Homework #2

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Question 5:

Solve the following questions from the Discrete Math zyBook.

Part A.

Exercise 1.12.2:

Section B:

1.	$\neg q$	*Hypothesis*
2.	$\neg q \lor \neg r$	*Addition* (1)
3.	$\neg(q \land r)$	*De Morgan* (2)
4.	$p \to (q \land r)$	*Hypothesis*
5.	$\neg p$	*Modus Tollens* (3, 4)

Section E:

1.	$p \lor q$	*Hypothesis*
2.	$\neg q$	*Hypothesis*
3.	p	*Disjunctive Syllogism* (1, 2)
4.	$\neg p \lor r$	*Hypothesis*
5.	r	*Disjunctive Syllogism* (3, 4)

Exercise 1.12.3:

Section C:

1.	$p \lor q$	*Hypothesis*
2.	$\neg p$	*Hypothesis*
3.	$\neg p \land (p \lor q)$	*Conjunction* (1, 2)
4.	$(\neg p \land p) \lor (\neg p \land q)$	*Distributive* (3)
5.	$F \lor (\neg p \land q)$	*Complement* (4)
6.	$\neg p \land q$	*Identity* (5)
7.	q	*Simplification* (6)

Exercise 1.12.5:

For this we can make three statements, as in the examples from sections (a) and (b).
j: I will get a job
c: I will buy a new car
h: I will buy a new house

Section C:

The form of the argument is:

$$\begin{array}{c} (c \wedge h) \rightarrow j \\ \neg j \\ \hline \therefore \neg c \end{array}$$

This argument is invalid, if we do the truth table we can see that the fourth line invalidates it:

c	h	j	$(c \wedge h) \rightarrow j$	$\neg j$	$\neg c$
T	T	T	F	F	F
T	T	F	T	T	F
T	F	T	T	F	F
T	F	F	T	T	F
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	F	T
F	F	F	T	T	T

Section D:

The form of the argument is:

$$\begin{array}{c} (c \wedge h) \rightarrow j \\ \neg j \\ h \\ \hline \therefore \neg c \end{array}$$

This argument is valid because of $c = j = F$ and $h = T$, all three hypothesis are true and the conclusion is true.

Part B.

Exercise 1.13.3:

Section B:

The form of the argument is:

$$\begin{array}{c} \exists_x (P(x) \vee Q(x)) \\ \exists_x \neg Q(x) \\ \hline \therefore \exists_x P(x) \end{array}$$

We can make a truth table for this and check:

$P(x)$	$Q(x)$
F	F
F	T

We can see that the if $P(x)$ is false, and $Q(x)$ is true, then $\exists_x (P(x) \vee Q(x))$ is true and we also see if both $P(x)$ and $Q(x)$ are both false, then $\exists_x \neg Q(x)$ is true. However the statement $\exists_x P(x)$ is false in both, and therefore this argument is invalid.

Exercise 1.13.5:

The domain for each problem is the set of students in a class.

Section D:

The two equations for this:
$M(x)$: x missed class.
$D(x)$: x got a detention.

We can set up equations for this as:

$$\begin{array}{c} \forall_x (M(x) \to D(x)) \\ \neg M(Penelope) \\ \hline \therefore \neg D(Penelope) \end{array}$$

This argument is invalid when $M(Penelope) = T$ and $D(Penelope) = F$, as the first hypothesis is thus false, but the conclusion is true due to universal instantiation.

Section E:

We can set up equations for this as, we also define $A(x)$ in addition to above to indicate if x got an A.

$$\begin{array}{c} \forall_x (M(x) \vee D(x)) \to \neg A \\ Penelope\ is\ a\ student. \\ A(p) \\ \hline \therefore \neg D(Penelope) \end{array}$$

This is valid and we can use the rules of inference to prove it:

1.	$\forall_x (M(x) \vee D(x)) \to \neg A$	*Hypothesis*
2.	*Penelope is a student.*	*Hypothesis*
3.	$M(p) \vee D(p) \to \not A(p)$	*Universal Instillation* $(1, 2)$
4.	$(\neg((M(p) \vee D(p))) \vee \not A(p)$	*Conditional Identity* (3)
5.	$(\neg(M(p) \wedge \neg(D(p)) \vee \neg A(p)$	*De Morgan* (4)
6.	$\neg A(p) \vee (\neg M(p) \wedge \neg D(P))$	*Commutative Law* (5)
7.	$A(p) \to (\neg M(p) \wedge \neg D(p))$	*Conditional Identity* (6)
8.	$A(p)$	*Hypothesis*
9.	$\neg M(p) \wedge \neg D(p)$	*Modus Ponens* $(7, 8)$
10.	$\neg D(p) \wedge \neg M(p)$	*Commutative* (9)
11.	$\neg D(p)$	*Simplification* (10)

Question 6:

Solve the following questions from the Discrete Math zyBook.

Exercise 2.2.1:

C.

If x is a real number and $x \leq 3$, then $12 - 7x + x^2 \geq 0$.

$$\begin{array}{cc} x \leq 3 & Hypothesis \\ x - x \leq 3 - x = 0 \leq 3 - x & Subtract\ x \\ 0 \leq 3 - x \text{ and } 0 \leq 4 - x & If\ left\ one\ is\ true,\ so\ is\ right. \\ (3-x)(4-x) \geq 0 & Expand\ this\ out\ to\ get\ the\ following. \\ 12 - 7x + x^2 \geq 0 & \end{array}$$

D.

The product of two odd integers is an odd integer.
Assume that both x and y are both odd integers. Because x and y are both odd, then having two integers a and b then $x = 2a + 1$ and $y = 2b + 1$. Then we can see that:

$$xy = (2a + 1)(2b + 1)$$
$$xy = 2(2ab + a + b) + 1$$

Because any number divided by 2 is even, then $2(2ab + a + b)$ is even. So adding 1 to this must make xy odd.

Question 7:

Solve the following questions from the Discrete Math zyBook.

Exercise 2.3.1:

D.

For every integer n, if $n^2 - 2n + 7$ is even, then n is odd.
Assuming that n is even, we can represent every even n as $2k$, where k is any arbitrary integer. Then it follows that:

$$n^2 - 2n + 7 = (2k)^2 - 2(2k) + 7$$
$$n^2 - 2n + 7 = 2(2k^2 - 2k) + 7$$

Because k is an integer then $2(2k^2 - 2k)$ is an even integer. Therefore adding 7 to that makes it an odd integer. Hence it proves the theorem, when n is even, the equation is odd, so the contrapositive must be true, in that the equation is odd when the n is odd.

F.

For every non-zero real number x, if x is irrational, then $\frac{1}{x}$ is also irrational.
Assume that $\frac{1}{x}$ is rational and x is nonzero. There must then be integers a and b where $\frac{1}{x} = \frac{a}{b}$. Then cross multiplying we have:

$$b = ax$$
$$x = \frac{b}{a}$$

x is in the form of a rational number. Therefore the theorem is true due to the contrapositive, since x is irrational, the reciprocal is also irrational.

G.

For every pair of real numbers x and y, if $x^3 + xy^2 \leq x^2y + y^3$, then $x \leq y$.
We assume that $x > y$, therefore $x^3 > x^2y$ and $xy^2 > y^3$. If we add these two statements up, we obtain that $x^3 + xy^2 > x^2y + y^3$. Which proves the theorem by being the contrapositive.

L.

For every pair of real numbers x and y, if $x + y > 20$, then $x > 10$ or $y > 10$.
Assume that when $x \leq 10$ and $y \leq 10$ then $x + y \leq 20$. Then we assume that $x = y = 10$. Thus it follows that:

$$x + y \leq 20$$
$$10 + 10 \leq 20$$
$$20 \leq 20$$

Which ends up being true, therefore the theorem is true due to the contrapositive.

Question 8:

Solve the following questions from the Discrete Math zyBook.

Exercise 2.4.1:

C. *The average of three real numbers is greater than or equal to at least one of the numbers.*

Proving this by contradiction, we can assume that there are three real numbers, x, y and z such that the average of the them is less than each of the 3 numbers. So we have: $(x+y+z)/3 < x$, $(x+y+z)/3 < y$, $(x+y+z)/3 < z$. Adding up the three equations together, we have: $(x+y+z) < x+y+z$, which cannot be true, since a number cannot be less than itself, therefore the theorem must be true.

E. *There is no smallest integer.*

Proving this by contradiction, let us assume there is a smallest integer and we will call it n. If n is the smallest integer, then nothing can be smaller/lesser than it. If we subtract 1 from an integer, we also get an integer. Then we get that there must exist an integer $n-1$ and we know that $n-1 < n$, therefore we know that there is no integer n smaller than itself less one. Therefore the theorem must be true.

Question 9:

Solve the following questions from the Discrete Math zyBook.

Exercise 2.5.1:

Section C:

If integers x and y have the same parity, then x + y is even. The parity of a number tells whether the number is odd or even. If x and y have the same parity, they are either both even or both odd.

Case 1: Even Integers

If we assume both x and y are both even, then integers a and b exist such that $x = 2a$ and $y = 2b$. Therefore:

$$x + y = 2a + 2b$$
$$x + y = 2(a + b)$$

Because all integers when multiplied by 2 are even, then $x + y$ is also even, since that is equal to $2(a + b)$.

Case 2: Odd Integers

If we assume both x and y are both odd, then integers a and b exist such that $x = 2a + 1$ and $y = 2b + 1$. Therefore:

$$x + y = 2a + 2b + 1 + 1$$
$$x + y = 2a + 2b + 2$$
$$x + y = 2(a + b + 1)$$

Because all integers when multiplied by 2 are even, then $x + y$ is also even, since that is equal to $2(a + b + 1)$.