Foundations of Computer Science (COMP109)

Tutorial VI, Week 23.11.2020 – 27.11.2020

*A reasonable attempt at answering Question (VI.2.) should be submitted on Canvas by **23:59 on Tuesday 24.11.2020** either as a text entry, a text file (txt), a pdf file, or a photo of the handwritten answer. This assignment makes up 1% of your final mark. We would like to encourage you to discuss the questions with your fellow students in person or on the Canvas discussion board, but do not copy your answer from anybody else.*

VI.1. Let $25\mathbb{Z}$ be the set of all integers that are multiple of 25, $25\mathbb{Z} = \{n \in \mathbb{Z} \mid n = 25k, \text{ for some integer } k\}$. Prove that $25\mathbb{Z}$ has the same cardinality as the set of even integers.

VI.2. Use the pigeonhole principle to give solutions to the following problems:

(a) How many times must a single die be rolled to guarantee that some number is obtained at least twice?

(b) How many times must two dice be rolled to guarantee that the same total score is obtained at least twice?

(c) How many times must two dice be rolled to guarantee that the same total score is obtained at least three times?

VI.3. A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.

(a) How many socks must he take out to be sure that he has at least two socks of the same colour?

(b) How many socks must he take out to be sure that he has at least two black socks?

VI.4. Let $A = \{1, 2, 3\}$ and $B = \{a, b\}$. Determine the set $A \times B$.

VI.5. Let A be a set. Determine the set $A \times \emptyset$.