

Foundations of Computer Science (COMP109)

Tutorial VII, Week 30.11.2020 – 04.12.2020

A reasonable attempt at answering Question (VII.2.) should be submitted on Canvas by 23:59 on Tuesday 01.12.2020 either as a text entry, a text file (txt), a pdf file, or a photo of the handwritten answer. This assignment makes up 1% of your final mark. We would like to encourage you to discuss the questions with your fellow students in person or on the Canvas discussion board, but do not copy your answer from anybody else.

VII.1. Describe the set $(\mathbb{R} \times \mathbb{Z}) \cap (\mathbb{Z} \times \mathbb{R})$.

VII.2. Let $X = \{1, 2, 3, 4\}$ and $Y = \{a, b, c\}$. Let $R \subseteq X \times X$ be a relation on X given by xRy if, and only if, $x < y$. Let $S \subseteq X \times Y$ be a relation between X and Y given by the matrix:

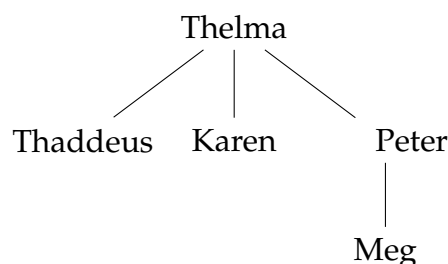
$$M_S = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

- (a) Represent R as a set of ordered pairs;
- (b) Represent S as a set of ordered pairs;
- (c) Represent S in graphical form;
- (d) Find the matrix M_R representing the relation R
- (e) Use Boolean matrix multiplication to find the matrix $M_{S \circ R}$ representing the composition $S \circ R$.

VII.3. Let R be the relation on $\{1, 2, 3, 4\}$ given by xRy if, and only if, $x - y = 0$. Represent R in the following ways:

- as a set of ordered pairs;
- in graphical form;
- in matrix form.

VII.4. Consider the family tree



Let $A = \{\text{Thelma, Thaddeus, Karen, Peter, Meg}\}$ be the set of all family members and $B = \{\text{Thaddeus, Peter}\}$ be the set of male members.

Let R be the relation between the sets A and B consisting of pairs (a, b) , where a is a parent of b . Let S be the relation between sets B and A consisting of pairs (a, b) , where a and b are siblings (brothers or sisters).

- Represent relations R and S in the matrix form.
- Use the matrix forms to compute $R \circ S$ and $S \circ R$.