# Homework 1

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**1. Please remove noise form Figure 1.**

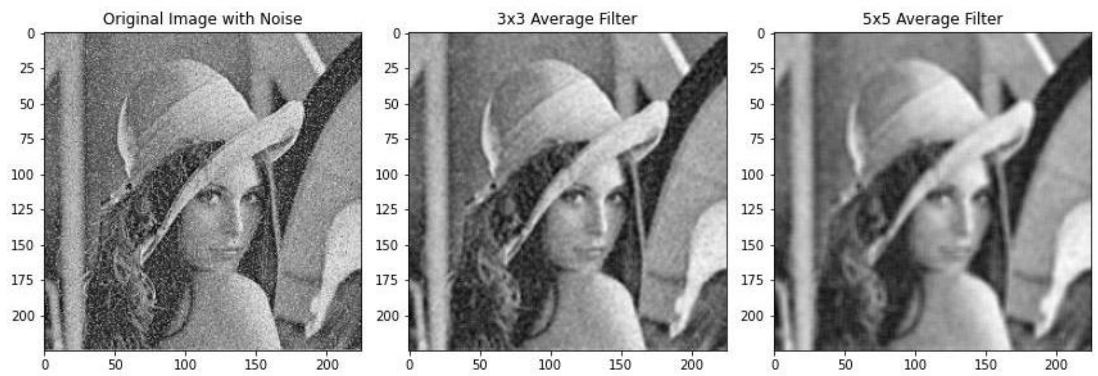
**(a) Use the average filter and the median filter, and compare their results.**

**(b) Use the median filter to remove noise.**

**(c) Please compare the results of (a) and (b) in view of performance.**

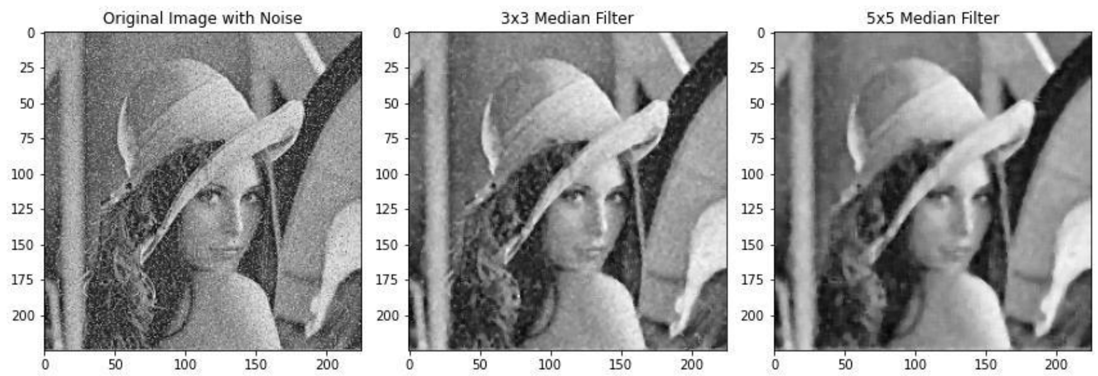
**Answer:**

a) average filter




原圖（左）、3*3 average filter（中）、5*5 average filter（右）

b) median filter




c) 原圖（左）、3*3 median filter（中）、5*5 median filter（右）

c)

在計算方面，average filter 是基於局部平均值的平滑 filter，而 median filter 則是基於局部中值的非線性 filter。

結果的部分，average filter 它對於移除均勻雜訊表現良好，但對於保留邊緣細節效果較差；而 median filter 它在移除椒鹽雜訊方面非常有效，並能很好地保留邊緣細節。

**2. Please sharp the Figure 2.**

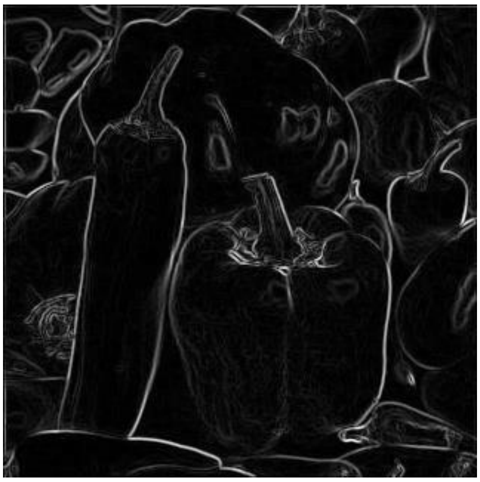
**(a) Use the Sobel mask to sharp this Figure 2.**

**(b) Use the Fourier transform to sharp this Figure 2.**

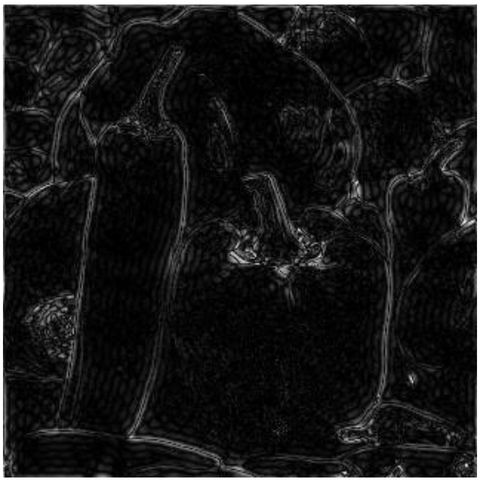
**(c) Compare the results of (a) and (b) in view of performance.**

**Answer:**

a) Sobel mask



b) Fourier transform



c)

根據結果可見，sobel mask 的表現較好，輪廓較清楚且完整，sobel 在平滑區域的處理保持不變，沒有過多的細節改變；相反地，fourier transform 在平滑區域的處理上，高頻增強會對整體圖像的所有區域產生影響包括背景，可能導致背景部分產生額外的紋理，或帶來一些不必要的細節，有些過渡區域出現小波紋效應。根據以上輸出圖片的差異，我認為如果需要突顯邊緣細節且保留背景自然感，會選擇 Sobel。

**3. Please design a Gaussian filter of 3*3 mask and use this mask to low-pass filter of Figure 1.**

**Answer:**

我設計一個 3×3 的 Gaussian mask，其權重來自 2D Gaussian 分佈：

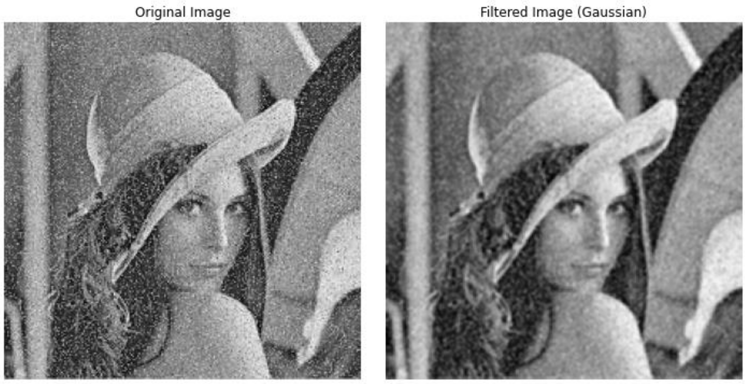
$$G(x,y)=\frac{1}{2\pi\sigma^2}e^{-\frac{x^2+y^2}{2\sigma^2}}$$

取標準差 $\sigma=1$，且中心在 (0,0) 的值，計算如下：

| x,y | -1,-1 | -1,0 | -1,1 | 0,-1 | 0,0 | 0,1 | 1,-1 | 1,0 | 1,1 |
|---|---|---|---|---|---|---|---|---|---|
| $G(x,y)$ | 0.0585 | 0.0965 | 0.0585 | 0.0965 | 0.1592 | 0.0965 | 0.0585 | 0.0965 | 0.0585 |

為簡化並取近似值，我們將其權重標準化為整數，並除以總和 16：

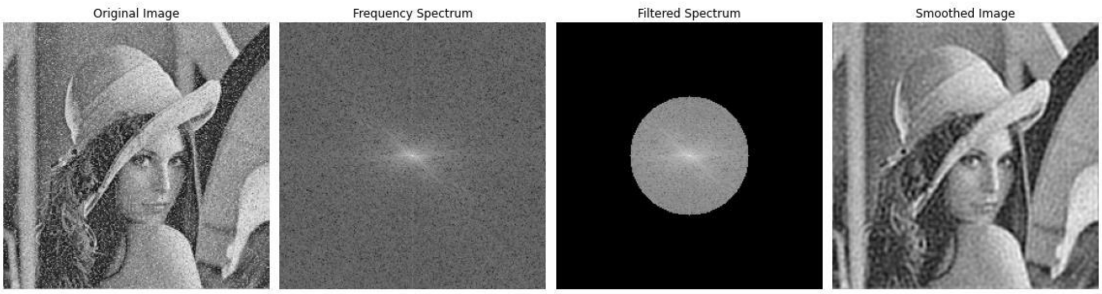
$$Gaussian\ mask=\frac{1}{16}\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix}$$




**4. Please design a lower-pass Fourier filter corresponding designed one by the 3rd to smooth the Figure 1.**

**Answer:**

當進行傅立葉變換後，遮罩大小等於頻域圖像的大小。我設計半徑為 50 的低通濾波器，會以頻譜中心為圓心，畫一個半徑為 50 的圓。根據實驗結果，較大的設計半徑會允許更多頻率通過，影像平滑程度較低，細節保留較多，而較小的設計半徑會允許較少頻率通過，影像平滑程度較高，細節和雜訊都會被大幅濾除。如設計半徑為 30，產生的圖片會更有顆粒感。



| | | |
|---|---|---|
| 1 | 0 | 7 |
| 5 | 1 | 8 |
| 4 | 0 | 9 |

Figure 3. Mask

**5. Please compute the corresponding phase angle and Fourier spectrum of Figure 3.**

**Answer:**

Step 1: 進行傅立葉變換 (Fourier Transform)

$$F(0,0)=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)e^{-j2\pi}=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)(\cos\theta-i\sin\theta)$$

$$=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)=f(0,0)+f(0,1)+f(0,2)+f(1,0)+f(1,1)+f(1,2)+f(2,0)+f(2,1)+f(2,2)$$

$$=1+7+5+1+8+4+9=35.$$

$$f(1,0)=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)\left[e^{-j2\pi\left[\frac{1\cdot x}{3}+\frac{0\cdot y}{3}\right]}\right]=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)e^{-j\cdot\frac{2\pi}{3}x}$$

$$=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)\left[\cos\frac{2\pi}{3}x-i\sin\frac{2\pi}{3}x\right]=f(0,0)\left[\cos0^\circ-i\sin0^\circ\right]+f(0,1)\left[\cos0^\circ+i\sin0^\circ\right]$$

$$+f(0,2)\left[\cos0^\circ-i\sin0^\circ\right]+f(1,0)\left[\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right]+f(1,1)\left[\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right]+$$

$$f(1,2)\left[\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right]+f(2,0)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]+f(2,1)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]+$$

$$f(2,2)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]=1+7+14\times\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)+13\times\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$

$$=8-7-7\sqrt{3}i-\frac{13}{2}+\frac{13\sqrt{3}}{2}i=-5.5-\frac{\sqrt{3}}{2}i$$

$$f(2,0)=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)\left[e^{-j2\pi\left[\frac{2x}{3}+\frac{0y}{3}\right]}\right]=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)e^{-j\cdot\frac{4\pi}{3}x}=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)\left[\cos\frac{4\pi}{3}x-i\sin\frac{4\pi}{3}x\right]$$

$$=f(0,0)\left[\cos0^\circ-i\sin0^\circ\right]+f(0,1)\left[\cos0^\circ-i\sin0^\circ\right]+f(0,2)\left[\cos0^\circ-i\sin0^\circ\right]+$$

$$f(1,0)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]+f(1,1)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]+f(1,2)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]+$$

$$f(2,0)\left[\cos\frac{8\pi}{3}-i\sin\frac{8\pi}{3}\right]+f(2,1)\left[\cos\frac{8\pi}{3}-i\sin\frac{8\pi}{3}\right]+f(2,2)\left[\cos\frac{8\pi}{3}-i\sin\frac{8\pi}{3}\right].$$

$$=1+7+14\times\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)+13\times\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=8-7-\frac{13}{2}+7\sqrt{3}i-\frac{13}{2}\sqrt{3}i=-5.5+\frac{\sqrt{3}}{2}i$$

$$f(0,1)=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)\left[e^{-j2\pi\left[\frac{0\cdot x}{3}+\frac{1\cdot y}{3}\right]}\right]=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)e^{-j\cdot\frac{2\pi}{3}y}$$

$$=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)\left[\cos\frac{2\pi}{3}y-i\sin\frac{2\pi}{3}y\right]=f(0,0)\left[\cos0^\circ-i\sin0^\circ\right]+f(0,1)\left[\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right]$$

$$+f(0,2)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]+f(1,0)\left[\cos0^\circ-i\sin0^\circ\right]+f(1,1)\left[\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right]+$$

$$f(1,2)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]+f(2,0)\left[\cos0-i\sin0\right]+f(2,1)\left[\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right]+$$

$$f(2,2)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]=10+-\frac{1}{2}-\frac{\sqrt{3}}{2}i+24\times\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)=10-\frac{1}{2}-12+12\sqrt{3}i-\frac{\sqrt{3}}{2}i$$

$$=-2.5+\frac{23}{2}\sqrt{3}i$$

$$f(1,1)=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)\left[e^{-j2\pi\left[\frac{1x}{3}+\frac{1y}{3}\right]}\right]=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)e^{-j2\pi\frac{x+y}{3}}=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)\left[\cos\left(2\pi\frac{x+y}{3}\right)-i\sin\left(2\pi\frac{x+y}{3}\right)\right]$$

$$=f(0,0)\left[\cos0^\circ-i\sin0^\circ\right]+f(0,1)\left[\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right]+f(0,2)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]+$$

$$f(1,0)\left[\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right]+f(1,1)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]+f(1,2)\left[\cos2\pi-i\sin2\pi\right]+$$

$$f(2,0)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]+f(2,1)\left[\cos2\pi-i\sin2\pi\right]+f(2,2)\left[\cos\frac{8\pi}{3}-i\sin\frac{8\pi}{3}\right].$$

$$=1+5\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)+12\times\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)+8+9\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)=-4-\sqrt{3}i.$$

$$f(2,1)=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)\left[e^{-j2\pi\left[\frac{2x}{3}+\frac{1\cdot y}{3}\right]}\right]=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)e^{-j2\pi\frac{2x+y}{3}}=\sum_{x=0}^{2}\sum_{y=0}^{2}f(x,y)\left[\cos\left(2\pi\frac{2x+y}{3}\right)-i\sin\left(2\pi\frac{2x+y}{3}\right)\right]$$

$$=f(0,0)\left[\cos0^\circ-i\sin0^\circ\right]+f(0,1)\left[\cos\frac{2\pi}{3}-i\sin\frac{2\pi}{3}\right]+f(0,2)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]+$$

$$f(1,0)\left[\cos\frac{4\pi}{3}-i\sin\frac{4\pi}{3}\right]+f(1,1)\left[\cos2\pi-i\sin2\pi\right]+f(1,2)\left[\cos\frac{8\pi}{3}-i\sin\frac{8\pi}{3}\right]+$$

$$f(2,0)\left[\cos\frac{8\pi}{3}-i\sin\frac{8\pi}{3}\right]+f(2,1)\left[\cos\frac{10\pi}{3}-i\sin\frac{10\pi}{3}\right]+f(2,2)\left[\cos4\pi-i\sin4\pi\right]$$

$$=1+1+9+4\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)+5\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)+8\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right)+7\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$

$$=-1.$$

$$f(0,2) = \sum_{x=0}^{2}\sum_{y=0}^{2} f(x,y)\left[e^{-j2\pi\left[\frac{0x}{3}+\frac{2y}{3}\right]}\right] = \sum_{x=0}^{2}\sum_{y=0}^{2} f(x,y)\left[\cos\frac{4\pi}{3}y - i\sin\frac{4\pi}{3}y\right]$$

$$= f(0,0)[\cos 0^\circ - i\sin 0^\circ] + f(0,1)\left[\cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}\right] + f(0,2)\left[\cos\frac{8\pi}{3} - i\sin\frac{8\pi}{3}\right] +$$

$$f(1,0)[\cos 0^\circ - i\sin 0^\circ] + f(1,1)\left[\cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}\right] + f(1,2)\left[\cos\frac{8\pi}{3} - i\sin\frac{8\pi}{3}\right] +$$

$$f(2,0)[\cos 0^\circ - i\sin 0^\circ] + f(2,1)\left[\cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}\right] + f(2,2)\left[\cos\frac{8\pi}{3} - i\sin\frac{8\pi}{3}\right]$$

$$= 10 + 1\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) + 24\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) = -2.5 - \frac{23}{2}\sqrt{3}i$$

$$f(1,2) = \sum_{x=0}^{2}\sum_{y=0}^{2} f(x,y)\left[e^{-j2\pi\left[\frac{x}{3}+\frac{2y}{3}\right]}\right] = \sum_{x=0}^{2}\sum_{y=0}^{2} f(x,y)e^{-j2\pi\frac{x+2y}{3}} = \sum_{x=0}^{2}\sum_{y=0}^{2} f(x,y)\left[\cos\left(2\pi\frac{x+2y}{3}\right) - i\sin\left(2\pi\frac{x+2y}{3}\right)\right]$$

$$= f(0,0)[\cos 0^\circ - i\sin 0^\circ] + f(0,1)\left[\cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}\right] + f(0,2)\left[\cos\frac{8\pi}{3} - i\sin\frac{8\pi}{3}\right] +$$

$$f(1,0)\left[\cos\frac{2\pi}{3} - i\sin\frac{2\pi}{3}\right] + f(1,1)[\cos 2\pi - i\sin 2\pi] + f(1,2)\left[\cos\frac{10\pi}{3} - i\sin\frac{10\pi}{3}\right] +$$

$$f(2,0)\left[\cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}\right] + f(2,1)\left[\cos\frac{8\pi}{3} - i\sin\frac{8\pi}{3}\right] + f(2,2)[\cos 4\pi - i\sin 4\pi]$$

$$= 1+1+9+5\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) + 4\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) + 7\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) + 8\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$

$$= -1$$

$$f(2,2) = \sum_{x=0}^{2}\sum_{y=0}^{2} f(x,y)\left[e^{-j2\pi\left[\frac{2(x+y)}{3}\right]}\right] = \sum_{x=0}^{2}\sum_{y=0}^{2} f(x,y)e^{-j2\pi\frac{2x+2y}{3}} = \sum_{x=0}^{2}\sum_{y=0}^{2} f(x,y)\left[\cos\left(2\pi\frac{2x+2y}{3}\right) - i\sin\left(2\pi\frac{2x+2y}{3}\right)\right]$$

$$= f(0,0)[\cos 0 - i\sin 0] + f(0,1)\left[\cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}\right] + f(0,2)\left[\cos\frac{8\pi}{3} - i\sin\frac{8\pi}{3}\right] +$$

$$f(1,0)\left[\cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}\right] + f(1,1)\left[\cos\frac{8\pi}{3} - i\sin\frac{8\pi}{3}\right] + f(1,2)[\cos 4\pi - i\sin 4\pi] +$$

$$f(2,0)\left[\cos\frac{8\pi}{3} - i\sin\frac{8\pi}{3}\right] + f(2,1)[\cos 4\pi - i\sin 4\pi] + f(2,2)\left[\cos\frac{16\pi}{3} - i\sin\frac{16\pi}{3}\right]$$

$$= 9 + 5\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right) + 12\left(-\frac{1}{2}-\frac{\sqrt{3}}{2}i\right) + 9\left(-\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$$

$$= -4 + \sqrt{3}i.$$

| | | |
|---|---|---|
| 35 | $-2.5+\frac{23\sqrt{3}}{2}i$ | $-2.5-\frac{23\sqrt{3}}{2}i$ |
| $-5.5-\frac{\sqrt{3}}{2}i$ | $-4-\sqrt{3}i$ | $-1$ |
| $-5.5+\frac{\sqrt{3}}{2}i$ | $-1$ | $-4+\sqrt{3}i$ |

Step 2: 計算傳立葉頻譜（Fourier Spectrum）

$$\text{Fourier Spectrum} = |F(u,v)| = \sqrt{\text{Re}^2 + \text{Im}^2}$$

| | | |
|---|---|---|
| 35 | 20.0748599 | 20.0748599 |
| 5.56776436 | 4.35889894 | 1 |
| 5.56776436 | 1 | 4.35889894 |

Step 3: 計算相位角（Phase Angle）

| | | |
|---|---|---|
| $\arctan(0)$ | $\arctan(\frac{23\sqrt{3}}{-5})$ | $\arctan(\frac{23\sqrt{3}}{5})$ |
| $\arctan(\frac{\sqrt{3}/2}{5.5})$ | $\arctan(\frac{\sqrt{3}}{4})$ | $\arctan(0)$ |
| $\arctan(\frac{\sqrt{3}/2}{-5.5})$ | $\arctan(0)$ | $\arctan(\frac{\sqrt{3}}{-4})$ |

換算成角度（大約/約等於）

| | | |
|---|---|---|
| $0°$ | $-82.85°$ | $82.85°$ |
| $8.95°$ | $23.41°$ | $0°$ |
| $-8.95°$ | $0°$ | $-23.41°$ |

Figure 1

Figure 2