Artificial Intelligence (COMP111)
Exercise 6

To answer the questions, please watch the videos *Knowledge Representation 2* and *Reasoning under Uncertainty 1*.

Your answers to Question 1(a), 3(d), and 3(e) should be submitted on canvas for assignment *Exercise 6* either as a text entry, a text file (txt), a pdf file, or a photo of the handwritten solution. The deadline is Monday, 23rd of November, at 6pm. You should also attempt to answer the other questions before your tutorial (but not submit them).

You obtain 1 point (1 percent of the final mark) if you make a reasonable attempt to answer Question 1(a), 3(d), and 3(e) *and* actively participate in your tutorial in the week starting Monday 23rd of November.

1. Which of the following are true? Check using truth tables or defining relevant interpretations directly.

    (a) $\{p_1, (p_1 \Rightarrow p_2), (p_2 \Rightarrow p_3)\} \models p_3$.

    (b) $\{(p_1 \Rightarrow p_2)\} \models (p_2 \Rightarrow p_1)$.

    (c) $\{(p_1 \vee \neg p_2)\} \models p_1$.

    (d) $\{(p_1 \wedge \neg p_1)\} \models p_2$.

2. Let (S, P) be a probability space with $S = \{a, b, c, d, e\}$ and

    $$P(a) = 0.1, \quad P(b) = 0.2, \quad P(c) = 0.3, \quad P(d) = 0.2, \quad P(e) = 0.2$$

    Let $E_1 = \{b, c, d\}$ and $E_2 = \{d, e\}$. Compute the following probabilities:

    - $P(E_1 \cap E_2)$,
    - $P(\neg E_1)$,
    - $P(E_1 \cup E_2)$.
    - $P(E_1 \mid E_2)$.
    - $P(E_1 \mid E_1)$.

3. Suppose that I have four fair coins with values $1p$, $2p$, $5p$ and $10p$. Call the $1p$ coin and the $2p$ coin "low-value" coins and the other two coins "high-value coins".

(a) What is the probability that, when I flip the coins, the $1p$ and the $5p$ come up heads, and the other two coins come up tails? To answer the question, first define the probability space (S, P).

(b) What is the probability that at least one of the low-value coins comes up tails?

(c) What is the probability that at least three of the coins come up tails?

(d) What is the probability that at least three of the coins come up tails, conditioned on the fact that at least one of the low-value coins comes up tails?

(e) Is the event that at least three of the coins come up tails independent of the event that at least one of the low-value coins comes up tails?

4. What probability should be assigned to the outcome of heads when a biased coin is tossed, if heads is three times as likely to come up as tails? What probability should be assigned to the outcome of tails?