CENG 384 - Signals and Systems for Computer Engineers
Spring 2024
Homework 1

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March 12, 2024

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1. (a) Multiply numerator and denominator by $2 - 2\sqrt{3}j$.

   $z = \frac{\sqrt{2}+\sqrt{2}j}{2+2\sqrt{3}j} = \frac{2\sqrt{2}-2\sqrt{6}+(2\sqrt{2}-2\sqrt{6})j}{4-12j^2} = \frac{(2\sqrt{2}-2\sqrt{6})(1+j)}{16} = \frac{\sqrt{2}-\sqrt{6}}{8} + \frac{\sqrt{2}-\sqrt{6}}{8}j.$

   $Re\{z\} = \frac{\sqrt{2}-\sqrt{6}}{8} \approx -0.13.$

   $Im\{z\} = \frac{\sqrt{2}-\sqrt{6}}{8} \approx -0.13.$

   (b) We can express z as $z = a + bj$, where

   $a = \frac{\sqrt{2}-\sqrt{6}}{8}$, $b = \frac{\sqrt{2}-\sqrt{6}}{8}$.

   The magnitude is:

   $r = \sqrt{a^2+b^2} = \sqrt{2a^2} = |a|\sqrt{2} = |\frac{\sqrt{2}-\sqrt{6}}{8}| \cdot \sqrt{2} = |\frac{2-2\sqrt{3}}{8}| = |\frac{1-\sqrt{3}}{4}| \approx 0.18.$

   The phase is:

   $\theta = arctan(\frac{b}{a}) = arctan1 = \frac{\pi}{4}.$

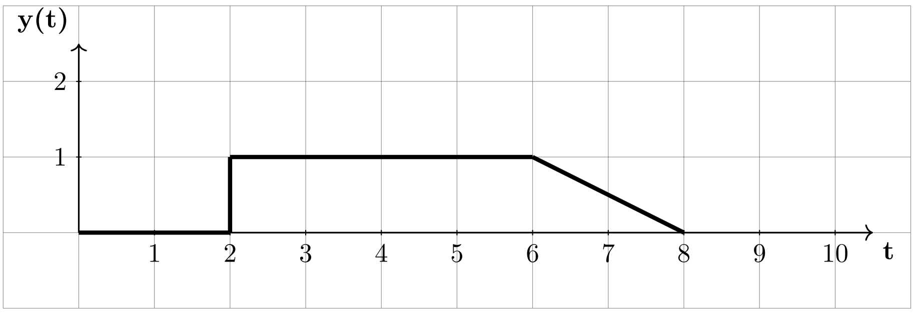
2. $t = 0 \rightarrow y(0) = x(-2).$

   $t = 2 \rightarrow y(2) = x(-1).$

   $t = 4 \rightarrow y(4) = x(0).$

   $t = 6 \rightarrow y(6) = x(1).$

   $t = 8 \rightarrow y(8) = x(2).$



3. (a) $x[n] = \delta[n+3] - \delta[n+2] - \delta[n+1] - \delta[n] + \delta[n-1] + 2\delta[n-2] + \delta[n-3].$

   (b) For $n <= -5,\ y[n] = 0.$

   $n = -4 \rightarrow y[-4] = x[-6] + x[5] = 0.$

$n = -3 \rightarrow y[-3] = x[-4] + x[4] = 0.$

$n = -2 \rightarrow y[-2] = x[-2] + x[3] = 0.$

$n = -1 \rightarrow y[-1] = x[0] + x[2] = 1.$

$n = 0 \rightarrow y[0] = x[2] + x[1] = 3.$

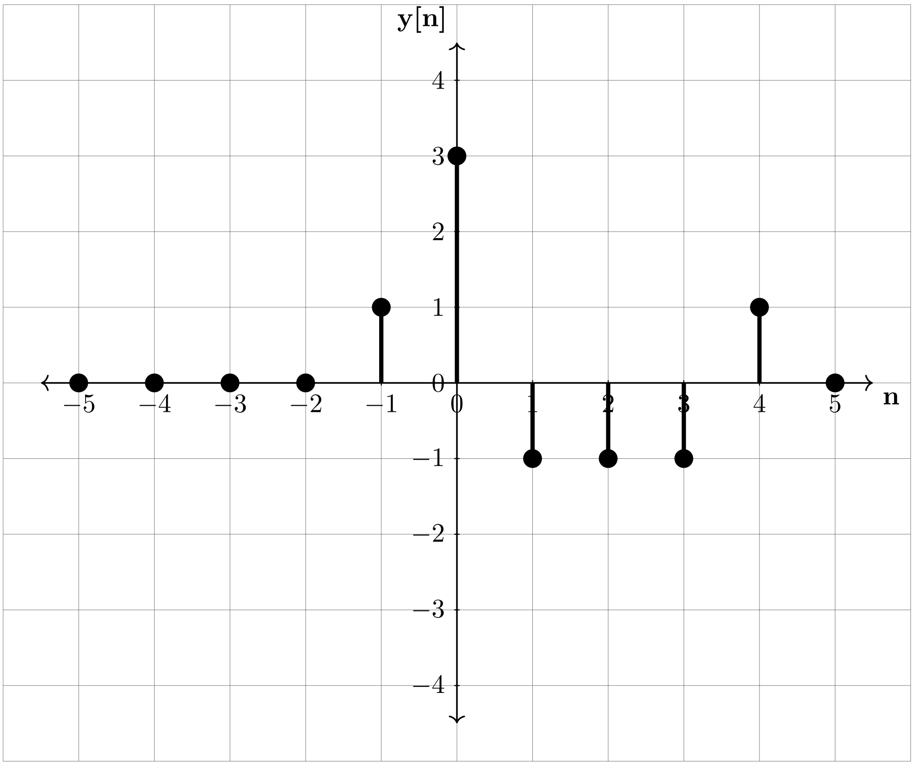
$n = 1 \rightarrow y[1] = x[4] + x[0] = -1.$

$n = 2 \rightarrow y[2] = x[6] + x[-1] = -1.$

$n = 3 \rightarrow y[3] = x[8] + x[-2] = -1.$

$n = 4 \rightarrow y[4] = x[10] + x[-3] = 1.$

For $n >= 5,\ y[n] = 0.$



(c) $y[n] = \delta[n+1] + 3\delta[n] - \delta[n-1] - \delta[n-2] - \delta[n-3] + \delta[n-4].$

4. (a) If $x_1[n]$ is periodic, then

$$x_1[n] = x_1[n+N] \rightarrow \cos\left(\frac{5\pi n}{2}\right) = \cos\left(\frac{5\pi n}{2} + \frac{5\pi N}{2}\right)$$

The equation above implies the following equation:

$$\frac{5\pi n}{2} + 2\pi k_1 = \frac{5\pi n}{2} + \frac{5\pi N}{2} + 2\pi k_2$$

where $k_1, k_2 \in Z$

$$\frac{5\pi N}{2} = 2\pi(k_2 - k_1)$$

Define $k_2 - k_1 = k_3$. Then,

$$\frac{N}{k_3} = \frac{4}{5}$$

where $k_3 \in Z$. Since we obtained two integers N and k_3, the function is periodic, and its fundamental period is $N = 4$ for $k_3 = 5$.

(b) If $x_2[n]$ is periodic, then

$$x_2[n] = x_2[n+N] \rightarrow \sin(5n) = \sin(5n+5N)$$

The equation above implies the following equation:

$$5n + 2\pi k_1 = 5n + 5N + 2\pi k_2$$

where $k_1, k_2 \in Z$.

$$5N = 2\pi(k_2 - k_1)$$

Define $k_2 - k_1 = k_3$. Then,

$$\frac{N}{k_3} = \frac{2\pi}{5}$$

where $k_3 \in Z$. We cannot find any integer values for both N and k_3 satisfying the equation above. Thus, the function is not periodic.

(c) If $x_3(t)$ is periodic, then

$$x_3(t) = x_3(t+T) \rightarrow 5\sin(4t + \frac{\pi}{3}) = 5\sin(4(t+T) + \frac{\pi}{3})$$

The above equation directly implies that

$$4t + \frac{\pi}{3} + 2k_1\pi = 4t + 4T + \frac{\pi}{3} + 2k_2\pi$$

where $k_1, k_2 \in Z$.

$$4T = 2\pi(k_1 - k_2)$$

Define $k_3 = k_1 - k_2$. Hence,

$$T = \frac{\pi k_3}{2}$$

where $k_3 \in Z$.The function is periodic, and its fundamental period is $T = \frac{\pi}{2}$ for $k_3 = 1$.

5. First, notice that $\delta(\alpha t)$ is an even function since it is symmetric with respect to the y axis. This directly implies that

$$\delta(t) = \delta(-t)$$

That is, the sign of α doesn't matter in the given expression. Then, we know the following equation

$$\int_{-\infty}^{\infty} x(\tau)\delta(\tau)d\tau = x(0)$$

Using a similar logic, let's prove the given expression.

$$\int_{-\infty}^{\infty} x(\tau)\delta(a\tau)d\tau = \int_{-\infty}^{\infty} x(\frac{\tau'}{a})\delta(\tau')\frac{1}{a}d\tau'$$

if τ is replaced with $\frac{\tau'}{a}$ by the rule of integration by substitution. Then,

$$\int_{-\infty}^{\infty} x(\frac{\tau'}{a})\delta(\tau')\frac{1}{a}d\tau' = \frac{1}{a}\int_{-\infty}^{\infty} x(\frac{\tau'}{a})\delta(\tau')d\tau' = \frac{1}{a}x(\frac{0}{a}) = \frac{1}{a}x(0)$$

Notice that

$$\frac{1}{a}x(0) = \frac{1}{a}\int_{-\infty}^{\infty} x(\tau)\delta(\tau)d\tau$$

Therefore, since $\delta(at)$ and $\frac{1}{|a|}\delta(t)$ have the same effect on any function $x(t)$, $\delta(at) = \frac{1}{|a|}\delta(t)$ for $a \neq 0$.

6. (a) To find the difference equation for S, let's first find $y_1[n-2]$

$$y_1[n-2] = 4x_1[n-2] + 2x_1[n-3]$$

Then, we need to calculate $y_2[n]$ in terms of $x_1[n]$.

$$y_2[n] = 4x_1[n-2] + 2x_1[n-3]$$

This gives us the difference equation for the overall system S as the following:

$$S : y[n] = 4x[n-2] + 2x[n-3]$$

(b) If the order of the series connection of $S1$ and $S2$ were reversed, then

$$S2 : y_1[n] = x_1[n-2]$$
$$S1 : y_2[n] = 4x_1[n-2] + 2x_1[n-3]$$

Therefore, the overall system S would be

$$S : y[n] = 4x[n-2] + 2x[n-3]$$

That equation is the same as the one found in part a. Therefore, it can be concluded that the series connection of the subsystems $S1$ and $S2$ is commutative.

(c) A system is called linear if and only if it satisfies the superposition property. Let's check the superposition property on the system given.

$$y_1[n] = 4x_1[n-2] + 2x_1[n-3]$$
$$y_2[n] = 4x_2[n-2] + 2x_2[n-3]$$
$$\alpha y_1[n] + \beta y_2[n] = 4\alpha x_1[n-2] + 2\alpha x_1[n-3] + 4\beta x_2[n-2] + 2\beta x_2[n-3]$$

The superposition of the two inputs is as the following:

$$x = \alpha x_1 + \beta x_2$$

Now, let's give the superposition of the inputs found above to the system as the input, and check whether it is linear or not.

$$4(\alpha x_1[n-2] + \beta x_2[n-2]) + 2(\alpha x_1[n-2] + \beta x_2[n-2]) = 4\alpha x_1[n-2] + 2\alpha x_1[n-3] + 4\beta x_2[n-2] + 2\beta x_2[n-3]$$

Since the expression found above is equal to the expression $\alpha y_1[n] + \beta y_2[n]$, the system is linear.

(d) To check whether the system is time invariant, let's shift the input of the system by the time amount of n_0 to define a new input $x[n] := x[n-n_0]$. Then, the corresponding output becomes

$$4x[n-n_0-2] + 2x[n-n_0-3]$$

Notice that the output above is equal to the expression, which is $y[n-n_0] = 4x[n-n_0-2] + 2x[n-n_0-3]$. Consequently, the provided system is time invariant.

7. (a)
```
import sympy as sp

n = sp.Symbol("n")
alpha = sp.Symbol("alpha")
beta = sp.Symbol("beta")

x1 = sp.Function("x1", real=False)(n)
x2 = sp.Function("x2", real=False)(n)
x3 = alpha * x1 + beta * x2

y1 = x1 * n
y2 = x2 * n
y3_1 = x3 * n
y3_2 = alpha * y1 + beta * y2


if y3_1.equals(y3_2):
    print("The given system is a Linear system")
else:
    print("The given system is a Non-Linear system")
```

(b)
```
import sympy as sp

n = sp.Symbol("n")
alpha = sp.Symbol("alpha")
beta = sp.Symbol("beta")
```

```python
x1 = sp.Function("x1", real=False)(n)
x2 = sp.Function("x2", real=False)(n)
x3 = alpha * x1 + beta * x2

y1 = x1 ** 2
y2 = x2 ** 2
y3_1 = x3 ** 2
y3_2 = alpha * y1 + beta * y2

if y3_1.equals(y3_2):
    print("The given system is a Linear system")
else:
    print("The given system is a Non-Linear system")
```