Tutorial for Week 3 - Answers

Ben Weston

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4. Prove by mathematical induction that the sum of the first n positive integers is $n^2 + n$.

Proof:

Base Case:

$$\begin{aligned} n &= 0 \\ 0 &= 0^2 + 0 = 0 \end{aligned} \tag{1}$$

Inductive Step:

Suppose for $n = m$ the property holds.
Then,

$$0 + 2 + 4 + \ldots + 2(m) = m^2 + m \tag{2}$$

Consider the statement for $n = m + 1$:

$$0 + 2 + 4 + \ldots + 2(m) + 2(m+1) \tag{3}$$

By induction hypothesis,

$$0 + 2 + 4 + \ldots + 2(m) = m^2 + m \tag{4}$$

So,

$$\begin{aligned} 0 + 2 + 4 + \ldots + 2(m) + 2(m+1) &= m^2 + m + 2(m+1) \\ &= m^2 + m + m + 1 + (m+1) \\ &= m^2 + 2m + 1 + (m+1) \\ &= (m=1)^2 + (m+1) \end{aligned} \tag{5}$$

By the principle of mathematical induction $0 + 2 + 4 + \ldots + 2(n) = n^2 + n$ for all natural n. ■