Problem Set 6
ECE 590 Fall 2019

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Problem 1: Bayesian Network

Student X has registered for a deep learning course and wants to know his/her odds of obtaining a letter of recommendation from the instructor. In order to do this, the student can use the Bayesian network shown in Fig. 1.

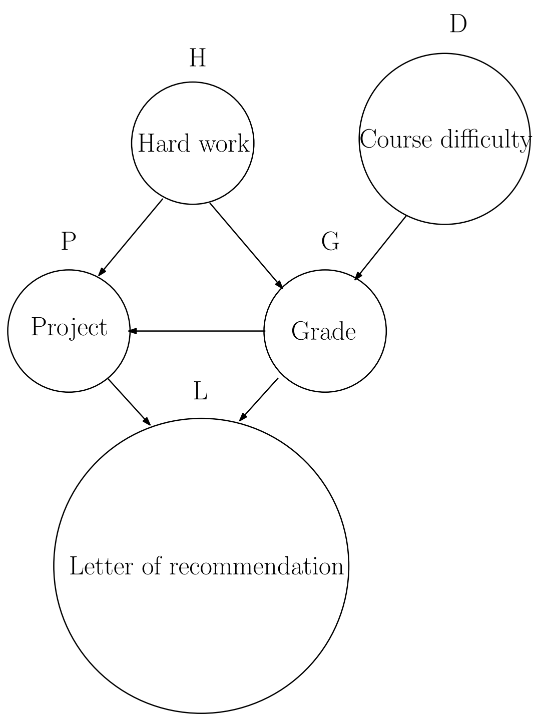

Figure 1: Bayesian network.

Indicate whether the following statements are true or false and explain why.

- $(H \perp\!\!\!\perp L)|P$
- $(H \perp\!\!\!\perp L)|P, G$
- $(P \perp\!\!\!\perp D)|G$
- $(H \perp\!\!\!\perp D)|L$

Write down the factorized form of the joint distribution $\mathbb{P}(H, D, P, G, L)$.

Write down the expression for the probability of obtaining the letter of recommendation $\mathbb{P}(L = 1)$ using $G \in \{A, B\}$ and $P \in \{A, B\}$. Compute $\mathbb{P}(L = 1)$ using the following probability tables.

Table 1: Joint probability distribution of P and G

$\mathbb{P}(P = A, G = A) = 0.4$
$\mathbb{P}(P = A, G = B) = 0.3$
$\mathbb{P}(P = B, G = A) = 0.1$
$\mathbb{P}(P = B, G = B) = 0.2$

Table 2: Conditional probability distribution of L given P and G

$\mathbb{P}(L = 1 \mid P = A, G = A) = 0.8$
$\mathbb{P}(L = 1 \mid P = A, G = B) = 0.6$
$\mathbb{P}(L = 1 \mid P = B, G = A) = 0.3$
$\mathbb{P}(L = 1 \mid P = B, G = B) = 0.1$

Problem 2: Image Denoising using Singular Value Decomposition

A closely related problem to Principal Component Analysis (PCA) is image denoising using Singular Value Decomposition (SVD). In this exercise you are asked to reconstruct (i.e., denoise) a given noisy image using the largest K singular values and the corresponding singular vectors. To assess the quality of reconstruction, you are required to use *peak signal-to-noise ratio* (PSNR). Specifically, for an $m \times n$ monochrome noisy image I and its approximation $\hat{I}$, the PSNR is given by

$$PSNR = 10 \cdot \log_{10} \left(\frac{MAX_I^2}{MSE} \right)$$

where

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} \left[I(i,j) - \hat{I}(i,j) \right]^2$$

and MAX_I is the maximum possible pixel value of the image I.

Write a Python program that plots the $PSNR$ vs. K (defined above), for $K = 1, \ldots, 15$.

You are allowed to use built-in functions for computing the SVD.

Problem 3: Feature Extraction using Autoencoder

In this problem you are asked to do a classification task based on the features extracted using various approaches such as autoencoder (AE), contractive autoencoder (CAE) and PCA (i.e., linear autoencoder). The data set is normalized MNIST (as provided in the Jupyter notebook template that will be posted on Sakai).

AE-based feature extraction

Use the following architecture for the encoder:

- Four fully connected layers with 128, 64, 12 and 3 output neurons in each layer;
- The first three layers use ReLU activation functions while the last layer uses linear activation function (i.e., no activation)

The last layer of the decoder uses $\tanh(\cdot)$ activation function. The loss function for DAE is the MSE (see the lecture notes). You should implement forward pass by yourself. For backward pass you can use autograd.

CAE-based feature extraction

Use the same architecture as the AE with contractive loss function (see lecture notes). You should implement forward pass by yourself. For backward pass you can use autograd.

PCA-based feature extraction

Use three components. You can use the built-in function in sklearn.

Classification method

Use multiclass logistic regression with default parameters in sklearn. You can use the built-in function.

Report the train and test classification accuracies for all three feature extraction methods.