# Tutorial for Week 8 - Answers

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1. (a) $R$ can be represented by the following graph:

$$\begin{tikzcd} 1 \arrow[r] & 3 \arrow[r] & 2 \arrow[ll, bend left] \\ & 4 \arrow[loop below] & \end{tikzcd}$$

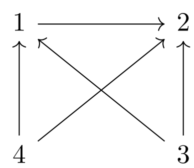
- Functional: True as each element in $A$ has $\leq 1$ assignment.
- Reflexive: False as there are not links from every node to itself.
- Symmetric: False as there are not return arrows for all nodes.
- Anti-symmetric: True as there are no return arrows. Loops don't count.
- Transitive: False as there is no link for $1 \to 2$.

$S$ can be represented by the following graph:

$$\begin{tikzcd} 1 \arrow[r] & 2 \arrow[r] & 4 \\ & 3 \arrow[loop below] & \end{tikzcd}$$

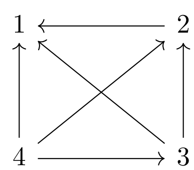
- Functional: True as each element in $A$ has $\leq 1$ assignment.
- Reflexive: False as there are not links from every node to itself.
- Symmetric: False as there are not return arrows for all nodes.
- Anti-symmetric: True as there are no return arrows. Loops don't count.
- Transitive: False as there is no link for $4 \to 1$.

$T$ can be represented by the following graph:



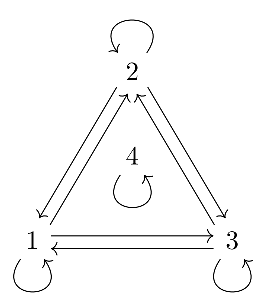

- Functional: False as nodes have multiple assignments.
- Reflexive: False as there are not links from every node to itself.
- Symmetric: False as there are not return arrows for all nodes.
- Anti-symmetric: True as there are no return arrows.
- Transitive: True as for every link $x$ to $y$ and $y$ to $z$ there is a link from $x$ to $z$.

$U$ can be represented by the following graph:




- Functional: False as nodes have multiple assignments.
- Reflexive: False as there are not links from every node to itself.
- Symmetric: False as there are not return arrows for all nodes.
- Anti-symmetric: True as there are no return arrows.
- Transitive: True as for every link $x$ to $y$ and $y$ to $z$ there is a link from $x$ to $z$.

(b) The following graph represents the transitive closure of $R$:




(c) An equivalence relation is described as a relation which is reflective, transitive and symmetric. In $R^*$ ever node has a loop to itself, making it reflective, and it is a transitive closure making it transitive.

Additionally, for every arrow there is a return arrow; this makes it symmetric. As a result of these three properties, $R^*$ is an equivalence relation.

I'm not too sure about this one. As, in $R^*$, $1, 2$ and $3$ are in a closed loop and $4$ is in a loop then $A$ is split into two equivalence classes: $\{1, 2, 3\}$ and $\{4\}$.