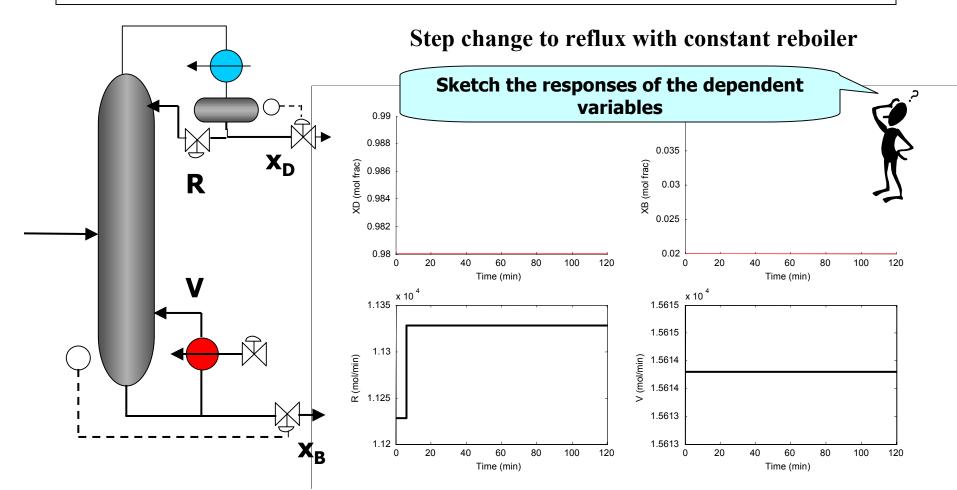
#### How can we determine how much interaction exists?

## One way - Fundamental modelling



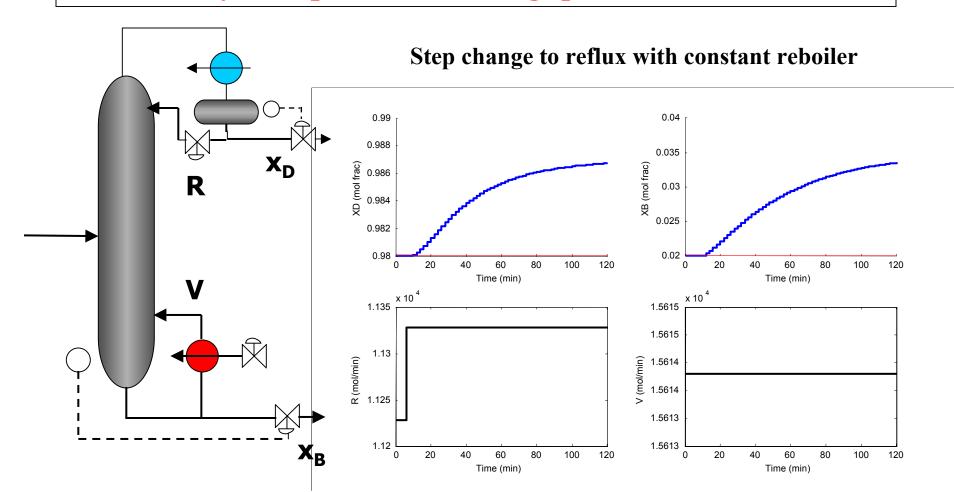
How can we determine how much interaction exists?

Second way - Empirical modelling (process reaction curve)



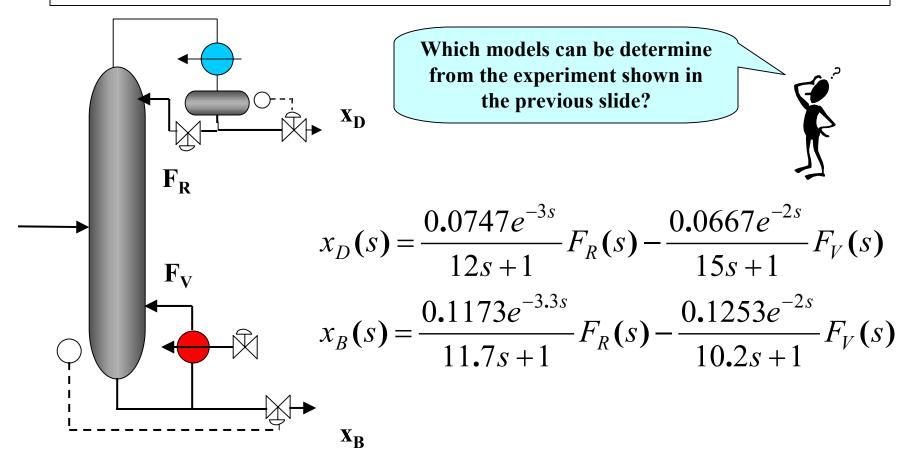
#### How can we determine how much interaction exists?

Second way - Empirical modelling (process reaction curve)



#### How can we determine how much interaction exists?

Second way - Empirical modelling (process reaction curve)



How can we determine how much interaction exists?

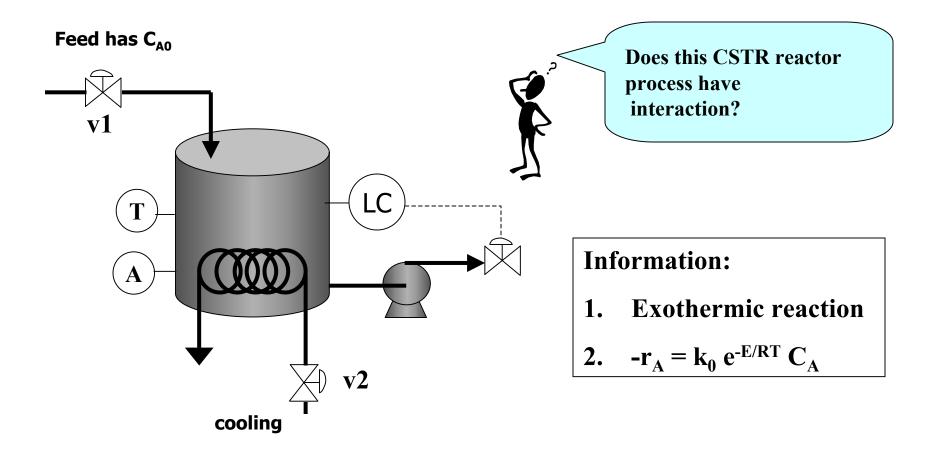
Use the model; if model can be arranged so that it has a "diagonal" form, no interaction exists.

$$\begin{bmatrix} CV_1 \\ ... \\ CV_n \end{bmatrix} = \begin{bmatrix} K_{11} & 0 & 0 \\ 0 & K_{22} & 0 \\ 0 & 0 & K_{nn} \end{bmatrix} \begin{bmatrix} MV_1 \\ MV_2 \\ MV_3 \end{bmatrix}$$

If any off-diagonal are non-zero, interaction exists.

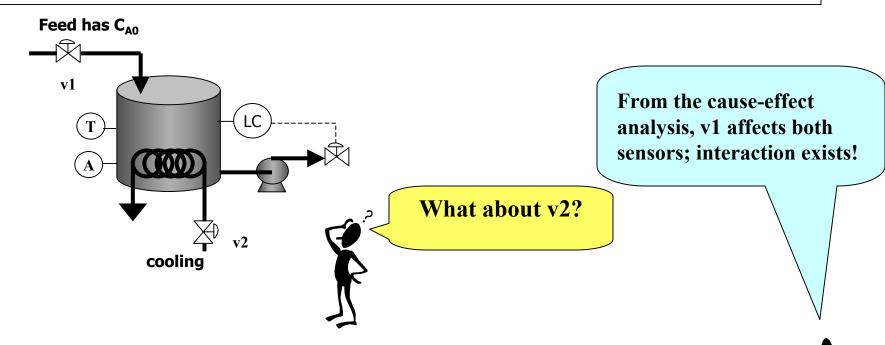
How can we determine how much interaction exists?

Let's use qualitative understanding to answer the question.



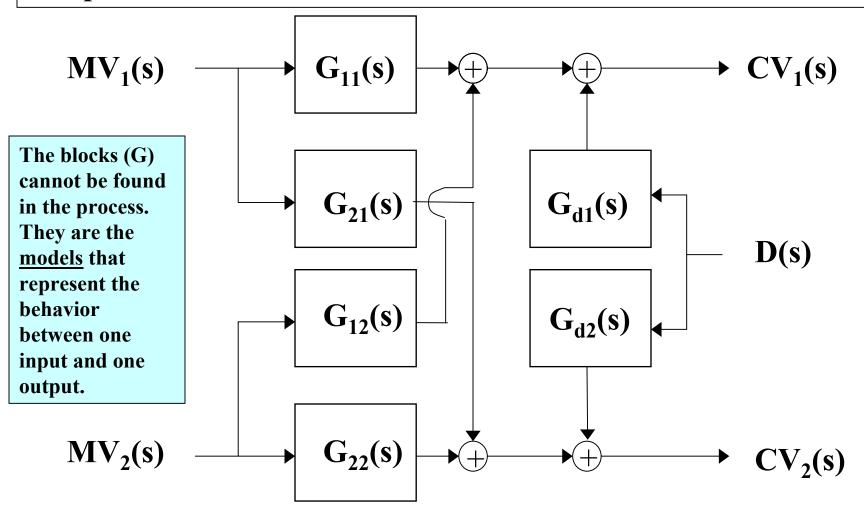
How can we determine how much interaction exists?

Let's use qualitative understanding to answer the question.

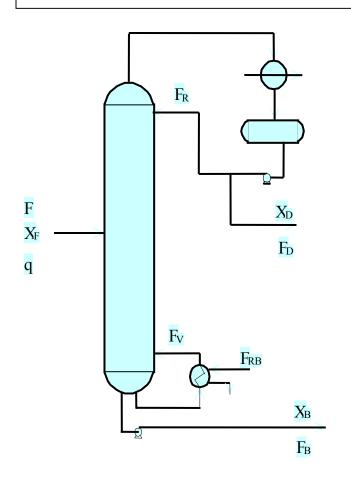


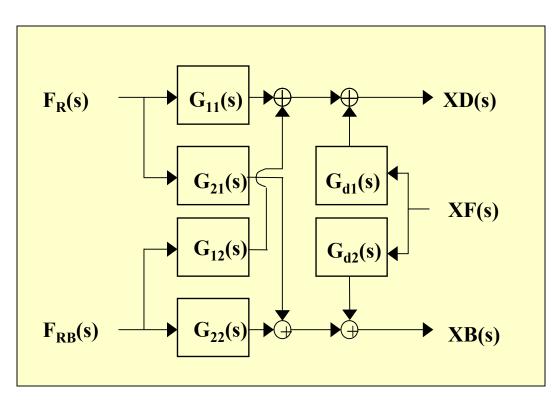
 $v1 \rightarrow \text{ feed flow rate} \rightarrow \text{residence time} \rightarrow \text{conversion} \rightarrow \text{``heat generated''} \rightarrow T$   $\rightarrow Q/(F\rho C_P) \rightarrow \qquad \rightarrow T$   $v2 \rightarrow \text{coolant flow} \rightarrow \text{heat transfer} \rightarrow \text{conversion} \qquad \rightarrow \text{Analyzer}$ 

We will use a block diagram to represent the dynamics of a 2x2 process, which involves multivariable control.

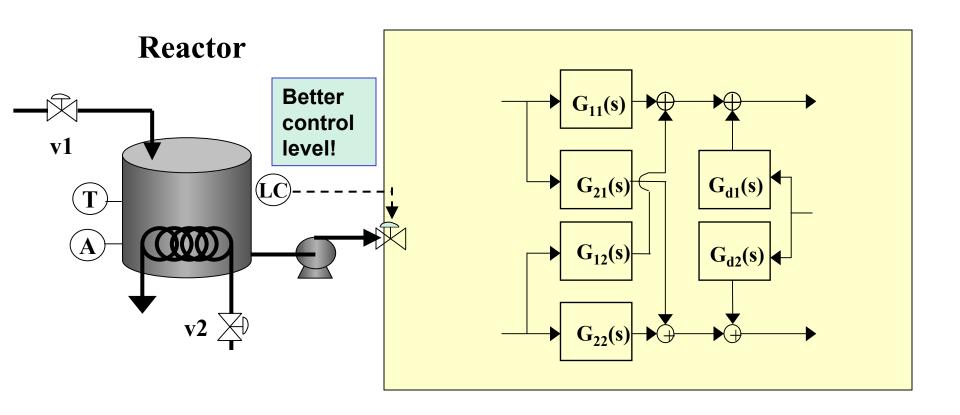


Let's relate the block diagram to a typical physical process. What are the MVs, CVs, and a disturbance, D?

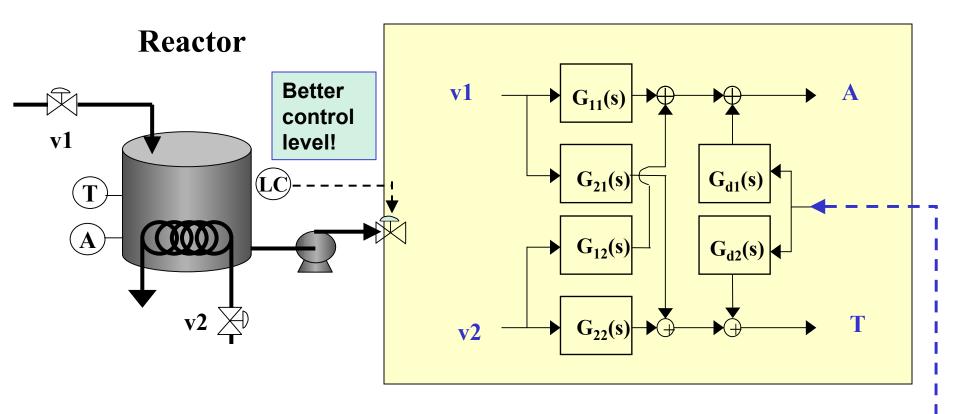




Let's relate the block diagram to a typical physical process. What are the MVs, CVs, and a disturbance, D?



Let's relate the block diagram to a typical physical process. What are the MVs, CVs, and a disturbance, D?



Disturbances = feed composition, feed temperature, coolant temperature, ...

Some key questions whose answers help us design a multiloop control system.



## **IS INTERACTION PRESENT?**

- If no interaction  $\Rightarrow$  All single-loop problems

## 2. <u>IS CONTROL POSSIBLE</u>?

- How many degrees of freedom exist?
- Can we control CVs with MVs?

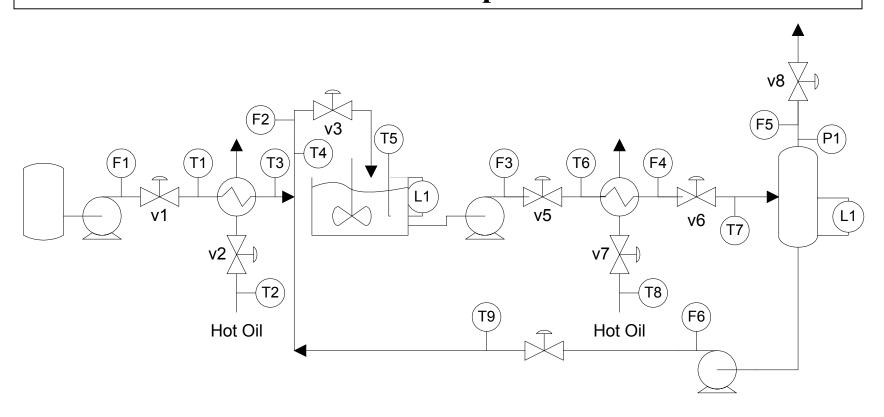


## 3. WHAT IS S-S AND DYNAMIC BEHAVIOR?

- Over what range can control keep CVs near the set points?

#### **DEGREES OF FREEDOM**

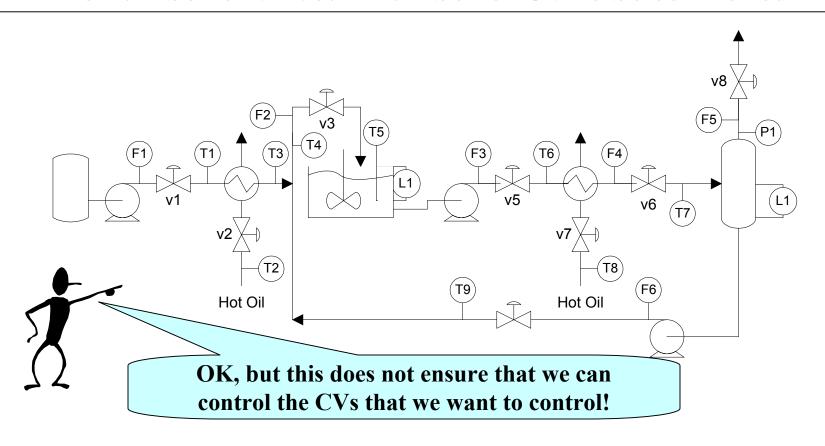
How do we determine the maximum # variables that be controlled in a process?



#### **DEGREES OF FREEDOM**

A requirement for a successful design is:

The number of valves  $\geq$  number of CV to be controlled

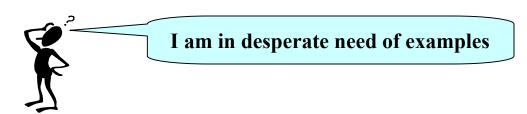


#### **CONTROLLABILITY**

A system is controllable if its CVs can be maintained at their set points, in the steady-state, in spite of disturbances entering the system.

$$\begin{bmatrix} CV_1 \\ CV_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} \begin{bmatrix} MV_1 \\ MV_2 \end{bmatrix} + \begin{bmatrix} K_{d1} \\ K_{d2} \end{bmatrix} D$$

A system is controllable when the matrix of process gains can be inverted, i.e., when the determinant of  $K \neq 0$ .



Let's do the toy autos first; then, do some processes

## For the autos in the figure

- Are they independently controllable?
- Does interaction exist?

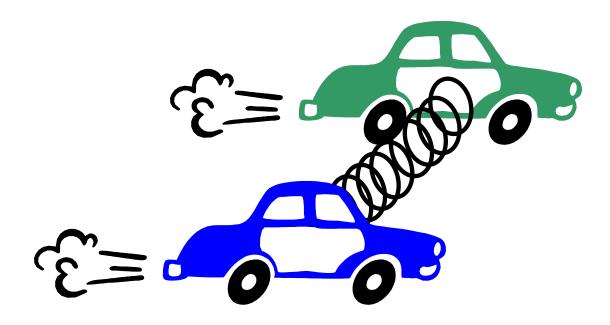






## For the autos in the figure

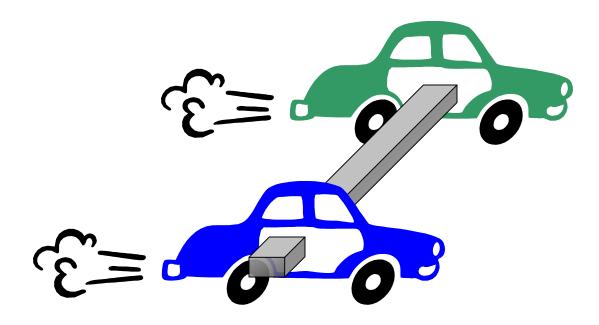
- Are they independently controllable?
- Does interaction exist?



**Connected by spring** 

## For the autos in the figure

- Are they independently controllable?
- Does interaction exist?



Connected by beam

## For process Example #1: the blending process

- Are the CVs independently controllable?
- Does interaction exist?

$$F_{A}, x_{A}$$

$$F_{S}, x_{AS} = 0$$

$$F_{M}, x_{AM}$$

$$\mathbf{F'_M} = \mathbf{F'_A} + \mathbf{F'_S}$$

$$\mathbf{x'_{AM}} = \left[ \frac{\mathbf{F_S}}{(\mathbf{F_S} + \mathbf{F_A})^2} \right]_{ss} \mathbf{F'_A} + \left[ \frac{-\mathbf{F_A}}{(\mathbf{F_S} + \mathbf{F_A})^2} \right]_{ss} \mathbf{F'_S}$$

## For process Example #1: the blending process

- Are the CVs independently controllable?
- **Does interaction exist?**

$$F_{A}, x_{A} \longrightarrow F_{M}, x_{AM}$$

$$F_{S}, x_{AS} = 0 \longrightarrow F_{M}, x_{AM}$$

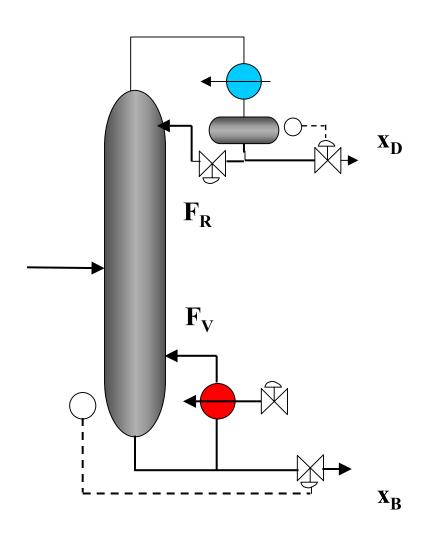
$$F_{A}, x_{A} \longrightarrow F_{M}, x_{AM}$$

$$F'_{M} = F'_{A} + F'_{S}$$

$$x'_{AM} = \left[\frac{F_{S}}{(F_{S} + F_{A})^{2}}\right]_{SS} F'_{A} + \left[\frac{-F_{A}}{(F_{S} + F_{A})^{2}}\right]_{SS} F'_{S}$$

Det(K) = 
$$\frac{-\mathbf{F}_{A}^{2}}{(\mathbf{F}_{A} + \mathbf{F}_{S})^{2}} - \frac{\mathbf{F}_{S}^{2}}{(\mathbf{F}_{A} + \mathbf{F}_{S})^{2}} \neq 0$$

Yes, this system is controllable!

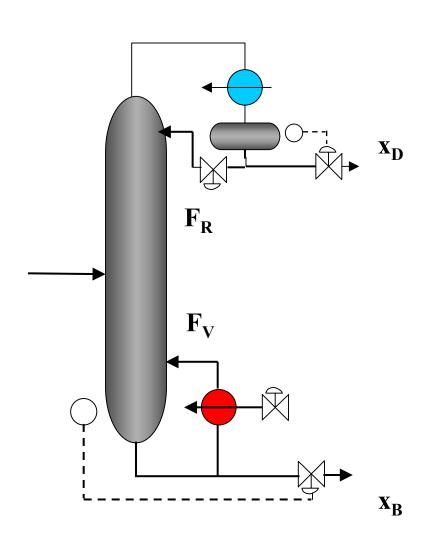


## For process Example #2: the distillation tower

- Are the CVs independently controllable?
- **Does interaction exist?**

$$x_D(s) = \frac{0.0747e^{-3s}}{12s+1} F_R(s) - \frac{0.0667e^{-2s}}{15s+1} F_V(s)$$
$$x_B(s) = \frac{0.1173e^{-3.3s}}{11.7s+1} F_R(s) - \frac{0.1253e^{-2s}}{10.2s+1} F_V(s)$$

$$x_B(s) = \frac{0.1173e^{-3.3s}}{11.7s+1} F_R(s) - \frac{0.1253e^{-2s}}{10.2s+1} F_V(s)$$



## For process Example #2: the distillation tower

$$x_D(s) = \frac{0.0747e^{-3s}}{12s+1}F_R(s) - \frac{0.0667e^{-2s}}{15s+1}F_V(s)$$

$$x_B(s) = \frac{0.1173e^{-3.3s}}{11.7s+1} F_R(s) - \frac{0.1253e^{-2s}}{10.2s+1} F_V(s)$$

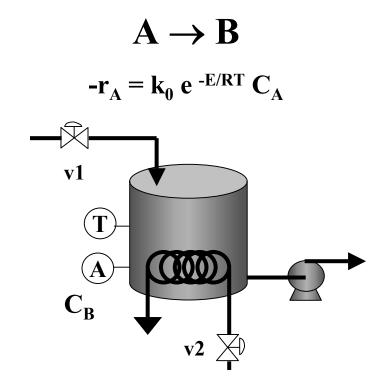
Det (K) =  $1.54 \times 10^{-3} \neq 0$ 

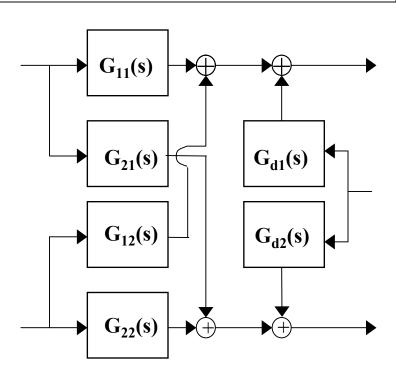
Small but not zero (each gain is small)

The system is controllable!

## For process Example #3: the non-isothermal CSTR

- Are the CVs independently controllable?
- Does interaction exist?



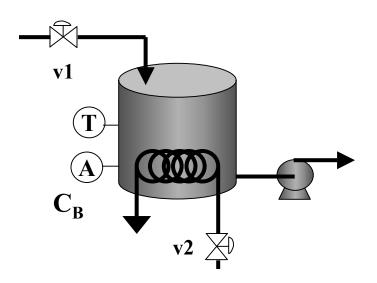


## For process Example #3: the non-isothermal CSTR

- Are the CVs independently controllable?
- Does interaction exist?

$$A \rightarrow B$$

$$-\mathbf{r}_{\mathbf{A}} = \mathbf{k}_{\mathbf{0}} \mathbf{e}^{-\mathbf{E}/\mathbf{R}\mathbf{T}} \mathbf{C}_{\mathbf{A}}$$



The interaction can be strong

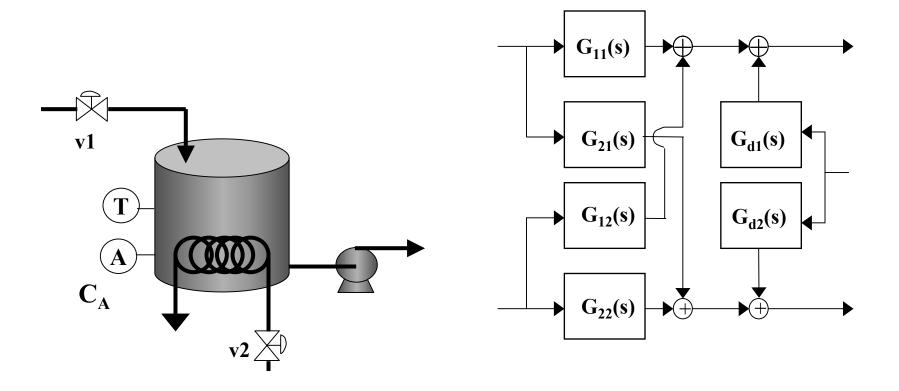
In general, the temperature and conversion (extent of reaction) can be influenced.

The system is controllable.

(See Appendix 3 for examples)

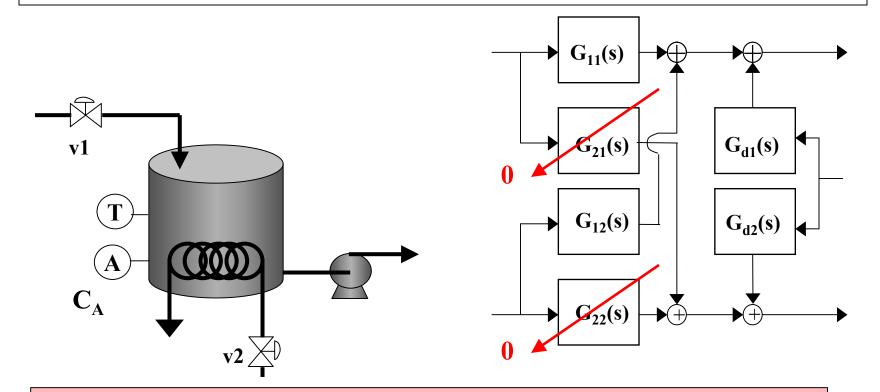
## For process Example #4: the mixing tank

- Are the CVs independently controllable?
- Does interaction exist?



## For process Example #4: the mixing tank

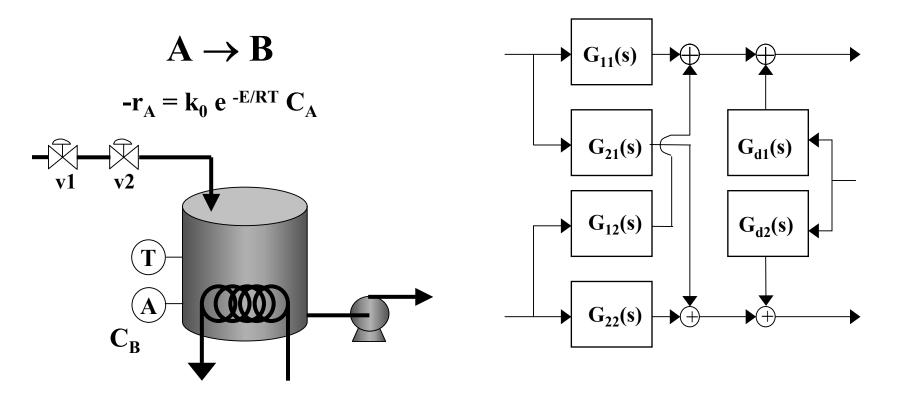
- Are the CVs independently controllable?
- Does interaction exist?



Nothing affects composition at S-S; the system is NOT controllable.

## For process Example #5: the non-isothermal CSTR

- Are the CVs independently controllable?
- Does interaction exist?



## For process Example #5: the non-isothermal CSTR

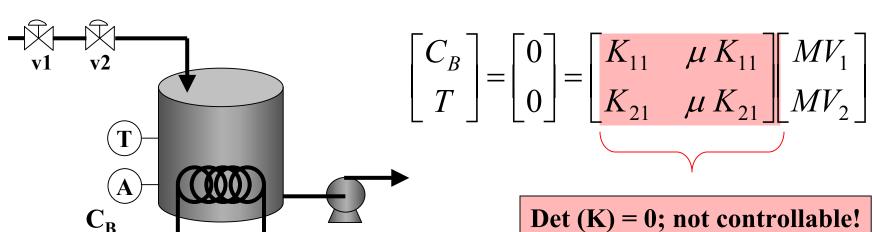
- Are the CVs independently controllable?
- Does interaction exist?

Solution continued on next slide

$$\mathbf{A} \to \mathbf{B}$$

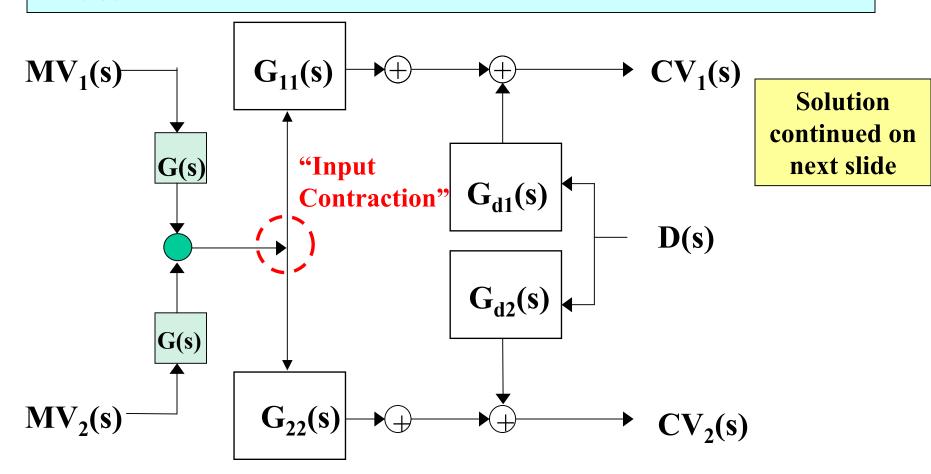
$$-\mathbf{r}_{\mathbf{A}} = \mathbf{k}_{0} \ \mathbf{e}^{-\mathbf{E}/\mathbf{R}T} \ \mathbf{C}_{\mathbf{A}}$$

Both valves have the same effects on both variables; the only difference is the magnitude of the flow change ( $\mu$  = constant).



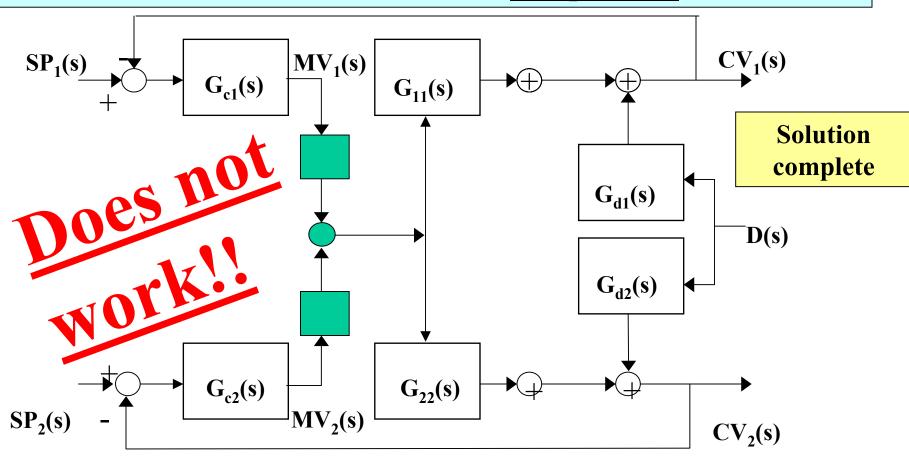
#### For process Example #5: the non-isothermal CSTR

In this case, both MVs affect ONE common variable, and this common variable affects both CVs. We can change both CVs, but we cannot move the CVs to <u>independent</u> values!



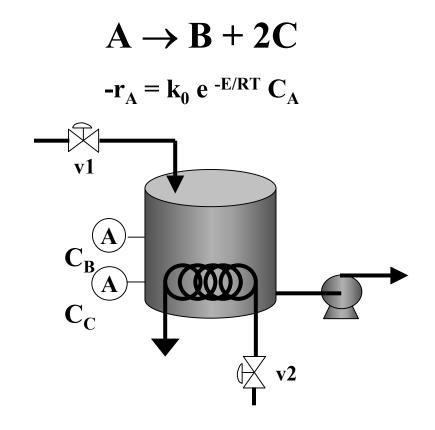
#### For process Example #5: the non-isothermal CSTR

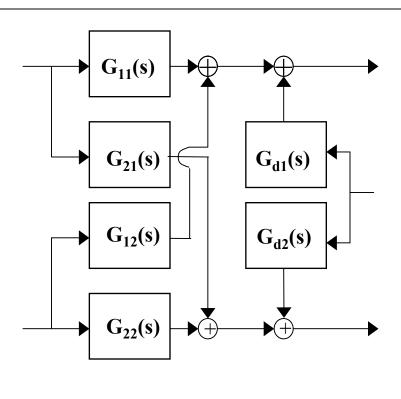
For input contraction, multivariable feedback control is not possible; the system is not controllable! We can change both CVs, but we cannot move the CVs to <u>independent</u> values!



#### For process Example #6: the non-isothermal CSTR

- Are the CVs independently controllable?
- Does interaction exist?





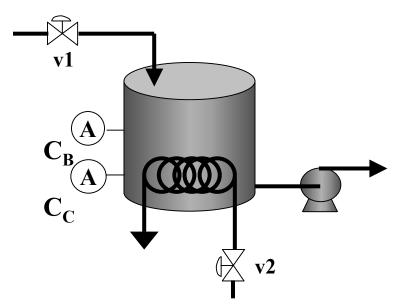
#### For process Example #6: the non-isothermal CSTR

- Are the CVs independently controllable?
- Does interaction exist?

Solution continued on next slide

$$A \rightarrow B + 2C$$

$$-\mathbf{r}_{\mathbf{A}} = \mathbf{k}_{\mathbf{0}} \mathbf{e}^{-\mathbf{E}/\mathbf{R}\mathbf{T}} \mathbf{C}_{\mathbf{A}}$$



Using the symbol  $N_i$  for the number of moles of component "i" that reacts, we have the following.

$$N_B = -N_A$$
  $N_C = -2N_A$ 

Because of the stoichiometry,

$$N_C = 2 N_B$$

and the system is not controllable!

#### For process Example #6: the non-isothermal CSTR

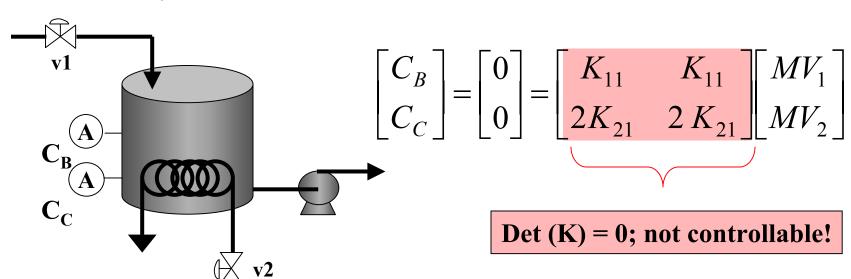
Are the CVs independently controllable?

Solution continued on next slide

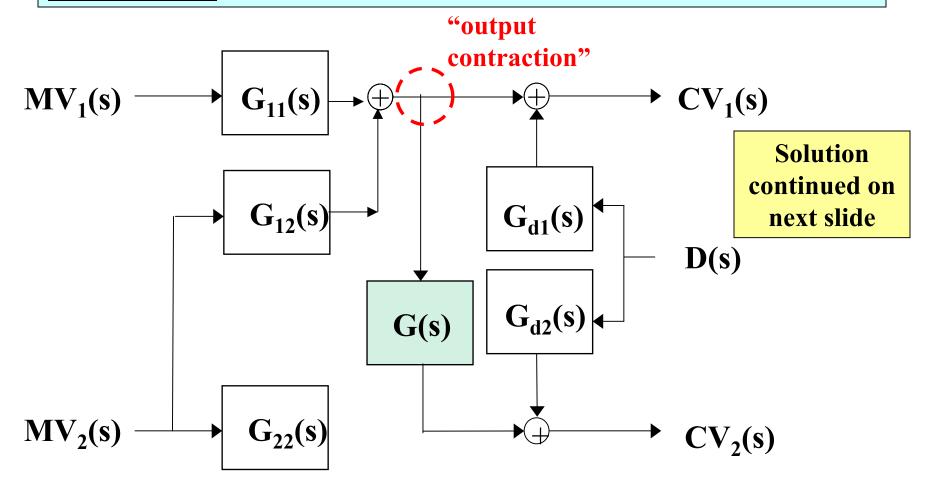
Does interaction exist?

$$A \rightarrow B + 2C$$

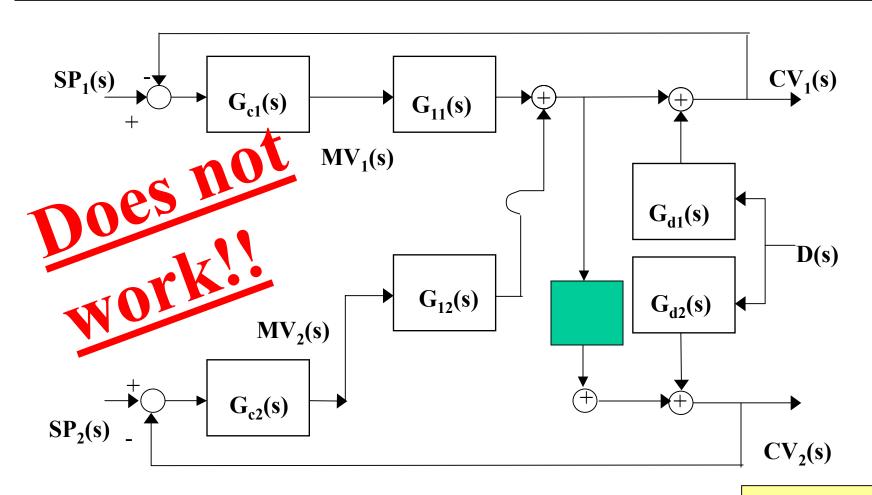
$$-\mathbf{r}_{\mathbf{A}} = \mathbf{k}_{\mathbf{0}} \mathbf{e}^{-\mathbf{E}/\mathbf{R}\mathbf{T}} \mathbf{C}_{\mathbf{A}}$$



For output contraction, both MVs affect both CVs, but the CVs are related through the physics and chemistry. We can change both CVS, but we cannot move the CVs to <a href="independent">independent</a> values!



## In this case, multivariable feedback control is not possible; the system is uncontrollable!



Solution complete

#### **CONTROLLABILITY**

Conclusions about determining controllability

Lack of controllability when

1. One CV cannot be affected by any valve

This is generally easy to determine.

This requires care and



process insight or modelling to determine.

3. Lack of <u>independent</u> effects.

Look for "contractions"



Some key questions whose answers help us design a multiloop control system.



### **IS INTERACTION PRESENT?**

- If no interaction  $\Rightarrow$  All single-loop problems



#### IS CONTROL POSSIBLE?

- How many degrees of freedom exist?
- Can we control CVs with MVs?

Let's see how good the performance can be

### 3. WHAT IS S-S AND DYNAMIC BEHAVIOR?

- Over what range can control keep CVs near the set points?

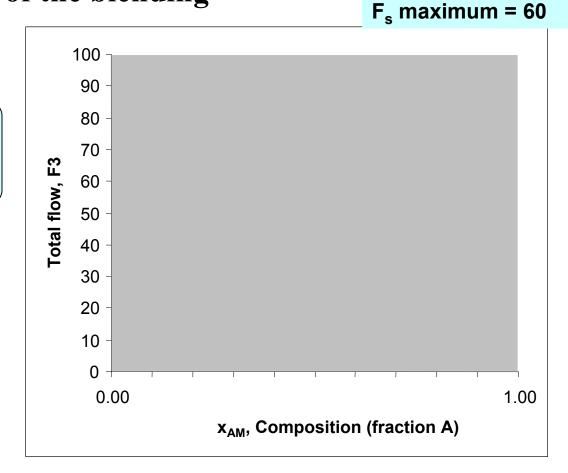
How does interaction affect the steady-state behavior of the blending

 $F_{A} \text{ maximum} = 30$   $F_{A}, x_{A} \xrightarrow{\text{P}} F_{M}, x_{AM}$   $F_{S}, x_{AS} = 0 \xrightarrow{\text{P}} F_{M}, x_{AM}$ 

process?

Please sketch the achievable values

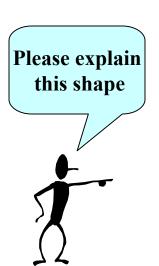


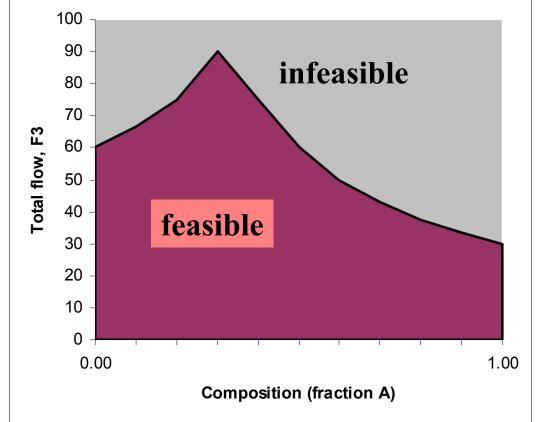


How does interaction affect the steady-state behavior of the blending

 $F_{A} \text{ maximum} = 30$   $F_{A}, X_{A} \longrightarrow F_{M}, X_{AM}$   $F_{S}, X_{AS} = 0 \longrightarrow F_{M}, X_{AM}$   $F_{S} \text{ maximum} = 60$ 

process?





#### Note:

This shows a range of set points that can be achieved (without disturbances).

#### **STEADY-STATE OPERATING WINDOW**

Conclusions about the steady-state behavior of a multivariable process

Summarize your conclusions here.

1. Shape of the Window

The operating window is not generally a simple shape, like a rectangle.

2. What influences the size of the Window?

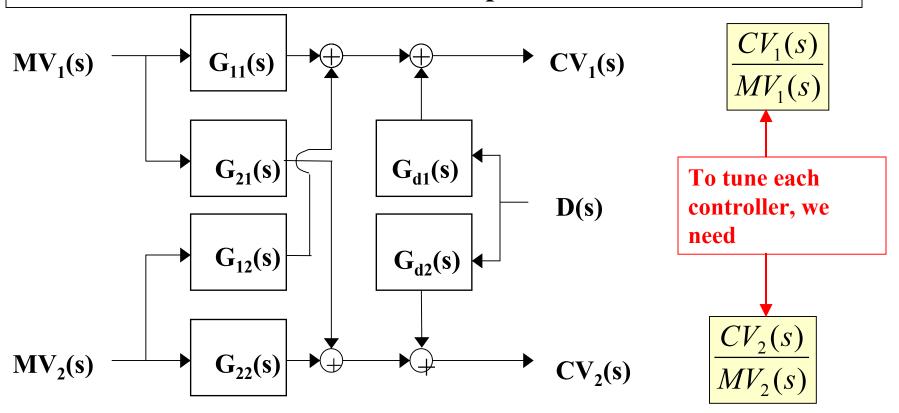
The operating window is influenced by process chemistry, equipment capacity, disturbances ...

3. How does Window relate to Controllability?

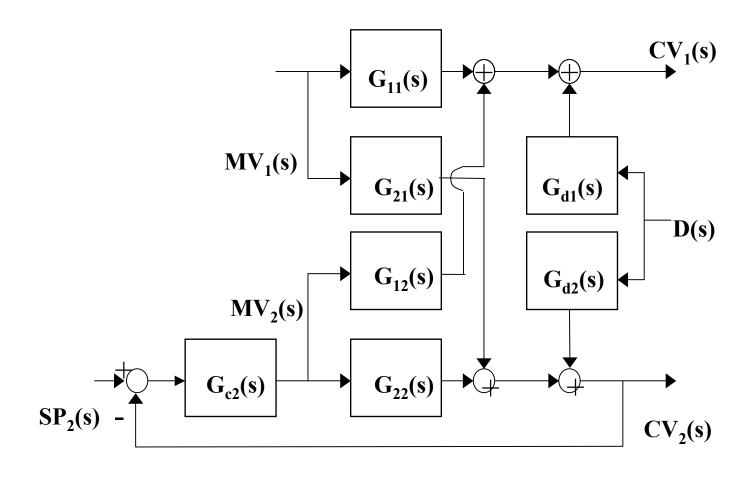
Process cannot be "moved" (controlled) outside of the window. If operating window is empty, the system is not controllable.

#### NOW, LET'S LOOK AT THE DYNAMIC BEHAVIOR

- 1. How many experiments are needed to tune controllers?
- 2. Which controller should be implemented first?

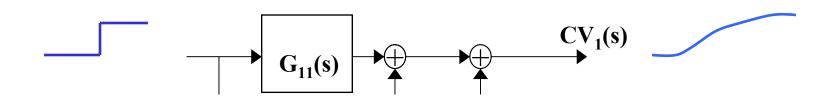


3. We have implemented one controller. What do we do now?

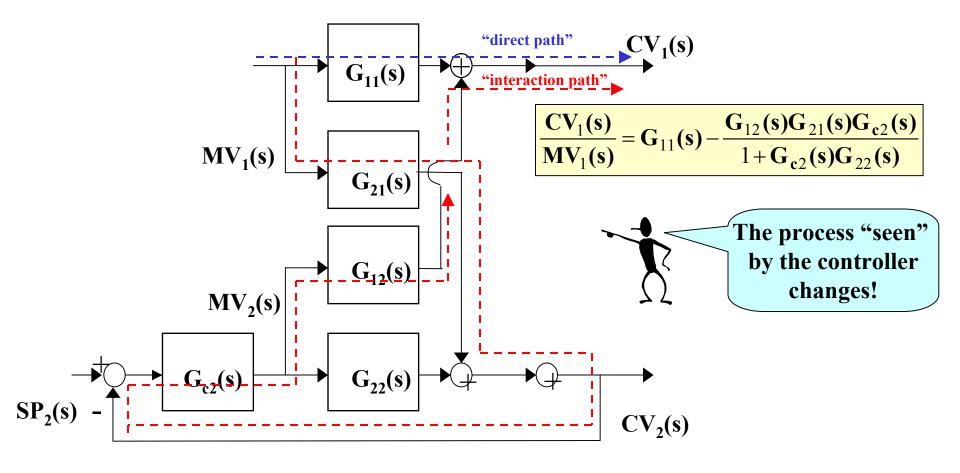


4. What if we perform another experiment to learn the dynamics between  $MV_1$  and  $CV_1$ , with controller #2 in automatic?

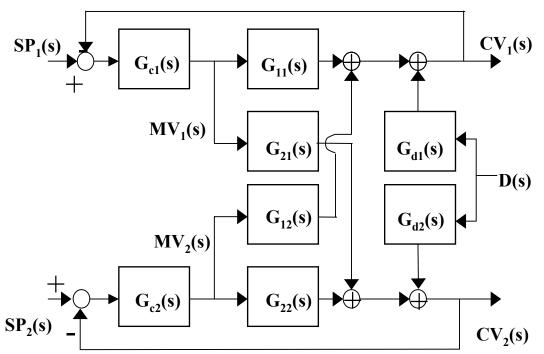
Is the dynamic behavior different from without second controller  $(G_{C2})$ ? What elements does the behavior depend upon?



4. Is the dynamic behavior different from without second controller  $(G_{C2})$ ? What elements does the behavior depend upon?



In general, the behavior of one loop depends on the interaction and the <u>tuning of the other loop(s)</u>.



- Tuning that is stable for each loop might not be stable when both are in operation!
- We need to tune loops iteratively, until we obtain good performance for all loops!



I think that I need an example again!

In general, the behavior of one loop depends on the interaction and the tuning of the other loop(s).

Let's look at a simple example with interaction,

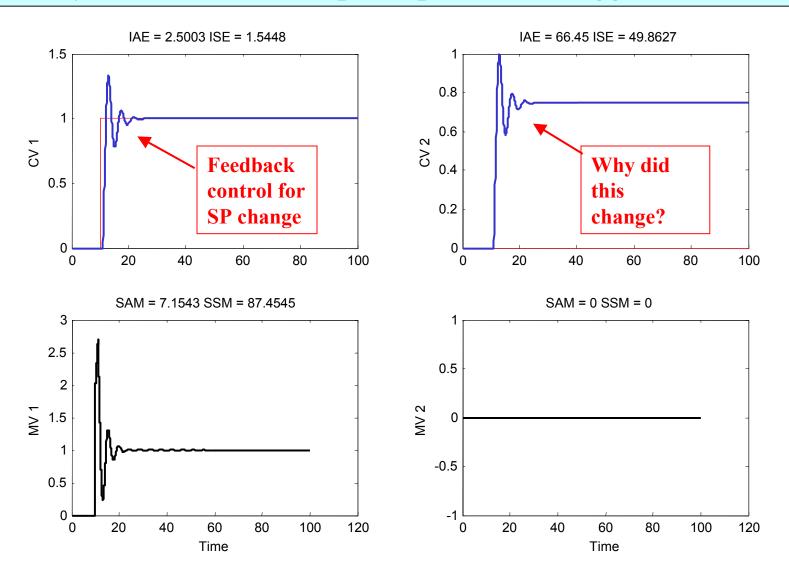
$$\begin{bmatrix} \mathbf{CV}_{1}(\mathbf{s}) \\ \mathbf{CV}_{2}(\mathbf{s}) \end{bmatrix} = \begin{bmatrix} \frac{1.0\mathbf{e}^{-1.0\mathbf{s}}}{1+2\mathbf{s}} & \frac{0.75\mathbf{e}^{-1.0\mathbf{s}}}{1+2\mathbf{s}} \\ \frac{0.75\mathbf{e}^{-1.0\mathbf{s}}}{1+2\mathbf{s}} & \frac{1.0\mathbf{e}^{-1.0\mathbf{s}}}{1+2\mathbf{s}} \end{bmatrix} \begin{bmatrix} \mathbf{MV}_{1}(\mathbf{s}) \\ \mathbf{MV}_{2}(\mathbf{s}) \end{bmatrix}$$

We will pair the loops on the "strongest" gains,

$$MV_1(s) \Rightarrow CV_1(s)$$
 and  $MV_2(s) \Rightarrow CV_2(s)$ 

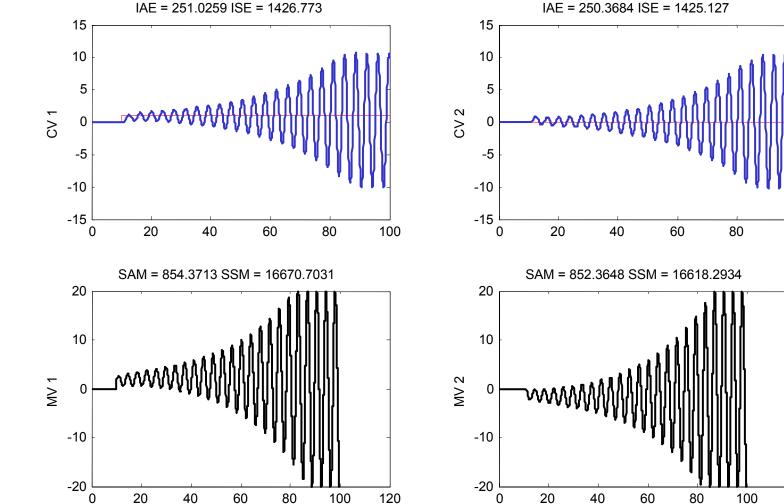
#### Results with <u>only one controller</u> in automatic ( $K_{C1} = 2.0$ , $T_{I1} = 3$ )

#### This system is stable and perhaps a bit too aggressive.



#### Results with <u>both controllers</u> in automatic (Kc = 2.0, TI = 3)

#### This system is unstable!! Each was stable by itself!!



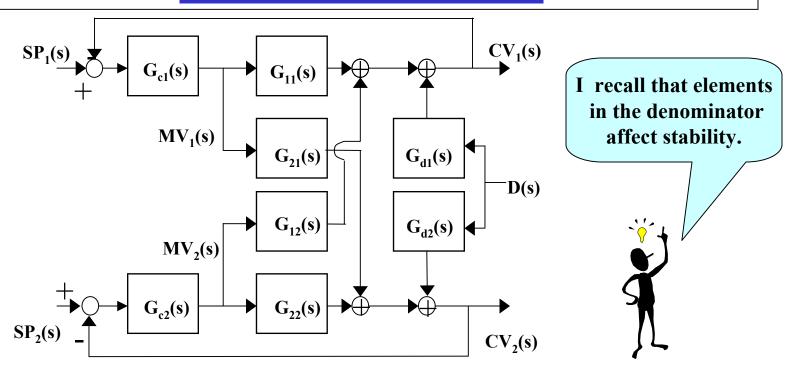
Time

100

120

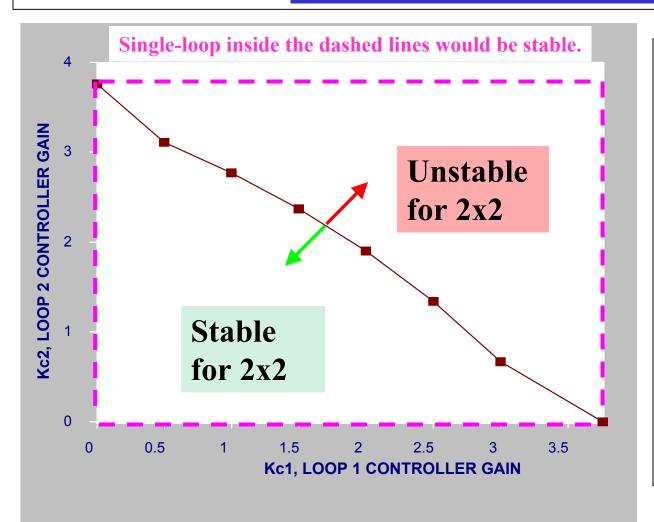
Time

In general, the behavior of one loop depends on the interaction and the <u>tuning of the other loop(s)</u>.



$$\frac{\mathbf{CV}_{1}(s)}{\mathbf{SP}_{1}(s)} = \frac{numerator(s)}{1 + \mathbf{G}_{c1}(s)\mathbf{G}_{11}(s) + \mathbf{G}_{c2}(s)\mathbf{G}_{22}(s) + \mathbf{G}_{c1}(s)\mathbf{G}_{c2}(s)[\mathbf{G}_{11}(s)\mathbf{G}_{22}(s) - \mathbf{G}_{12}(s)\mathbf{G}_{21}(s)]}$$

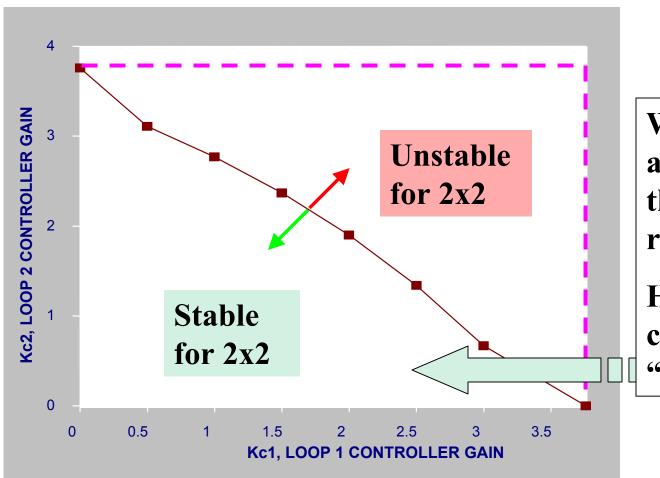
In general, the behavior of one loop depends on the interaction and the <u>tuning of the other loop(s)</u>.



#### **Notes:**

- 1. TI = 3 for both controllers (reasonable = t63%)
- 2. KC < 3.75 stable for single-loop feedback
- 3. KC = 2.0 stable for one loop
- 4. KC = 2.0 unstable for two loops!!
- 5. These numerical results are for the example only; concepts are general

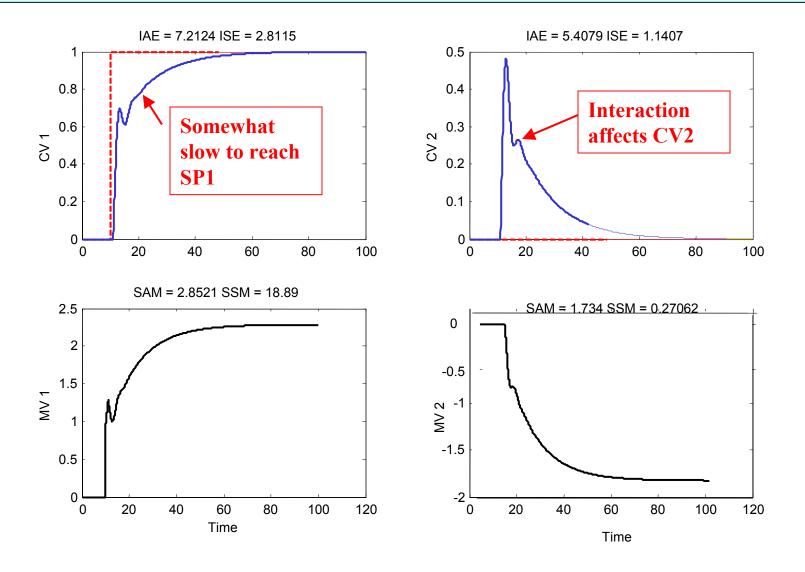
In general, the behavior of one loop depends on the interaction and the <u>tuning of the other loop(s)</u>.



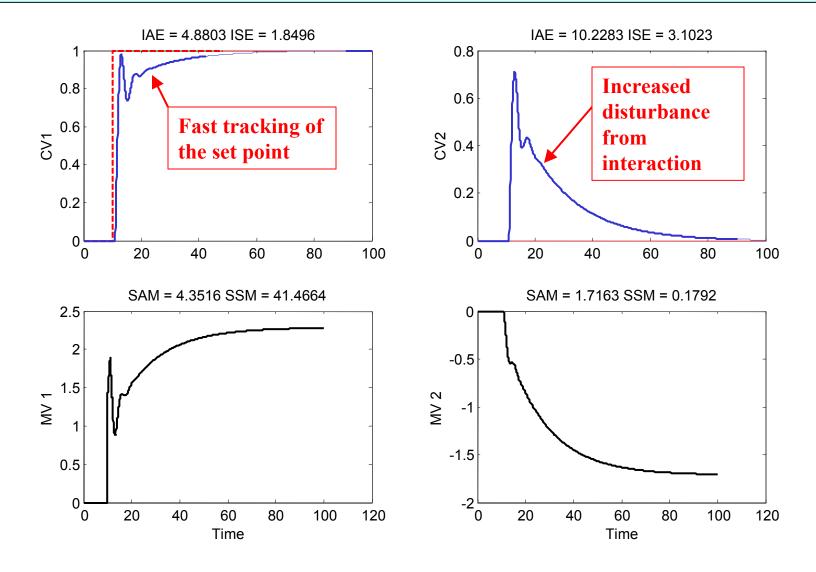
We can have any values in the stable region.

How do we choose the "best".

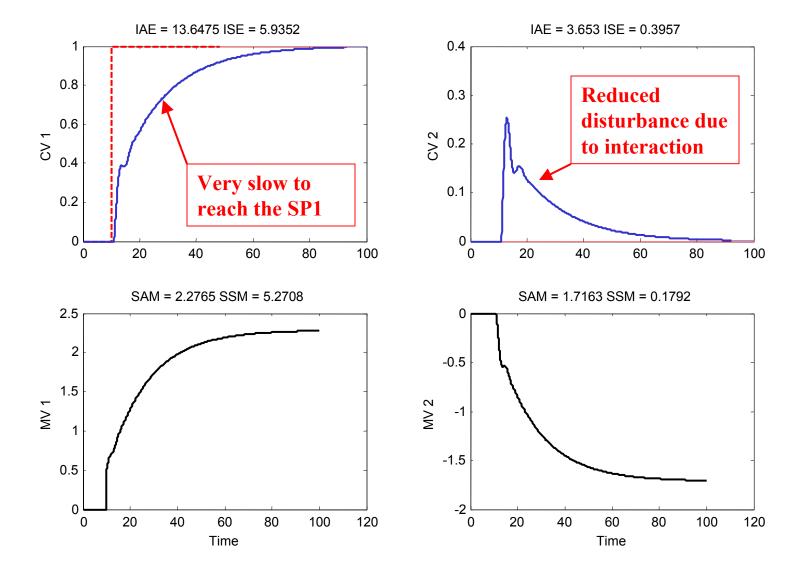
## If both CVs are of equal importance, we would detune both controllers equally. (Kc1 = Kc2 = 0.95; TI1 = TI2 = 3.0)



### If CV1 is more important, we would make Gc1 aggressive and detune Gc2 more. (Kc1 = 1.40 and Kc2 = 0.50; TI1 = TI2 = 3.0)



## If CV2 is more important, we would make Gc2 aggressive and detune Gc1 more. (Kc1 = 0.50 and Kc2 = 1.40; TI1 = TI2 = 3.0)



In general, the behavior of one loop depends on the interaction and the <u>tuning of the other loop(s)</u>.

#### Some conclusions for multiloop PID tuning:

- 1. For multiloop, we generally have to tune the controllers in a less aggressive manner than for single-loop.
- 2. Textbook gives tuning approach,  $(Kc)_{ml} \approx 1/2 (Kc)_{sl}$
- 3. We can tune important loop tightly, if we also detune (make less aggressive) other loops.

# Summary of key questions for multiloop control system

- 1. IS INTERACTION PRESENT?
  - No interaction ⇒ All single-loop problems
  - We use models to determine interaction Fundamental or empirical
- 2. IS CONTROL POSSIBLE?
- 3. WHAT IS S-S AND DYNAMIC BEHAVIOR?

# Summary of key questions for multiloop control system

- 1. IS INTERACTION PRESENT?
- 2. IS CONTROL POSSIBLE?
  - DOF: The # of valves  $\geq$  # of controlled variables
  - Controllable: Independently affect every CV

Check if det  $[K] \neq 0$ 

Look for "contractions"

3. WHAT IS S-S AND DYNAMIC BEHAVIOR?

# Summary of key questions for multiloop control system

- 1. IS INTERACTION PRESENT?
- 2. IS CONTROL POSSIBLE?
- 3. WHAT IS S-S AND DYNAMIC BEHAVIOR?
  - The operating window is:

strongly affected by interaction strongly affected by equipment capacities

not a rectangle or symmetric

# Summary of key questions for multiloop control system

- 1. IS INTERACTION PRESENT?
- 2. IS CONTROL POSSIBLE?
- 3. WHAT IS S-S AND DYNAMIC BEHAVIOR?
  - Interaction affects loop stability
  - We have to detune to retain stability margin
  - We can improve some loops by tight tuning, but we have to detune and degrade other loops

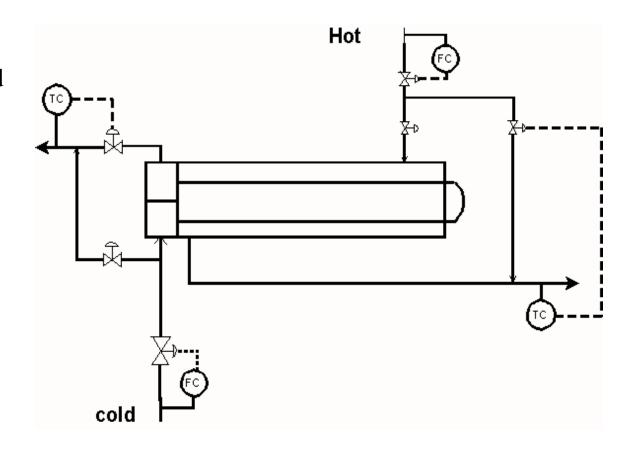
#### Workshop in Interaction in Multivariable Control Systems



#### Workshop Problem # 1

You have been asked to evaluate the control design in the figure.

Discuss good and poor aspects and decide whether you would recommend the design.

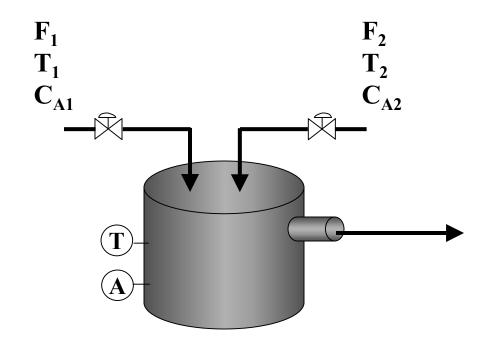


#### Workshop Problem # 2

We need to control the mixing tank effluent temperature and concentration.

You have been asked to evaluate the design in the figure.

Discuss good and poor aspects and decide whether you would recommend the design.



#### Workshop Problem # 3

Summarize the underlying principles that can lead to a "contraction" and to a loss of controllability. These principles will be applicable to many process examples, although the specific variables could be different.

Hint: Review the examples in the lecture and identify the root cause for each.