

# Kinematic Model of a Four Mecanum Wheeled Mobile Robot

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## ABSTRACT

This paper introduces omnidirectional Mecanum wheels and discusses the kinematic relations of a platform used four Mecanum wheels. Forward and Inverse kinematic is been derived in this paper. Experimental and analytically results are obtained and 8 different motions without changing the robot's orientation is achieved.

## Keywords

Omnidirectional, four Mecanum wheeled robot, mobile robot, kinematic.

## 1. INTRODUCTION

Omni-differential locomotion is being used in current mobile robots in order to obtain the additional maneuverability and productivity. These features are expanded at the expense of improved mechanical complication and increased complexity in control mechanism. Omni-differential systems work by applying rotating force of each individual wheel in one direction similar to regular wheels with a difference in the fact that Omni-differential systems are able to slide freely in a different direction, in other word, they can slide frequently perpendicular to the torque vector. The main advantage of using Omni-drive systems is that translational and rotational motions are decoupled for simple motion although in making an allowance for the fastest possible motion this is not essentially the case.

### 1.1 Mecanum Wheels

Mecanum wheel also called Ilon wheel and Swedish wheel is a more common omnidirectional wheel designs, invented in 1973 by Bengt Ilon a Swedish engineer [1]. In this design, similar to the Omni wheel, There are a series of free moving rollers attached to the hub but with an  $45^\circ$  of angle about the hub's circumference but still the overall side profile of the wheel is circular. See Figure 1[2].

Omnidirectional motion can be reached by mounting four Mecanum wheels on the corners of a four-sided base. Because of the angled rollers, the mechanical design is much more difficult, but due to the smoother transfer of contact surfaces a higher loads can be supported [3].

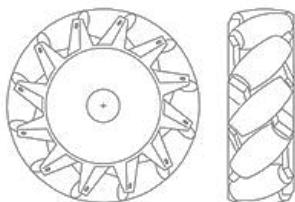


Fig 1: Mecanum wheel design

## 2. KINEMATIC

Figure 2 shows the configuration of a robot with four omnidirectional wheels.

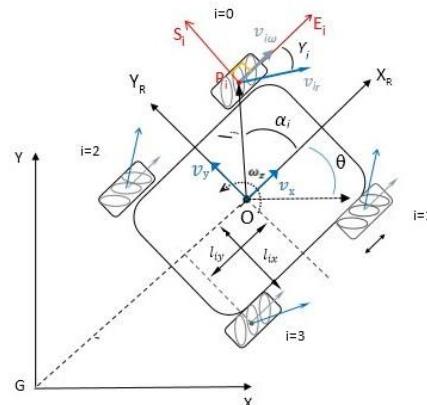
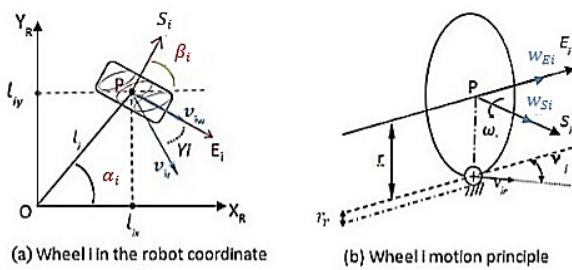


Fig 2: Wheels Configuration and Posture definition

The configuration parameters and system velocities are defined as follows:

- $x, y, \theta$ , robot's position ( $x, y$ ) and its orientation angle  $\theta$  (The angle between X and  $X_R$ );
- $X G Y$ , inertial frame;  $x, y$  are the coordinates of the reference point O in the inertial basis;
- $X_R O Y_R$ , robot's base frame; Cartesian coordinate system associated with the movement of the body center;
- $S_i P_i E_i$ , coordinate system of  $i$ th wheel in the wheel's center point  $P_i$  ;
- $O, P_i$ , the inertial basis of the Robot in Robot's frame and  $P_i = \{X_{P_i}, Y_{P_i}\}$  the center of the rotation axis of the wheel  $i$  ;
- $\overrightarrow{OP_i}$ , is a vector that indicates the distance between Robot's center and the center of the wheel  $i$ th;
- $l_{ix}, l_{iy}$ ,  $l_{ix}$ , half of the distance between front wheels and  $l_{iy}$  half of the distance between front wheel and the rear wheels.
- $l_i$ , distance between wheels and the base (center of the robot O);
- $r_i$ , denotes the radius of the wheel  $i$  (Distance of the wheel's center to the roller center)
- $r_r$ , denotes the radius of the rollers on the wheels.
- $\alpha_i$ , the angle between  $OP_i$  and  $X_R$  ;

- $\beta_i$ , the angle between  $S_i$  and  $X_R$ ;
- $\gamma_i$ , the angle between  $v_{ir}$  and  $E_i$ ;
- $\omega_i$  [rad/s], wheels angular velocity;
- $v_{i\omega}$  [m/s], ( $i = 0,1,2,3 \in R$ ) is the velocity vector corresponding to wheel revolutions
- $v_{ir}$ , the velocity of the passive roller in the wheel  $i$ ;
- $[w_{Si} \ w_{Ei} \ \omega_i]^T$ , Generalized velocity of point  $P_i$  in the frame  $S_iP_iE_i$ ;
- $[v_{S_i} \ v_{E_i} \ \omega_i]^T$ , Generalized velocity of point  $P_i$  in the frame  $X_R OY_R$ ;
- $v_x, v_y$  [m/s] - Robot linear velocity;
- $\omega_z$  [rad/s] - Robot angular velocity;



**Fig 3: Parameters of ith wheel**

According to Figure 3(b), we can calculate the velocity of the wheel  $i$  and the tangential velocity of the free roller attached to the wheel touching the floor:

$$v_{ir} = \frac{1}{\cos 45} r_i \omega_i, \quad w_{Ei} = r_i \omega_i \quad [4], \quad i = 0,1,2,3. \quad \text{eq. 1}$$

According to Figure 3(b) and considering the equations (eq.1), the velocity of the wheel  $i$  in the frame  $S_iP_iE_i$ , can be derived by:

$$v_{S_i} = v_{ir} \sin \gamma_i.$$

$$v_{E_i} = \omega_i r_i + v_{ir} \cos \gamma_i.$$

$$\begin{bmatrix} v_{S_i} \\ v_{E_i} \end{bmatrix} = \begin{bmatrix} 0 & \sin \gamma_i \\ r_i & \cos \gamma_i \end{bmatrix} \begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix} = {}^{w_i}T_{P_i} \begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix}. \quad \text{eq. 2}$$

The transformation matrix from velocities of the  $i$ th wheel to its center:

$${}^{w_i}T_{P_i} = \begin{bmatrix} 0 & \sin \gamma_i \\ r_i & \cos \gamma_i \end{bmatrix}. \quad \text{eq. 3}$$

According to Figure 3(a) and Figure 2, the velocity of the wheel's center translated to the  $X_R OY_R$  coordinate system can be achieved by equation 7.

$$\begin{bmatrix} v_{ix_R} \\ v_{iy_R} \end{bmatrix} = \begin{bmatrix} \cos \beta_i & -\sin \beta_i \\ \sin \beta_i & \cos \beta_i \end{bmatrix} \begin{bmatrix} v_{S_i} \\ v_{E_i} \end{bmatrix} = {}^{w_i}T_{P_i} {}^{P_i}T_R \begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix}. \quad \text{eq. 4}$$

Then, the transformation matrix from the  $i$ th wheel's center to the robot coordinate's system can be obtained from equation 5.

$${}^{P_i}T_R = \begin{bmatrix} \cos \beta_i & -\sin \beta_i \\ \sin \beta_i & \cos \beta_i \end{bmatrix}. \quad [5] \quad \text{eq. 5}$$

Since the robot's motion is planar, we also have:

$$\begin{bmatrix} v_{ix_R} \\ v_{iy_R} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -l_{iy} \\ 0 & 1 & l_{ix} \end{bmatrix} \begin{bmatrix} v_X \\ v_Y \\ \omega \end{bmatrix} = T' \begin{bmatrix} v_{X_R} \\ v_{Y_R} \\ \omega_R \end{bmatrix}. \quad \text{eq. 6}$$

Where:

$$T' = \begin{bmatrix} 1 & 0 & -l_{iy} \\ 0 & 1 & l_{ix} \end{bmatrix}. \quad \text{eq. 7}$$

From (eq.3) and (eq.5), the inverse kinematic model can be obtained:

$${}^{w_i}T_{P_i} {}^{P_i}T_R \begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix} = T' \begin{bmatrix} v_{X_R} \\ v_{Y_R} \\ \omega_R \end{bmatrix}, \quad i = 0,1,2,3. \quad \text{eq. 8}$$

As  $r_i \neq 0$ ,  $0 < |\gamma_i| < \pi/2$ ,  $\det({}^{P_i}T_R) \neq 0$ ,  $\det({}^{w_i}T_{P_i}) \neq 0$  hence, by merging equations 4 and 6 the robot's base velocity (at point O) related to the rotational velocity of the  $i$ th wheel can be obtained from eq. 9.

$$\begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix} = {}^{w_i}T_{P_i}^{-1} \cdot {}^{P_i}T_R^{-1} \cdot T' \begin{bmatrix} v_{X_R} \\ v_{Y_R} \\ \omega_R \end{bmatrix}, \quad i = 0,1,2,3. \quad \text{eq. 9}$$

According to eq.3 and eq.4 there is a relationship between variables in each robot's wheels frames and its center. And with the inverse kinematic, the velocity of the system can be obtained by implementing  $v_{ir}$  the linear velocity and  $\omega_i$  the rotational speed of wheel  $i$ th in eq.10 and the contrary in eq.11.

$$\begin{bmatrix} v_{X_R} \\ v_{Y_R} \\ \omega_z \end{bmatrix} = T^+ \begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix} \quad \text{eq. 10}$$

$$\begin{bmatrix} \omega_i \\ v_{ir} \end{bmatrix} = T \begin{bmatrix} v_{X_R} \\ v_{Y_R} \\ \omega_R \end{bmatrix} \quad \text{eq. 11}$$

Where  $T = {}^{w_i}T_{P_i}^{-1} \cdot {}^{P_i}T_R^{-1} \cdot T'$ ,  $T^+ = (T^T T)^{-1} T^T$ .

$$T = \begin{bmatrix} \cos \beta_i & -\sin \beta_i \\ \sin \beta_i & \cos \beta_i \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 & \sin \gamma_i \\ r_i & \cos \gamma_i \end{bmatrix}^{-1} \cdot \begin{bmatrix} 1 & 0 & -l_{iy} \\ 0 & 1 & l_{ix} \end{bmatrix}$$

Considering the fact that  $l_{ix} = l_i \cos \alpha_i$  and  $l_{iy} = l_i \sin \alpha_i$ , and assuming that the wheels are in a same size, the transformation matrix is:

$$T := \frac{1}{-r} \begin{bmatrix} \frac{\cos(\beta_i - y_i)}{\sin(y_i)} & \frac{\sin(\beta_i - y_i)}{\sin(y_i)} & \frac{l_i \sin(-\alpha_i + \beta_i - y_i)}{\sin(y_i)} \\ -\frac{r \cos(\beta_i)}{\sin(y_i)} & -\frac{r \sin(\beta_i)}{\sin(y_i)} & -\frac{l_i \sin(-\alpha_i + \beta_i) r}{\sin(y_i)} \end{bmatrix}; \quad \text{eq. 12}$$

$$T^+ = \frac{1}{l_i^2 + 1} \begin{bmatrix} -\frac{1}{2} (l_i^2 \sin(\beta_i) - l_i^2 \sin(-\beta_i + 2\alpha_i) + 2 \sin(\beta_i)) r & \frac{1}{2} l_i^2 \sin(y_i - \beta_i + 2\alpha_i) - \frac{1}{2} \sin(-y_i + \beta_i) l_i^2 - \sin(-y_i + \beta_i) \\ \frac{1}{2} r (l_i^2 \cos(\beta_i) - l_i^2 \cos(-\beta_i + 2\alpha_i) + 2 \cos(\beta_i)) & -\frac{1}{2} l_i^2 \cos(y_i - \beta_i + 2\alpha_i) + \frac{1}{2} \cos(-y_i + \beta_i) l_i^2 + \cos(-y_i + \beta_i) \\ \cos(\alpha_i - \beta_i) l_i r & \cos(\alpha_i - \beta_i + y_i) l_i \end{bmatrix} \quad \text{eq. 13}$$

Since there is a relation between independent variables  $v_{ir}$  and  $\omega_i$  in each joint and the systems angular and linear velocity, assuming that there is no wheel slipping on the ground, the system inverse kinematic can be obtained by eq.14.

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{-1}{r} \begin{bmatrix} \cos(\beta_1 - \gamma_1) & \sin(\beta_1 - \gamma_1) & l_1 \sin(\beta_1 - \gamma_1 - \alpha_1) \\ \sin \gamma_1 & \sin \gamma_1 & \sin \gamma_1 \\ \cos(\beta_2 - \gamma_2) & \sin(\beta_2 - \gamma_2) & l_2 \sin(\beta_2 - \gamma_2 - \alpha_2) \\ \sin \gamma_2 & \sin \gamma_2 & \sin \gamma_2 \\ \cos(\beta_3 - \gamma_3) & \sin(\beta_3 - \gamma_3) & l_3 \sin(\beta_3 - \gamma_3 - \alpha_3) \\ \sin \gamma_3 & \sin \gamma_3 & \sin \gamma_3 \\ \cos(\beta_4 - \gamma_4) & \sin(\beta_4 - \gamma_4) & l_4 \sin(\beta_4 - \gamma_4 - \alpha_4) \\ \sin \gamma_4 & \sin \gamma_4 & \sin \gamma_4 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix}. \quad \text{eq.14}$$

eq.15 shows the Jacobian matrix for the system's inverse kinematic:

$$T = \frac{-1}{r} \begin{bmatrix} \cos(\beta_1 - \gamma_1) & \sin(\beta_1 - \gamma_1) & l_1 \sin(\beta_1 - \gamma_1 - \alpha_1) \\ \sin \gamma_1 & \sin \gamma_1 & \sin \gamma_1 \\ \cos(\beta_2 - \gamma_2) & \sin(\beta_2 - \gamma_2) & l_2 \sin(\beta_2 - \gamma_2 - \alpha_2) \\ \sin \gamma_2 & \sin \gamma_2 & \sin \gamma_2 \\ \cos(\beta_3 - \gamma_3) & \sin(\beta_3 - \gamma_3) & l_3 \sin(\beta_3 - \gamma_3 - \alpha_3) \\ \sin \gamma_3 & \sin \gamma_3 & \sin \gamma_3 \\ \cos(\beta_4 - \gamma_4) & \sin(\beta_4 - \gamma_4) & l_4 \sin(\beta_4 - \gamma_4 - \alpha_4) \\ \sin \gamma_4 & \sin \gamma_4 & \sin \gamma_4 \end{bmatrix}. \quad \text{eq. 15}$$

And for the forward kinematic according to the eq.10, we have:

$$\begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} = T^+ \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} \quad \text{eq. 16}$$

## 2.1 The Relation between Motions and the Translation MATRIX

Analyzing the motion of a four Mecanum wheeled robot brings out the following conclusion: According to the inverse kinematic, there is a relationship between velocities in each joint and the robot's center velocity, thus, the velocity of the robot's center is reflected by and obtained from an individual wheels velocity. According to the robot kinematic, inverse kinematics can be achieved when the rank of the system is less than the rank of the Jacobian matrix for each wheel of the robot that reduces the degree of freedom of the robot's joints. Hence in a four Omni-differential design, the kinematic works with following conditions:

- R Jacobian full column rank, i.e. if rank (R) = 3, the robot performs a better movement.
- The rank of the Jacobian matrix column dissatisfaction, i.e. if the rank (R) < 3, the robot can only move in a singular form and cannot achieve all-directional movement.

## 3. FOUR MECANUM OMNIDIRECTIONAL SOLUTION

Typical Mecanum four system shown in Figure 2; the parameters of this configuration are shown in table 1. In this configuration wheels sizes are the same.

Table 1. Robot Parameters

i	Wheels	$\alpha_i$	$\beta_i$	$\gamma_i$	$l_i$	$l_{ix}$	$l_{iy}$
0	1sw	$\pi/4$	$\pi/2$	$-\pi/4$	l	$l_x$	$l_y$
1	2sw	$-\pi/4$	$-\pi/2$	$\pi/4$	l	$l_x$	$l_y$
2	3sw	$3\pi/4$	$\pi/2$	$\pi/4$	l	$l_x$	$l_y$
3	4sw	$-3\pi/4$	$-\pi/2$	$-\pi/4$	l	$l_x$	$l_y$

By replacing the parameters of Table 1 in matrix (eq. 15) and considering eq.14 we have come up with:

$$T = \frac{1}{r} \begin{bmatrix} 1 & -1 & -(l_x + l_y) \\ 1 & 1 & (l_x + l_y) \\ 1 & 1 & -(l_x + l_y) \\ 1 & -1 & (l_x + l_y) \end{bmatrix}, \quad \text{eq. 17}$$

$$T^+ = \frac{r}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ \frac{1}{(l_x+l_y)} & \frac{1}{(l_x+l_y)} & -\frac{1}{(l_x+l_y)} & \frac{1}{(l_x+l_y)} \end{bmatrix}; \quad \text{eq. 18}$$

According to equations (10) and (11) for *Forward* and *Inverse* kinematics there is:

$$\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} = \frac{1}{r} \begin{bmatrix} 1 & -1 & -(l_x + l_y) \\ 1 & 1 & (l_x + l_y) \\ 1 & 1 & -(l_x + l_y) \\ 1 & -1 & (l_x + l_y) \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix}. \quad \text{eq. 19}$$

$$\begin{cases} \omega_1 = \frac{1}{r}(v_x - v_y - (l_x + l_y)\omega), \\ \omega_2 = \frac{1}{r}(v_x + v_y + (l_x + l_y)\omega), \\ \omega_3 = \frac{1}{r}(v_x + v_y - (l_x + l_y)\omega), \\ \omega_4 = \frac{1}{r}(v_x - v_y + (l_x + l_y)\omega). \end{cases} \quad \text{eq. 20}$$

And

$$\begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} = \frac{r}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & 1 & -1 \\ \frac{1}{(l_x+l_y)} & \frac{1}{(l_x+l_y)} & -\frac{1}{(l_x+l_y)} & \frac{1}{(l_x+l_y)} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ \omega_4 \end{bmatrix} \quad \text{eq. 21}$$

Longitudinal Velocity:

$$v_x(t) = (\omega_1 + \omega_2 + \omega_3 + \omega_4) \cdot \frac{r}{4} \quad \text{eq. 22}$$

Transversal Velocity:

$$v_y(t) = (-\omega_1 + \omega_2 + \omega_3 - \omega_4) \cdot \frac{r}{4} \quad \text{eq. 23}$$

Angular velocity:

$$\omega_z(t) = (-\omega_1 + \omega_2 - \omega_3 + \omega_4) \cdot \frac{r}{4(l_x+l_y)} \quad \text{eq. 24}$$

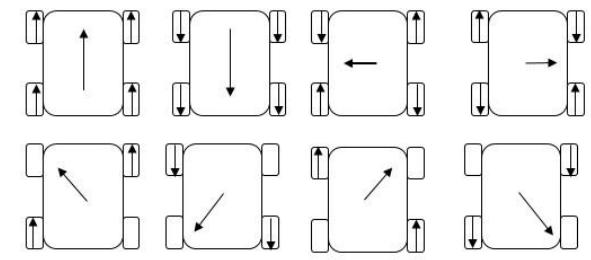
The resultant velocity and its direction in the stationery coordinate axis (x, y, z) can be achieved by the following equations (eq. 25, 26):

$$\rho = \tan^{-1} \left( \frac{v_y}{v_x} \right), \quad \text{eq. 25}$$

$$v_R = \sqrt{v_x^2 + v_y^2}. \quad \text{eq. 26}$$

## 4. EXPERIMENTAL RESULTS

We used four Mecanum wheels in our project. The wheel topology was the same as figure 2. The direction and the velocity of the diagonal wheels were set independently. Using the same speed in each wheel at the same time during the operation led us to get eight directions for the robot's motion without changing its orientation. By changing the velocities of the diagonal wheels we achieved a motion between  $0^\circ$  to  $360^\circ$ . For example, to accomplish a transversal motion to the right, the right wheels were rotated against each other inwardly, while the left wheels were rotated against each other outwardly (See Figure 4). By using the same technique we achieved all eight different motions shown in Figure 4.



**Fig 4: Motions of Omnidirectional platform**

In our test we assumed that there was no wheel sleep and by driving the robot to the left we achieved 0.046 rad per second, which was multiplied by each wheel's velocity. The table (table 2) shows the results of driving the robot in 8 different directions without changing its orientation. In order to achieve the same speed for diagonal directions the velocities of two of the wheels were greater than other cases since the velocity in the two other wheels was 0.

**Table 2. Experimental and Analytical Results**

Direction	$v_x$	$v_y$	$\omega_z$	Wheel1	Wheel2	Wheel3	Wheel4
Forward	5	0	0	4.87	4.87	4.87	4.87
Backward	-5	0	0	-4.87	-4.87	-4.87	-4.87
Left	0	5	0	-4.87	4.87	4.87	-4.87
Right	0	-5	0	4.87	-4.87	-4.87	4.87
Left diagonal forward	5	5	0	0	9.74	9.74	0
Left diagonal	-5	5	0	-9.74	0	0	-9.74

backward							
Right diagonal forward	5	-5	0	9.74	0	0	9.74
Right diagonal backward	-5	-5	0	0	-9.74	-9.74	0

## 5. CONCLUSION

A mobile platform with four omnidirectional wheels was introduced in this paper. The results were systematically obtained by using kinematic equations that were similar to those achieved from the experimental results. The results show that the platform performs full omnidirectional motions. This shows that by using Mecanum wheels in the platform the robot can achieve any direction between  $0^\circ$  to  $360^\circ$  without changing its orientation.

## 6. REFERENCES

- [1] O. Diegel, A. Badve, G. Bright, J. Potgieter, and S. Tlale, “Improved Mecanum Wheel Design for Omni-directional Robots,” no. November, pp. 27–29, 2002.
- [2] I. Doroftei, V. Grosu, and V. Spinu, *Omnidirectional Mobile Robot - Design and Implementation, Bioinspiration and Robotics Walking and Climbing Robots*, no. September. I-Tech, 2007.
- [3] R. P. A. van Haendel, “Design of an omnidirectional universal mobile platform,” National University of Singapore, 2005.
- [4] T. A. Baede, “Motion control of an omnidirectional mobile robot,” 2006.
- [5] X. Li and A. Zell, “Motion control of an omnidirectional mobile robot,” 2006.