Artificial Intelligence (COMP111)

Exercise 8

To answer the questions, please watch the videos *Reasoning under Uncertainty 2*.

Your answer to Question 1 should be submitted on canvas for assignment *Exercise 8* either as a text entry, a text file (txt), a pdf file, or a photo of the handwritten solution. The deadline is Monday, 7th of December, at 6pm. You should also attempt to answer the other questions before your tutorial (but not submit them).

You obtain 1 point (1 percent of the final mark) if you make a reasonable attempt to answer Question 1 *and* actively participate in your tutorial in the week starting Monday 7th of December.

We would like to encourage you to discuss the questions with your fellow students, but do not copy your answer from anybody else.

1. Based on the populations 60776228, 5116900 and 2980700 of England (E), Scotland (S), and Wales (W), the prior probability that a randomly selected person from these countries is living in England, Scotland, or Wales is approximately 0.88, 0.08, and 0.04, respectively. Consider the random variable *Country* with values E, S, and W and the probability distribution given by

$$\mathbf{P}(\mathit{Country}) = (0.88, 0.08, 0.04)$$

   Consider the random variable *Mothertongue* that takes values *Eng*, *Scot*, and *Welsh*. Consider the following fictitious conditional probabilities:

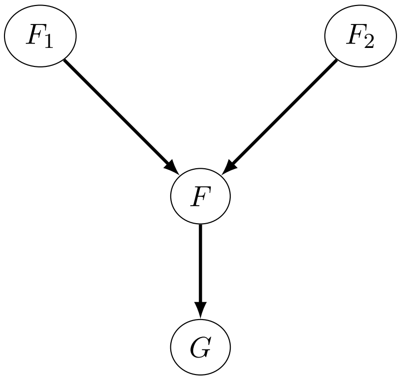
   - $P(\mathit{Mothertongue} = \mathit{Eng} \mid \mathit{Country} = E) = 0.95$;
   - $P(\mathit{Mothertongue} = \mathit{Eng} \mid \mathit{Country} = S) = 0.7$;
   - $P(\mathit{Mothertongue} = \mathit{Eng} \mid \mathit{Country} = W) = 0.6$;
   - $P(\mathit{Mothertongue} = \mathit{Scot} \mid \mathit{Country} = E) = 0.04$;
   - $P(\mathit{Mothertongue} = \mathit{Scot} \mid \mathit{Country} = S) = 0.3$;
   - $P(\mathit{Mothertongue} = \mathit{Scot} \mid \mathit{Country} = W) = 0.0$;
   - $P(\mathit{Mothertongue} = \mathit{Welsh} \mid \mathit{Country} = E) = 0.01$;

- $P(\mathit{Mothertongue} = \mathit{Welsh} \mid \mathit{Country} = S) = 0.0$;
- $P(\mathit{Mothertongue} = \mathit{Welsh} \mid \mathit{Country} = W) = 0.4$.

Compute

- the joint probability distribution $\mathbf{P}(\mathit{Country}, \mathit{Mothertongue})$.
- the marginal distribution $\mathbf{P}(\mathit{Mothertongue})$.

2. Consider the following belief network with Boolean random variables F_1, F_2, F, G:



Assume we know the following probabilities:

(a) The probability distributions $\mathbf{P}(F_1)$ and $\mathbf{P}(F_2)$ are given by $P(F_1 = 1) = 0.4$ and $P(F_2 = 1) = 0.3$. Recall that then $P(F_1 = 0) = 0.6$ and $P(F_2 = 0) = 0.7$.

(b) The conditional probability distribution $\mathbf{P}(F \mid F_1, F_2)$ is given as

- $P(F = 1 \mid F_1 = 1, F_2 = 1) = 0.5$;
- $P(F = 1 \mid F_1 = 1, F_2 = 0) = 0.1$;
- $P(F = 1 \mid F_1 = 0, F_2 = 1) = 0.2$;
- $P(F = 1 \mid F_1 = 0, F_2 = 0) = 0$.

Recall that we obtain the probabilities:

- $P(F = 0 \mid F_1 = 1, F_2 = 1) = 0.5$;
- $P(F = 0 \mid F_1 = 1, F_2 = 0) = 0.9$;
- $P(F = 0 \mid F_1 = 0, F_2 = 1) = 0.8$;
- $P(F = 0 \mid F_1 = 0, F_2 = 0) = 1$.

(c) The conditional probability distribution $\mathbf{P}(G \mid F)$ is given as

- $P(G = 1 \mid F = 1) = 0.4$;
- $P(G = 1 \mid F = 0) = 0.1$.

Recall that we obtain the probabilities

- $P(G = 0 \mid F = 1) = 0.6$;
- $P(G = 0 \mid F = 0) = 0.9$.

Determine the joint probability distribution $\mathbf{P}(F_1, F_2, F, G)$. For this, you will have to compute 16 probabilities. As this is rather boring, only show how you compute the values for

$$P(F_1 = 1, F_2 = 1, F = 1, G = 1)$$

and

$$P(F_1 = 1, F_2 = 1, F = 0, G = 0)$$

3. The Chain Rule is a generalization of the Product Rule from the Comp111 slides and states that for all events $A_1, \ldots, A_n$:

   $$P(A_1 \cap \cdots \cap A_n) = P(A_1)P(A_2 \mid A_1)P(A_3 \mid A_1 \cap A_2) \cdots P(A_n \mid A_1 \cap \cdots \cap A_{n-1})$$

   Prove the Chain Rule by induction on n.

4. (Only if you enjoy to prove things.) Assume a random variable F from the probability space (S, P) and a real number λ are given. Denote by F' the random variable from the probability space (S, P) defined by setting $F'(s) = \lambda F(s)$ for all $s \in S$.

   Show the following equation for the expected value $E[F']$ of the random variable F':

   $$E[F'] = \lambda E[F]$$

5. Consider the probability space (S, P) defined by setting

   $$S = \{1, 2, 3, 4, 5, 6\}^{10}$$

   and $P(a_1 \cdots a_n) = \frac{1}{6^{10}}$ for every $a_1 \cdots a_{10} \in S$. What is the expected value $E(F)$ of the random variable F from S defined by

   $$F(a_1 \cdots a_{10}) = 10(a_1 + \cdots + a_{10})$$

   Do not just give the answer but show how you compute $E(F)$.