Artificial Intelligence (COMP111)

Exercise 7

To answer the questions, please watch the videos *Reasoning under Uncertainty 1 and the first three videos of Reasoning under Uncertainty 2*.

Your answer to Question 2 should be submitted on canvas for assignment *Exercise 7* either as a text entry, a text file (txt), a pdf file, or a photo of the handwritten solution. The deadline is Monday, 30th of November, at 6pm. You should also attempt to answer the other questions before your tutorial (but not submit them).

You obtain 1 point (1 percent of the final mark) if you make a reasonable attempt to answer Question 2 *and* actively participate in your tutorial in the week starting Monday 30th of November.

We would like to encourage you to discuss the questions with your fellow students, but do not copy your answer from anybody else.

1. Let (S, P) be a probability space with $S = \{\text{swim}, \text{run}, \text{walk}, \text{sleep}\}$ and

   $$P(\text{swim}) = 0.5, \quad P(\text{run}) = 0.1, \quad P(\text{walk}) = 0.2, \quad P(\text{sleep}) = 0.2$$

   Let F_1 be a random variable with

   $$F_1(\text{swim}) = 2, \quad F_1(\text{run}) = 2, \quad F_1(\text{walk}) = 0, \quad F_1(\text{sleep}) = 1$$

   and let F_2 be a random variable with

   $$F_2(\text{swim}) = 1, \quad F_2(\text{run}) = 2, \quad F_2(\text{walk}) = 3, \quad F_2(\text{sleep}) = 4$$

   Compute the following probabilities:

   - $P(F_1 = 2)$;
   - $P(F_1 = 2, F_2 = 2)$;
   - $P(F_1 + F_2 = 3)$.

2. Let *BuysBeer* be a random variable with values 0 and 1 for false and true, respectively, and let *CustomerType* be a random variable with values *single*, *couple*, and *family*. The joint probability distribution for *BuysBeer* and *CustomerType* is given by:

	BuysBeer = 1	*BuysBeer* = 0
CustomerType = *single*	0.02	0.30
CustomerType = *couple*	0.14	0.32
CustomerType = *family*	0.10	0.12

Compute the following probabilities:

1. $P(\textit{BuysBeer} = 1)$
2. $P(\textit{CustomerType} = \textit{couple})$
3. $P(\textit{CustomerType} = \textit{couple}, \textit{BuysBeer} = 1)$
4. $P(\textit{CustomerType} = \textit{couple} \mid \textit{BuysBeer} = 1)$

3. Let the probability distribution $\mathbf{P}(\textit{Weather})$ be given by

   - $P(\textit{Weather} = \textit{sunny}) = 0.7$;
   - $P(\textit{Weather} = \textit{rain}) = 0.2$
   - $P(\textit{Weather} = \textit{cloudy}) = 0.08$;
   - $P(\textit{Weather} = \textit{snow}) = 0.02$.

   Let the joint probabality distribution of the random variables *Toothache*, *Cavity*, *Catch* be given by the following table:

	Toothache = 1		*Toothache* = 0	
	Catch = 1	*Catch* = 0	*Catch* = 1	*Catch* = 0
Cavity = 1	0.108	0.012	0.072	0.008
Cavity = 0	0.016	0.064	0.144	0.576

   Assume that the joint probability distribution

   $$\mathbf{P}(\textit{Weather}, \textit{Toothache}, \textit{Cavity}, \textit{Catch})$$

   equals

   $$\mathbf{P}(\textit{Weather}) \times \mathbf{P}(\textit{Toothache}, \textit{Cavity}, \textit{Catch})$$

   Compute the probability of the event that the weather is sunny, you don't have a cavity, the steel probe catches, but you have toothache.

4. Let (S, P) be a probability space and let A and $A_1, \ldots, A_n$ be events in S such that

   - $A_i \cap A_j = \emptyset$ for all $1 \leq i < j \leq n$;
   - $A_1 \cup \cdots \cup A_n = S$.

Prove that

$$P(A) = \sum_{i=1}^{n} P(A \mid A_i) \times P(A_i)$$

Hint. First show that $P(A) = \sum_{i=1}^{n} P(A \cap A_i)$.

5. (Only if you enjoy probability puzzles.) Suppose you are a witness to a nighttime hit-and-run accident involving a taxi in Madrid. All taxis in Madrid are blue or green. Assume that 9 out of 10 taxis are green. You swear, under oath, that the taxi was blue. Extensive testing shows that, under the dim lighting conditions, discrimination between blue and green is 75% reliable.

   Compute the most likely colour of the taxi. In your computation, distinguish carefully between the event that the taxi is blue and the event that it appears blue.