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Guest Editorial

Discrete Math First!

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Introduction

Recently, this *Journal* published several intriguing editorials on calculus and modeling. James Cargal established the bounds of modeling as a science and the limitations of mathematical problem solving [2007]. I certainly agree with his points on the art and science of modeling. Paul Campbell [2006] and Underwood Dudley [2008] debated the viability of the calculus course. I guess I am in an agreeable mood, since I also support Campbell's statement that we really do need to change the way we teach calculus.

I agree so much with Dudley's points made in rebuttal that I will be foolish (his word) and advocate for

discrete mathematics as the standard first-year college mathematics course

(vs. a crappy or even a superb calculus course).

First, let me be clear: I love calculus (both as a liberal art and as a professional tool). It is wonderful mathematics that can enrich and empower one's life. I also agree with Dudley that even though some students do not fully understand the concepts and theories of calculus, it should still be taught—and we should continue to reform, refine, improve, and enhance its teaching. And I strongly agree with him that mathematics is good for students because it can and does develop thinking and problem solving skills. I believe we (mathematics educators) should be pleased by what we are doing and confident we are having a positive impact on students in

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calculus and in modeling and other math courses as well.

Here is where I go a step further than Dudley: I strongly believe that we would have even greater success if we taught discrete modeling as the standard course to our first-year students. Such a course should include copious modeling and modern interdisciplinary problem solving. If we do this well and prepare and motivate our students to take the calculus, that course can be more like the one that Campbell advocates—full of rich ideas and fuller applications, resulting in improved student understanding of mathematics.

Background and Perspective

Before I provide my case, let me give a little background and perspective. Mathematics, as a tool for humans, has always been useful. Benjamin Franklin highlighted this in his 1735 essay "Of the usefulness of mathematics" [1735]:

> There has not been any science so much esteemed and honored as this of mathematics, nor with so much industry and vigilance become the care of great men, and labored in by the potentates of the world, viz. emperors, kings, princes, etc.

Students who learn mathematics can understand, interpret, and predict the behavior of real-life phenomena, and then share what they learn with people all over the world. Likewise, mathematics as a liberal art has always developed thinking skills. Mathematics requires one to think abstractly, conceptually, and systematically. Alfred North Whitehead in his Preface to *Universal Algebra* wrote "The whole of mathematics consists in the organization of a series of aids to the imagination in the process of reasoning" [1898, as quoted in Moritz 1914, 6].

In today's world, many people face thinking, reasoning, and quantitative challenges. Today's college-educated managers and professionals are required to process data and synthesize information, use and understand information technology, optimize elaborate plans, confront complexity, think through difficult challenges, and leverage new technologies. To meet these challenges and to insure that our future citizens anticipate and respond effectively to the uncertainties of a changing world, college core mathematics programs need to develop students as

> *creative, confident, competent problem-solvers and clear, critical thinkers.*

The essential components of modern undergraduate mathematics are

- *modeling* (forming and analyzing problems, using technical tools, and implementing solutions) and
- *inquiry* (formulating questions, moving toward answers and more questions, generalizing, seeking understanding, connecting topics and ideas).

College graduates need to use technological tools to solve problems from every facet of life (physical sciences, life sciences, social sciences, behavioral sciences, political sciences, technology, and humanities). Our students need to study mathematics because of its importance in the everyday world and to develop their way of thinking. Undergraduate mathematics must challenge the mind to dream, to hope, to believe, and then provide the skills and the tools needed to achieve those dreams.

Beyond Just Mathematics

Also needed are interdisciplinary experiences that give students the opportunity to connect their mathematics to real problems involving aspects of many disciplines. I believe what Descartes wrote:

> Hence we must believe that all the sciences are so interconnected, that it is much easier to study them all together than to isolate one from all the others. If, therefore, anyone wishes to search out the truth of things in serious earnest, he ought not select one special science, for all the sciences are cojoined with each other and interdependent.
>
> —Descartes [1629]

It is imperative that our nation's colleges design and implement courses that integrate important topics and connect to other disciplines, along with developing skills in using technology, and solving problems. The curriculum needs to be tied together with student-growth threads of important attitudes and skills in student development as life-long learners who are able to formulate questions, research answers, reach logical conclusions, and make informed decisions.

I believe that *discrete modeling is best suited* to prepare students for success in the future era of the information age, where new concepts like complexity, network science, and information science will be prevalent. Such a course is most appropriate in scope and complexity to give students an awareness of the discipline of mathematics.

The basic concept in discrete dynamical modeling is that the future is predicted by understanding the present and adding to it the hypothesized change over the interval of interest. Discrete dynamical models (difference equations) are solvable numerically by iteration, so students are not restricted by solution techniques but are free to think, model, and analyze problems. The prerequisite mathematics to learn and perform elementary discrete dynamical modeling is algebra. Therefore, this topic is accessible for first-year college students without an investment in learning the more-sophisticated calculus concepts needed to study continuous dynamics (differential equations). Many discrete mathematics topics, especially the modeling, reasoning, and computing, that are traditionally covered in higher-level courses are accessible to freshmen taking an introductory discrete dynamical modeling course. Through a first-year discrete modeling course, the foundations of our new sciences are available to all students at the core level.

A valuable set of goals for a core mathematics course might include:

- students acquiring fundamental knowledge for future application;
- students developing sound, logical thought processes relevant to future science; and
- students learning how to solve problems.

By achieving these goals, successful students could formulate intelligent questions, reason and research solutions using scientific principles, and be confident and independent in their future work.

A discrete modeling course can accomplish these goals via study of

- linear and nonlinear difference equations;
- systems of equations, along with the matrix-algebra concepts of eigenvalues and eigenvectors;
- analytic, numeric, and graphic solution methods and analysis;
- conjecturing;
- long-term behavior through determination of equilibria and stability;
- proportionality modeling; and
- applied problem solving.

Throughout such a course, major mathematical themes can be studied, including functions, limits, dynamics, accumulation, vectors, and modeling. The COMAP-sponsored team-written textbook *Principles and Practice of Mathematics* [Meyer 1997] (of which I was a co-author) presents these ideas; other books also cover this subject at a first-year level.

Mathematics is like life. Both are rewarding, challenging, offer great gifts, inspire great dreams, and hold great promise. I believe that discrete modeling is the best course to deliver that promise to our first-year students.

References

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About the Author

Chris Arney graduated from West Point and became an intelligence officer. His studies resumed at Rensselaer Polytechnic Institute with an M.S. (computer science) and a Ph.D. (mathematics). He spent most of his military career as a mathematics professor at West Point, before becoming Dean of the School of Mathematics and Sciences and Interim Vice President for Academic Affairs at the College of Saint Rose in Albany, NY. Chris has authored 20 books, written more than 100 technical articles, and given more than 200 presentations and 30 faculty development workshops. His technical interests include mathematical modeling, cooperative systems, and the history of mathematics and science; his teaching interests include using technology and interdisciplinary problems to improve undergraduate teaching and curricula; his hobbies include reading and mowing his lawn. Chris is Director of the Mathematical Sciences Division of the Army Research Office, where he researches cooperative systems, particularly in information networks, pursuit-evasion modeling, and robotics. He is co-director of COMAP's Interdisciplinary Contest in Modeling (ICM)® and the editor for the *Journal*'s ILAP (Interdisciplinary Lively Applications Project) Modules. In August 2009, he will rejoin the faculty at West Point, where his daughter Kristin also teaches.

Modeling Forum

Results of the 2009 Interdisciplinary Contest in Modeling

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Introduction

A total of 374 teams from four countries spent a weekend in February working in the 11th Interdisciplinary Contest in Modeling (ICM)®. This year's contest began on Thursday, Feb. 5 and ended on Monday, Feb. 9, 2009. During that time, teams of up to three undergraduate or high school students researched, modeled, analyzed, solved, wrote, and submitted their solutions to an open-ended interdisciplinary modeling problem involving marine ecology. After the weekend of challenging and productive work, the solution papers were sent to COMAP for judging. Two of the top papers, which were judged to be Outstanding by the expert panel of judges, appear in this issue of *The UMAP Journal*.

COMAP's Interdisciplinary Contest in Modeling (ICM), along with it sibling contest, the Mathematical Contest in Modeling (MCM)®, involves students working in teams to model and analyze an open problem. Centering its educational philosophy on mathematical modeling, COMAP supports the use of mathematical tools to explore real-world problems. It serves society by developing students as problem solvers in order to become better informed and prepared as citizens, contributors, consumers, workers, and community leaders. The ICM and MCM are examples of COMAP's efforts in working towards its goals.

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This year's environmental sciences problem was challenging in its demand for teams to utilize many aspects of science, mathematics, and analysis in their modeling. The problem required teams to understand the complexity of marine ecology and aquaculture systems and to model that complexity to reverse current environmental destruction while retaining financial prosperity of the aquaculture. To accomplish their tasks, the teams had to consider many difficult and complex issues. The problem also included the requirements of the ICM to use thorough analysis, research, creativity, and effective communication. The author of the problem was marine biology researcher Melissa Garren of the Scripps Institute of Oceanography.

All members of the 374 competing teams are to be congratulated for their excellent work and dedication to modeling and problem solving. The judges remarked that this year's problem was challenging and demanding in many aspects of modeling and problem solving.

Next year, we will continue the environmental science theme for the contest problem. Teams preparing for the 2010 contest should consider reviewing interdisciplinary topics in the area of environmental issues.

Creating Food Systems: Rebalancing Human-Influenced Ecosystems

Background

Less than 1% of the ocean floor is covered by coral. Yet 25% of the ocean's biodiversity is supported in these areas. Thus, conservationists are concerned when coral disappears, since the biodiversity of the region disappears shortly thereafter.

Consider an area in the Philippines located in a narrow channel between Luzon Island and Santiago Island in Bolinao, Pangasinan, that used to be filled with coral reef and supported a wide range of species (**Figure 1**). The once-plentiful biodiversity of the area has been dramatically reduced with the introduction of commercial milkfish (*Chanos chanos*) farming in the mid 1990s. It's now mostly muddy bottom, the once living corals are long since buried, and there are few wild fish remaining, due to overfishing and loss of habitat.

While it is important to provide enough food for the human inhabitants of the area, it is equally important to find innovative ways of doing so that allow the natural ecosystem to continue thriving; that is, establishing a desirable polyculture system that could replace the current milkfish monoculture. The ultimate goal is to develop a set of aquaculture practices that would not only support the human inhabitants financially and nutritionally, but simultaneously improve the local water quality to a point where reef-building corals could recolonize the ocean floor and co-exist with the farms.

A desirable polyculture is a scenario where multiple economically valuable species are farmed together and the waste of one species is the food for another. For example, the waste of a fin-fish can be eaten by filter feeders, and excess nutrients from both fish and filter feeders can be absorbed by algae which can also be sold, either as food or commercially useful byproducts. Not only does this reduce the amount of nutrient input from the fish farming into the surrounding waters, it also increases the amount of profit a farmer can make by using the fish waste to generate a greater quantity of usable products (mussels, seaweed, etc.)

For modeling purposes, the primary animal organisms involved in these biodiverse environments can be partitioned into

- predatory fish (phylum *Chordata*, subphylum *Vertebrata*);
- herbivorous fish (phylum *Chordata*, subphylum *Vertebrata*);
- molluscs (such as mussels, oysters, clams, and snails) (phylum *Mollusca*);
- crustaceans (such as crabs, lobsters, barnacles, and shrimp) (phylum *Arthropoda*, subphylum *Crustacea*);
- echinoderms (such as starfish, sea cucumbers, and sea urchins) (phylum *Echinodermata*); and
- algae.

By feeding type, there are

- primary producers (photosynthesizers—these can be single-cell phytoplankton, cyanobacteria, or multicellular algae);
- filter feeders (they strain plankton, organic particles, and sometimes bacteria out of the water);
- deposit feeders (they eat mud and digest the organic molecules and nutrients out of it);
- herbivores (they eat primary producers); and
- predators (carnivores).

Just as on land, most of the carnivores eat herbivores or smaller carnivores, but in the ocean they can also eat many of the filter feeders and deposit feeders. Most animals have growth efficiencies of 10–20%, so 80–90% of what they ingest ends up as waste in one form or another (some dissipated heat, some physical waste, etc.).

The role of coral in this biodiverse environment is largely to partition the space and allow species to condense and coexist by giving a large number of species each its own chance at a livable environment in a relatively small space: the aquatic analogue of high-rise urbanization. Coral also provides some filter feeding, which helps clean the water.

The ability of an area to support coral depends on many factors, the most important of which is water quality. For example, corals in Bolinao are able to live and reproduce in waters that contain half a million to a million bacteria per milliliter and 0.25 μg chlorophyll per liter (a proxy for phytoplankton biomass). The fish-pen channel currently sees levels upwards of 10 million bacteria per milliliter and 15 μg chlorophyll per liter. Excess nutrients from the milkfish farms encourage fast-growing algae to choke out coral growth, and particulate influx from the milkfish farms reduces corals ability to photosynthesize. Therefore, before coral larvae can begin to grow, acceptable water quality must be established. Other threats to coral include degradation from increasing ocean acidity due to increased atmospheric CO_2, and degradation from increasing ocean temperature due to global warming. These can be considered second-order threats, which we will not specifically address in this problem.

Problem Statement

The challenge for this problem is to come up with viable polyculture systems to replace the current monoculture farming of milkfish, so as to improve water quality sufficiently that coral larvae could begin settling and recolonizing the area. Your polyculture scenario should be economically interesting and environmentally friendly both in the short and long term.

1. Model the Original Bolinao Coral Reef Ecosystem before Fishfarm Introduction

Develop a model of an intact coral reef foodweb containing the milkfish as the only predatory fish species, one particular herbivorous fish (of your choice), one mollusc species, one crustacean species, one echinoderm species, and one algae species. Specify the numbers of each species present in a way that you find reasonable; cite the sources you use or show the estimates you make in arriving at these population numbers. In articulating your model, specify how each species interacts with the others. Show how your model predicts a steady-state level of water quality sufficient for the continued healthy growth of your coral species. If your model does not yield a high-enough level of water quality, then adjust your number of each species in a way that you find most reasonable until you do achieve a satisfactory quality level, and describe clearly which species numbers you adjusted and why your changes were reasonable.

2. Model the Current Bolinao Milkfish Monocutlure

a. First examine the impact if milkfish farming were to suppress other animal species. Do this by removing (setting the population to zero of) all herbivorous fish, all molluscs, all crustaceans, and all echinoderms. Set all other populations to be the same as in your full model above. Since you have removed the milkfish's natural food supply, you will need to introduce a constant term that models farmer-feeding of the penned milkfish; choose this term to keep your model in equilibrium. What steady-state level of water quality does your model now predict? Is water quality sufficient for the continued healthy growth of your coral species? Compare and describe how your result compares to observations.

b. Milkfish farming does not totally suppress all other animal species and water quality is probably not as bad as your results from part 2a. suggest, so use your model to simulate the current Bolinao situation by reintroducing all deleted species and adjust only those populations until water quality matches that currently observed in Bolinao. Compare your populations with those currently observed in Bolinao and discuss what changes to your model could bring your population predictions into closer agreement with observations.

3. Model the Remediation of Bolinao via Polyculture

You now strive to replace the current monoculture with a polyculture industry, seeking to make the water clear enough that the original reef ecosystem that you modeled in part 1 can re-establish itself without any help from humans. The idea is to introduce an interdependent set of species such that, whatever feed the milkfish farmer puts in, the combination of all of the "livestock" will use it entirely so that there are no (or only minimal) leftover nutrients and particles (feed and feces) falling onto the newly-growing reef habitat below. Additionally, you seek to commercially harvest edible biomass from this polyculture in order to feed humans and increase value.

a. Develop a commercial polyculture to remediate Bolinao. Do this by starting with your "current" penned model from part 2b, and introduce into it additional species that both help clean the water and yield valuable, harvestable biomass. For example, you could line the pens with mussels, oysters, clams, or other economically-valuable filter feeder to remove some of the waste from the milkfish. Economically-valuable algae could be grown on the sides of the pens near the surface (where they get enough light), and some of these could feed the small herbivorous fish that feed the milkfish. Clearly present your model and its steady-state populations.

b. Report on the outputs of your model. What did you optimize, what constraints did you enforce, and why? What water quality does your model yield? How much harvest does your model yield, and what is its economic value? How much does it cost you to further improve water quality? In other words, from your optimal scenario, how many dollars of harvest does it cost to improve water quality by one unit?

4. Science

Discuss the harvesting of each species for human consumption. How do we use your model for predicting or understanding harvesting for human consumption? Does a harvested pound of carnivorous fish count the same as a harvested pound of seaweed, so that we seek to maximize the total weight harvested; or do we differentiate by value (as measured by price of each harvested species), so that we seek to maximize the value of the harvest? Or do we seek to maximize the total value of harvest minus cost of milkfish feed? Should we define the value of edible biomass as the sum of the values of each species harvested, minus the cost of milkfish feed?

5. Maximize the Value of the Total Harvest

We now wish to maintain an acceptable (maximal) level of water quality while harvesting a high (maximal) value of marketable biomass from all living species in the model for human consumption (edible and saleable byproducts are equally legitimate ways to maximize value). Change your model to harvest a constant amount from each species. What is the total value of biomass (as defined above) that you can harvest and the corresponding water quality? Try different harvesting strategies and different levels of milkfish feeding (always choosing values that will keep your model in equilibrium), and graph water quality as a function of harvest value. What strategy is optimal and what is the optimal harvest?

6. Call to Action

Write an information paper to the director of the Pacific Marine Fisheries Council summarizing your findings on the relationship between biodiversity and water quality for coral growth. Include a strategy for remediating an area like Bolinao and how long it will take to remediate. Present your optimal harvesting/feeding strategy from part 5 above along with persuasive justification, and present suggested fishing/harvest quotas that will implement your plan. Show the leverage of your strategy by presenting the ratio of the harvest value under your plan to the harvest value under the current Bolinao scenario. Discuss the pros and cons from an ecological perspective of implementing your polyculture system.

Getting Started References

`http://en.wikipedia.org/wiki/Integrated_Multi-trophic_Aquaculture`
`http://en.wikipedia.org/wiki/Coral_reef`
`http://www.seaworld.org/infobooks/Coral/home.html`

Supplementary Information

Tables 1–3 are representative of the data that you will be able to find through public searches. These data may not be complete for your purposes and are intended only to help give you ideas on how to get started. You should use the best-suited and most complete data that you find.

References for Information found in the Tables

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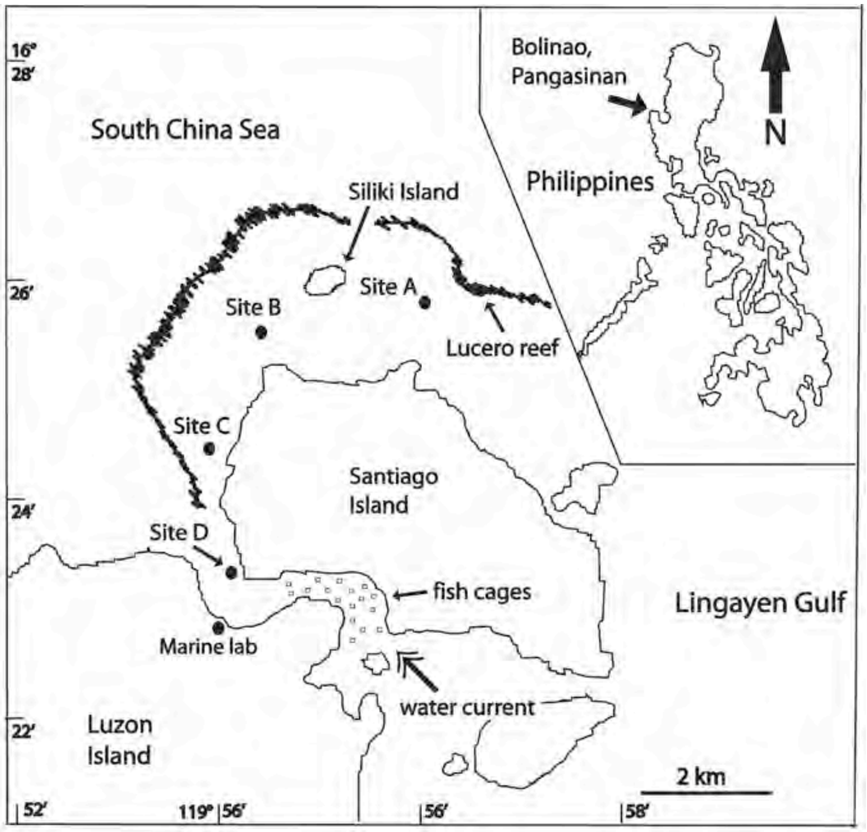

Figure 1. Map of the Bolinao area and the sites sampled for water quality data listed in **Tables 1** and **2**. Sites A and B have fairly healthy coral reefs, while Site C has fairly degraded reefs, Site D has a few corals still holding on but is mostly dead coral and algae at this point in time, and the area under the fish pens no longer has live coral at all. In the fish pen channel, farmers employ nets measuring roughly 10 m × 10 m × 8 m with stocking densities of 50,000 fish per pen and 10 pens per hectare. (Source: Garren et al. [2008]).

Table 1.
Water characteristics of Bolinao sites (from Garren et al. [2008]).

Site	Dissolved Organic Carbon (DOC) (μM)	Total Nitrogen (dissolved) (μM)	Chl *a* (μg/L)	Particulate Organic Carbon (POC) (μg/L)	Total Nitrogen (particulate) (μg/L)
A	69.7 ± 1.3	7.4 ± 0.4	0.25 ± 0.03	106 ± 4	9 ± 15
B	80.4 ± 2.9	8.0 ± 0.2	0.28 ± 0.03	196 ± 57	39 ± 15
C	89.6 ± 1.7	14.2 ± 0.77	0.38 ± 0.03	662 ± 68	54 ± 17
D	141 ± 2.9	30.5 ± 1.3	4.5 ± 0.2	832 ± 338	86 ± 45
Fish pens	162 ± 18.5	39.8 ± 2.7	10.3 ± 0.2	641 ± 60	86 ± 18

Table 2.

Bacteria and particle abundances in Bolinao (from Garren et al. [2008]).

Site	Virus-like particles abundance (#/ml) $\times 10^7$	Free-living bacteria abundance (cells/ml) $\times 10^5$	Particle-attached bacteria abundance (cells/ml) $\times 10^2$	% of total bacteria attached to particles (%)	# of particles per ml (particle defined as larger than 3 μm)		Avg particle size (μm^2)
					Detritus (#) $\times 10^3$	Phytoplankton cells (#) $\times 10^2$	
A	1.0 ± 0.07	5.4 ± 0.3	5.3 ± 2.2	< 0.1	3.4 ± 0.2	1.6 ± 0.2	42.7
B	0.8 ± 0.04	4.2 ± 0.6	3.9 ± 0.6	< 0.1	4.4 ± 0.2	1.0 ± 0.1	19.7
C	1.7 ± 0.1	3.0 ± 0.04	113.7 ± 3.6	3.7	9.6 ± 0.8	1.1 ± 0.1	65.8
D	7.0 ± 0.3	6.1 ± 0.6	144.5 ± 5.6	2.3	14.4 ± 0.1	9.7 ± 0.7	576.1
Fish pens	6.1 ± 0.7	9.9 ± 0.3	583.2 ± 28.1	5.6	11.3 ± 0.5	78.4 ± 5.5	280.8

Fox, Rebecca J., and David R. Bellwood. 2008. Direct versus indirect methods of quantifying herbivore grazing impact on a coral reef. *Marine Biology* 154 (2) (April 2008): 325–334.

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Table 3.
Organism information.

Organism	Data source	Trophic classification	What it eats	How much it eats	What it excretes	Value when harvested
Milkfish	Homer et al. [2002]	predator	fish feed or smaller fish	In pens: 6.58 kg/m^2 of pen/5 months	242–493 g dry weight of sediment/m^2/day*	$1,278 USD/metric ton (from *Agribusiness Weekly*)
Herbivorous fish (*Siganus doliatus*, a rabbitfish as representative)	Fox and Bellwood [2008]	herbivore	macro algae (fleshy algae)	18–22 cm^3 of algae material/m^2 of reef/month		
Crustaceans (data averaged over one crab (*Menaethius monoceros*) and one amphipod (*Cymadusa imbroglio*))	Cruz-Rivera and Paul [2006]	herbivore	macro algae and cyanobacteria	10–20 mg wet weight of food/individual/day		Values on the Web
Molluscs (averaged over 5 species of mussels and oysters)	Hawkins et al. [1998]	filter feeder	particles 1–16 μm in diameter	They clear 5–7 L of water/hr of particles and absorb 4–15 mg organic material/g dry soft tissue weight/hr		Values on the Web
Echinoderm (urchin, *Tripneustes gratilla*, from the Philippines as representative	Dy et al. [2002]	herbivore	fleshy algae	0.05 g wet weight algae/g dry weight urchin/hr, where average dry weight of an individual was 6.9 g	0.2–11.5 mg dry weight feces/g dry weight urchin	
Algae	Yokoya and Oliveira [1992]	primary producer	sunlight, carbon dioxide, nitrogen, phosphorus	**	***	

*This sediment is approximately 10% carbon, 0.4% nitrogen, and 0.6% phosphorus dry weight.

**Depending on temperature, ecanomically important red algae can double their mass (wet weight) in as little as 2.8 days (*Hypnea cornuta*) and as long as 50.0 days (*Pterocladia capillacea*).

***These organisms can extrude excess photosynthate in the form of dissolved organic carbon but this is a difficult number to quantify. Simply keep in mind that this process is occurring as you think about the ecological perspective in part 6.

The Results

The 374 solution papers were coded at COMAP headquarters so that names and affiliations of the authors were unknown to the judges. Each paper was then read preliminarily by "triage" judges at the U.S. Military Academy at West Point, NY. At the triage stage, the summary, the model description, and overall organization are the primary elements in judging a paper. Final judging by a team of modelers, analysts, and subject-matter experts took place in April. The judges classified the 374 submitted papers as follows:

	Outstanding	Meritorious	Honorable Mention	Successful Participation	Total
Coral reef	2	36	144	192	374

The two papers that the judges designated as Outstanding appear in this special issue of *The UMAP Journal*, together with a commentary by the judges. We list those two Outstanding teams and the 36 Meritorious teams (and advisors) below. The complete list of all participating schools, advisors, and results is provided in the **Appendix**.

Outstanding Teams

Institution and Advisor	**Team Members**
"Rebalancing Human-Influenced Ecosystems"	
China University of Mining and Technology	YuanSi Zhang
Xuzhou, Jiangsu, China	ShuoPeng Wang
Xingyong Zhang	Ning Cui
"Striving for Balance: Why Reintroducing More Species to Fish Farm Ecosystems Yields Bigger Profits"	
United States Military Academy	Sean Clement
West Point, NY	Timothy Newlin
Kristen Arney	Joseph Lucas

Meritorious Teams (36)

Asbury College, Mathematics and Computer Science, Wilmore, KY (Duk Lee)
Asbury College, Mathematics and Computer Science, Wilmore, KY (Kenneth P. Rietz)
Bandung Institute of Technology, Mathematics, Bandung, West Java, Indonesia (Agus Yodi Gunawan)
Beijing University of Posts and Telecommunications, Computer Science and Technology, Beijing, China (Hongxiang Sun)
California State University Monterey Bay, Mathematics, Seaside, CA (Hongde Hu)

Carroll College, Mathematics, Engineering, and Computer Science, Helena, MT (Kelly Cline)
Fudan University, Mathematical Sciences, Shanghai, China (Yuan Cao)
Harbin Institute of Technology, Mathematics, Harbin, Heilongjiang, China (Qi Guo)
Harbin Institute of Technology, Mathematics, Harbin, Heilongjiang, China (Yong Wang)
Harvey Mudd College, Mathematics, Claremont, CA (Zach Dodds)
Humboldt State University, Environmental Resources Engineering, Arcata, CA (Brad Finney)
Jinan University, Mathematics, Guangzhou, Guangdong, China (Daiqiang Hu)
National University of Defense Technology, Applied Mathematics, Changsha, Hunan, China (Lizhi Cheng)
National University of Defense Technology, Mathematics and System Science, Changsha, Hunan, China (Mengda Wu)
Northwestern Polytechnical University, Applied Mathematics, Xi'an, Shaanxi, China (Huayong Xiao)
Northwestern Polytechnical University, Applied Mathematics, Xi'an, Shaanxi, China (Min Zhou)
Olin College, Needham, MA (Burt S. Tilley)
Peking University, Health Science Center, Beijing China (Zhiyu Tang)
Peoples' Liberation Army University of Science and Technology, Command Automation, Nanjing, Jiangsu, China (Zhao Ying)
Shandong University at Weihai, Mathematics and Statistics, Weihai, Shandong, China (Yang Bing and Cao Zhulou)
Simpson College, Biology, Indianola, IA (Pat Singer)
Simpson College, Mathematics, Indianola, IA (Debra Czarneski)
Southeast University, Mathematics, Nanjing, Jiangsu, China (Zhizhong Sun)
Southeast University, Mathematics, Nanjing, Jiangsu, China (Jun Huang)
Southeast University, Mathematics, Nanjing, Jiangsu, China (Feng Wang)
Southwest University, Mathematics, Chongqing, China (Lin Wei)
University of International Business and Economics, International Trade and Economics, Beijing, China (Baomin Dong)
University of Science and Technology of China, Electronic Engineering and Information Science, Hefei, Anhui, China (Yu He)
Xidian University, Mathematics, Xi'an, Shaanxi, China (Xiaogang Qi)
Xidian University, Science, Xi'an, Shaanxi, China (Hanwen Yu)
Zhejiang University, Mathematics, Hangzhou, China (Biao Wu)
Zhejiang University, Mathematics, Hangzhou, China (Yong Wu)
Zhejiang University, Mathematics, Hangzhou, China (Zhongfei Zhang)
Zhengzhou Information Engineering Institute, Zhengzhou, Henan, China (Jian Ping Du)
Zhuhai College of Jinan University, Mathematical Modeling Innovative Practice Base, Zhuhai, Guangdong, China (Advisor Team)
Zhuhai College of Jinan University, Mathematical Modeling Innovative Practice Base, Zhuhai, Guangdong, China (Yuanbiao Zhang)

Awards and Contributions

Each participating ICM advisor and team member received a certificate signed by the Contest Directors and the Head Judge. Additional awards were presented to the team from the China University of Mining and Technology by the Institute for Operations Research and the Management Sciences (INFORMS).

Judging

Contest Directors

Chris Arney, Division Chief, Mathematical Sciences Division,
Army Research Office, Research Triangle Park, NC

Joseph Myers, Computing Sciences Division, Army Research Office, Research Triangle Park, NC

Associate Director

Rodney Sturdivant, Dept. of Mathematical Sciences,
U.S. Military Academy, West Point, NY

Judges

John Kobza, Dept. of Industrial Engineering, Texas Tech University,
Lubbock, TX

Sheila Miller, Dept. of Mathematical Sciences, U.S. Military Academy,
West Point, NY

Melissa Garren, Scripps Institution of Oceanography, La Jolla, CA

Frank Wattenberg, Dept. of Mathematical Sciences, U.S. Military Academy,
West Point, NY

Triage Judges

Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY:

Amanda Beecher, Randy Boucher, Robert Burks, Pete Charbonneau, Eric Drake, Aaron Elliott, Bill Fehlman, Douglas Fletcher, Andy Glen, Tina Hartley, Alex Heidenberg, Donald Outing, Jon Roginski, Rodney Sturdivant, Frank Wattenberg, and Brian Winkel

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- IBM for their support for the contest;
- all the ICM judges and ICM Board members for their valuable and unflagging efforts;
- the staff of the U.S. Military Academy, West Point, NY, for hosting the triage and final judgings.

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each of the team papers here is the result of undergraduates working on a problem over a weekend; allowing substantial revision by the authors could give a false impression of accomplishment. So these papers are essentially au naturel. Light editing has taken place: minor errors have been corrected, wording has been altered for clarity or economy, style has been adjusted to that of *The UMAP Journal*, and the papers have been edited for length. Please peruse these student efforts in that context.

To the potential ICM Advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

Appendix: Successful Participants

KEY:
P = Successful Participation
H = Honorable Mention
M = Meritorious
O = Outstanding (published in this special issue)

C denotes the ICM Problem

INSTITUTION	DEPT.	CITY	ADVISOR	C
CALIFORNIA				
Calif. State U. Monterey Bay	Math	Seaside	Hongde Hu	M
Harvey Mudd C.	Math	Claremont	Zach Dodds	M
Harvey Mudd C.	Math	Claremont	Francis Su	H
Humboldt State U.	Env'l Res. Eng.	Arcata	Brad Finney	M
IOWA				
Simpson C.	Biology	Indianola	Clinton Meyer	H
Simpson C.	Biology	Indianola	Pat Singer	M
Simpson C.	Math	Indianola	Debra Czarneski	M
Simpson C.	Math	Indianola	William Schellhorn	H
KENTUCKY				
Asbury C.	Math & CS	Wilmore	David L. Coulliette	H
Asbury C.	Math & CS	Wilmore	Duk Lee	M
Asbury C.	Math & CS	Wilmore	Kenneth P. Rietz	M
MASSACHUSETTS				
Frontier Regional Sch.	Biology	South Deerfield	Bill Canaday	H
Frontier Regional Sch.	Biology	South Deerfield	Bill Canaday	P
Frontier Regional Sch.	Math	South Deerfield	Steve Blinder	H
Frontier Regional Sch.	Math	South Deerfield	Steve Blinder	H
Frontier Regional Sch.	Math	South Deerfield	Garrett Deane	H
Frontier Regional Sch.	Math	South Deerfield	Garrett Deane	P
Frontier Regional Sch.	Math	South Deerfield	Bev MacLeod	H
Frontier Regional Sch.	Math	South Deerfield	Dave Mako	P
Frontier Regional Sch.	Math	South Deerfield	Dave Mako	P
Frontier Regional Sch.	Math	South Deerfield	Carol Pike	H
Frontier Regional Sch.	Math	South Deerfield	Carol Pike	P
Frontier Regional Sch.	Sci.	South Deerfield	Chevy Seney	P
Frontier Regional Sch.	Sci.	South Deerfield	Chevy Seney	P
Olin College		Needham	Burt S. Tilley	M
MINNESOTA				
Bemidji State U.	Math & CS	Bemidji	Colleen Livingston	P
MONTANA				
Carroll C.	Math, Eng., & CS	Helena	Kelly Cline	M
NEW JERSEY				
Princeton U.	Ops. Res. & Fin. Eng.	Princeton	Birgit Rudloff	H
NEW YORK				
U.S. Military Acad.	Math	West Point	Kristin Arney	O
U.S. Military Acad.	Math	West Point	Janet Braunstein	P
WISCONSIN				
Beloit C.	Math & CS	Beloit	Paul J. Campbell	H

INSTITUTION	DEPT.	CITY	ADVISOR	C
CHINA				
Anhui				
Anhui U.	Electron. Sci. & Tech.	Hefei	Zhixiang Huang	H
Anhui U.	Appl. Math	Hefei	Xuejun Wang	H
Anhui U.	Stats	Hefei	Ligang Zhou	H
Anqing Teachers College	Math & CS	Anqing	Ben Yue Su	P
Hefei U. of Tech.	Math	Hefei	Xueqiao Du	H
Hefei U. of Tech.	Appl. Math	Hefei	Huaming Su	H
Hefei U. of Tech.	Appl. Math	Hefei	Huaming Su	P
U. of Sci. & Tech. of China	Electron. Eng. & Info.	Hefei	Yu He	M
Beijing				
Beihang U.	Advanced Eng.	Beijing	Wei Feng	P
Beihang U.	Instr. Sci. & Opto-electron. Eng.	Beijing	Haifeng Dong	P
Beihang U.	Sci.	Beijing	Hongying Liu	P
Beijing Forestry U.	Sci.	Beijing	Li Hong Jun	P
Beijing Forestry U.	Sci.	Beijing	Mengning Gao	P
Beijing Inst. of Tech.	Math	Beijing	Huafei Sun	P
Beijing Inst. of Tech.	Math	Beijing	Chunlei Cao	P
Beijing Inst. of Tech.	Math	Beijing	Gui-Feng Yan	P
Beijing Inst. of Tech.	Math	Beijing	Yan Dong	P
Beijing Jiaotong U.	Chemistry	Beijing	Yongsheng Wei	P
Beijing Jiaotong U.	CS	Beijing	Xun Chen	H
Beijing Jiaotong U.	CS	Beijing	Xun Chen	P
Beijing Jiaotong U.	Math	Beijing	Dan Xue	H
Beijing Jiaotong U.	Math	Beijing	Dan Xue	P
Beijing Jiaotong U.	Physics	Beijing	Bingli Fan	P
Beijing Jiaotong U.	Physics	Beijing	Qiao Wang	H
Beijing Jiaotong U.	Traffic Eng.	Beijing	Wen Deng	P
Beijing Jiaotong U.	Traffic Eng.	Beijing	Wen Deng	P
Beijing Lang. & Cult. U.	CS	Beijing	Guilong Liu	H
Beijing Lang. & Cult. U.	CS	Beijing	Guilong Liu	P
Beijing Lang. & Cult. U.	CS	Beijing	Xiaoxia Zhao	P
Beijing Lang. & Cult. U.	CS	Beijing	Xiwen Zhang	P
Beijing Lang. & Cult. U.	CS	Beijing	Yanbing Feng	H
Beijing U. of Chemical Tech.	Math & Info. Sci.	Beijing	Guangfeng Jiang	H
Beijing U. of Posts & Telecomm.	Appl. Math.	Beijing	Zuguo He	H
Beijing U. of Posts & Telecomm.	Appl. Math.	Beijing	Zuguo He	H
Beijing U. of Posts & Telecomm.	Appl. Phys.	Beijing	Jinkou Ding	H
Beijing U. of Posts & Telecomm.	Appl. Phys.	Beijing	Wenbo Zhang	H
Beijing U. of Posts & Telecomm.	Comm. Eng.	Beijing	Lixia Wang	P
Beijing U. of Posts & Telecomm.	CS & Tech.	Beijing	Hongxiang Sun	M
Beijing U. of Posts & Telecomm.	CS & Tech.	Beijing	Lixia Wang	H
Beijing U. of Posts & Telecomm.	CS & Tech.	Beijing	Lixia Wang	H
Beijing U. of Posts & Telecomm.	CS & Tech.	Beijing	Wenbo Zhang	H
Beijing U. of Posts & Telecomm.	CS & Tech.	Beijing	Xiaoxia Wang	H
Beijing U. of Posts & Telecomm.	CS & Tech.	Beijing	Xiaoxia Wang	H
Beijing U. of Posts & Telecomm.	CS & Tech.	Beijing	Xinchao Zhao	P
Beijing U. of Posts & Telecomm.	CS & Tech.	Beijing	Xinchao Zhao	P
Beijing U. of Posts & Telecomm.	CS & Tech.	Beijing	Zuguo He	P
Beijing U. of Posts & Telecomm.	Electron. Eng.	Beijing	Jianhua Yuan	H
Beijing U. of Posts & Telecomm.	Electron. Eng.	Beijing	Qing Zhou	H
Beijing U. of Posts & Telecomm.	Electron. Info. Eng.	Beijing	Xueli Wang	P
Beijing U. of Posts & Telecomm.	Communication Eng.	Beijing	Zuguo He	P

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Capital U. of Econ. & Business	Econ.	Beijing	Xue Li	H
Capital U. of Econ. & Business	Econ.	Beijing	Xue Li	H
Capital U. of Econ. & Business	Info. Mgmt	Beijing	Wei Shen	P
Capital U. of Econ. & Business	Stats	Beijing	Quan Zhang	H
Central U. of Finance & Econ.	Appl. Math	Beijing	Xianjun Yin	P
Central U. of Finance & Econ.	Appl. Math	Beijing	Xiaoming Fan	P
Central U. of Finance & Econ.	Appl. Math	Beijing	Xiuguo Wang	H
Central U. of Finance & Econ.	Appl. Math	Beijing	Zhaoxu Sun	H
Central U. of Finance & Econ.	Appl. Math	Beijing	Donghong Li	P
Central U. of Finance & Econ.	Appl. Math	Beijing	Huiqing Huang	H
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Central U. of Finance & Econ.	Appl. Math	Beijing	Zongze Chai	H
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Central U. of Finance & Econ.	Appl. Math	Beijing	Xiaoming Fan	H
Central U. of Finance & Econ.	China Econ. & Mgmt Acad.	Beijing	Yuanzhu Lu	P
China Agricultural U.	Sci.	Beijing	GuoHui Li	P
China U. of Geosciences	Info. Eng.	Beijing	Jiegen Feng	P
China U. of Geosciences	Info. Eng.	Beijing	Baozeng Chu	P
China U. of Geosciences	Info. Tech.	Beijing	Haiying Wang	P
China U. of Geosciences	Math	Beijing	Cuixiang Wang	P
China U. of Geosciences	Math	Beijing	Linlin Zhao	P
North China Electr. Power U.	Math	Changping	Zhang Keming	H
Peking U.	Electron. Eng. &CS	Beijing	Zhiwei Tong	H
Peking U.	Guanghua Schl of Mgmnt	Beijing	Xiao Fu	H
Peking U.	Math	Beijing	Yulong Liu	H
Peking U.	Physics	Beijing	Liqiang Sun	H
Peking U.	Physics	Beijing	Liqiang Sun	H
Peking U.	Physics	Beijing	Xiaodong Hu	P
Peking U. Health Sci. Ctr		Beijing	Zhiyu Tang	M
Peking U. Health Sci. Ctr	Math	Beijing	Donghong Gao	H
Peking U. Health Sci. Ctr	Math	Beijing	Dongqi He	P
Peking U. Health Sci. Ctr	Math	Beijing	Jinbing An	H
Peking U. Health Sci. Ctr	Math	Beijing	Qiang Wang	H
Peking U. Inst. Condensed Matter	Physics	Beijing	Hongli Wang	H
Tsinghua U.	Math	Beijing	Jun Ye	P
Tsinghua U.	Math	Beijing	Mei Lu	P
Tsinghua U.	Math	Beijing	Zhiming Hu	H
U. of Int'l Business & Econ.	Info. Tech. & Mgmnt Eng.	Beijing	Wei Guo	P
U. of Int'l Business & Econ.	Info. Tech. & Mgmnt Eng.	Beijing	Junlin Hao	P
U. of Int'l Business & Econ.	Info. Tech. & Mgmnt Eng.	Beijing	Yanling Su	H
U. of Int'l Business & Econ.	Int'l Trade & Econ.	Beijing	Baomin Dong	M
U. of Int'l Business & Econ.	Int'l Trade & Econ.	Beijing	Hongyu Pan	H
U. of Int'l Business & Econ.	Int'l Trade & Econ.	Beijing	Jin Zhang	H
U. of Int'l Business & Econ.	Int'l Trade & Econ.	Beijing	Qiang Wang	H
U. of Int'l Business & Econ.	Int'l Trade & Econ.	Beijing	Qiang Wang	P
U. of Int'l Business & Econ.	Int'l Trade & Econ.	Beijing	Ye Dongya	P
U. of Int'l Business & Econ.	Int'l Trade & Econ.	Beijing	Ye Dongya	P
U. of Int'l Business & Econ.	Int'l Trade & Econ.	Beijing	Yiping Xu	P
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U. of Sci. & Tech.	Math	Beijing	Jin Zhu	H

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Chongqing U.	Info. & Comp'l Sci.	Chongqing	Jian Xiao	P
Chongqing U.	Info. & Comp'l Sci.	Chongqing	Luosheng Wen	P
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Chongqing U.	Stats & Act'l Sci.	Chongqing	Tengzhong Rong	P
Southwest U.	Math	Chongqing	Lin Wei	M
Southwest U.	Stats	Chongqing	Jianjun Yuan	P
Southwest U.	Stats	Chongqing	Xuegao Zheng	H
Fujian				
Fujian Agri. & Forestry U.	Food Sci.	Fuzhou	Yongxue Chen	P
Guangdong				
Jinan U.	Math	Guangzhou	Shizhuang Luo	P
Jinan U.	Math	Guangzhou	Daiqiang Hu	H
Jinan U.	Math	Guangzhou	Shizhuang Luo	H
Jinan U.	Math	Guangzhou	Chuanlin Zhang	P
Jinan U.	Math	Guangzhou	Daiqiang Hu	M
Shenzhen Poly.	Electron. & Info. Eng.	Shenzhen	Jianlong Zhong	H
Shenzhen Poly.	Mech'l & Electr. Eng.	Shenzhen	Kanzhen Chen	P
South China Agri. U.	Math	Guangzhou	Shaomei Fang	H
South China Agri. U.	Math	Guangzhou	Qingmao Zeng	P
South China Normal U.	Math	Guangzhou	Hunan Li	H
South China Normal U.	Math	Guangzhou	Xiuxiang Liu	H
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South China U. of Tech.	Appl. Math	Guangzhou	Weijian Ding	H
South China U. of Tech.	Appl. Math	Guangzhou	Yi Hong	H
Xiamen U.	Math & Appl. Math	Xiamen	Jianguo Qian	H
Zhuhai C. of Jinan U.	Math Modeling Innov. Pract.	Zhuhai	Advisor Team	M
Zhuhai C. of Jinan U.	Math Modeling Innov. Pract.	Zhuhai	Advisor Team	P
Zhuhai C. of Jinan U.	Math Modeling Innov. Pract.	Zhuhai	Yuanbiao Zhang	M
Zhuhai C. of Jinan U.	Packaging Eng. Inst.	Zhuhai	Zhi-wei Wang	H
Zhuhai C. of Jinan U.	Packaging Eng. Inst.	Zhuhai	Zhi-wei Wang	H
Hebei				
North China Electr. Power U.	Math & Phys.	Baoding	Po Zhang	P
North China Electr. Power U.	Math & Phys.	Baoding	Yagang Zhang	P
Heilongjiang				
Harbin Eng. U.	Sci.	Harbin	Liyan Xu	H
Harbin Eng. U.	Sci.	Harbin	Jue Wang	P
Harbin Eng. U.	Sci.	Harbin	Lei Zhu	P
Harbin Eng. U.	Sci.	Harbin	Liyan Xu	H
Harbin Eng. U.	Sci.	Harbin	Xiaowei Zhang	H
Harbin Eng. U.	Sci.	Harbin	Xuguang Yang	H
Harbin Inst. of Tech.	Electron. Eng.	Harbin	Lin Li	H
Harbin Inst. of Tech.	Electron. Eng.	Harbin	Lin Li	H
Harbin Inst. of Tech.	Electron. Eng. department	Harbin	Liwei Song	P
Harbin Inst. of Tech.	Env'l Sci. & Eng.	Harbin	Tong Zheng	H
Harbin Inst. of Tech.	Management Sci. & Eng.	Harbin	Hong Ge	H
Harbin Inst. of Tech.	Management Sci. & Eng.	Harbin	Wei Shang	P
Harbin Inst. of Tech.	Math	Harbin	Chiping Zhang	H

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Harbin Inst. of Tech.	Math	Harbin	Qi Guo	M
Harbin Inst. of Tech.	Math	Harbin	Xianyu Meng	H
Harbin Inst. of Tech.	Math	Harbin	Yong Wang	M
Harbin Inst. of Tech.	Math	Harbin	Zhenfeng Shi	H
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Inst. of Tech.	Math	Harbin	Guanghong Gao	P
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Northeast Agri. U.	Food Sci. & Eng.	Harbin	Yueying Yang	P
Northeast Agri. U.	Life Sci.	Harbin	Fangge Li	H
Henan				
Henan Inst. of Sci. & Tech.	Math	Xinxiang	Donge Bao	P
Zhengzhou Info. Eng. Inst.	Dept. 5	Zhengzhou	Jian Ping Du	M
Hubei				
Huazhong U. of Sci. & Tech.	Math & Stats	Wuhan	Zhibin Han	H
Wuhan U.	Math & Stats	Wuhan	Liuyi Zhong	P
Wuhan U.	Math & Stats	Wuhan	Zhuangchu Luo	P
Hunan				
Central South U.	Info. Sci. & Eng.	Changsha	Hongyan Zhang	H
Central South U.	Metal. Sci. & Eng.	Changsha	Muzhou Hou	H
Hunan U.	Math & Econometrics	Changsha	Chuanxiu Ma	H
Hunan U.	Math & Econometrics	Changsha	Han Luo	P
Hunan U.	Math & Econometrics	Changsha	Yueping Jiang	P
Hunan U.	Sftwr	Changsha	Zhiqiang You	P
National U. of Defense Tech.	Appl. Math	Changsha	Lizhi Cheng	M
National U. of Defense Tech.	Appl. Math	Changsha	Meihua Xie	P
National U. of Defense Tech.	Appl. Math	Changsha	Yi Wu	H
National U. of Defense Tech.	Math & Sys. Sci.	Changsha	Dan Wang	H
National U. of Defense Tech.	Math & Sys. Sci.	Changsha	Mengda Wu	H
National U. of Defense Tech.	Math & Sys. Sci.	Changsha	Mengda Wu	M
National U. of Defense Tech.	Math & Sys. Sci.	Changsha	Wenqiang Yang	P
Inner Mongolia				
Inner Mongolia U.	Math	Hohhot	HaiTao Han	H
Jiangsu				
China Pharmaceutical U.	Basic Sci.	Nanjing	Yan Fangrong	H
China U. of Mining & Tech.	Math	Xuzhou	Hu Shao	P
China U. of Mining & Tech.	Math	Xuzhou	Miao Han	P
China U. of Mining & Tech.	Math	Xuzhou	Shengwu Zhou	H
China U. of Mining & Tech.	Math	Xuzhou	Xingyong Zhang	O
China U. of Mining & Tech.	Math	Xuzhou	Xinli Suo	P
China U. of Mining & Tech.	Math	Xuzhou	Zongxiang Wu	H
China U. of Mining & Tech.	Info. & Electr. Eng.	Xuzhou	Dunwei Gong	P

INSTITUTION	DEPT.	CITY	ADVISOR	C
Nanjing U.	Earth Sci.	Nanjing	Huiqun Zhou	P
Nanjing U. of Info. Sci. & Tech.	Math	Nanjing	Guosheng Cheng	P
Nanjing U. of Posts & Telecomm.	Math & Phys.	Nanjing	LiWei Xu	P
Nanjing U. of Posts & Telecomm.	Math & Phys.	Nanjing	Jin Xu	H
Nanjing U. of Posts & Telecomm.	Math & Phys.	Nanjing	Ye Jun	P
Nanjing U. of Sci. & Tech.	Appl. Math	Nanjing	Chungen Xu	P
Nanjing U. of Sci. & Tech.	Appl. Math	Nanjing	Jun Zhang	P
Nanjing U. of Sci. & Tech.	Appl. Math	Nanjing	Wei Xiao	P
PLA U. of Sci. & Tech.	Comm. & Automation	Nanjing	Ying Zhao	M
PLA U. of Sci. & Tech.	Comm. Eng.	Nanjing	Kui Yao	H
PLA U. of Sci. & Tech.	Eng. Crops	Nanjing	Zuowei Tian	P
Southeast U.	Math	Nanjing	Dan He	H
Southeast U.	Math	Nanjing	Dan He	P
Southeast U.	Math	Nanjing	Daoyuan Zhu	H
Southeast U.	Math	Nanjing	Daoyuan Zhu	H
Southeast U.	Math	Nanjing	Feng Wang	M
Southeast U.	Math	Nanjing	Feng Wang	P
Southeast U.	Math	Nanjing	Jun Huang	H
Southeast U.	Math	Nanjing	Jun Huang	M
Southeast U.	Math	Nanjing	Rui Du	P
Southeast U.	Math	Nanjing	Rui Du	P
Southeast U.	Math	Nanjing	Zhizhong Sun	H
Southeast U.	Math	Nanjing	Zhizhong Sun	M
Jilin				
Jilin U.	Math	Changchun	Chunling Chao	P
Jilin U.	Math	Changchun	Mingji Liu	H
Jilin U.	Math	Changchun	Peichen Fang	P
Jilin U.	Math	Changchun	Wenrui Zheng	P
Jilin U.	Math	Changchun	Xiuling Yao	P
Liaoning				
Dalian Maritime U.	Math	Dalian	G. Chen	P
Dalian Maritime U.	Math	Dalian	Shuqin Yang	P
Dalian Maritime U.	Math Department	Dalian	Yunjie Zhang	P
Dalian Nationalities U.	Dean's Office	Dalian	Xiaoniu Li	P
Dalian Nationalities U.	Innovation College	Dalian	Rixia Bai	P
Dalian Nationalities U.	Sci.	Dalian	Jinzhi Wang	P
Dalian Nationalities U.	Sci.	Dalian	Liming Wang	P
Dalian Nationalities U.	Sci.	Dalian	Rendong Ge	P
Dalian U.	Info. & Eng.	Dalian	Jiatai Gang	H
Dalian U.	Info. & Eng.	Dalian	Xiangyu Dong	H
Dalian U.	Info. & Eng.	Dalian	Guangzhi Liu	P
Dalian U.	Info. & Eng.	Dalian	Zixin Liu	P
Dalian U.	Info. & Eng.	Dalian	Zixin Liu	P
Dalian U.	Info. & Eng.	Dalian	Xinxin Tan	H
Dalian U.	Info. & Eng.	Dalian	Cheng Zhang	P
Dalian U. of Tech.	Appl. Math	Dalian	Lin Feng	H
Dalian U. of Tech.	Appl. Math	Dalian	Mingfeng He	P
Dalian U. of Tech.	Appl. Mathematica	Dalian	Liang Zhang	H
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Rebalancing Human-Influenced Ecosystems

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Summary

In Task 1, we establish a Volterra predator-prey model with three biological populations, and we specify the steady-state numbers of the three populations. Then, based on the Analytic Hierarchy Process and a competition model, we obtain the ratio of different species in the second population, predict that the steady-state level of water quality is not high, and make the water quality satisfactory by adjusting the numbers of six species.

In Task 2, when milkfish farming suppresses other animal species, we set up a logistic model, and predict that the water quality at steady-state is awful, the same as in the fish pens—insufficient for the continued healthy growth of coral species. When other species are not totally suppressed, with an improved predator-prey model we simulate the water quality of Bolinao (making it match current quality), obtain predicted numbers of populations, and discuss changes to the predator-prey model aimed at making the numbers of the populations agree more closely with observations.

In Task 3, we establish a polyculture model that reflects an interdependent set of species, introduce mussels and seaweed growing on the sides of the pens, and obtain the numbers of populations in steady state and the outputs of our model.

In Tasks 4 and 5, we differentiate the monetary values of different kinds edible biomass and define the total value as the sum of the values of each species harvested, minus the cost of milkfish feed. Under circumstances

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of acceptable water quality, we build a nonlinear equilibrium optimization model, from which we obtain an optimal strategy and harvest.

In Task 6, we put forward a strategy to improve the water quality in Bolinao. With the ratio between feed cost and net income as the index, the index value of the model is smaller than that of Bolinao area, which signifies the leverage of the strategy. Also, we analyze the polyculture system in terms of ecology.

Introduction

To improve the situation in Bolinao, we need to establish a practicable polyculture system and introduce it gradually. So our goal is pretty clear:

- Model the original Bolinao coral reef ecosystem before fish farm introduction.
- Model the current Bolinao milkfish monoculture.
- Model the remediation of Bolinao via polyculture.
- Discuss the outputs and economic values of species.
- Write a brief to the director of the Pacific Marine Fisheries Council summarizing the relationship between biodiversity and water quality for coral growth.

Our approach is:

- Deeply analyze data in the problem, gradually establishing a model of the coral reef foodweb.
- With available data as evaluation criteria, confirm the water quality based on elements in the sediment.
- Establish models, and interpret the actual situation with data, with the purpose of improving water quality.
- Do further discussion based on our work.

Solutions

Task 1

Aiming toward a coral reef foodweb model, we assume that all the species grow in the same fish pen. We divide the species into three populations:

- one alga species (Population 1);

- one herbivorous fish, one mollusc species, one crustacean species, and one echinoderm species (Population 2); and
- the sole predator species, milkfish (Population 3).

The interrelationships among the species are presented in **Figure 1**.

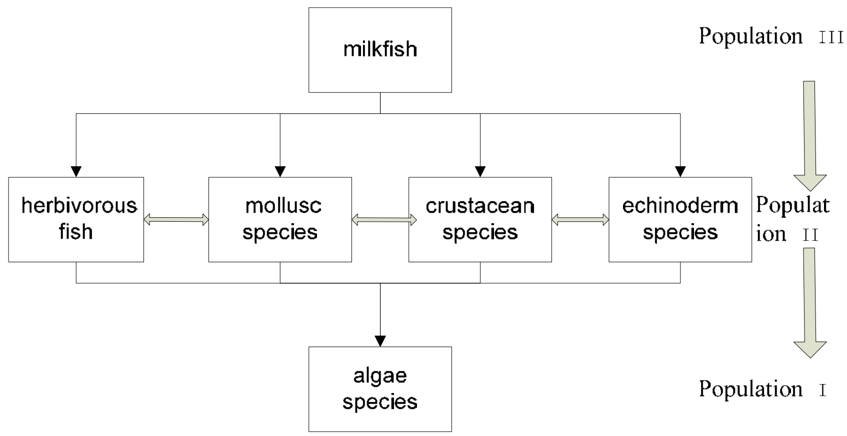

Figure 1. Interrelationships among three populations.

On this basis, we can establish a Volterra predator-prey model with three populations [Shan and Tang 2007]. Let the number of the ith population be $x_i(t)$. If we do not take into consideration the restrictions of natural resources, the algae species of Population 1 growing in isolation will follow an exponential growth law with relative growth rate r_1, so that $\dot{x}(t) = r_1 x_1$. However, species of Population 2 feeding on the alga species will decrease the growth rate of the algae, so the revised model of the alga species is

$$\dot{x}_1(t) = x_1(r_1 - \lambda_1 x_2),$$

where the proportionality coefficient λ_1 reflects the feeding capability of species in Population 2 for the alga species.

Assume that the death rate of the species in Population II is r_2 when existing in isolation; then $\dot{x}_2(t) = -r_2 x_2$, so based on the foodweb we conclude that

$$\dot{x}_2(t) = x_2(-r_2 + \lambda_2 x_1),$$

where the proportionality coefficient λ_2 reflects the support capability of the alga species for Population 2—which in turn provide food for the milkfish. The milkfish reduce the growth rate of the species in Population 2, so we must subtract their feeding effect to get

$$\dot{x}_2(t) = x_2(-r_2 + \lambda_2 x_1 - \mu x_3).$$

Likewise, the model for the milkfish is

$$\dot{x}_3(t) = x_3(-r_3 + \lambda_3 x_2).$$

Altogether, we have an interdependent and mutually-restricting mathematical model of the three populations:

$$\begin{aligned}\dot{x}_1(t) &= x_1(r_1 - \lambda_1 x_2),\\ \dot{x}_2(t) &= x_2(-r_2 + \lambda_2 x_1 - \mu x_3),\\ \dot{x}_3(t) &= x_3(-r_3 + \lambda_3 x_2).\end{aligned}$$

Since this system of differential equations has no analytic solution, we need to use Matlab to get its numerical solution.

Ecologists point out that a periodic solution cannot be observed in most balanced ecosystems; in a balanced ecosystem, there is an equilibrium. In addition, some ecologists think that the long-existing and periodically-changing balanced ecosystems in nature tend toward a stable equilibrium; that is, if the system diverges from the former periodic cycle because of disturbance, an internal control mechanism will restore it. However, the periodically-changing state described by the Volterra model is non-structured stability, and even subtle adjustments to the parameters will change the periodic solution.

So we improve the model by letting the alga species follow logistic growth if in isolation:

$$\dot{x}_1(t) = r_1 x_1 \left(1 - \frac{x_1}{N_1}\right),$$

where N_1 is the maximum population of the alga species allowed by the environmental resources. The alga species provides food for the species of Population 2, so the model for the algae species is

$$\dot{x}_1(t) = x_1 r_1 \left(1 - \frac{x_1}{N_1} - \sigma_1 \frac{x_2}{N_2}\right),$$

where N_2 is the maximum capacity of the species in Population 2 and σ_1 refers to the quantity of the algae (compared to N_1) eaten by the unit quantity species in Population 2 (compared to N_2).

Without the algae, the species in Population 2 will perish; let its death rate be r_2, so that in isolation we will have:

$$\dot{x}_2(t) = -r_2 x_2.$$

The algae provide food for Population 2, so we should add that effect; and the growth of the species in Population 2 is also influenced by internal blocking action; so we get

$$\dot{x}_2(t) = r_2 x_2 \left(-1 - \frac{x_2}{N_2} + \sigma_2 \frac{x_1}{N_1}\right),$$

where σ_2 is analogous to σ_1. Analogously, we can get a full model of the species in Population 2 via

$$\dot{x}_2(t) = r_2 x_2 \left(-1 - \frac{x_2}{N_2} + \sigma_2 \frac{x_1}{N_1} - \sigma_3 \frac{x_3}{N_3} \right).$$

Without the species in Population 2, milkfish will disappear; we set their death rate as r_3. The species in Population 2 provide food for the milkfish, and the growth of milkfish is also restricted by internal blocking action. Here the model is

$$\dot{x}_3(t) = r_3 x_3 \left(-1 - \frac{x_3}{N_3} + \sigma_4 \frac{x_2}{N_2} \right).$$

Summarizing, we have simultaneous equations constituting an interdependent mathematical model for the three populations:

$$\begin{aligned}
\dot{x}_1(t) &= x_1 r_1 \left(1 - \frac{x_1}{N_1} - \sigma_1 \frac{x_2}{N_2} \right), \\
\dot{x}_2(t) &= r_2 x_2 \left(-1 - \frac{x_2}{N_2} + \sigma_2 \frac{x_1}{N_1} - \sigma_3 \frac{x_3}{N_3} \right), \\
\dot{x}_3(t) &= r_3 x_3 \left(-1 - \frac{x_3}{N_3} + \sigma_4 \frac{x_2}{N_2} \right).
\end{aligned}$$

We obtain the values of some parameters in the model, and through nonlinear data fitting of the original data of the local three populations [Shan and Tang 2007; Sumagaysay-Chavoso 1998; Chen and Chou 2001], we get their natural growth rates:

$$\begin{array}{llll}
\sigma_1 = 0.6, & \sigma_2 = 0.5, & \sigma_3 = 0.5, & \sigma_4 = 2; \\
N_1 = 150 \times 10^3, & N_2 = 30 \times 10^3, & N_3 = 2.2 \times 10^3. &
\end{array}$$

According to the volume of local fish pens and relevant materials, we get the original numbers of the three populations:

$$x_1(0) = 121.5 \times 10^3, \qquad x_2(0) = 27 \times 10^3, \qquad x_3(0) = 2 \times 10^3.$$

Then we use Matlab to implement the model, with the results of **Figure 2**, where we see that can see that with the passage of time, the $x_i(t)$ tend to the steady-state values 69,027, 27,015, and 1,760.

The number 27,015 of the species in Population 2 is made up of herbivorous fish, molluscs, crustaceans, and echinoderms. Now we confirm the numbers of all the species in Population 2, which stay at the same trophic level, coexisting and mutually competing.

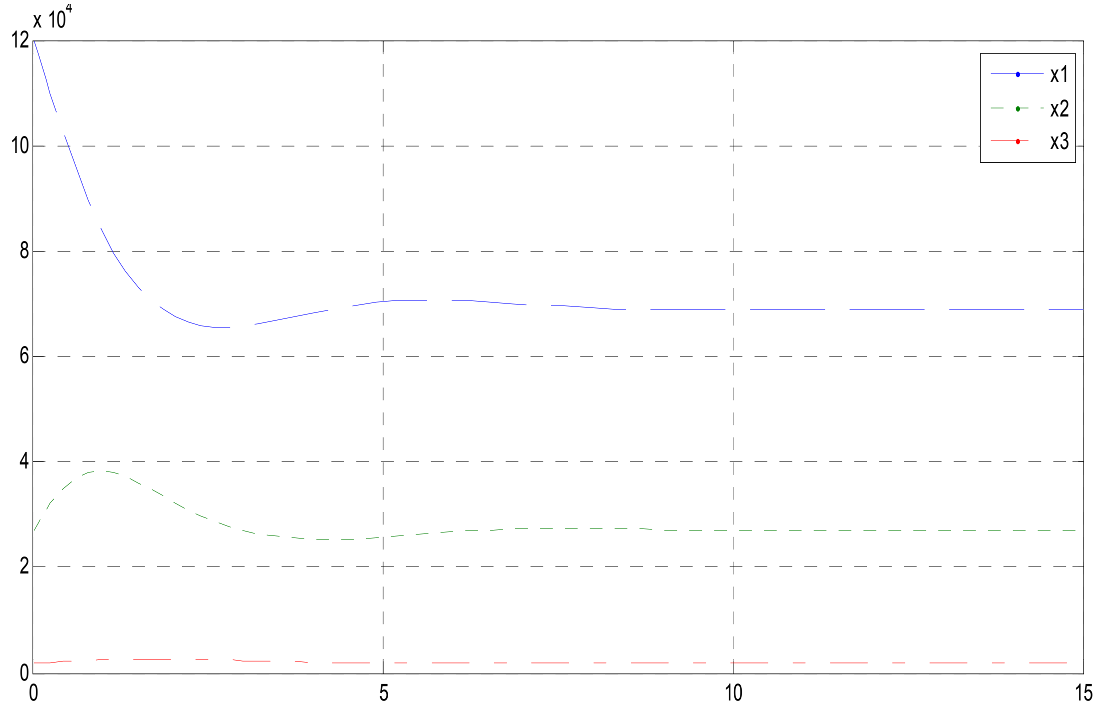

Figure 2. Numerical solutions for $x_i(t)$.

We apply expert system and group decision theory to determine the weights of the species in Population 2. We have a multi-attribute decision problem, where the aim is to select the optimal solution from many alternatives or to sort the available alternatives.

Assume that the finite solution set is $Y = \{y_1, \ldots, y_n\}$, the attribute set is $C = \{c_1, \ldots, c_q\}$, and the decision expert set is $E = \{e_1, \ldots, e_m\}$. Let $S = \{s_1, \ldots, s_g\}$ be a predefined set consisting of odd-chain elements. Expert e_k selects one element from S as the value of solution y_i under attribute c_j; let it be denoted as $p_{ij}^k \in S$, and let

$$p^k = \left(p_{ij}^k\right)_{n \times q}$$

denote the judgment matrix of expert e_k on all the solutions for all the attributes. The attribute weight vector in evaluating information given by expert e_k is

$$W^k = \left(w_1^k, \ldots, w_q^k\right)^T,$$

where w_j^k is the weight of attribute c_j selected by expert e_k from set S, $w_j^k \in S$.

This theory can be actualized through the Analytical Hierarchy Process (AHP), first put forward by American operational researcher T.L. Saaty in the 1970s. AHP is a method for decision-making analysis that combines qualitative and quantitative methods. Using this method, decision-makers can separate complex problems into several levels and factors, and compare and find the weights for different solutions, and provide the basis for the optimum solution.

AHP first classifies the problem into different levels based on the nature and the purpose of the problem, constructing a multilevel structure model ranked as the lowest level (program for decision making, measures etc.), compared with the highest level (the highest purpose). Based on AHP, we can establish the stratification diagram shown in **Figure 3**.

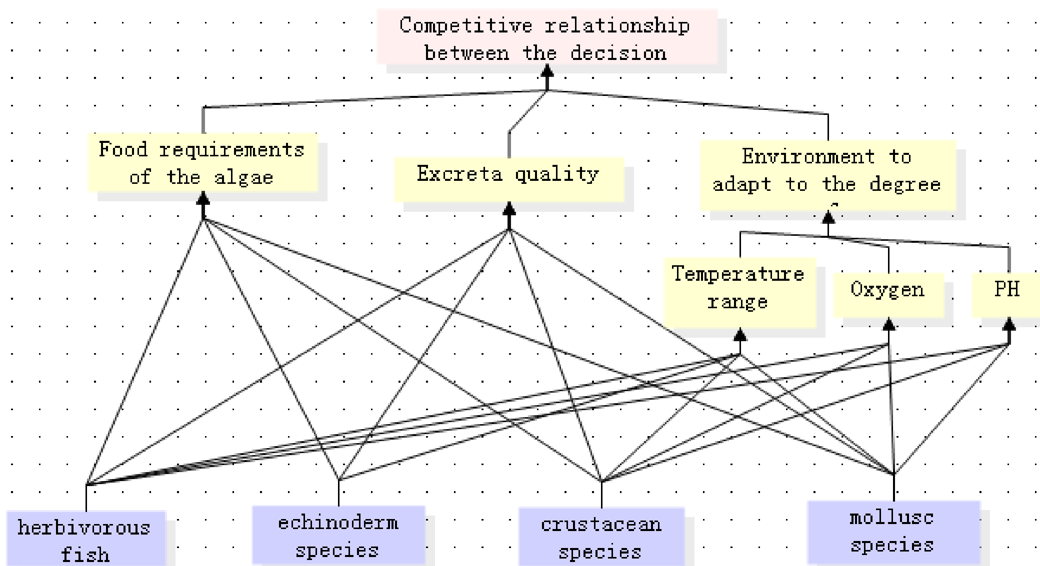

Figure 3. AHP stratification diagram.

At last, we make consistency check of the result, finding that the consistency ratio of each expert's judgment matrix is below 1, so the consistency of the judgment matrix is acceptable. Finally we figure out the weight of the numbers of all the species in Population 2, as shown in **Table 1**:

Table 1.
Weight of each species in Population 2 as measured by AHP.

Species	Weight
Herbivorous fish	.21
Crustaceans	.23
Molluscs	.31
Echinoderms	.24

Here we adopt population competition model to confirm the weight of each species in Population 2:

$$\dot{N}_1 = N_1(\varepsilon_1 + \gamma_1 N_2),$$
$$\dot{N}_2 = N_2(\varepsilon_2 + \gamma_2 N_1),$$

where ε_i are birthrates and γ_i are coefficients of species interaction.

According to these equations, we find that the ratio between different species is almost consistent with that obtained by AHP, which also confirms the correctness of our method.[1]

In this way, we find that herbivorous fish, crustaceans, molluscs, and echinoderms can coexist and also compete. So the number of each species can be figured out based on the data in the steady state from the previous models, as shown in **Table 2**.

Table 2.

Number per pen of each species in steady state.

Organism	Number
Algae	69,027
Herbivorous fish	5,638
Crustaceans	6,305
Molluscs	8,483
Echinoderms	6,589
Milkfish	1,760

Now we use the model to check the water quality, and make clear whether it is suitable for the continued healthy growth of the coral. First, we calculate the current concentration of chlorophyll in a fish pen. With help of relevant references, we find the regression equation between the number of algae and chlorophyll:

$$N = 1.2785 + 0.7568C, \tag{1}$$

where the units are 10^4/ml for N (algae) and μg/L for C (chlorophyll). For $N = 6.9027$ (from **Table 2**), we get $C = 7.43$, a concentration of chlorophyll that is far beyond 0.25 μg/L, the highest suitable concentration for the growth of coral.

From the available data in the problem, we figure out the mass of organic particles in the fish pen, and then work out the mass of each element.

- The dry weight of echinoderms in the pen is 45.5 kg, the dry weight of milkfish excrement is 0.4–0.9 kg, so the total dry weight of excrement in the pen is 1.0–1.4 kg.
- The pen is 10 m $\times$ 10 m $\times$ 8 m, for a volume of 800 m^3 = 800 $\times$ 10^3 L.
- Finally, we get the concentration of organic particles is 1186–1738 μg/L. Based on the percentage of elements given in the problem, we figure out then concentration of carbon C (10%), nitrogen N (0.4%), and phosphorus P (0.6%) (**Table 3**).

[1] EDITOR'S NOTE: The authors' paper does not give further details of the AHP calculation nor of the population competition model.

Table 3.
Concentrations of elements in a pen.

Element	Concentration (μg/L)
C (10%)	119–174
N (0.4%)	5– 7
P (0.6%)	7– 10

Comparing the water quality in Sites A, B, C, and D, we find that the concentration of organics is between A and B, which is suitable for the growth of coral (here the concentration of elements is calculated only based on the excrement of milkfish and echinoderm), so the concentration of the microbes meets the reproduction needs of the coral. But the concentration of chlorophyll is seriously out of limits. So we have to adjust the numbers of some species to make the concentration of chlorophyll reach the standard.

We reason backward from the desired concentration (0.25 μg/L) of chlorophyll suitable for the growth of the coral, using the regression equation **(1)**. With the estimated steady-state value, we can assume the initial values as: (10000, 5500, 350). From relevant references, we get the maximum volume of fish pens : $(N_1, N_2, N_3) = (30000, 6000, 400)$, and through resimulation finally find the positive revised results for the steady-state values: (13732, 5432, 320).

We work out the estimated steady-state number of algae $N = 1.4677$, and then derive the numbers of the three populations: (14677, 5744, 350). After revision, we get the actual steady-state number of the algae: $N = 1.3732$. Putting this value into the regression equation, we get $C = 0.125$, that is, the concentration of chlorophyll is 0.125 μg/L, which means that the water quality after adjustment completely meets the standard demanded. Moreover, the total number of milkfish and echinoderm is smaller than that before revision, so the index of the organics can certainly reach the growing demands of the coral, as shown in **Figure 4**.

In the retroregulation process, which is the feedback mechanism of this model, with known water quality, we reason backwards to the estimated steady-state numbers of all the species, make positive simulation after estimating the initial introducing value of all the species, and get the revised steady-state values. With this mechanism, we can find out the steady-state number of each species based on water quality, which provides great convenience to the solution to the following problems.

Task 2a: Establishment of Logistic Model

In this task, with all the herbivorous fish, crustaceans, molluscs, and echinoderms excluded, we are required to find out the changes to the species and the circumstances of water quality. Based on our analysis, we make

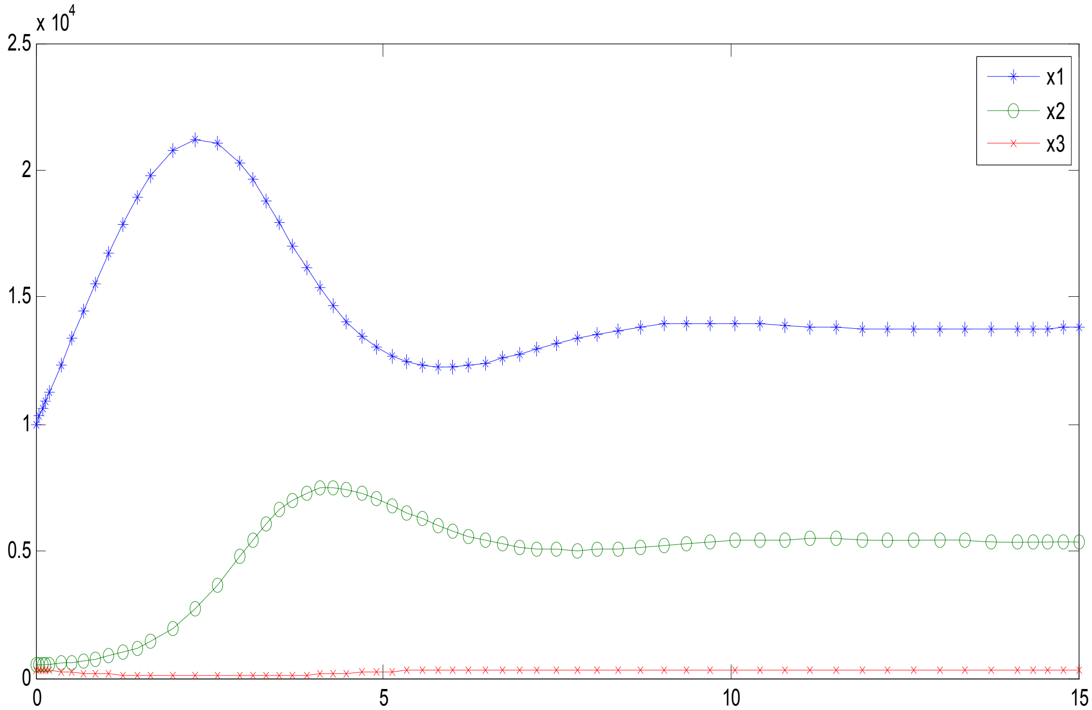

Figure 4. The numbers of species meeting the demands after adjustment.

clear the reasons why the growth rate will decrease after the milkfish increase. Factors such as natural resources and environmental conditions restrict the growth of milkfish; and with their growth, the blocking effect will become greater and greater. The blocking effect is expressed in terms of the influence on the growth rate r of milkfish, making r decrease with the increase in the number x of milkfish. If we express r as $r(x)$, a function of x, it should be a decreasing function, so we have:

$$\dot{x} = r(x), \qquad x(0) = x_0.$$

The simplest assumption of that $r(x)$ is a linear function:

$$r(x) = r - sx \qquad (r > 0,\ s > 0),$$

where r is the intrinsic growth rate. To confirm the meaning of the coefficient s, we introduce the maximum quantity x_m that is allowed by natural resources and environmental conditions, which we regard as the milkfish capacity. When $x = x_m$, then x will stop increasing, that is, the growth rate $r(x)$ will be 0. That occurs for $s = r/x_m$, so that we have

$$r(x) = r\left(1 - \frac{x}{x_m}\right). \tag{2}$$

Another interpretation of **(2)** is that the growth rate $r(x)$ is in direct proportion to the unsaturated part of the milkfish capacity $x = (x_m - x)/x_m$, where the proportionality coefficient is the intrinsic growth rate r. Putting **(2)** into **(1)**, we get

$$\dot{x} = rx\left(1 - \frac{x}{x_m}\right), \qquad x(0) = x_0. \tag{3}$$

The factor rx on the right side expresses the internal growing tendency of the milkfish, and the factor $(1 - x/x_m)$ expresses the blocking effect of resources and environment on milkfish growth. Obviously, the bigger x is, the bigger rx is, and the smaller $(1 - x/x_m)$ is. The growth of milkfish is the result of the co-action of the two factors. Equation **(3)** can be solved by separation of variables to yield

$$x(t) = \frac{x_m}{1 + \left(\dfrac{x_m}{x_0} - 1\right)e^{-rt}}. \tag{4}$$

We use linear least squares to estimate the parameters r and x_m of this model, and express **(3)** as

$$\frac{\dot{x}}{x} = r - sx, \qquad s = \frac{r}{x_m}.$$

We consult relevant data in Sumagaysay-Chavoso [1998] (where the amount of milkfish is the amount harvested over the entire Philippines), insert these data into Matlab, and get $r = 0.5$ and $x_m = 1.9 \times 10^5$. Putting these into **(4)**, we get the changes to the function shown in **Figure 5**.

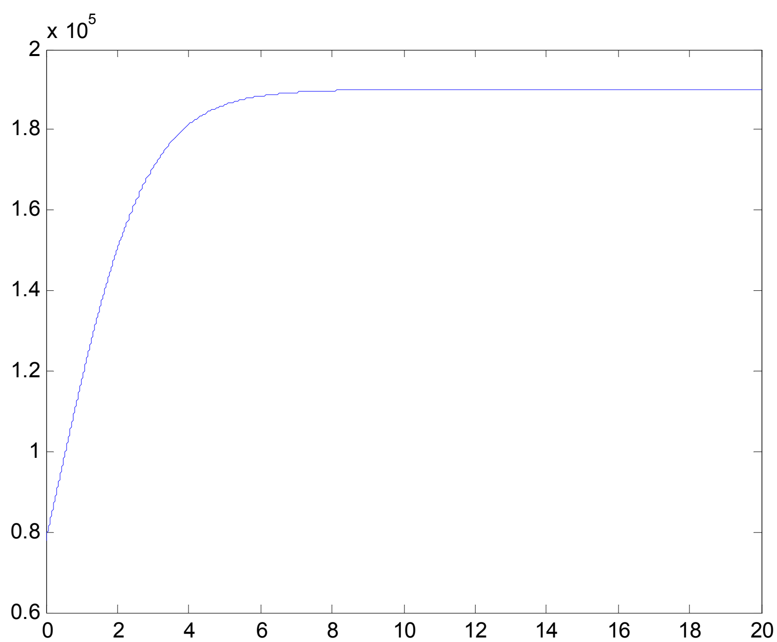

Figure 5. Milkfish changes.

Further, we get the weight and number of the milkfish respectively as 172×10^6 kg and 25–34 $\times 10^6$.

The land area of the Philippines is 300,000 km^2, the sea area is 27.6 mi^2. The Philippines is surrounded by the sea, and has lots of islands; the depth of the sea between islands is mostly within 50 m.

Based on the sea area, we calculate the sediment per square meter to be 0.12–0.33 g/m^2. Since the sediment is usually not very thick, we assume that the depth is 0.1 m, so that the sediment per cubic meter is 1.2–3.3 g/m^3. Then based on the information given in the problem, we get the results of **Table 4**.

Table 4.

Element concentrations.

Element	Concentration (μg/L)
C (10%)	117–333
N (0.4%)	47–133
P (0.6%)	70–200

From the table, we can see eutrophication is very serious, and the coral cannot grow. The water quality is very poor, which almost matches the environment in the pens.

Task 2b: Simulating Comparison of the Current Situation

In Task 2a, we discussed the independent farming of the milkfish; but actually in the pen, there are more than just milkfish and algae. So here we have to introduce the removed species as the middle strata, and according to the requirements of the problem, adjust the numbers of the species in the middle strata to simulate the water quality in the Bolinao area until the water quality matches the one currently observed.

The concrete practices are as follows: Simulate the water quality (in Site D, for example) and solve the problem according to the model in Task 1. It is easier to find out the water quality from the initial values of algae, milkfish, and other species than vice versa.

We adopt brute-force random search:

- Set the initial values of algae, other species, and milkfish to 100×10^3, 10×10^3, and 1.3×10^3.
- According to the introductory ratio between the milkfish and the algae, and the requirements for the capacity of the pen obtained from Task 2a, we introduce the algae and the milkfish respectively as 72×10^3 and 1.3×10^3, and at the same time have the introductory numbers of other species come from a random distribution between 8×10^3 and 10×10^3, with the aim of searching for the theoretical value matching the observed water quality.

- Simulate the model in Task 1 1,000 times, and finally output the water quality in steady state that is consistent with the actually observed value.

We set out the criteria for judging water quality:

- Chlorophyll a $\equiv (0.0001x_1 - 1.2785)/0.7568$.
- Total concentration of organics =

$$x_2 \times 0.2438 \times 6.9 \times [0.2, 11.5] + x1 \times [242, 493].$$

- Percentage of different elements in the excrement: C 10%, N 0.4%, P 0.6%.
- C meets $|c(1) - c1(1)| \leq 100$ and $|c(2) - c1(2)| \leq 100$.
- N meets $|n(1) - n1(1)| \leq 10$ and $|n(2) - n1(2)| \leq 10$.
- Chlorophyll meets $|c_a - 4.5| \leq 0.15$.

We sort out results meeting the above requirements, that is, the numbers of three species when the water quality obtained through simulation similar to the observed one, and show the result in **Table 5**.[2]

Table 5.
Simulation results.

		Pop. 1	Pop. 2	Pop. 3
Simulation results	Initial number $\times 10^3$	70.0	[8.01,9.00]	1.10
	Number in steady state $\times 10^3$	46.1	9.0	1.04
Estimated from data	Number in steady state $\times 10^3$	45.7	9.3	0.9

To make the numbers of the species close to those predicted in the model, we compare the numbers of existing species with those observed in Bolinao area. Here we take into account that the added feedstuff for milkfish can revise the model in Task 1, that is, we can add a constant λ to the the third equation of the model in Task 1 to express the influence of feedstuff on the numbers of the species. The revised model is:

$$\begin{aligned}
\dot{x}_1(t) &= x_1 r_1 \left(1 - \frac{x_1}{N_1} - \sigma_1 \frac{x_2}{N_2}\right), \\
\dot{x}_2(t) &= r_2 x_2 \left(-1 - \frac{x_2}{N_2} + \sigma_2 \frac{x_1}{N_1} - \sigma_3 \frac{x_3}{N_3}\right), \\
\dot{x}_3(t) &= r_3 x_3 \left(-1 - \frac{x_3}{N_3} + \sigma_4 \frac{x_2}{N_2}\right) + \lambda.
\end{aligned}$$

We set initial values (70000, [8008,8995], 1100), and calculate the steady-state numbers of all the species: (46062, 8989, 1051), as shown in **Figure 6**.

[2]EDITOR'S NOTE: The accompanying Matlab code does not impose the constraints indicated above on N and C.

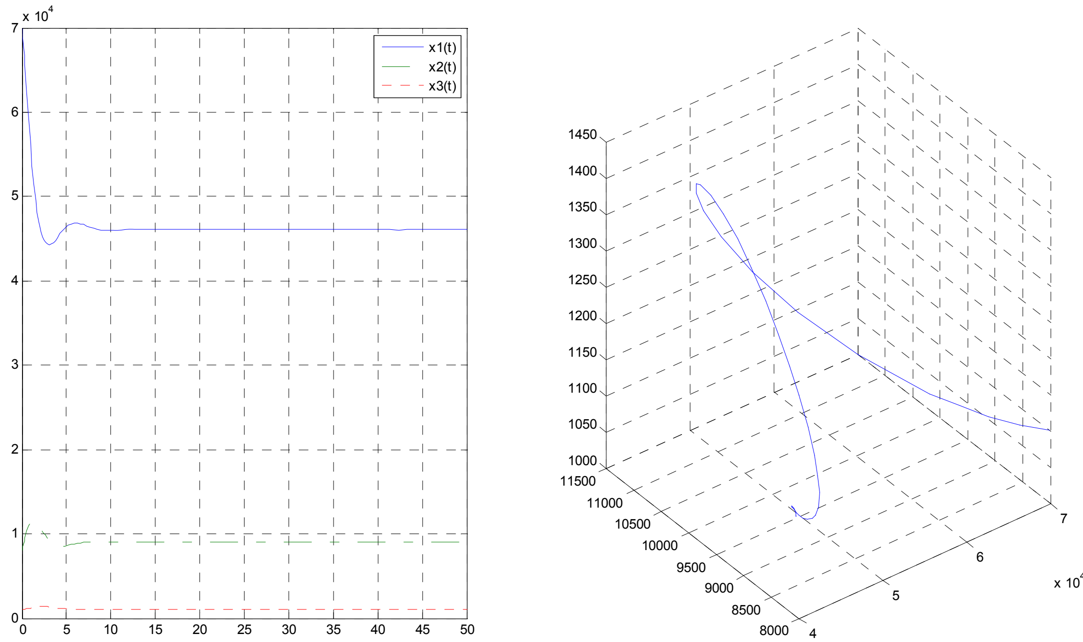

Figure 6. Comparison between observed values and simulated values.

Task 3

Task 3a: Develop a commercial polyculture to remediate Bolinao

We start from the model of Task 1 (the Bolinao coral reef ecosystem model before farming), introduce filter feeders, and revise the model. We renumber the species, with algae as 1, filter feeders as 2, herbivores as 3, and milkfish as 4. Following the same modeling principles as earlier, we arrive at the system:

$$
\begin{aligned}
\dot{x}_1(t) &= x_1 r_1 \left(1 - \frac{x_1}{N_1} - \sigma_{12}\frac{x_2}{N_2} - \sigma_{13}\frac{x_3}{N_3}\right),\\
\dot{x}_2(t) &= r_2 x_2 \left(-1 - \frac{x_2}{N_2} + \sigma_2\frac{x_1}{N_1} - \sigma_7\frac{x_4}{N_4}\right),\\
\dot{x}_3(t) &= r_3 x_3 \left(-1 - \frac{x_3}{N_3} + \sigma_3\frac{x_1}{N_1} - \sigma_8\frac{x_4}{N_4}\right),\\
\dot{x}_4(t) &= r_4 x_4 \left(-1 - \frac{x_4}{N_4} + \sigma_4\frac{x_2}{N_2} + \sigma_6\frac{x_3}{N_3} + \sigma_5 k\right),
\end{aligned}
\tag{5}
$$

where we now use k for the constant of feedstuff.

We solve this system in Matlab to obtain the numbers of algae, filter feeders, herbivorous fish, and milkfish: (14314, 6092, 6129, 6979). **Figure 7** shows the system tending toward equilibrium.

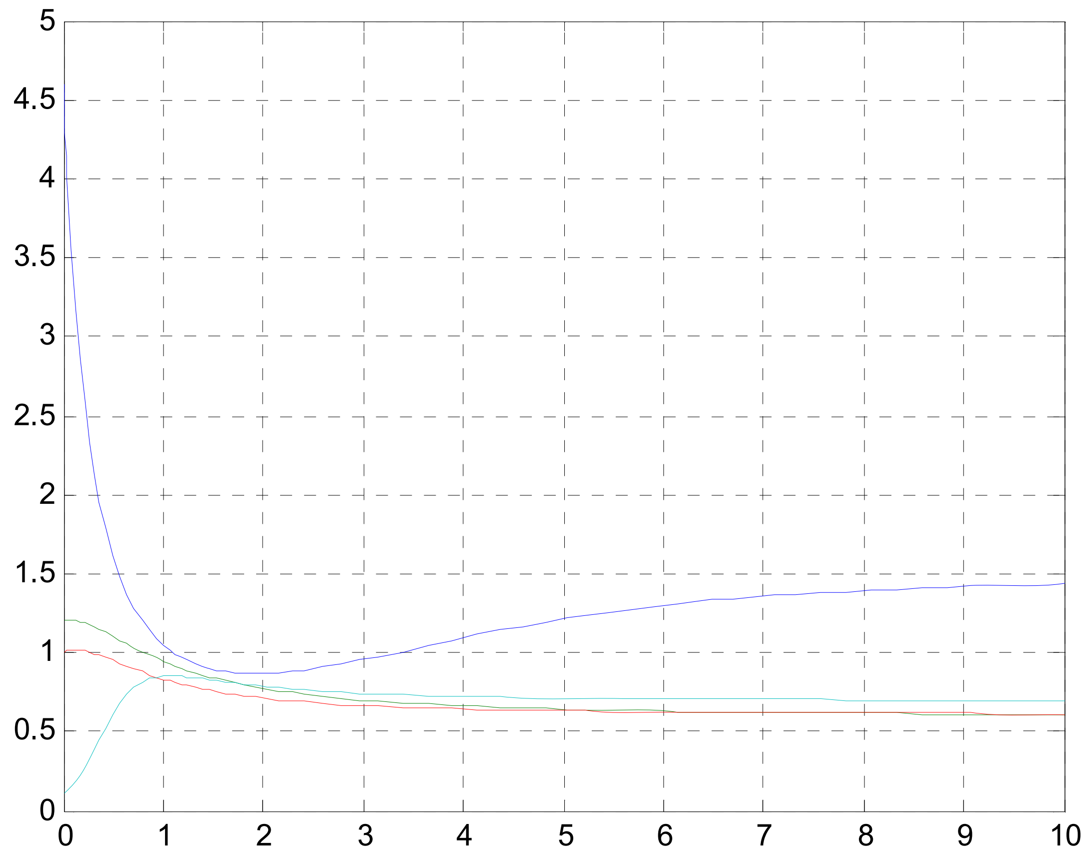

Figure 7. The changes in the numbers of algae, filter feeders, herbivorous fish, and milkfish.

Report on the outputs of the model

Based on **(6)**, we find:

- This model optimize the water quality, since only when the water quality reaches a certain standard, can it provide the ideal growing environment for a species, and only in the viable environment, is it meaningful to talk about the number of each species.
- We establish a newly-born coral reef habitat without the help of man, that is, without feedstuff casting, with least leftover nutriment and particles (foodstuff and excrement) sediment.
- According to Task 3a, we get the steady-state numbers of algae, filter feeders, herbivorous fish, and milkfish. We regard those as the initial values and determine the concentration of chlorophyll as 0.202 μg/L. Based on the information about the elements percentage given in problem, we calculate the content of different elements, as shown in **Table 6**.
- Assume that the total income is $K = \sum x_i v_i$, where v_i is the market value of a unit of species i.
- Based on market investigation and relevant online data, we get the aver-

Table 6.
Concentrations of elements in a pen.

Element	Concentration (μg/L)
C (10%)	35 –72
N (0.4%)	1.4– 2.9
P (0.6%)	2.1– 4.3

age weight and price of each species, and finally figure out the income: $K = \$114 \times 10^3/\text{pen}$.

- To calculate the cost of improving water quality, assume that we introduce 1,000 mussels into the pen. We investigate such factors as weight and market price of mussels, and put them into the model in Task 1 to figure out all the indexes.

Table 7.
Steady-state numbers ($\times 10^4$) of species before adjustment.

	Algae	Molluscs (mussels)	Herbivorous Fish	Milkfish
Before adjustment	1.43	0.61	0.61	0.70
After adjustment	1.37	0.62	0.61	0.70

Table 8.
Concentrations of elements (μg/L) before and after adjustment.

	Chlorophyll	C	N	P
Before adjustment	0.202	35–71	1.4–2.9	2.1–4.3
After adjustment	0.125	33–69	1.3–2.8	2.0–4.1

From **Table 8**, it is easy to see that the water quality has improved. For one thing, the introduced mussels feed on the algae for one thing, and for another they decompose the organic particles.

- The 1,000 introduced mussels cost \$361 or so, scarcely making a dent in the income.

Task 4

From Task 3a, we know the numbers of algae, filter feeders, herbivorous fish, and milkfish: (14314, 6092, 6129, 6979). The algae are the most numerous, and the numbers of the other species are roughly equal. In such a steady state:

- According to the relationship between supply, demand, the price of milkfish is higher that that of seaweed. In addition, although the amount of seaweed is large, it is light, so we cannot pursue maximizing weight.
- Measuring harvest with the price of each species harvested, we have to differentiate the values of the species. Since it costs to feed the milkfish, we should take these costs into consideration when calculating the values of each species. We define the value of edible biomass as the sum of the values of each species harvested, minus the cost of milkfish feed.

Task 5

When evaluating a commercial polyculture scheme, we usually consider not only the economic benefits of farming, but also try to ensure reaching a win-win between economy and environment under the premiss of keeping the ecological environment and water quality in good condition.

Hence, we establish the following optimal model to pursue the maximum commercial benefits, with the premiss of not having water quality worsen. Combined with the previous polyculture system model, we establish the following nonlinear optimization model of balance to maximize the total values of harvest. It is a complex nonlinear single-objective optimization model, since nonlinear differential equations are embedded into the constraints:

Objective function: $\max f = ax_1 + bx_2 + cx_3 + dx_4 - \mu$,
where a, b, c, d are the unit market prices of the species and μ is the feedstuff price.

The constraints on water quality are:

- concentration of of chlorophyll ≤ 0.28 mg/mL,
- concentration of C ≤ 196 μg/L, and
- concentration of N ≤ 39 μg/L.

We can express these conditions in the equations involving the x_i as follows:

$$\begin{aligned} \frac{0.0001x_1 - 1.2785}{0.756} &\leq 0.28, \\ 1.68222x_2[0.2, 11.5] + 0.1x_4[242, 493] &\leq 196, \\ 1.68222x_2[0.2, 11.5] + 0.004x_4[242, 493] &\leq 39. \end{aligned}$$

In addition, we have the equality relations among the x_i in **(6)**.

Such a complex optimization problem cannot be solved directly with any software, so first we make a cycle simulation search (actually still a brute-force search) to find enough solutions meeting water quality conditions, and obtain intervals for the steady-state numbers of the species that meet the demands of water quality, as shown in **Table 9**.

Table 9.
Steady-state numbers ($\times 10^4$) of species.

	Algae	Molluscs (mussels)	Herbivorous Fish	Milkfish
Maximum	1.3922	0.6249	0.6233	0.7061
Minimum	1.3286	0.6152	0.6174	0.7018

Therefore, we can replace the equality conditions among the x_i by intervals for the steady-state numbers:

$$
\begin{aligned}
1.3286 \leq x_1 \leq 1.3922, \\
0.6152 \leq x_2 \leq 0.6249, \\
0.6174 \leq x_3 \leq 0.6233, \\
0.7108 \leq x_4 \leq 0.7061.
\end{aligned}
$$

We can now use Lingo to solve the equivalent model, with the results of **Table 10**.

Table 10.
Optimal steady-state numbers ($\times 10^4$) of species.

	Algae	Molluscs (mussels)	Herbivorous Fish	Milkfish
Optimal	1.39	0.62	0.62	0.71

The corresponding the maximum harvest value is \$115 $\times 10^3$, and the corresponding water quality is shown in **Table 11**.

Table 11.
Concentrations of elements (μg/L) after optimization.

	Chlorophyll	C	N	P
At optimal	0.15	17–36	0.7–1.4	1.0–2.2

Compared to the water quality required by coral growth, the water quality obtained here is obviously satisfactory, and we reap relatively high economic benefits at the same time.

In order to prove the results of our model are correct, we define:

$$
\text{fishing/harvest index} = \frac{\text{feed cost}}{\text{net income}}
$$

Then the result we obtained is: fishing/harvest index = 0.06%. The actual result is: fishing/harvest index = 2.8%.

Based on analyses of the model, for the optimal solution we find the feeding cost for one unit of net income is obviously less than the current cost, so our feeding strategy can produce better harvest.

Ecological Perspective on Polyculture

Adding herbivorous fish as the middle stratum

- contributes to the decomposition of solid particles,
- can suppress the over-multiplication of the algae,
- can improve water quality,
- can enable the coral to grow normally, and
- thereby can restore the ecosystem and biodiversity.

However, in our model we don't take into account the soluble POC released by the algae, the accumulation of which is likely to hinder the improvement of water quality. In view of this, some may doubt the restorative ability of our polyculture system. But bacteria in the waters can process POC and rational measures can be taken to control the concentration of microbes, thus ensuring the improvement of water quality. So, in terms of ecology, our polyculture system bears the potential of improving water quality and promoting favorable development of the ecosystems.

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Striving for Balance: Why Reintroducing More Species to Fish Farm Ecosystem Yields Bigger Profits

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Summary

Demand for animal protein is the root problem that the people of Bolinao, Philippines have experienced over the last 15 years. Past solutions focused on harvesting large quantities of one type of fish using large cages. Unfortunately this approach failed to meet the demand for protein, ruined local water quality, and destroyed the coral reef.

Future technological innovations such as self-powered fish cages, algae-based biodiesel fuel, and radio-frequency identification tracking offer great potential for waste reduction and improved open-water fish harvesting. However, the people of Bolinao cannot wait; change must begin now. We must assist the transition, but ultimately the people of Bolinao are the greatest stakeholders in the future quality of life there.

Mathematics-based models show the various stages of this deterioration by demonstrating how the ecosystem in Bolinao once functioned before demand for fish grew dramatically in the early 1990s. We demonstrate the dangers to water quality of the current practice of farming only milkfish. Finally, we show how introducing other species into commercial fish pens will allow equilibrium to recur, reducing levels of waste in the water and allowing the coral reef (a catalyst for growth) to return.

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Combining the balanced ecosystem with market pricing formulas demonstrates how alternative fish-harvesting practices will lead to higher income for the local population and provide the protein that they need. Fish is the most efficient source of animal protein for humans because it requires less feed to obtain the same amount of protein as chicken, beef, or pork.

A key limit of our models is scant data on prices and on ratios of species necessary to recreate a balanced ecosystem. Still, our results demonstrate to the Bolinao people both the environmental and the economic value of transitioning from producing only milkfish to a more diverse aquaculture.

Finally, we suggest policy changes designed so that the people of Bolinao don't have to choose between getting enough food to eat now and having a healthy environment in the future.

Table 1.

Symbol key.

Symbol	Meaning	Formula
a	algae	
l	blue mussels	
m	milkfish	
r	rabbitfish	
s	starfish	
t	giant tiger prawn	
P_x	current population of species X	
P_{x-1}	population of species X the previous month	
B_x	birth rate of species X	
S_x	survivability rate of species X	$G_x - D_x$
G_x	growth rate of species X	
D_x	death rate of species X	
E_x	rate at which species X is eaten by a predator	
P_a	current population of algae	$S_a P_{a-1} + E_a P_{r-1}$
P_l	current population of mussels	$S_l P_{l-1} + E_l P_{s-1}$
P_m	current population of milkfish	$P_{my} + P_{mo}$
P_{my}	current population of juvenile (young) milkfish	$B_m P_{mo-1} + 0.066 P_{my-1}$
P_{mo}	current population of breeding (old) milkfish	$S_m P_{mo-1} + 0.066 P_{my-1}$
P_r	current population of rabbitfish	$S_r P_{r-1} - E_r P_{m-1}$
P_s	current population of starfish	$S_s P_{s-1}$
P_t	current population of giant tiger prawns	$S_t P_{t-1} - E_t P_{m-1}$
C_d	level of carbon dissolved	$Y_m P_m + Y_t P_t - Y_l P_l$
N_d	level of nitrogen dissolved	$Y_m P_m + Y_r P_r - Y_l P_l - Y_a P_a$
Chl	level of chlorophyll	$Y_a P_a$
C_p	level of particulate carbon	$Y_m P_m + Y_r P_r + Y_s P_s - Y_l P_l$
N_p	level of particulate nitrogen	$Y_m P_m + Y_t P_t - Y_a P_a$
W_x	level of bacteria created by individual species X	
M_x	market price for species X	

Problem Approach

Task 1

To model water quality before milkfish dominated the local ecosystem, we create formulas that model the interactions among the species in the ecosystem. This model focuses on a steady-state equilibrium of water quality. We first establish how to measure the change in water quality, as the sum of the waste products of each species. Some species, such as the blue mussel, which consumes the waste of other species, contribute negative waste and thus help improve water quality. We develop functions to describe the population of each species at any given time; the population determines the waste produced by that species and thus the water quality. The formula for each species calculates the change in the population by adding the number of new individuals (based on the determined growth rate) and subtracting the number eaten by other species as well as the number that die naturally.

We determine a steady state by running the whole model for several iterations until the level of the water quality stabilizes. Adjusting the number of each species in the system while keeping the ratios among species constant should allow prediction of population levels before the disruption of overfishing that led to the commercial milkfish monoculture.

Task 2

We set to zero the populations of all species except milkfish and algae and run the model to determine water quality. Based on the known current water quality, we attempt to determine the current populations of a variety of species.

Task 3

Setting the water quality to an acceptable desired constant, we run simulations of adjusting the populations of species in different combinations that would reestablish an equilibrium polyculture. This polyculture would consume the waste products of the milkfish and keep the growth of algae under control. We expect to determine different combinations for how many of various species would need to be introduced to the sites in the Bolinao region to reestablish acceptable water quality and create coral growth.

Task 4

We determine from data the dollar values for each species.

Task 5

Based on the values from Task 4, we assess which combinations from Task 3 are likely to create the most economic value for owners.

Task 6

We address policy changes that the Pacific Marine Fisheries Council can adopt to assist the Philippines in implementing long-term viability of a self-sustaining ecosystem. These policies center on harvesting all species at rates that keep the milkfish population under control and thus maintain the polyculture.

Assumptions

- The growth rates of species are constant.
- Variability of amount of eggs laid by species is normally distributed.
- Humans are the only predator of milkfish.
- The channel is not a closed system; excess population can emigrate to other reef locations.
- The algae are a mix of cyan bacteria and red varieties (this assumption provides more-realistic results).
- Milkfish stop being omnivores when they mature, after which they eat only other animals.
- It takes five years for milkfish to become sexually mature [Luna 2009].
- An adult milkfish is capable of eating an adult rabbitfish.
- The fish pens currently hold approximately 58.5 million fish.
- Milkfish weigh 500–600 g [Hambrey 1999].
- None of the other five species in the ecosystem model eats starfish.
- Rabbitfish waste has the same composition as milkfish waste.
- The prices found in Task 4 are estimates assumed from solitary sources.
- Giant tiger prawns spawn nightly at a rate of 7.6% to 9% but only half of spawn hatch [Bray and Lawrence 1998].
- Giant tiger prawns have a mortality rate of 10% to 40% and an average weight of 106 g [Bray and Lawrence 1998].
- Rabbitfish double in population every 1.4 to 4.4 years.

- Prawns excrete 0.028 mg of ammonia per gram of body weight per hour [Burford and Williams 2001].
- Molluscs urinate up to 45% of their body weight per day.
- Each year, 55% of blue mussels die.
- Female mussels release 1 million eggs semi-annually, of which 30% hatch.
- Japanese starfish release 10 million to 25 million eggs per year.
- A starfish has an average lifespan of 3 years.
- A starfish eats 36 g of mussels each month.

Task 1: Water Quality before Disruption

For a long time, the amount of fish in the area was more than adequate to meet the needs of the population. However, as people sought better nutrition by eating more fish protein, they fished more intensively, using dynamite and sodium cyanide, until the local population of wild fish was no longer large enough to sustain itself. These techniques killed off not only milkfish but other species that kept the ecosystem in balance. The resulting uncontrollable growth of algae, in combination with the destruction caused by explosives, destroyed parts of the coral reef by depriving it of the nutrients and sunlight needed for it to grow. The people built the milkfish population back up by introducing them in large numbers and keeping them in large cages where they could be fed until they were large enough to harvest. Using better-quality fish feed allowed the milkfish population to grow more quickly but also increased pollution in the local waters as a result of the fish waste. Previously, other species, such as the blue mussel mollusc (which feeds on the waste of milkfish), kept water pollution in check. Other herbivorous fish, such as the rabbitfish, and echinoderms, such as the starfish, helped contain algae growth. The starfish also ate the blue mussels. As seen in **Figure 1**, the food web of this ecosystem allowed for different species to coexist in certain ratios to one another, which kept the water clean and allowed the coral reef to grow.

By allowing special feed to replace the natural diet of the milkfish, the people unknowingly depleted the quality of the local water supply while simultaneously destroying the coral reef. This coral reef had served as a catalyst for the growth of the overall system by providing shelter for certain species from their predators.

By modeling the earlier stability, it is possible to show what levels of different populations were previously required to maintain a balanced ecosystem. These ratios can then serve as a helpful starting point for re-establishing a new balance within commercial milkfish farms.

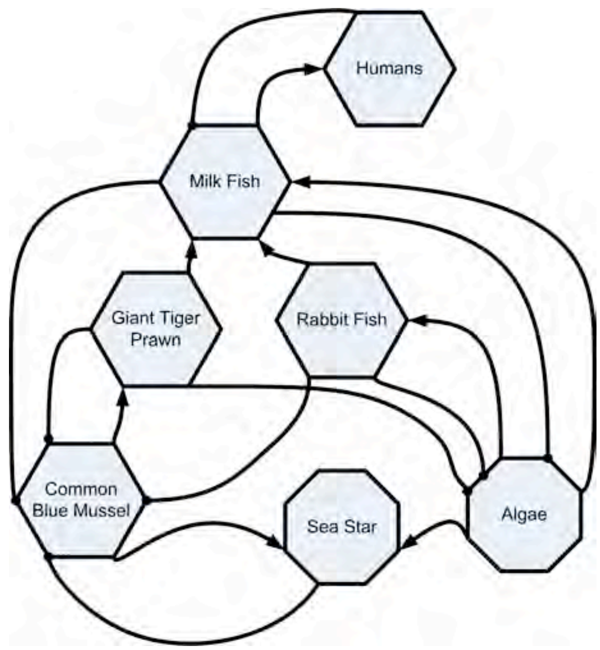

Figure 1. Food web.

To produce this model, we researched the relationships among the various species and determined appropriate rates of population growth patterns. We use a general formula to calculate the current population P_x of species X, given the population P_{x-1} of X in the previous month, the growth rate G_x, the death rate D_x, and the amount $E_y P_y$ of X eaten by each other species y in the system:

$$P_x = P_{x-1} + P_{x-1} G_x - P_{x-1} D_x - \sum E_y P_y.$$

We obtain the overall bacterial level in the water as the sum over all species of its population P_x times its rate W_x of bacteria waste production. The same calculation applies to calculating levels of all waste products (C_d, N_d, Chl, C_p, N_p).

Our model, executed for enough iterations, should have converged to an equilibrium for water quality; but it did not. The main reason was that our model set the growth rates and death rates to remain constant, which does not occur in nature due to the conservation of mass. An example of the more natural trend of this relationship is depicted in **Figure 2**. As the fish population increases, the rate at which they are eaten increases, so the rate at which they survive decreases.

In any closed system, the overall mass of the system must stay the same. Thus, the addition of any new member to the system precludes the growth of something else either immediately or in the future. An example is that when the fish population is larger, the death rate should be greater at some point because fish are more easily caught by their predators.

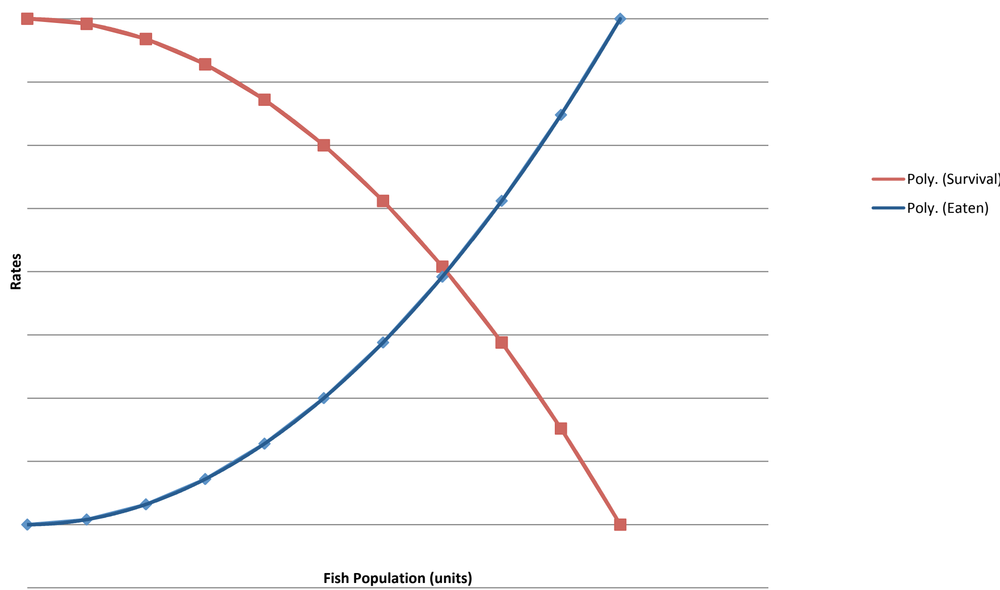

Figure 2. Change in rates due to population change.

Our model did not include any upper limit on the population of any of the species within the ecosystem. So over time, the population of all organisms continued to grow at similar rates, and water quality never reached an equilibrium value. In reality, there has to be a natural limit, if for no other reason than that if the fish waste grows uncontrollably, it will eventually occupy all of the space, choking off nutrient access.

One possibility would be to introduce an assumed limit to the ecosystem by confining the space to the Bolinao region. The water area of Bolinao covers 1170 ha. Based on the limit in the problem statement that the farmers currently use 50,000 milkfish to a pen and operate 10 pens per hectare, a natural limit is 585 million milkfish (500,000 milkfish/ha $\times$ 1170 ha). Assuming this upper bound, we can base the growth rate from a factor of the difference between the current population of milkfish and the upper limit of 585,000,000, via the formula $G_x(585000000 - P_m)$.

Despite the difficulty in achieving steady-state equilibrium of water quality, we still produce a model that demonstrates the general trend that should have been present in the ecosystem before mass-farming of milkfish.

Task 2: Current Water Quality

Poor water quality and the destruction of coral don't really seem like problems to people who are trying to meet basic needs and keep their children healthy. It is difficult to show people how their actions now are ultimately leading to greater problems for them and their children in the future. The current thought process is that growing just one type of fish (milkfish) and feeding them specially formulated fishmeal creates the larger amounts of fish necessary to meet growing demand. Doing so also doesn't require the sustenance of a variety of different creatures. Why is it not possible simply to apply modern agriculture methods to aquaculture? Why shouldn't Filipinos continue to increase the yield of milkfish with specially-designed fishmeal, just as a farmer in America's Midwest increases the yield of a soybean or corn harvest by using specially-formulated seed and fertilizer?

Initial observations may lead to the conclusion that such an approach is both viable and desirable. After all, why not simply remove the excess fish waste and sell it as fertilizer for local farmers? That might be possible. However, just as land farmers eventually realized that growing certain crops year after year leads to decreased yields because of nutrient depletion in the soil, fish farmers encounter the threat of decreased overall yield because growing only milkfish depletes water quality by causing algae and waste to grow uncontrollably. The excess algae reduce coral growth in the same way that lack of crop rotation depletes the soil of nitrogen. Both conditions appear to offer better results in the short term but destroy the longer-term viability of the system. Still, for people to change behavioral practices, it is important to demonstrate the limiting effects of the current system. For our model, this requires showing that farming only milkfish causes water quality and the amount of harvestable fish to decline.

To model the current system, we took our model from Task 1 and set the values for the populations of everything but milkfish and algae to zero. **Figure 3** shows the decline in water quality over time. The rise in algae population chokes off the viability of the milkfish because of the increased oxygen demanded by the algae and consequently the decreased quantity available to the fish.

However, it is unrealistic to assume the current system consists only of milkfish and algae. We know that the current system has a water quality of 10^{10} bacteria/ml and 15 μg/l of chlorophyll, both of which are much greater than the suggested 0.5–1.0 $\times 10^6$ bacteria/ml and 0.25 μg/l of chlorophyll suggested to be acceptable for adequate coral growth.

Coral growth acts like a skyscraper in that it allows more fish to grow in a given space through vertical partitioning. Therefore, we gradually adjust the populations of the various species in our model to achieve the level of current water pollution in Bolinao. Again, our model is unable to produce a steady-state equilibrium of water quality when the ecosystem consists of only milkfish and algae, because the algae do not entirely dispose of the waste from the milkfish; without another species such as blue mussels to reduce the waste of the milkfish, the milkfish grow uncontrollably, even if the 20% that mature each year are removed by humans after reproducing. If humans harvest also immature milkfish, the level of milkfish will drop below sustainability. This human harvesting can reduce the level of waste in the water somewhat, although it is insufficient to achieve a steady state

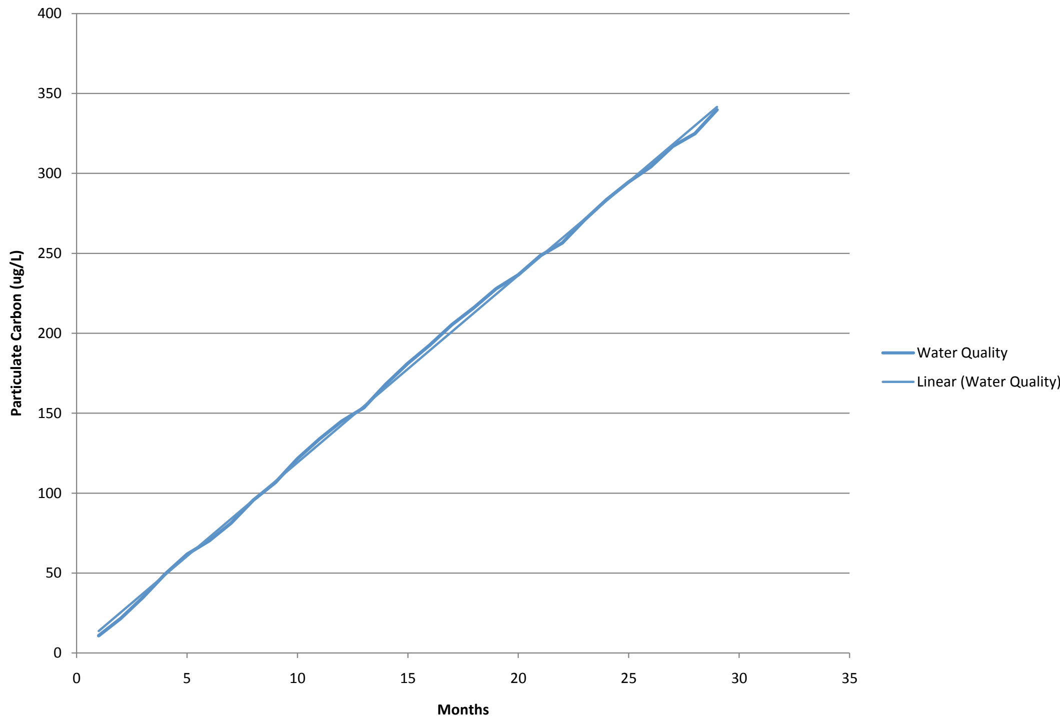

Figure 3. Water quality when only milkfish are present.

because there is still nothing to reduce the waste except the algae—which will grow uncontrollably to consume the milkfish waste, thus raising the level of chlorophyll to the point where it chokes off the sunlight and nutrients needed for the coral reef to grow [Environmental Protection Agency 2004]. While it is possible to reduce the levels of waste through harvesting, doing so will only reduce the rate at which the waste level of bacteria grows (a more gradual slope), not cause it to decline.

Task 3: Water Quality of a Polyculture

Before the farming of massive quantities of milkfish in pens, there was a balanced ecosystem of a variety of species that coexisted in ratios that allowed the waste of certain animals to serve as food for others. However, the demand for milkfish led to a disruption of this balance. The ecosystem is not as ideal as it once was, as we modeled in Task 1; but it is not as bleak a situation as the milkfish monoculture that we modeled in Task 2. The second model in that task shows that the quantities of other species in the current system are insufficient to reach target levels of water quality—ones that would maximize the value of biomass available for harvest by restoring the natural catalyst of coral growth. The coral serves as protective shelter for all of these species. Coral grows very slowly, on average only 80 mm / yr [Roth 1979].

By determining the quantities of species required to reach the desired water quality of 0.5–1.0 $\times 10^6$ bacteria/ml and 0.25 μg/l of chlorophyll, it is possible to increase the overall yield of fish available for harvest while recreating a polyculture that is sustainable. Through modeling this process, we determine how to recreate the stable ecosystem present before commercial milkfish farming. This process will also reduce the cost of overall feed for the milkfish, since they can eat some of the other species.

By fixing the goals of acceptable water quality as the output of this model, we determine what combinations of populations of the species could be self-sustaining. Still, this practice requires guidelines for harvesting only a portion of any species, so as to prevent recreating the overfishing problem that was the cause for the rise of commercial fish-farming, which created the issues with water quality and coral reef destruction in the first place.

Re-establishing the balance that occurred in the region under the conditions present in the model from Task 1 is difficult. It requires introducing other species into the commercial fish pens that help to keep the other populations under control. However, our model demonstrates the pattern of what would occur to waste levels over time if such a combination is attempted. This process was possible by taking data from Internet sources to determine sustainability rates for each of the species and then adjusting the populations of each species to achieve the desired water quality levels. The results of this model rely heavily on increasing the population of blue mussel to control the waste levels of bacteria from the growing milkfish population. The downward trend in the level of bacteria present in the water is depicted in **Figure 4**.

In a few years, the population of blue mussels almost entirely eliminates the bacteria waste. Similarly, rabbitfish reduce the level of chlorophyll through consumption of algae, a process that provides more sunlight and nutrients for coral to grow again [Capuli and Kesner-Reyes 2008]. The milkfish keep the rabbitfish under control, and tiger prawns provide the milkfish an alternative food source so that the milkfish don't wipe out the rabbitfish population. Moreover, starfish consume the mussels to keep them from growing uncontrollably.

The reproductive rate of starfish can vary widely. If an overpopulation of starfish occurs before blue mussels can grow sufficiently, the waste levels of bacteria can grow upward exponentially because the blue mussel is not yet able to sustain its own survivability. Thus, the process requires a reduced presence of starfish early in the biodiversity effort and a greater number of blue mussels. After about six to eight months, the mussels have grown enough that more starfish can gradually be introduced. If the starfish reproduce too quickly early, it may be necessary to add more blue mussels periodically, because there is no effective control on the starfish population.

Our model requires introduction of certain quantities of starfish, rabbitfish, blue mussels, and giant tiger prawn to re-establish a sustainable polyculture that would support the milkfish while improving water qual-

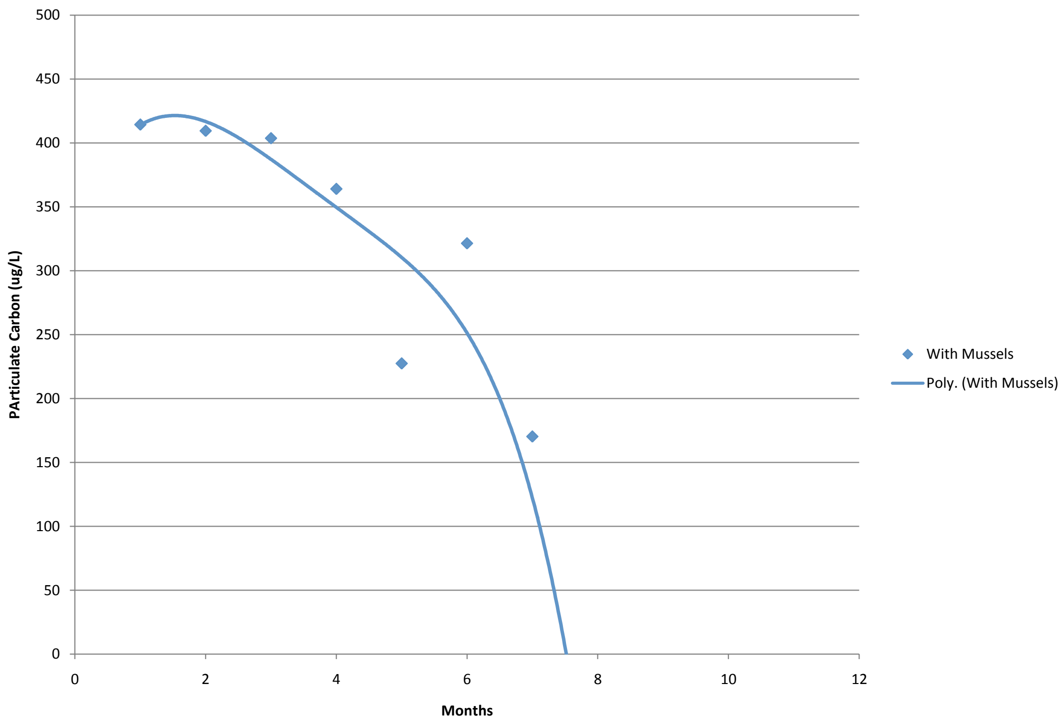

Figure 4. Water quality with mussels.

ity and coral growth. It also requires harvesting guidelines so that the system can maintain itself naturally. The goal is to keep the harvesting guidelines above the demand for milkfish, so that overfishing would become economically undesirable by creating excess supply above the level of demand.

The specific equations used to determine the water quality levels are given in **Table 1** on p. 142.

Task 4: Valuing Polyculture for Human Consumption

Showing how a milkfish monoculture is undesirable for the long term is an insufficient argument to change a population's practices. We must also demonstrate how it benefits the population economically to change their practices now.

As part of Task 3, we modeled and demonstrated what input quantities of other species would establish a self-sustaining polyculture that yields more harvestable biomass over the long term. However, those inputs come with short-term up-front monetary costs, in addition to longer-term costs in the form of restrained harvesting guidelines.

To demonstrate the benefits of these changed practices, it is important to clarify the time required for water quality to improve and coral to grow again. It is also necessary to demonstrate how this growth will lead to more money for the population than continuing to farm only milkfish. This process requires setting a value on coral growth as well as on the harvestable fish in the system. We therefore sought to explain the values for different types of species and why these various species as a whole could produce a greater overall value of income for the population than simply growing milkfish.

On the simplest level, besides being unsustainable over the long term because of depletion of the natural resources in an area, growing only milkfish is undesirable because an excessive supply of milkfish only makes the value of each additional fish worth less. By harvesting a polyculture of species with economic value to both the local and global population, the people of the Bolinao region have the potential to make more money and raise their standard of living over both the short term and the long term. Through diversification of risk, this policy also reduces the likelihood of a farmer losing an entire stock to disease.

We established that coral reef growth creates a value of \$52,000/km^2 and that each square kilometer of coral reef could produce 20 tons of fish biomass overall [White et al. 2007]. Giant tiger prawn are worth \$6,400 per ton [Bray and Lawrence 1998]. Blue mussels yield much less at \$1,000/ton. Starfish yield \$2,200 and rabbitfish \$4,600 a ton—although it is difficult to believe that such an herbivorous fish would be more valuable than the \$1,280 for milkfish. Unfortunately, the pricing of most of these products was very difficult to obtain and estimates vary greatly.

Our model from Task 3 was only able to yield general combinations of the ratios required in a biologically-diverse polyculture ecosystem. A pie chart of the combination that worked well to achieve acceptable water quality is depicted in **Figure 5**.

It is difficult to produce the exact optimal market value of the new system and thus conclusively show the desirability of transitioning from the current system. However, the high price of giant tiger prawn over milkfish makes it an attractive alternative. Growing additional blue mussels, while they may not be worth as much as milkfish, is desirable because the reduction in waste levels they creates allows for more milkfish to be grown in the same area. Algae can be sold in smaller quantities to produce what is now \$18 to \$30 a gallon biodiesel [Morton 1998].

Hopefully, the global production of a wider variety of seafood produce would create pressure for a more transparent and standardized market for seafood commodities similar to the markets that already exist for cattle and grain. Such a market would allow for better research on the desirability of making certain adjustments to various species.

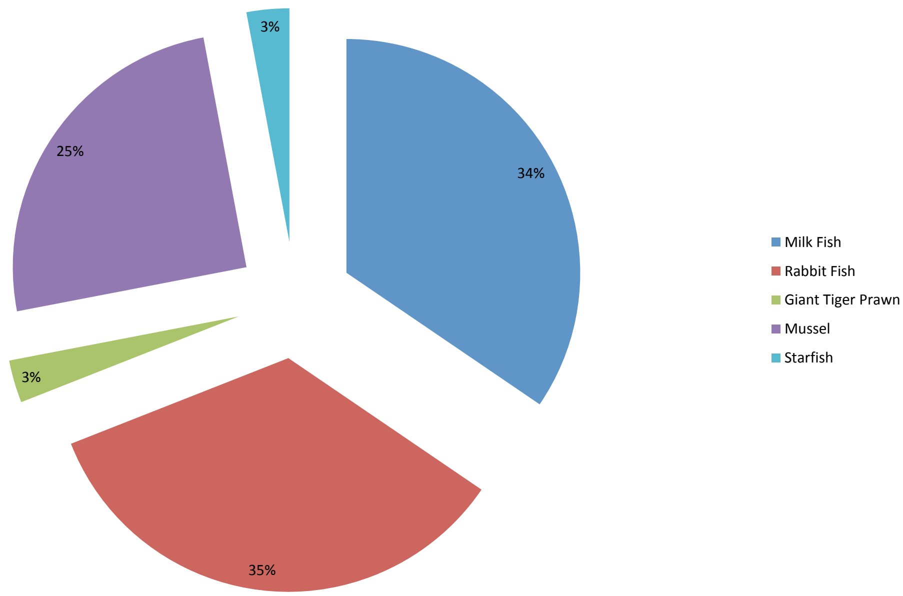

Figure 5. Optimal polyculture proportions (without algae).

Task 5: Maximizing Bioproduce

One of the great difficulties in getting the population of Bolinao and commercial fish farmers to change their milkfish monoculture is to show tangibly how a polyculture would not only improve water quality and coral growth but would also give them greater income. Different proportions of the populations of species in a polyculture may produce the same water quality, but they are not all equal in economic value. In Task 4 we established estimates for each species in the polyculture. By taking the proportions of these populations from the model in Task 3 and multiplying the harvestable population of each by its price from Task 4, we can estimate the revenue from that polyculture. By subtracting the associated input costs for establishing that polyculture, we can compare various polyculture combinations that meet the desired water quality levels and choose one that ultimately maximizes profit for the fish farmer.

Unfortunately, although we were able to develop such a model, its accuracy is questionable, because of uncertainties in current and future market prices.

Ultimately, with greater understanding of price changes and levels of pollution at different sites, it would be possible to use our models from Task 3 to determine an optimal harvest strategy is to achieve a unit of water quality. Because water quality is a product of both the level of bacteria and the level of chlorophyll, certain sites may produce more value when the combination of different species is tailored to reduce more of one of those two water contaminants.

Task 6: Changes to Reestablish a Balance

Our policy recommendations result from understanding the interactions of the various inputs in the polyculture ecosystem. Principles such as inclusion of additional blue mussels to reduce waste levels are helpful to improve initial conditions and reduce waste. Even if the Bolinao people ultimately reject a transition to a polyculture, at the very least they should attempt to improve the water quality of the milkfish pens by using scoops to remove fish waste, which can be recycled and sold to local land farmers. Simple ideas and principles the value of local community-based education and policing efforts are helpful in improving local quality of life for all Filipinos regardless of the approach they choose. We have highlighted many of these pragmatic practices in a letter addressed to the Pacific Marine Fisheries Council, which we believe is the best avenue to suggest these ideas to the people in the region.

Conclusion

At first glance, Bolinao appears to have a problem of coral-reef destruction and water-quality deterioration based on the overproduction of a single type of fish. However, when examined more closely, the real issues are much more personal.

Filipinos are not farming large amounts of milkfish because they seek to destroy their living environment. Their milkfish monoculture practices stem from a growing need for animal protein, of which fish is the most economical and accessible source for the people in their country to produce. Fish offers the highest yield of food for raw weight at 65% while simultaneously requiring the lowest amount of feed input to achieve a kilogram of animal protein. What Bolinao is really struggling to deal with is the very human problem of meeting a growing need for better nutrition and quality of life for their children.

To break this cycle of short-term economic gain at the expense of gradual environmental destruction of both accessible water quality and coral growth, it must be demonstrated to local farmers that their current milkfish monoculture does more harm than good and that an alternative polyculture offers not only a better long-term stability for the environment in the Bolinao region (through improved water quality and coral growth) but also a better economic situation for the local population.

Our solution involves a series of models to explain the past system, the current system, and what a transition in aquaculture practices could create if a future system is adopted. It then focuses on explaining the economic value of the current system and comparing it to the better potential economic value of a polyculture system based on the harvesting of a variety of species, as opposed to the current monoculture focused on harvesting only milkfish.

Our solution offers the double benefit of a more sustainable ecosystem that reduces bacteria and chlorophyll to better levels while allowing for greater coral growth and economic benefit to the local population through greater revenue from the variety of species harvestable from a polyculture.

At first glance, a monoculture of milkfish seems to be the type of specialization that offers the highest economic profit for fish farmers by reducing the unit cost of each fish. However, the long-term sustainability cost of the byproduct waste of a milkfish monoculture is not taken into consideration. Neither is the greater profit that can be obtained through the introduction and harvesting of other species that naturally reduce the economic cost of raising milkfish by reducing the effect of milkfish waste and creating more space to grow additional milkfish. These other species eat the byproduct of the milkfish, which provides more harvestable biomass produce per unit of effort. Finally, a variety of combinations of proportions of different populations in a polyculture produce the same level of water quality.

However, not all combinations yield the same economic profit for the farmer, because certain fish offer a better profit than others and can be raised in larger quantities than in other scenarios. While our model was unable to demonstrate multiple scenarios that provide greater economic profit than the current system, such scenarios do exist and could be demonstrated by our model given a larger data set.

We developed simulations to determine the varying water quality based on the conditions of different quantities of the various species and accounted for the harvesting rates required in order to make these polycultures obtainable. By applying population quantities for polyculture combinations that achieved the appropriate water quality levels to a formula that produced an profit value for that combination, we could have determined which polyculture could provide the most profit to the fish farmer for the desired level of water quality that would allow for successful coral growth and long term sustainability of the polyculture. Furthermore, this increased profit could then be used in global trade to create a wider variety of diet than would be otherwise available.

Our model could be improved through the use of a more complete data set to improve the ratio of relationship between the species to levels closer to what is observable in nature. A more-developed data set would not fundamentally change any of the relationships among the variables in the models we developed. Additionally, more accurately accounting for the human population in the model, and adjusting the harvesting rate of the milkfish based on this inclusion, would also provide more accurate results than trying to extrapolate what the human population should harvest on a periodic basis in order to bring the ecosystem back into balance. This inclusion of the human population into the growth model of the population of the milkfish is necessary because in our ecosystem humans are the only predator of the milkfish, thus making them a requirement for equilibrium to be achieved.

Finally, our model of the economic benefits for the people of Bolinao would be more accurate if we had been able to obtain more complete and recent pricing data for the market value and cost inputs of introducing the other species into the commercial fish farms next to the milkfish.

Despite the shortcomings of our models, we were still able to adequately show the economic and environmental benefit to the region by transitioning from a monoculture of milkfish to a polyculture of biological diversity. One of the biggest contributors to changing this system and reducing bacterial waste was the growth of the blue mussel molluscs. Through this process, our models should convince the people of the Bolinao region of the Philippines to transition from the current monoculture of raising and harvesting only milkfish to a polyculture where they raise and harvest a wider variety of species to obtain the maximum sustainable yield from the ecosystem. With this optimal combination, implemented through the introduction of better farming practices and other species of aquatic life, it is possible to achieve a better result for both the environmental and economic quality of life for the Bolinao people over both the short and long term.

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Dr. Edward Swim (MCM/ICM coordinator at West Point) with team members Sean Clement, Timothy Newlin, and Joseph Lucas receiving their ICM certificates.

Authors' Commentary: The Outstanding Coral Reef Papers

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Introduction

According to the Food and Agriculture Organization of the United Nations, aquaculture is the fastest growing sector of animal-based food production for human consumption. As the global population increases, pressure on coastal ecosystems and the need to produce food also grow. More than half of the world's population lives within 200 km (120 mi) of a coast, and many natural fisheries are already fished at or over capacity. Within this context, the influence of aquaculture on coastal ecosystems is a topic of social, environmental and scientific concern and the subject for this year's problem in the Interdisciplinary Contest in Modeling (ICM)®.

Coral reefs are delicate and valuable ecosystems that only thrive in shallow, tropical, nutrient-poor waters. They cover less than 1% of the ocean's floor but harbor 25% of marine biodiversity. Many people depend on these ecosystems for food, trade, tourism, shoreline protection, and new sources of medicinal compounds. The majority of coral reefs on this planet grow along inhabited tropical coastlines of developing countries. Thus, as an ever-growing number of aquaculture facilities are installed in coastal waters, the interactions between coral reef ecosystems and fish farms are of particular interest.

There are many forms of aquaculture practices, but the more environmentally compatible versions tend to be more costly to set up and operate than their

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less compatible counterparts. Developing methods that are both cost-effective and have a low impact on the surrounding ecosystem is an important issue and a complex and timely challenge. A common method is simply to raise one species of carnivorous fish in pens set directly in coastal waters. Unfortunately, this method causes several environmental problems:

- There is no real barrier between the captive and wild populations, so any disease that occurs in the densely packed pens will flow directly into contact with wild populations.
- No filtration of effluent exists—all excess feed, fish feces, and microbial populations mix directly with natural waters.
- Living organisms can only use 10–20% of the energy they consume, so the other 80–90% goes to waste—raising an organism higher up the food chain (a carnivore) means that several rounds of 80–90% loss occurred to simply make the food that the target species will eat.

These practices are currently happening on and adjacent to many coral reefs. A growing body of scientific literature is demonstrating that these fish farms have a significant negative impact on the corals, and thus major improvements are needed to attain a viable industry and a sustainable coral reef ecosystem.

Formulation and Intent of the Problem

The goal of this year's ICM problem was for student teams to tackle the ecological and technological challenges of improving such practices within the tractable confines of one specific case study of milkfish (*Chanos chanos*) aquaculture directly next to coral reefs in Boliano, Philippines. There are many possible approaches to improving the current situation, but we asked teams specifically to come up with a polyculture scenario that would improve water quality sufficiently for corals to recolonize the areas close to the fish pens where they currently cannot survive. By adding more than one species to the industry, energy inputs can be reduced by growing food for the milkfish locally and water quality can be improved by filter feeders and algae that absorb excess nutrients without requiring major gear or technology shifts. This particular method of more environmentally responsible aquaculture also emphasizes the ecological links between different species and trophic levels. There are a number of potentially negative impacts associated with introducing new species into an ecosystem, so teams were also asked to evaluate the potential risks associated with their polyculture solution.

Teams were first asked to model the original, healthy coral reef ecosystem before the introduction of fish farms. For the purpose of modeling, the complex ecosystem was simplified to one member from each major trophic and phylogenetic guild. The purpose was to identify how the natural system's organisms interact to control water quality in the area.

The second task was to model the current system with the monoculture of milkfish present. Since the natural milkfish food supply was removed by placing the animals in pens, feed must be purchased and added to the system. The idea was to see the effect of exogenous feeding on water quality. They compared the results of their model to actual observed water quality data from Bolinao. Next, teams were asked to model a remediation scenario. They chose the species they wanted to include in their polyculture system and modeled the effects on water quality, harvest, and economic value. They were asked to discuss the harvesting of each species and what parameters they would use to determine the value of the harvest. The last modeling challenge was to maximize the value of the total harvest while maintaining sufficient water quality levels for corals to grow.

The end result of modeling was to write recommendations to the Pacific Marine Fisheries Council regarding the management of the Bolinao milkfish aquaculture industry. This is where the teams evaluated the ecological pros and cons of the species chosen for their particular polyculture system, the economic trade-offs of improving water quality, and how long the remediation of Bolinao coral reefs can be expected to take.

A major goal of this contest problem was for teams to relate the modeling choices they made to realistic ecological and biological processes. Teams were asked to use realistic parameters for their models based on actual ecological and physiological data and to justify any assumptions made. Fundamental understandings of primary production, trophic interactions, and energy transfer were essential for building and critiquing their own models.

This year's ICM problem is based on research being done by the World Bank and Global Environment Facility's Coral Disease Working Group. This international group of scientists has been working to understand the ecological consequences of this fish farm industry on coral health. The first phase of the project was to identify some of the mechanisms by which fish pens are negatively impacting corals. This is the final year of phase one, and much progress has been made. As we enter phase two of the project, we move forward with a goal of testing and implementing alternative methods of farming in this area. Polyculture is one of the alternatives currently being discussed.

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About the Authors

Melissa Garren is currently a Ph.D. candidate in marine biology at Scripps Institution of Oceanography in La Jolla, CA. She earned her B.S. in molecular biology from Yale University and her M.S. in marine biology from Scripps Institution of Oceanography. Her research focuses on the ecological response of microbes to organic coastal pollution, particularly in coral reef environments, with the aim of understanding the effects on coral health and disease.

Joe Myers has served for two decades in the Dept. of Mathematical Sciences at the United States Military Academy. He holds degrees in Applied Mathematics and other disciplines and is a licensed Professional Engineer. He currently serves as a Professor, having directed freshman calculus, sophomore multivariable calculus, the electives program, and the research program. He has been involved in several major initiatives to improve teaching and learning, including building interdisciplinary activities and programs under the NSF-sponsored Project Intermath; integrating technology and student laptop computers into the classroom; and weaving modeling, history, and writing threads into the mathematics curriculum. He enjoys modeling and problem solving, has posed and guided the research of dozens of math majors, and has been involved in several research projects with the Army Research Laboratory.

Judges' Commentary: The Outstanding Coral Reef Papers

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Introduction

The Interdisciplinary Contest in Modeling (ICM)® is an opportunity for teams of students to tackle challenging real-world problems that require a wide breadth of understanding in multiple academic subjects. This year's problem required a particularly deep understanding of ecology to model a solution effectively. Due to the rapid growth of aquaculture facilities currently being installed in or adjacent to many sensitive coastal ecosystems, research into sustainable culturing methods is an active area of investigation. Seven judges gathered in late March to select the most successful entries of this challenging competition out of an impressive set of submissions.

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The Problem

The primary goal of this year's ICM was to develop an aquaculture scenario that incorporated species from multiple trophic levels to reduce the level of effluent leaving the fish pens for a specific case study in the Philippines. These fish farms are adjacent to coral reefs, thus the target was to improve water quality such that corals could thrive in the area while an economically viable aquaculture industry could also be maintained. The main tasks expected of the teams were as follows:

1. Model the original Bolinao coral reef ecosystem before the introduction of fish farms.
2. Model the current Bolinao milkfish monoculture.
3. Observe the remediation of Bolinao via aquaculture.
4. Maximize the value of the total harvest.
5. Call to action.

Overall, the judges were impressed by both the strength of many of the submissions of individual teams and the variety of approaches that teams used to address the questions posed by the ICM problem.

Judges' Criteria

In order to ensure that the individual judges assessed submissions on the same criteria, we developed a rubric. The framework used to evaluate submissions is described below.

- **Executive Summary:** It was important that teams succinctly and clearly explained the highlights of their submissions. These executive summaries needed to include modeling approach(es) used for both the current monoculture and in remediation using polyculture. Further, the summary needed to answer the most pressing questions posed in the problem statement, namely recommendations for remediation and the impact on water quality and optimizing the harvest. Truly Outstanding papers were those that communicated their approach and recommendations in well-connected and concise prose.
- **Domain Knowledge and Science:** The problem this year was particularly challenging for teams in terms of the science.
  - To address the requirements effectively, teams needed first to *establish an ecological frame of reference*. Many teams were able to do this reasonably well; teams that excelled clearly did a great deal of research. Often, what distinguished the top teams was the ability not just to describe the

  ecosystem in a single section of the paper, but to integrate this domain knowledge throughout the modeling process.
  - A second important facet of the problem was the ability to *understand issues that impact water quality*. Many teams created reasonable models of the species and their interactions but very few effectively modeled the water quality.

- **Modeling and Assumptions:** The most popular models used were differential equations—usually linear for the simple cases and then expanding to include nonlinear terms. Simulation was also a popular approach to the problem. Often the models appeared appropriate but neglected any discussion of important assumptions. Additionally, many papers lacked a reasonable discussion of model development, instead presenting a series of equations and parameter values without support. Finally, the very best papers not only formulated the models well, but were able to use the models to produce meaningful results to address the problem and to make recommendations.

- **Solution (Optimization):** Perhaps the most distinct difference between the best papers and others was the ability to utilize their models to develop an actual solution to the problems. Many teams failed to address the most important portions of the problem in any substantive way—what should be done to remediate Bolinao and how to balance the water quality while maximizing harvesting. As a result, the judges put additional emphasis on the actual solution presented, in addition to the modeling approach.

- **Analysis/Reflection:** Successful papers utilized the models developed in early sections of the paper to draw conclusions about the important issues in addressing problems with the Bolinao ecosystem. For example, the important parameters were identified in terms of their impact on the water quality and the harvest available. In the best papers, trade-offs were discussed and, in truly exceptional cases, some sensitivity analysis conducted to identify potential issues with the solutions presented.

- **Communication:** The challenges of the modeling in this problem may have contributed to the difficulty many teams had in clearly explaining their solutions. Papers that were clearly exposited distinguished themselves significantly, emphasizing that it is not only good science that is important, but also the presentation of the ideas.

Discussion of the Outstanding Papers

The two Outstanding papers each had features that distinguished them from the other submissions. Working under the time constraint, both teams were impressive in their ability to research the ecological issues, propose reasonable models, and to present their work in a clear and readable manner. This year, in particular, the judges felt that the Outstanding papers each demonstrated particular strengths in one of the important dimensions discussed in the previous section. No submission was able to dominate every area, but these two teams were clearly superior in different ways.

China University of Mining and Technology

The China University of Mining and Technology submission was notable for the impressive array of modeling techniques utilized in attacking the problems. There were other papers with a similar level of modeling, but this group not only described the modeling process clearly but connected the models coherently to the problem at hand. As with many of the teams, the principal models used were differential equations (Volterra models). The team also used the Analytical Hierarchy Process (AHP), as well as nonlinear optimization to improve their models and to address the later requirements of the problem. They propose strengthening the "middle strata of the foodweb" by introducing herbivorous species to the polyculture. They also propose a strategy for harvesting various species while still satisfying the constraint to maintain good water quality in Bolinao. While extremely strong on the modeling, the paper could have been further improved with more depth on the ecological issues and the overall quality of the writing.

U.S. Military Academy

The paper from the U.S. Military Academy included perhaps the clearest understanding and presentation of the ecological problem and issues among all submissions. The paper was extremely well written and researched. Unlike many teams, the group chose discrete models (difference equations) as the primary tool for their analysis and then employed simulations to help with the optimization tasks. The team did an exceptional job of showing how their model output support the move from a mono- to poly-culture—they added blue mussel mollusks to the system to show the positive effects of such a change. They also proposed an optimal harvesting strategy involving multiple species. This paper could have been strengthened by adding detail about the models and modeling process.

Why Some Other Teams Weren't Outstanding

In addition to the two Outstanding papers, the judges noted several other papers of equal merit in terms of the modeling effort but excluded from award due to issues with proper documentation. The issue was not the fact that material from Websites or books was included—within reason, quotations properly cited are appropriate. Rather, some teams used material taken directly from such sources (sometimes as much as one or more pages of text) in place of their own ideas, failed to document a quoted passage as a quotation, or both.

Conclusion

The judges extend their congratulations to all who participated in the contest. It is a pleasure to see the variety of approaches taken by the different teams; some of these were novel and interesting. The number of excellent papers made the judging both enjoyable and difficult. The problem this year was extremely challenging and the ability to both research and then model in a short period of time was impressive.

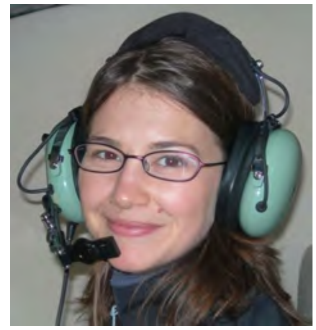
Two facets of this year's ICM are worth noting:

- The importance of understanding the underlying science in formulating mathematical models. In the practice of modeling, assumptions should be carefully thought out and checked and, whenever possible, experts should be consulted.
- How critical communication skills are to the analyst. A great mathematical model is not likely to be used if not clearly and concisely explained.

Recommendations for Future Participants

- Not even ingenious solutions are a substitute for clear exposition.
- Ensure that the assumptions you make are clear to the reader, and address them in your conclusions and recommendations.
- Address all aspects of the problem that are asked.
- Between two equally-clear explanations, the shorter one is better.
- Properly citing sources is critical. Judges notice plagiarized material and disqualify papers that contain it; cite as you go, not at the end.
- The recommendations and sensitivity analysis are often as important as the model itself. Frequently, it is better to have a well-analyzed model that accounts for slightly less than a comprehensive but untested one.
- Team members should work to integrate their final submissions. Your paper should read as though it has only one author.

About the Authors



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On Jargon

Ptolemy to Fourier: Epicycles

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Introduction

This remark highlights the remarkable resemblance of Fourier series to Ptolemy representation of the motion of a planet. We may say that Ptolemy anticipated Fourier analysis some sixteen centuries ago [Zebrowski 2000]. Perhaps Lagrange was the first to recognize this connection [Goldstein 1977, 171].

Fourier series is the Fourier representation

$$f(x) = \sum_{-\infty}^{\infty} F[k] e^{2\pi i k x/p}, \qquad -\infty < x < \infty, \tag{1}$$

for a p-periodic function f with $F[k]$ being the amount of the exponential $e^{2\pi i k x/p}$ that we must use in **(1)** for f. With Euler's identity, we also can obtain the alternative representation:

$$f(x) = \frac{a_0}{2} + \sum_{k=1}^{\infty} \left\{ a_k \cos\left(\frac{2\pi k x}{p}\right) + b_k \sin\left(\frac{2\pi k x}{p}\right) \right\},$$

with

$$a_k = \frac{2}{p} \int_{x=0}^{p} f(x) \cos\left(\frac{2\pi k x}{p}\right) dx, \qquad b_k = \frac{2}{p} \int_{x=0}^{p} f(x) \sin\left(\frac{2\pi k x}{p}\right) dx.$$

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Old Ideas: Hipparchus—Ptolemy Model

Claudius Ptolemy created a mathematical model of the Aristotelian universe in which the planet moved on a small circle called an *epicycle* that in turn moved on a larger circle called the *deferent* (**Figure 1**).

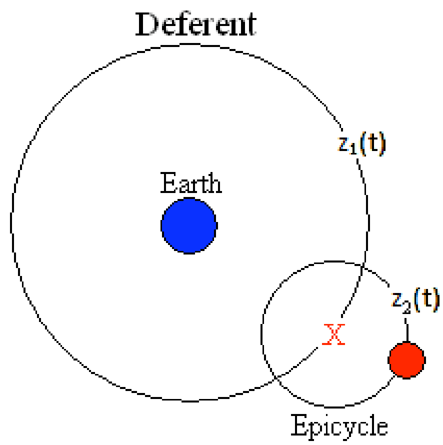

Figure 1. The basic epicycle-deferent system.

The word "planet" comes from the Greek word for "wanderer," referring to the eastward motion of the planets against the background of the fixed stars [Arny 2006]. The planets did not, however, move at a constant rate; and they could occasionally stop and move westward for a few months before resuming their eastward motion. This backward motion is called *retrograde motion*.

Modern Notation

Consider the uniform circular motion of a point X around the Earth E at the origin as shown in **Figure 1**. We represent the position of X by the equation

$$z_1(t) = r_1 e^{2\pi i t/p_1}, \qquad -\infty < t < \infty,$$

with period p_1 and radius of the orbit of the Earth $|r_1|$. Let P be a planet that moves about the point X with radius $|r_2|$ and period p_2, and let $z_2(t)$ be the position of the planet at time t with respect to the origin (Earth E). Then

$$\begin{aligned} z_2(t) &= z_1(t) + r_2 e^{2\pi i t/p_2}, \qquad -\infty < t < \infty \\ &= r_1 e^{2\pi i t/p_1} + r_2 e^{2\pi i t/p_2}. \end{aligned}$$

Thus, the planet is moving in a circular path around a point that undergoes uniform circular motion around the Earth E at the origin (**Figure 1**).

This two-circle model can produce the observed retrograde motion, but *it cannot fit the motion of the planets to observational accuracy*.

We build a more sophisticated model using $n-1$ moving *epicycles*:

$$z_n(t) = \sum_{k=1}^{n} r_k e^{2\pi i t/p_k}. \tag{2}$$

In **(2)**, if the p_k are integer multiples of some p, then the motion is periodic and **(2)** is a *finite Fourier series*. Kammler [2000] summarizes this history:

> Hipparchus and Ptolemy used a shifted four-circle construction of this type (with the Earth near but not at the origin) to fit the motion of each planet. These models were used for predicting the positions of the five planets of antiquity until Kepler and Newton discovered the laws of planetary motion some 1,300 years later.

Example: Earth and Mars orbit the Sun with periods $T_E = 1$ year, $T_M = 1.88$ year at mean distances 1 au $\approx 150 \times 10^6$ km, 1.52 au $= 228 \times 10^6$ km. We may use the simple approximations

$$z_E(t) \approx e^{2\pi i t/1 \text{ yr}}, \qquad z_M(t) \approx e^{2\pi i t/1.5 \text{ yr}}$$

to study the motion of Mars as seen from Earth [Kammler 2000]. **Figure 2** displays

$$z(t) = z_M(t) - z_E(t), \qquad 0 \le t \le 2 \text{ yr}, \tag{3}$$

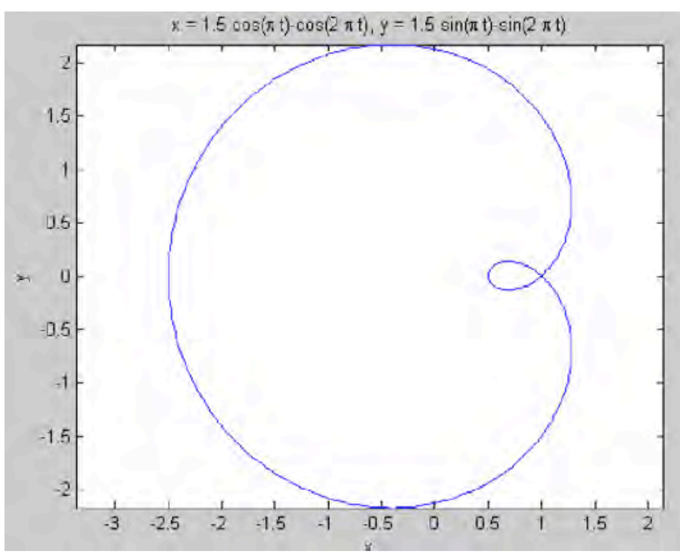

Figure 2. $z(t) = z_M(t) - z_E(t)$, $0 \le t \le 2$ yr, from **(3)**.

which shows the position of Mars as seen from Earth. This orbit corresponds to one of the two-circle approximations of Hipparchus and Ptolemy that we described.

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About the Author

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Reviews

Zill, Dennis G., and Patrick D. Shanahan 2008. *A First Course in Complex Analysis With Applications.* 2nd ed. Sudbury, MA: Jones and Bartlett; 480 pp, $129.95. ISBN–978–0–76375772–4.

It seems to be a foregone conclusion that a seasoned complex analyst will want to make love publicly to his/her subject by writing a book thereon. This could be an elementary text, or an advanced monograph, or something in between. I myself have fallen prey to this vagary of the spirit at least six times, and I may exhibit the weakness on additional occasions in the future.

And what better subject in which to exhibit the frailties of the flesh? For complex analysis is so elegant, so compelling, so full early on with exciting new results, that one can hardly resist the temptation to explain the subject to the world at large.

Now we have a new contribution to the fray, by Zill and Shanahan. This is a text for undergraduates. One of the distinguishing features of an undergraduate complex variables textbook—separating it markedly from a graduate text—is that the main audience is engineers (*not* mathematics majors). So both the content and the focus are special: There is certainly a de-emphasis of proofs and of rigor in general, and a special focus is given to differential equations and applications. There are lots of examples, and there are precise and compelling graphics. The cover of the book is liable to have purples and oranges and a dynamic design (because this is what the engineering market wants). The exposition is, likely as not, brisk and lively.

Certainly, the book under review conforms to the paradigm just described. And what makes it special or distinctive? First, there is an extraordinary number of well-selected exercises, covering both drill and solid thought problems. There are lots of examples in the text proper. The graphics, mostly in the vein of Mathematica figures, are well-drawn and accurate (though my gut feeling is that there could be many more of them). The book is full of textual explanation and comforting patter. Words are not wasted—they are used well—and points are made succinctly and clearly.

As already noted, a book of this type must have applications. And in fact each chapter of the present book has a substantive section called *Applications*. These applications are in no way surprising or innovative—there is the usual material on vector fields and the argument principle and approximation theory and boundary-value problems—but it is well-presented and

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coherent. The student will be kept gainfully employed in working through these sections, and he/she will get a good sense of "What is all this stuff good for?"

The book has several labor-intensive but certainly worthwhile features, such as a thorough glossary and a table of conformal mappings. It is clear that these authors are sensitive to, and indeed are plugged in to, a vast terrain of information about the points and mappings under consideration.

There are already a good many books out there on undergraduate complex variable theory, including the classic texts of Brown and Churchill [2009], Saff and Snider [2003], Derrick [1984], and many others. The present books fits comfortably into the lower end of this spectrum. It makes a conscious effort to downplay rigor and emphasize practicalities. On the one hand, it includes proofs of all the key results. But it presents those proofs in a clean and palatable fashion. It includes even tricky topics like the Schwarz-Christoffel formulas. But it shows real finesse in presenting these topics in an accessible and friendly manner.

As one might expect, a book like this does not present a complete proof of the Riemann mapping theorem. Fair enough. But it does give a fairly substantive coverage (with proofs) to the argument principle, to Rouché's theorem, and to the calculation of improper integrals using the calculus of residues.

If I had to say something critical—and I guess it is part of my job to find fault with *something*—I would note that the book does not have any sort of bibliography. This is really a shame. The lore of complex variables is vast and multitextured. The authors would be doing both students and teachers a great favor to provide a lexicon and concordance to the literature. Include books of applications, books of theory, books of history. This is part of teaching the subject.

In sum, I would call this a respectable and thorough introduction for undergraduates to the lore of complex variable theory. Although the orientation for engineers is evident, these authors do not give short shrift to the venerable mathematics of the discipline. A mathematics major would do well to study from this book and will be well-equipped to go on in the study of complex analysis. The instructor will find this text to be reliable, accurate, and comforting. It contains no surprises (pleasant or unpleasant), but it consistently pleases. It is a useful contribution to the didactics of complex analysis.

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Steven G. Krantz, Professor of Mathematics, Washington University in St. Louis, One Brookings Drive, St. Louis, MO 63130; `sk@math.wustl.edu`.

Krantz, Steven G. 2008. *A Guide to Complex Variables*. Washington, DC: Mathematical Association of America; xviii+182 pp, $49.95 ($39.95 for MAA members). ISBN 978-0-88385-338-2.

Frequent reviewer Steven Krantz has written over 50 books, and complex analysis is one of his core areas. This, his most recent book, can be described as a concise course on complex analysis or, more accurately, a summary of such a course. However, though it is not unique in being a concise overview, I find it surprisingly easy to read.

The cover suggests that the book is useful to undergraduates. As a rule, I find what is written on the covers of math books is often fiction of the fantasy category; but in this case, it is absolutely correct. The book is also a worthy aid for the graduate student preparing for qualifying exams.

Moreover, the cover says that this is MAA Guides #1. If later Guides live up to this precedent, I am keenly interested in them. I predict this slim volume will find its way into many institutional and personal libraries.

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Strang, Gilbert. 2007. *Computational Science and Engineering*. Wellesley, MA: Wellesley-Cambridge Press; xi + 716 pp, $90. ISBN 978-0-961408-81-7.

My second review for this journal [1986] was of Gilbert Strang's *Introduction to Applied Mathematics* (hereafter *IAM*). I have never been too happy with that review, where I said that it is a "wonderful book." True enough; but more appropriately, it is an important book, as is the book reviewed here, *Computational Science and Engineering* (hereafter *CSE*).

CSE is—and is not—a second edition of *IAM*. Apparently, it is the result of more than 20 years of Strang teaching his favorite course at MIT, presumably out of *IAM*. Since *CSE* does not contain everything in *IAM* and also contains topics not in *IAM*, it is a different text. *CSE* contains Strang's further ruminations on the nature of applied mathematics, and I view it as the superior text, but some individuals might prefer *IAM*. To some extent, either book represents Strang's philosophy of teaching applied mathematics—that we need a new approach—but this conviction is much more explicit in *CSE*.

In particular, Strang believes that *we should focus on both modeling and computation*. Many books are about one or the other, and he feels that *applied* mathematics is both. Furthermore, Strang believes that applied problems tend to have a common structure, and Chapter 2 is devoted to illustrating this principle through a wide variety of problems.

In my review of *IAM*, I tried to give an idea of the range of topics without enumerating the contents. *CSE* has the same difficulty: Enumerating the topics is tedious, but the titles of the chapters are informative (though listing them does not do justice to the sheer range of content):

1. Applied Linear Algebra
2. A Framework for Applied Mathematics
3. Boundary Value Problems
4. Fourier Series and Integrals
5. Analytic Functions
6. Initial Value Problems
7. Solving Large Systems
8. Optimization and Minimum Principles

Strang suggests that a course designed out of this text might follow the structure that he uses (p. v):

- Applied linear algebra
- Applied differential equations
- Fourier series

I have long been a champion of Strang's books. I have reviewed different editions of two texts on linear algebra, making clear that that I think he is the most influential author in linear algebra in the last 50 years. I have heaped high praise on his calculus text in my recent editorial on calculus [Cargal 2008]. I have done this for the exact reason that I have championed John Stillwell's books on geometry and algebra. These two authors, as well as a handful of others, write with authority leavened with the great enthusiasm of the born teacher. They are superb pedagogues.

What makes *IAM* and *CSE* so important is that they cover a great deal of applied mathematics, and there is nothing in the literature that compares to them. Pedagogical works, as opposed to dry tomes, are simply rarer in applied mathematics than they are in, say, calculus, linear algebra, geometry, and number theory. There are pedagogical works in differential equations and probability. But there is nothing that covers so much applied mathematics as these with comparative pedagogical skill and acumen.

Like *IAM*, *CSE* has a long first chapter that is a summary of applied linear algebra (86 pp in *IAM*, 97 pp in *CSE*). Linear algebra is a key to applied mathematics; it is the most important tool after calculus (this apparently is Strang's view). However, the first chapter is definitely a review. The reader needs to have had a course in linear algebra as well as the usual course in differential equations. These things are minimal. Courses in probability, numerical analysis, and so on certainly help. Knowledge of physics is a definite plus. These days, there are students of applied mathematics (computer science, statistics, operations research) who are physics-phobic. They would have problems with parts of the book. This necessity of a modicum of prior knowledge of applied mathematics means that the level of the book is for seniors and graduate students. The online comments about *IAM* are striking in their simplicity: Students who are not prepared despise the book, the others are enamored with it; there is no middle ground. The reader who is prepared should love this book. In particular, engineers and physicists should love this book.

People in industry, too, should love this book. Mathematicians and engineers in industry benefit particularly from a book such as this for a very simple reason. Mathematicians in academia tend to specialize because of the need to publish. However, mathematicians in industry are motivated to generalize. They don't have tenure; often they depend on contracts, so that specializing can limit opportunities to get work. If a book like *CSE* (or *AIM*) had been available when I went into industry more than 30 years ago, it would have changed my life; it certainly would have made those first years easier. In fact, one topic that Strang covers very nicely in both books is the Kalman filter, a topic that is very big in industry and that occupied me in my first job.

The most important thing I tell my students is the need to study if they go into industry. This is particularly true if the student has stopped at the bachelor's degree, since a bachelor's degree is essentially a learner's permit. Few students go to work for national labs (those who do, do not need my advice—I need theirs), which means that on-the-job training is unlikely or superficial. Of people who have technical degrees, only a small portion maintain their technical skills; most simply travel along and forget much of what they learned. People tend to learn or they forget; nobody remains in stasis. In industry, you should take some of your time on the job to study.

Is spending work time studying material that is not clearly work related to the work unethical? Typically, doing so does not create a problem (as long as one gets one's tasks done). However, if your supervisor sees you reading a newspaper, that could create a problem. On the other hand, if you are studying number theory, there is no problem; that number theory has nothing to do with your current job tasks will almost certainly not register. Moreover, the worker who studies number theory will tend to retain competence in differential equations far better than a worker who just lets technical skills dissipate. In fact, those few workers who develop good technical reputations almost always study widely while on the job. Their ability to quickly respond to new problems on the job is a result of having used work time not to do company tasks. I view this behavior as a survival skill. The fact is, if one "steals" company time to study mathematics and engineering—even topics that have nothing to do with the job—one is far more likely to be promoted because of it than to be reprimanded.

However, the young worker almost always would benefit not only from learning more number theory but—more urgently—needs to learn a lot more applied mathematics. The undergraduate curriculum can't cover it all. Key core areas are not just physics and differential equations, but probability, numerical analysis, and programming. For a worker in industry, *CSE* would be invaluable, and yet experienced engineers and mathematicians will also be impressed by this book.

Computational Science and Engineering should be in the library of every applied mathematician, not to mention engineers. As a textbook, it is well-suited for a senior or graduate course in applied mathematics.

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Daniel, James W. and Leslie Jane Federer Vaaler. 2009. *Mathematical Interest Theory*. 2nd ed. Washington, DC: Mathematical Association of America. xviii + 475 pp, $89.95 ($71.95 for MAA members). ISBN 978–0–88385–754–0.

This is an excellent book on interest theory, one of the four books currently recognized by the Society of Actuaries (SOA) [2009] and Casualty Actuary Society (CAS) [2009] as a basis of study for the interest theory component of their joint Financial Mathematics (FM) exam.

Additionally, the book has unique features that make it stand out among these four books and led me to use it for the Theory of Interest course at my institution prior to official recognition by the SOA and the CAS.

Before discussing the unique features, I point out that the book has many common features of good textbooks:

- abundant worked-out examples,
- ample expository discussions,
- numerous exercises,
- important formulae highlighted with boxes,
- a writing emphasis consisting of several dozen end-of-chapter exercises encouraging students to practice writing, as well as
- a flexible presentation allowing students without a calculus background to "skip" the few calculus-based sections.

To appreciate properly the unique features of the book, let us first introduce some (light) notation. A deposit of \$1 at time $t = 0$ in a bank with a constant interest rate of i compounded annually will accumulate to $1 + i$ at time $t = 1$, to $(1 + i)^2$ at time $t = 2$, and more generally to $(1 + i)^n$ at time $t = n$. Defining $v = 1/1 + i$, we immediately see that a deposit of v^n at time $t = 0$ accumulates to an amount of 1 at time $t = n$. The cool way of saying this in actuarial lingo is that "a deposit of 1 at time $t = n$ has present value v^n."

Using this notation, we might naïvely attempt to define the course content of the Theory of Interest as the study of finding the equivalent present value of a stream of payments of amounts a_i at time $t = i, i \in I$, with I some possibly parametrized index set.

The problem with such a formulation is that it does not necessitate a new course. Indeed, using the definition of v, we immediately see that the present value of amounts a_i at time $t_i, i \in I$, equals $\sum_{i \in I} a_i v^i$. Furthermore, the formulae for sums of geometric series admit simplified closed formulae when appropriate collections of a_i are constant.

So we need a more precise definition of the course content of the Theory of Interest. I would suggest the following:

> *The Theory of Interest is the study of finding the equivalent present value of a stream of payments of a_i at time $t = i, i \in I$, by skillfully using a core set of actuarial functions that facilitate quick and efficient computation of present values.*

This revised definition of the Theory of Interest explains why a separate course is needed for it:

- The Theory of Interest studies the *interaction* between a core set of actuarial functions and real-world financial problems.
- The Theory of Interest is not concerned with just meeting some pedantic standard of conformity with actuarial notation but rather seeks to use existing core actuarial functions skillfully to facilitate quick computation.

These two concerns—interaction and computation—*define* the subject and also enable us to evaluate the book.

- **Computation:** *Mathematical Interest Theory* has a short eight-page Chapter 0 guiding the student in the use of the specially designed Texas Instruments BA–II calculator. Furthermore, many of the several hundred worked-out problems in the book are accompanied with detailed calculator keystroke sequences illustrating computational technique.

  Special emphasis is placed on five worksheets available in the BA–II:

  - the time-value-of-money (TVM) worksheet,
  - the cash-flow worksheet,
  - the interest conversion worksheet,
  - the amortization worksheet, and
  - the bond worksheet.

  By emphasizing the computational utility of these worksheets, the authors ensure that students are proficient not only in ordinary calculator functions but in the special worksheets built into the calculator. Such an emphasis is consistent with the definition of the course content of the Theory of Interest presented above.

  A comparison of *Mathematical Interest Theory* with an approach used by Broverman [2004; 2008] will further enhance appreciation of the Daniel-Vaaler approach. Broverman wrote an extensive calculator manual [2005] as part of the book. Broverman's idea is to separate the theory and computation. Students can read the book, digest the theory, and then on the side learn any calculator functions that they need. In fact, I always list the Broverman calculator manual on my syllabus, since it is well-written and free. But my students rarely use it. Students want a one-stop textbook; something on the side frequently remains on the side. The Daniel-Vaaler approach is best: Students see inside the text what is needed.

- **Interaction:** What I particular like about *Mathematical Interest Theory* is that many problems are intrinsically multi-stepped, requiring use of several core functions. This is consistent with the definition of the course content of the Theory of Interest as the study of the interaction between core actuarial functions and real-world problems

  - The simplest level of textbook problems are *plug-ins*.
  - Many FM books also give *comparison problems*, which require comparing two basic subproblems, each of which is similar to a plug-in. For example:

    > *Xiang will pay Dmitry \$800 immediately and another \$200 at the end of three years. In return, Dmitry will pay Xiang \$K in exactly one year and again at the end of exactly two years. Find K if the transaction is base on compound interest at a nominal discount rate of 6% convertible monthly.*
    >
    > —[Daniel and Vaaler 2009, Chapter 2, 107]

– A typical Daniel-Vaaler, superior, real-world, *multi-step comparison* problem might be the following (paraphrased):

> *The 6% annual coupons of a $3,000 10-year par-value bond are reinvested in an account earning 4% annually. Given that the yield on the combined bond–savings-account investment is 5%, compute the price and yield of the bond.*
>
> — [Daniel and Vaaler 2009, Chapter 6, 297].

We invite the skeptical reader who thinks that course content of the Theory of Interest can be reduced to study of geometric sums to attempt to solve this problem.

By providing a multitude of superior problems, Daniel and Vaaler are able to familiarize the student not only with core actuarial functions but also develop their skills in studying the interaction between these functions and real-world problems.

Although officially both the SOA and CAS require a second textbook for the derivative component of the Financial Mathematics exam, many topics—including puts, calls, options, swaps, strips, stocks, and dividends—are covered in the Daniel and Vaaler book also.

Finally, a student solutions manual [Vaaler 2009] contains solutions to all of the odd-numbered exercises.

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MacKay, David J.C. 2009. *Sustainable Energy—Without the Hot Air*. Cambridge, UK: UIT Cambridge. xi + 370 pp, $79.95, $49.95 (P). ISBN 978–1–90686001–1, 978–0–9544529–3. Free download of the book at `http://www.withouthotair.com/download.html` and free 10-page synopsis at `http://www.withouthotair.com/synopsis10.pdf`.

In his review above, Reviews Editor Cargal laments "physics-phobic" students of applied mathematics. In their favor: They are the mathematically capable among the physics-avoiders in high school and college.

This *Journal* focuses on mathematical modeling and applications of mathematics. A prime arena for modeling and applications is, of course, physics.

This *Journal* often publishes sophisticated modeling efforts. Sometimes, though, vast insight can be obtained from basic physics concepts—length, area, volume, mass, density, force, energy, power, scaling, temperature—and fundamental modeling techniques—"back-of-the-envelope" calculation (estimation, rounding, bounding), good data, and sensitivity analysis.

This remarkable book uses just those simple tools to generate a "balance sheet" of energy consumption vs. energy production by sustainable means, for the UK in particular, but with remarks about other countries, too. Author MacKay assesses quantitatively all uses of energy and *every* conceivable sustainable energy source (you name it—it's definitely here).

The key to comparisons is measuring all energy usage and production in a single standard unit, the kilowatt-hour per day (kWh/d) per person. So, for example, a typical UK car driver uses 40 kWh/d, while "plausible production from on-shore windmills" in the UK is 20 kWh/d per person. MacKay even considers the energy used to produce imported goods.

The first 110 pp are devoted to the current balance sheet (final version on p. 109), the next 140 pp to options for sustainability, including construction costs for renewables and the potential for energy storage. (Mackay's recipe: electrify transport; heat via heat pumps; get electricity from renewables plus perhaps "clean coal" and nuclear). Then follow 70 pp of "technical" supplements (at the level of high school physics) and 20 pp of data.

Pertinent to the 2009 MCM Cellphone Charger Problem, MacKay pooh-poohs "every little bit helps": "All the energy saved in switching off your charger for one day is used up in one second of car-driving" (p. 68), and

> *If everyone does a little, we'll achieve only a little.* We must do a lot. What's required are *big* changes in demand and in supply. (p. 114)

MacKay keeps the presentation lively by displaying cumulative results in simple bar graphs, together with numerous photographs and figures.

This book is thorough, authoritative, passionate, exciting, and revolutionary.

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Makridakis, Spyros, Robin Hogarth, and Anil Gaba. 2009. *Dance with Chance: Making Luck Work for You*. Oxford, UK: Oneworld. x+287 pp, $22.95. ISBN 978–1–85168–697–7.

Devlin, Keith. 2008. *The Unfinished Game: Pascal, Fermat, and the Seventeenth-Century Letter that Made the World Modern*. New York: Basic Books. x+191 pp, $24.95. ISBN 978–0–465–00910-–7.

In 2001, Nassim Nicholas Taleb's *Fooled by Randomness: The Hidden Role of Chance in Markets and Real Life* became a best-seller among business books (the section of bookstores where it was usually placed). I discussed the second edition [Taleb 2004] in my omnibus review of books on investing [Cargal 2005]. Taleb has been a successful professional in finance and is mathematically sophisticated. Oddly enough, he doesn't mention Burton G. Malkiel's *A Random Walk Down Wall Street* [2003], which I also discussed in my review; it is a great book, which introduced to a general audience (in its original edition in 1973) the concept of a *randomly* unpredictable market.

Taleb's 2007 success, *The Black Swan: The Impact of the Highly Improbable*, has as its theme is the importance and inevitability of unexpected events. Black swans (or their discovery) are a metaphor for such events, and the term "black swan" has entered common usage.

Another book in the randomness vein, Leonard Mlodinow's *The Drunkard's Walk: How Randomness Rules our Lives* [2008], has also been a success; the book was awarded the Robert P. Balles Annual Prize in Critical Thinking given by the Committee for Skeptical Thinking for 2008.

Now we have *Dance with Chance: Making Luck Work for You* by Spyros Makridakis et al. Like Taleb's *Fooled by Randomness*, it is strongly oriented toward financial markets. Its contribution to the doctrine that chance is a huge part of our lives are claims that often our belief that we control our circumstances is pure illusion and that sometimes we make decisions simply to maintain that illusion. The book relies so much on anecdotes that it reminded me of the best-selling books by Malcolm Gladwell, such as *Blink: The Power of Thinking Without Thinking* [Gladwell 2005]. In fact, Makridakis et al. recapitulate the theme of *Blink* and credit Gladwell; similarly, a large section covers the random market as championed by Malkiel and the ideas of Taleb's *Black Swan*, and those authors are likewise credited.

The lead author, Makridakis, is a statistician, and some statistical topics show up in the book. At one point, the book refers to significant work of his in the early 1980s that shows surprising efficacy for exponential averages. Only at this point did it dawn on me that in the early 1980s I had spent a great deal of time reading works of Makridakis! In my review on books on investing, in fact, I discussed exponential averages.

The book mentioned so far are all very readable—and somewhat redundant. I think they would be of greater interest to the student than to the professor. Mlodinow's *The Drunkard's Walk* might be something of an exception; it covers a great deal of history of probability and statistics.

Nonetheless, it must be emphasized to students that books such as these are no substitute for actually studying probability and statistics.

Keith Devlin's *The Unfinished Game: Pascal, Fermat, and the Seventeenth-Century Letter that Made the World Modern* is not of the same cloth as the others and is not redundant. Devlin is a noted mathematician (logic and set theory) and a top popularizer of mathematics. His book is about a seminal event in the history of probability: the working out of the basic concepts of probability in the correspondence of Fermat and Pascal. He emphasizes that the invention (discovery?) of probability brought about a revolution in thinking; the concept of probability itself was revolutionary. Moreover, he demonstrates how difficult it was to develop these ideas, especially for Pascal, who struggled to keep up with Fermat.

I appreciated the book for filling in details about earlier work by Cardano on probability. Cardano, who may have been the single most important character in the development of algebra in the 16th century, wrote a book on probability that wasn't published until after the work of Pascal and Fermat. However, the manuscript—and Cardano's ideas—may have floated around. Similarly, Devlin fills in the details of Galileo's brief flirtation with probability. There is also a tour of some of the development of probability after its start with Fermat and Pascal. I consider the book a must-have for those interested in the history of probability (and statistics).

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