EE381 – Systems and Control	METU EEE – Fall 2023

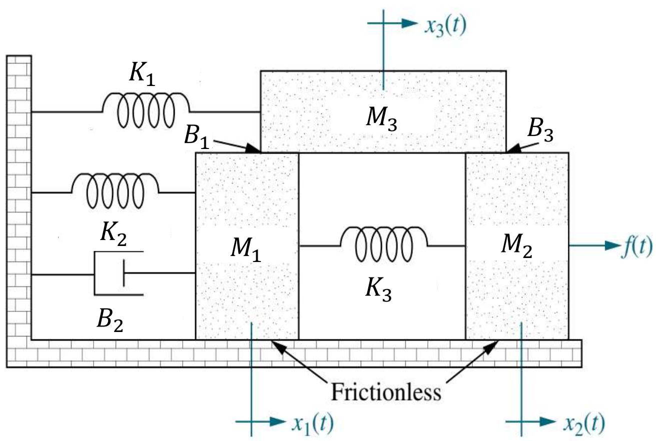
HOMEWORK ASSIGNMENT – I

Due @ 23:59 18/11/2023

Q1. Find the inverse Laplace transformations of the following functions.

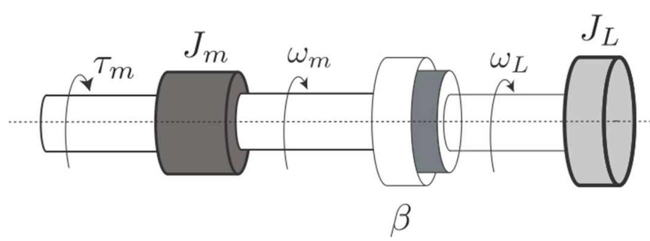
a. $F(s) = \frac{s^2-6s-19}{(s+3)^2(s^2+2s+5)}$ **b.** $F(s) = \frac{s^2}{(s+1)^3}$ **c.** $F(s) = \frac{s^3}{(s^2+1)^2}$

Q2. Consider the mechanical system given below, which was taken from Problem 29 in Chapter 2 of the course textbook. The input of the system is the force f applied onto the mass M_2 and the output is the displacement x_3 of the mass M_3. Find a state-space representation for the system (transfer function representation is not required).

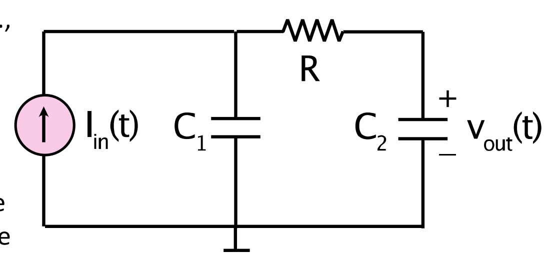


Q3. This problem has three parts.

a. In this part, your goal is to derive the transfer function representation for the rotational mechanical system composed of two inertias (J_m and J_L) and a damper (β) shown on the right. The input of the system is the external torque acting on the axis of J_m, i.e., $u(t) = \tau_m$, whereas the output of the system is the angular velocity of body J_L, i.e., $y(t) = \omega_L$.

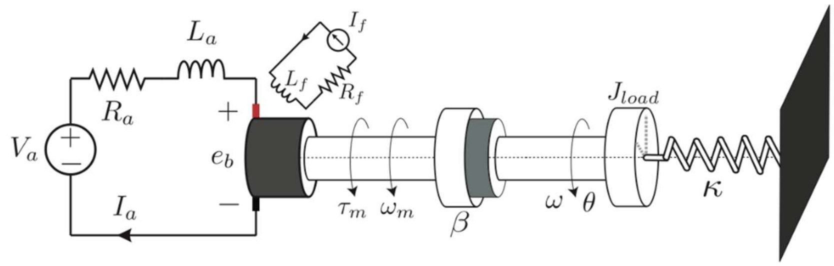


b. In this part, your goal is to derive the transfer function representation for the electrical system composed of two capacitors (C1 and C2) and a resistor (R) shown on the right. The input of the system is the current source i.e., $u(t) = I_{in}(t)$ whereas the output of the system is the voltage of the second capacitor, i.e., $y(t) = V_{out}(t)$.



c. Compare the transfer function derived structures in parts b and c, and comment on the similarities and differences (apart from the obvious fact that one is a mechanical system whereas the other one is an electrical system).

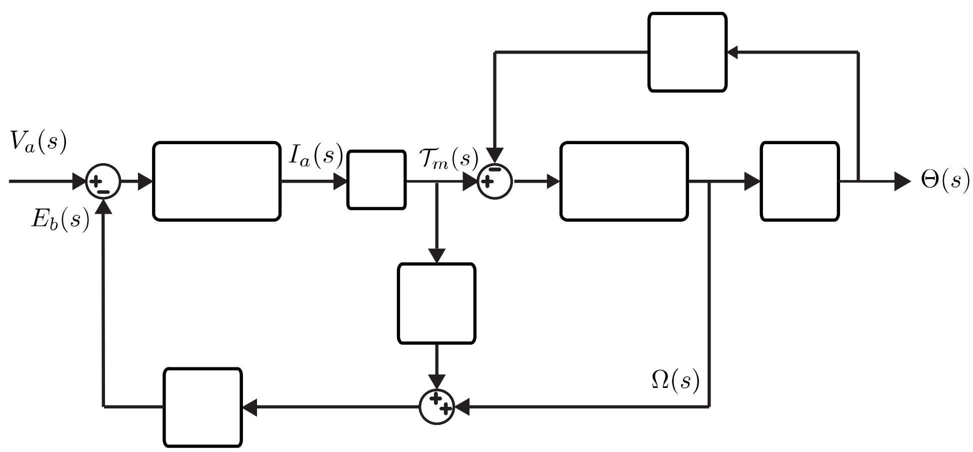
Q4. In this problem, you are going analyze the dynamics of the following electro-mechanical system. In this system an armature-controlled DC motor (electro-mechanical variable conversions are provided in the illustration) drives the rotatory mechanical part which is composed of an inertial load with an inertia of J_{load}, a rotational/torsional spring with a spring constant of κ (connected between the load and the ground), and a rotational damper with damping coefficient of β (connected between the motor shaft and the load.



$$\tau_m(t) = K \cdot I_a(t)$$
$$e_b(t) = K \cdot \omega_m(t)$$
$$K_a = K_b = K$$

The input of the system is the armature voltage, $u(t) = V_a(t)$, and output of the system is the angular displacement of the load, $y(t) = \theta(t)$.

a. The detailed block diagram structure (given below) belongs to the given electro-mechanical system. As you can see from the illustration, major signals and variables are labeled on the block-diagram, but individual transfer function blocks are left empty. Carefully analyze the input—output relations in the block diagram topology and complete the block diagram structure.



b. Let us define the state vector of the system as

$$x(t) = \begin{bmatrix} \theta(t) \\ \omega(t) \\ I_a(t) \end{bmatrix}.$$

Find the state-representation for the given state-vector, input, and output definitions