Foundations of Computer Science (COMP109)

Tutorial VIII, Week 07.12.2020 – 11.12.2020

A reasonable attempt at answering Question (VIII.1.) should be submitted on Canvas by ***23:59 on Tuesday 08.12.2020*** *either as a text entry, a text file (txt), a pdf file, or a photo of the handwritten answer. This assignment makes up 1% of your final mark. We would like to encourage you to discuss the questions with your fellow students in person or on the Canvas discussion board, but do not copy your answer from anybody else.*

VIII.1. Let $A = \{1, 2, 3, 4\}$ and let R, S, T and U be the following relations:

$R = \{(1, 3), (3, 2), (2, 1), (4, 4)\}$,
$S = \{(2, 1), (3, 3), (4, 2)\}$,
$T = \{(4, 1), (4, 2), (3, 1), (3, 2), (1, 2)\}$,
$U = \{(x, y) \mid x > y\}$.

(a) For each of R, S, T and U determine whether they are functional, reflexive, symmetric, anti-symmetric or transitive.
Explain your answer in each case, showing why your answer is correct.

(b) What is the transitive closure of R?

(c) Explain why R^*, the transitive closure of R, is an equivalence relation. Describe the equivalence classes E_x into which the relation partitions the set A.

VIII.2. Prove or give a counterexample to the following statement: for any relation R both R and $R \circ R$ always have the same transitive closure.

VIII.3. Is there a mistake in the following proof that any transitive and symmetric relation R is reflexive? If so, what is it?

> *Let aRb. By symmetry, bRa. By transitivity, if aRb and bRa, then aRa. This proves reflexivity.*

VIII.4. Determine for the following relations on the set of people if the relation is an equivalence relation, a partial order, both an equivalence relation and a partial order, or neither an equivalence relation nor a partial order.

(a) 'has the same parents (both) as'

(b) 'has at least one parent same as'

(c) 'is a brother of'

(d) 'is at least as clever as'.

VIII.5. Let R and S be relations on a set A. Use proof by contradiction to show that if R and S are partial orders then $R \cap S$ is also a partial order.