

$$\begin{aligned} & \nabla_{\beta_{yj}} \left(\sum_{c=1}^C y_{ic} \beta_c^T x_i + \log \left(\sum_{c'=1}^C \exp(\beta_{c'}^T x_i) \right) \right) \\ &= -y_{ic} x_j + \frac{x_j \exp(\beta_{y_j}^T x_j)}{\log \left(\sum_{c'=1}^C \exp(\beta_{c'}^T x_j) \right)} \end{aligned}$$