# Homework #6

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February 19, 2021

## Q5.

### Part A: $5n^3 + 2n^2 + 3n = \Theta(n^3)$

So we need to prove that $f = O(g)$ and $f = \Omega(g)$ in order to prove that $f = \Theta(g)$.
Proof of $f = O(g)$:

$$f(n) = 5n^3 + 2n^2 + 3n$$
$$g(n) = n^3$$

Select $c = 10$ and $n_0 = 1$. We will show that for any $n \geq 1$, $f(n) \leq c * g(n)$.
For $n \geq 1$, We know that $n \leq n^2 \leq n^3$:
$$f(n) = 5n^3 + 2n^2 + 3n \leq f(n) = 5n^3 + 2n^3 + 3n^3$$
$$f(n) = 5n^3 + 2n^3 + 3n^3 \leq f(n) = 10n^3 = 10 * g(n)$$
Then if we merge the inequalities we get that:
$$f(n) = 5n^3 + 2n^2 + 3n \leq 10n^3 = 10 * g(n)$$
Therefore we have proof that: $f \leq 10 * g(n)$ and as such $f = O(g)$.

Proof of $f = \Omega(g)$:

$$f(n) = 5n^3 + 2n^2 + 3n$$
$$g(n) = n^3$$

Select $c = 5$ and $n_0 = 1$. We will show that for any $n \geq 1$, $f(n) \geq c * g(n)$.
For $n \geq 1$, We know that $n \geq 1, 2n^2 + 3n \geq 0$:
If we add $5n^3$ to both sides we get that:
$$5n^3 + 2n^2 + 3n \geq 5n^3 = 5n^3 + 2n^2 + 3n \geq 5 * g(n).$$
Therefore we have proof that: $f \geq 5 * g(n)$ and as such $f = \Omega(g)$.

Since we proved both $f = O(g)$ and $f = \Omega(g)$, we have proved that $f = \Theta(g)$.

### Part B: $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

So we need to prove that $f = O(g)$ and $f = \Omega(g)$ in order to prove that $f = \Theta(g)$.
Proof of $f = O(g)$:

$$f(n)\sqrt{7n^2 + 2n - 8}$$

Select $c = 10$ and $n_0 = 1$. We will show that for any $n \geq 1$, $f(n) \leq c * g(n)$.
If we have $f(n)$ as above then we know that the following is true:
$$7n^2 + 2n - 8 \leq 7n^2 + 2n^2 = 7n^2 + 2n - 8 \leq 9n^2$$
Since square root is an increasing function, we can take square root of both sides, in order to get our $f(n)$
We can then take our $c * g(n)$ as $\sqrt{9n^2} = 3n$.
Since we already showed that $7n^2 + 2n - 8 \leq 9n^2$ it follows that $\sqrt{7n^2 + 2n - 8} \leq 3n$.
Which means that $f(n) \leq 3 * n$.
Therefore we have proof of $f = O(g)$.

Proof of $f = \Omega(g)$:

$$f(n) = \sqrt{7n^2 + 2n - 8}$$
$$g(n) = \sqrt{7n^2}$$

Select $c = 2$ and $n_0 = 4$. We will show that for any $n \geq 4$, $f(n) \geq c * g(n)$.
We know that since $f(n)$ has to be greater than $g(n)$ then we can take out terms in order to guarantee that.
So if $f(n) = \sqrt{7n^2 + 2n - 8}$ then we know that $7n^2 \geq 7n^2$ and $2n \geq 2n$ and $0 \geq -8$.
Using that we can create $g(n) = \sqrt{7n^2}$.
Since $2n - 8 \geq 0$ therefore $n \geq 4$.
Since we want the bound to be less than $\sqrt{7}$ so we round down to $c = 2$.
Therefore we have proof that $f = \Omega(g)$.

Since we proved both $f = O(g)$ and $f = \Omega(g)$, we have proved that $f = \Theta(g)$.