Homework #1

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Question 1:

A. Convert the following numbers to their decimal representation. Show your work.

In order to convert this to our decimal representation we go left to right, treating each space as a power of the base and multiplying it by the value in that spot. Thus for part **A** we have the following:

1. $10011011_2 =$

$$2^0 * 1 = 1$$
$$2^1 * 1 = 2$$
$$2^2 * 0 = 0$$
$$2^3 * 1 = 8$$
$$2^4 * 1 = 16$$
$$2^5 * 0 = 0$$
$$2^6 * 0 = 0$$
$$2^7 * 1 = 128$$

Then sum it up to get:

$$1 + 2 + 8 + 16 + 0 + 0 + 128 = 155_{10}$$

2. $456_7 =$

$$7^0 * 6 = 6$$
$$7^1 * 5 = 35$$
$$7^2 * 4 = 196$$

Then sum it up to get:

$$6 + 35 + 196 = 237_{10}$$

3. $38A_{16} =$

For this problem since it is in base 16, it is Hexadecimal and we know that letters A - F stand for $10 - 15$ respectively.

$$16^0 * A = 16^0 * 10 = 10$$
$$16^1 * 8 = 128$$
$$16^2 * 3 = 768$$

Then sum it up to get:

$$10 + 128 + 768 = 906_{10}$$

4. $2214_5 =$

$$5^0 * 4 = 4$$
$$5^1 * 1 = 5$$
$$5^2 * 2 = 50$$
$$5^3 * 2 = 250$$

Then sum it up to get:

$$4 + 5 + 50 + 250 = 309_{10}$$

B. Convert the following numbers to their binary representation:

For this problem we will be going from either decimal or hexadecimal to binary. The process from going to decimal to binary is essentially dividing by two continuously and checking the remainder each time, and then listing those from right to left, resulting in our number.

1. $69_{10} =$

$$
\begin{array}{l} 69_{10}: \end{array}
\left.\begin{array}{r|c}
2\lfloor\underline{69} & 1 \\
2\lfloor\underline{34} & 0 \\
2\lfloor\underline{17} & 1 \\
2\lfloor\underline{8} & 0 \\
2\lfloor\underline{4} & 0 \\
2\lfloor\underline{2} & 0 \\
2\lfloor\underline{1} & 1
\end{array}\right\} = \mathtt{1000101}_2
$$

2. $485_{10} =$

$$
\begin{array}{l} 485_{10}: \end{array}
\left.\begin{array}{r|c}
2\lfloor\underline{485} & 1 \\
2\lfloor\underline{242} & 0 \\
2\lfloor\underline{121} & 1 \\
2\lfloor\underline{60} & 0 \\
2\lfloor\underline{30} & 0 \\
2\lfloor\underline{15} & 1 \\
2\lfloor\underline{7} & 1 \\
2\lfloor\underline{3} & 1 \\
2\lfloor\underline{1} & 1
\end{array}\right\} = \mathtt{111100101}_2
$$

3. $6D1A_{16} =$

We use the same methodology from part **A** to get the decimal representation of the number, which is 27930_{10}, then we can do:

$$
\begin{array}{l} 27930_{10}: \end{array}
\left.\begin{array}{r|c}
2\lfloor\underline{27930} & 0 \\
2\lfloor\underline{13965} & 1 \\
2\lfloor\underline{6982} & 0 \\
2\lfloor\underline{3491} & 1 \\
2\lfloor\underline{1745} & 1 \\
2\lfloor\underline{872} & 0 \\
2\lfloor\underline{436} & 0 \\
2\lfloor\underline{218} & 0 \\
2\lfloor\underline{109} & 1 \\
2\lfloor\underline{54} & 0 \\
2\lfloor\underline{27} & 1 \\
2\lfloor\underline{13} & 1 \\
2\lfloor\underline{6} & 0 \\
2\lfloor\underline{3} & 1 \\
2\lfloor\underline{1} & 1
\end{array}\right\} = \mathtt{110110100011010}_2
$$

C. Convert the following numbers to their hexadecimal representation:

Can apply same method as part **B** to get the hexadecimal representation.

1. $1101011_2 =$

First we convert it to decimal like before, which gives us $107_{10} =$

$$\begin{array}{r|l} \multicolumn{2}{l}{107_{10}:} \\ 16\underline{\lfloor 107} & 11 \\ 16\underline{\lfloor 6} & 6 \end{array} \left.\vphantom{\begin{array}{l} 1 \\ 1 \end{array}}\right\} = \mathtt{6B}_{16}$$

2. $895_{10} =$

$$\begin{array}{r|l} \multicolumn{2}{l}{895_{10}:} \\ 16\underline{\lfloor 895} & 15 \\ 16\underline{\lfloor 55} & 7 \\ 16\underline{\lfloor 3} & 3 \end{array} \left.\vphantom{\begin{array}{l} 1 \\ 1 \\ 1 \end{array}}\right\} = \mathtt{37F}_{16}$$

Question 2:

Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 =$

For this, and the following problems we simply write it out long-hand and make sure to keep track of the carry digit.

$$\begin{array}{r} {\scriptstyle 11110} \\ 07566_8 \\ +\,4515_8 \\ \hline 14303_8 \end{array}$$

2. $10110011_2 + 1101_2 =$

$$\begin{array}{r} {\scriptstyle 01111110} \\ 10110011_2 \\ +\,1101_2 \\ \hline 11000000_2 \end{array}$$

3. $7A66_{16} + 45C5_{16} =$

$$\begin{array}{r} {\scriptstyle 1100} \\ 7A66_{16} \\ +\,45C5_{16} \\ \hline C02B_{16} \end{array}$$

4. $3022_5 - 2433_5 =$

This is essentially the reverse process of the above and you need to use a borrowed number, however I am not good enough at LaTeX to figure out how to show it, so I did it on paper and typed it up here.

$$\begin{array}{r} 3022_5 \\ -\,2433_5 \\ \hline 34_5 \end{array}$$

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation and show your work.

For this section we can initially ignore the negative sign and simply treat each number as a simple unsigned conversion from decimal to binary, which we handle as earlier.

1. $124_{10} =$

$$\begin{array}{l} 124_{10}: \\ \left.\begin{array}{r|l} 2\underline{\lfloor 124} & 0 \\ 2\underline{\lfloor 62} & 0 \\ 2\underline{\lfloor 31} & 1 \\ 2\underline{\lfloor 15} & 1 \\ 2\underline{\lfloor 7} & 1 \\ 2\underline{\lfloor 3} & 1 \\ 2\underline{\lfloor 1} & 1 \end{array}\right\} = \texttt{1111100}_2 \end{array}$$

Thus we then convert it to an 8-bit by adding a single 0 at the front to indicate that it is an 8-bit signed number. So the final result is $124_{10} = 01111100_2$.

2. $-124_{10} =$

In order to get the correct representation of this number, we take the result from above and inverse all the bits, and then add 1, to get the following: $-124_{10} = 10000011_2 + 1_2 = 10000100_2$.

3. $109_{10} =$

$$\begin{array}{l} 109_{10}: \\ \left.\begin{array}{r|l} 2\underline{\lfloor 109} & 1 \\ 2\underline{\lfloor 54} & 0 \\ 2\underline{\lfloor 27} & 1 \\ 2\underline{\lfloor 13} & 1 \\ 2\underline{\lfloor 6} & 0 \\ 2\underline{\lfloor 3} & 1 \\ 2\underline{\lfloor 1} & 1 \end{array}\right\} = \texttt{1101101}_2 \end{array}$$

Thus we then convert it to an 8-bit by adding a single 0 at the front to indicate that it is an 8-bit signed number. So the final result is $109_{10} = 01101101_2$.

4. $-79_{10} =$

First we must calculate 79_{10}.

$$\begin{array}{l} 79_{10}: \\ \left.\begin{array}{r|l} 2\underline{\lfloor 79} & 1 \\ 2\underline{\lfloor 39} & 1 \\ 2\underline{\lfloor 19} & 1 \\ 2\underline{\lfloor 9} & 1 \\ 2\underline{\lfloor 4} & 0 \\ 2\underline{\lfloor 2} & 0 \\ 2\underline{\lfloor 1} & 1 \end{array}\right\} = \texttt{1001111}_2 \end{array}$$

Thus we then convert it to an 8-bit by adding a single 0 at the front to indicate that it is an 8-bit signed number. So we get $79_{10} = 01001111_2$. Next we inverse the digits and add 1 to get the following final result: $-79_{10} = 10110000_2 + 1_2 = 10110001_2$.

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

The following numbers are all in 8-bit two's complement format. If the highest bit is 0 we don't need to worry about negatives, so we just use the general methodology in order to go from a binary number to a decimal. However if (the highest bit) is 0 the process is to flip all the bits, add 1 and convert to base 10, then report the negation of said number.

1. $00011110_2 =$

This number is not a negative, so we just follow the general process from **Question 1 - A** in order to get: $00011110_2 = 30_{10}$.

2. $11100110_2 =$

This number is a negative 11100110_2 so we flip the bits and get: 00011001_2 then we add one to get: 00011010_2. Lastly we convert this to base 10 and negate it to get: $26_{10} \rightarrow -26_{10}$.

3. $00101101_2 =$

This number is not a negative, so we just follow the general process from **Question 1 - A** in order to get: $00101101_2 = 45_{10}$.

4. $10011110_2 =$

This number is a negative 10011110_2 so we flip the bits and get: 01100001_2 then we add one to get: 01100010_2. Lastly we convert this to base 10 and negate it to get: $98_{10} \rightarrow -98_{10}$.

Question 4.

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4:

Section B:

p	q	$p \lor q$	$\neg(p \lor q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Section C:

p	q	r	$\neg q$	$p \land \neg q$	$r \lor (p \land \neg q)$
F	F	F	T	F	F
F	F	T	T	F	T
F	T	F	F	F	F
F	T	T	F	F	T
T	F	F	T	T	T
T	F	T	T	T	T
T	T	F	F	F	F
T	T	T	F	F	T

2. Exercise 1.3.4:

Section B:

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
F	F	T	T	T
F	T	T	F	T
T	F	F	T	F
T	T	T	T	T

Section D:

p	q	$p \leftrightarrow q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
F	F	T	F	T
F	T	F	T	T
T	F	F	T	T

Question 5.

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.7:

We are given the following pieces of identification for applying for a credit card:
B: Applicant presents a birth certificate.
D: Applicant presents a driver's license.
M: Applicant presents a marriage license.

Section B:

The applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license.
So we need to make sure we can treat it as having one of the items in conjunction with either of the other two, which when written out is same as:

$$((B \land (D \lor M)) \lor (D \land (B \lor M)) \lor (M \land (D \lor B))$$

and simplify to get:

$$(B \land (D \lor M)) \lor (D \land M)$$

Section C:

Applicant must present either a birth certificate or both a driver's license and a marriage license.
This one is slightly simpler than the last, since we only have two cases to consider, writing it out I got it as:

$$B \lor (D \land M)$$

2. Exercise 1.3.7:

We are given the following propositions:
s: a person is a senior
y: a person is at least 17 years of age
p: a person is allowed to park in the school parking lot

Section B:

A person can park in the school parking lot if they are a senior or at least seventeen years of age.

$$(s \lor y) \to p$$

Section C:

Being 17 years of age is a necessary condition for being able to park in the school parking lot.

$$p \to y$$

Section D:

A person can park in the school parking lot if and only if the person is a senior and at least 17 years of age.

$$(s \land y) \leftrightarrow p$$

Section E:

Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old.

$$p \to (s \lor y)$$

3. Exercise 1.3.9:

Use the definitions of the variables below to translate each English statement into an equivalent logical expression.
y: the applicant is at least eighteen years old
p: the applicant has parental permission
c: the applicant can enroll in the course

Section C:

The applicant can enroll in the course only if the applicant has parental permission.

$$c \to p$$

Section D:

Having parental permission is a necessary condition for enrolling in the course.

$$c \to p$$

Question 6.

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.3.6:

Give an English sentence in the form "If...then...." that is equivalent to each sentence. Question statements are in italics.

Section B:

Maintaining a B average is necessary for Joe to be eligible for the honors program.
If Joe is eligible for the honors program then he has a B average.

Section C:

Rajiv can go on the roller coaster only if he is at least four feet tall.
If Rajiv can go on the roller coaster then he is at least 4 feet tall.

Section D:

Rajiv can go on the roller coaster if he is at least four feet tall.
If Rajiv is at least four feet tall then he can go on the roller coaster.

2. Exercise 1.3.10:

The variable p is true, q is false, and the truth value for variable r is unknown. Indicate whether the truth value of each logical expression is true, false, or unknown.

Section C:

$$(p \vee r) \leftrightarrow (q \wedge r)$$

False. This always results in the same expression regardless of value of r.

Section D:

$$(p \wedge r) \leftrightarrow (q \wedge r)$$

Unknown. If r is false the expression is true, else if r is true then the expression is false.

Section E:

$$p \rightarrow (r \vee q)$$

Unknown. If r is false the expression is false, else if r is true then the expression is true.

Section F:

$$(p \wedge q) \rightarrow r$$

True. Regardless of the value of r the resulting expression is always true.

Question 7.

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.4.5:

Define the following propositions:
j: Sally got the job.
l: Sally was late for her interview
r: Sally updated her resume.
Express each pair of sentences using a logical expression. Then prove whether the two expressions are logically equivalent.

Section B:

If Sally did not get the job, then she was late for interview or did not update her resume.
If Sally updated her resume and was not late for her interview, then she got the job.

$$\neg j \rightarrow (l \vee \neg r)$$
$$(r \wedge \neg l) \rightarrow j$$

j	l	r	$\neg j \rightarrow (l \vee \neg r)$	$(r \neg l) \rightarrow j$
F	F	F	T	T
F	F	T	F	F
F	T	F	T	T
F	T	T	T	T
T	F	F	T	T
T	F	T	T	T
T	T	F	T	T
T	T	T	T	T

These **are** logically equivalent.

Section C:

If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.

$$j \rightarrow \neg l$$
$$\neg j \rightarrow l$$

j	l	$j \rightarrow \neg l$	$\neg j \rightarrow l$
F	F	T	F
F	T	T	T
T	F	T	T
T	T	F	T

These are **not** logically equivalent.

Section D:

If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

$$(r \vee \neg l) - > j$$
$$j \rightarrow (r \wedge \neg l)$$

j	l	r	$(r \vee \neg l) - > j$	$j \rightarrow (r \wedge \neg l)$
F	F	F	F	T
F	F	T	F	T
F	T	F	T	T
F	T	T	F	T
T	F	F	T	F
T	F	T	T	T
T	T	F	T	F
T	T	T	T	F

These are **not** logically equivalent.

Question 8.

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.5.2:

Use the laws of propositional logic to prove the following:

Section C:

$$(p \to q) \land (p \to r) \equiv p \to (q \land r)$$

Steps	*Laws Used*
$(p \to q) \land (p \to r)$	
$(\neg p \lor q) \land (p \to r)$	*Conditional*
$(\neg p \lor q) \land (\neg p \lor r)$	*Conditional*
$\neg p \lor (q \land r)$	*Distributive*
$p \to (q \land r)$	*Conditional*

Section F:

$$\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$$

Steps	*Laws Used*
$\neg(p \lor (\neg p \land q))$	
$\neg p \land \neg(\neg p \land q)$	*De Morgan*
$\neg p \land (\neg\neg p \lor \neg q)$	*De Morgan*
$\neg p \land (p \lor \neg q)$	*Double Negation*
$(\neg p \land p) \lor (\neg p \land \neg q)$	*Distributive*
$F \lor (\neg p \land \neg q)$	*Complement*
$\neg p \land \neg q$	*Identity*

Section I:

$$(p \land q) \to r \equiv (p \land \neg r) \to \neg q$$

Steps	*Laws Used*
$(p \land q) \to r$	
$\neg(p \land q) \lor r$	*Conditional*
$(\neg p \lor \neg q) \lor r$	*De Morgan*
$(\neg p \lor r) \lor \neg q$	*Associative*
$(\neg p \lor \neg\neg r) \lor \neg q$	*Double Negation*
$\neg(p \land \neg r) \lor \neg q$	*De Morgan*
$(p \land \neg r) \to \neg q$	*Conditional*

2. Exercise 1.5.3:

Section C:

$$\neg r \vee (\neg r \to p) \equiv T$$

Steps	*Laws Used*
$\neg r \vee (\neg r \to p)$	
$\neg r \vee (\neg\neg r \vee p)$	*Conditional*
$\neg r \vee (r \vee p)$	*Double Negation*
$(\neg r \vee r) \vee p$	*Associative*
$(r \vee \neg r) \vee p$	*Commutative*
$T \vee p$	*Complement*
$p \vee T$	*Commutative*
T	*Domination*

Section D:

$$\neg(p \to q) \to \neg q \equiv T$$

Steps	*Laws Used*
$\neg(p \to q) \to \neg q$	
$\neg(\neg p \vee q) \to \neg q$	*Conditional*
$(\neg\neg p \wedge \neg q) \to \neg q$	*De Morgan*
$(p \wedge \neg q) \to \neg q$	*Double Negation*
$\neg(p \wedge \neg q) \vee \neg q$	*Conditional*
$(\neg p \vee \neg\neg q) \vee \neg q$	*De Morgan*
$(\neg p \vee q) \vee \neg q$	*Double Negation*
$\neg p \vee (q \vee \neg q)$	*Associative*
$\neg p \vee T$	*Complement*
T	*Domination*

Question 9.

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.6.3:

Consider the following statements in English. Write a logical expression with the same meaning. The domain of discourse is the set of all real numbers. Problem statement in italics.

Section C:

There is a number that is equal to its square.

$$\exists_x (x = x^2)$$

Section D:

Every number is less than or equal to its square.

$$\forall_x (x \leq x^2)$$

2. Exercise 1.7.4:

In the following question, the domain of discourse is a set of employees who work at a company. Ingrid is one of the employees at the company. Define the following predicates:
S(x): x was sick yesterday
W(x): x went to work yesterday
V(x): x was on vacation yesterday
Translate the following English statements into a logical expression with the same meaning. Problem statements are in italics.

Section B:

Everyone was well and went to work yesterday.

$$\forall_x (\neg S(x) \wedge W(x))$$

Section C:

Everyone who was sick yesterday did not go to work.

$$\forall_x (S(x) \rightarrow \neg W(x))$$

Section D:

Yesterday someone was sick and went to work.

$$\exists_x (S(x) \wedge W(x))$$

Question 10.

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.7.9:

The domain for this question is the set a, b, c, d, e. The following table gives the value of predicates P, Q, and R for each element in the domain. For example, Q(c) = T because the truth value in the row labeled c and the column Q is T. Using these values, determine whether each quantified expression evaluates to true or false. *The truth table is on Zybooks.*

Section C:

$$\exists_x((x = c) \rightarrow P(x))$$

This evaluates to *true*.

Section D:

$$\exists_x(Q(x) \wedge R(x))$$

This evaluates to *true*.

Section E:

$$Q(a) \wedge P(d)$$

This evaluates to *true*.

Section F:

$$\forall_x((x \neq b) \rightarrow Q(x))$$

This evaluates to *true*.

Section G:

$$\forall_x(P(x) \vee (R(x))$$

This evaluates to *false*. Counter-example is $x = c$.

Section H:

$$\forall_x(R(x) \rightarrow P(x))$$

This evaluates to *true*.

Section I:

$$\exists_x(Q(x) \vee R(x))$$

This evaluates to *true*.

2. Exercise 1.9.2:

The tables below show the values of predicates P(x, y), Q(x, y), and S(x, y) for every possible combination of values of the variables x and y. The row number indicates the value for x and the column number indicates the value for y. The domain for x and y is 1, 2, 3. Indicate whether each of the quantified statements is true or false. *The truth tables are on Zybooks.*

Section B:

$$\exists_x \forall_y Q(x,y)$$

This evaluates to *true*. If $x = 2$ then $Q(x,1), Q(x,2), Q(x,3)$ are all true.

Section C:

$$\exists_x \forall_y P(y,x)$$

This evaluates to *true*, because for every y there exists some x which makes it true.

Section D:

$$\exists_x \exists_y S(x,y)$$

This evaluates to *false*. There is no combination of x and y which evaluates to true.

Section E:

$$\forall_x \exists_y Q(x,y)$$

This evaluates to *false*. There is no such y that $Q(1,y), Q(2,y), Q(3,y)$ are all true.

Section F:

$$\forall_x \exists_y P(x,y)$$

This evaluates to *true*. If $y = 1$ then $P(1,y), P(2,y), P(3,y)$ are all true.

Section G:

$$\forall_x \forall_y P(x,y)$$

This evaluates to *false*. Counter-example is $x = 2, y = 2$.

Section H:

$$\exists_x \exists_y Q(x,y)$$

This evaluates to *true*. There exists some x and y that $Q(x,y)$ is true. For example $x = 2$, $y = 1$.

Section I:

$$\forall_x \forall_y \neg S(x,y)$$

This evaluates to *true*. All combinations of $S(x,y)$ are false, hence $\neg S(x,y)$ is always true for any value of x and y.

Question 11.

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.10.4:

Translate each of the following English statements into logical expressions. The domain of discourse is the set of all real numbers. Problem statements in italics.

Section C:

There are two numbers whose sum is equal to their product.

$$\exists_x \exists_y (x + y = x * y)$$

Section D:

The ratio of every two positive numbers is also positive.

$$\forall_x \forall_y (((x > 0) \land (y > 0)) \rightarrow (x/y > 0))$$

Section E:

The reciprocal of every positive number less than one is greater than one.

$$\forall_x (((x > 0) \land (x < 1)) \rightarrow ((1/x) > 1))$$

Section F:

There is no smallest number.

$$\neg \exists_x \forall_y (x \leq y)$$

Section G:

Every number besides 0 has a multiplicative inverse.

$$\forall_x \exists_y ((x \neq 0) \rightarrow (xy = 1))$$

2. Exercise 1.10.7:

The domain of discourse is a group working on a project at a company. One of the members of the group is named Sam. Define the following predicates.
P(x, y): x knows y's phone number. (A person may or may not know their own phone number.)
D(x): x missed the deadline.
N(x): x is a new employee.
Give a logical expression for each of the following sentences. Problem statements are in italics.

Section C:

There is at least one new employee who missed the deadline.

$$\exists_x (N(x) \land D(x))$$

Section D:

Sam knows the phone number of everyone who missed the deadline.

$$\exists_x \forall_y (D(y) \to P(x = Sam, y))$$

Section E:

There is a new employee who knows everyone's phone number.

$$\exists_x \forall_y (N(x) \land P(x, y))$$

Section F:

Exactly one new employee missed the deadline.

$$\exists_x \forall_y (((D(x) \land N(x)) \land ((y \neq x) \to \neg(D(x) \land N(x)))))$$

3. Exercise 1.10.10:

The domain for the first input variable to predicate T is a set of students at a university. The domain for the second input variable to predicate T is the set of Math classes offered at that university. The predicate T(x, y) indicates that student x has taken class y. Sam is a student at the university and Math 101 is one of the courses offered at the university. Give a logical expression for each sentence.

Section C:

Every student has taken at least one class besides Math 101.

$$\forall_x \exists_y (y \neq Math\ 101) \land T(x, y)$$

Section D:

There is a student who has taken every math class besides Math 101.

$$\exists_x \forall_y (y \neq Math\ 101) \to T(x, y)$$

Section E:

Everyone besides Sam has taken at least two different math classes.

$$\forall_y \exists_x \exists_z T(x, y) \land ((y \neq Sam \land (z \neq y))T(x, z))$$

Section F:

Sam has taken exactly two math classes.

$$\exists_x \exists_y \forall_z (x \neq y \land T(Sam, y) \land T(Sam, z) \land (w \neq y \land w \neq z)) \to \neg T(Sam, w)$$

Question 12.

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.8.2:

In the following question, the domain of discourse is a set of male patients in a clinical study. Define the following predicates:
P(x): x was given the placebo
D(x): x was given the medication
M(x): x had migraines
Translate each statement into a logical expression. Then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's law until each negation operation applies directly to a predicate and then translate the logical expression back into English. Problem statement in italics.

Section B:

Every patient was given the medication or the placebo or both.
Statement: $\forall_x(D(x) \lor (P(x))$
Negation: $\neg\forall_x(D(x) \lor (P(x))$
Applying De Morgan's Law: $\exists_x(\neg D(x) \land \neg(P(x))$
English statement: There is a patient that was not given the medication and not given the placebo.

Section C:

There is a patient who took the medication and had migraines.
Statement: $\exists_x(D(x) \land (M(x))$
Negation: $\neg\exists_x(D(x) \land (M(x))$
Applying De Morgan's Law: $\forall_x(\neg D(x) \lor \neg(M(x))$
English statement: Every patient was either not given the medication or did not have migraines or both.

Section D:

Every patient who took the placebo had migraines.
Statement: $\forall_x(P(x) \to (M(x))$
Negation: $\neg\forall_x(P(x) \to (M(x))$
Applying De Morgan's Law: $\exists_x(P(x) \land \neg M(x))$. (Conditional Identity was used before De Morgan.)
English statement: There was a patient that was given the placebo and did not have migraines.

Section E:

There is a patient who had migraines and was given the placebo.
Statement: $\exists_x(M(x) \land P(x))$
Negation: $\neg\exists_x(M(x) \land P(x))$
Applying De Morgan's Law: $\forall_x(\neg M(x) \lor \neg P(x))$
English statement: Every patient either did not have migraines or was not given the placebo or both.

2. Exercise 1.9.4:

Write the negation of each of the following logical expressions so that all negations immediately precede predicates. In some cases, it may be necessary to apply one or more laws of propositional logic.
I have included only my final answers.

Section C:

$$\forall_x \exists_y (P(x, y) \wedge \neg Q(x, y))$$

Section D:

$$\forall_x \exists_y (((P(x, y) \wedge \neg P(y, x) \vee (P(y, x) \wedge \neg P(x, y)))$$

Section E:

$$\forall_x \forall_y \neg P(x, y) \wedge \exists_x \exists_y \neg Q(x, y)$$