Tutorial for Week 6 - Answers

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2. All of the following are solved by the pigeon-hole principle. Therefore the set A must have a cardinality $\geq k|B| + 1$ to satisfy the question.

   (a) A is the set of rolls and B is the set of pairs of numbers:

   $$\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)\},$$

   such that the statement is satisfied. $|B| = 6$, therefore $|A| = |B|+1 = 7$ rolls.

   (b) Two die can give any combination between 2 and 12. Therefore $|B| = 11$. The number of rolls required is $|A|$, as $|A| = |B| + 1$, $|A| = 11 + 1 = 12$ rolls.

   (c) This is similar to the last question but k is increased from 1 to 2, as there are now three matches required. This gives $k = n - 1 = 3 - 1 = 2$. As the number of rolls required is $|A|$ and $|A| = k|B| + 1$, $|A| = 2 \times 11 + 1 = 23$.