Student Information

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Answer 1

$\sum_{n=2}^{\infty} a_n x^n = 3\sum_{n=2}^{\infty} a_{n-1}x^n + 4\sum_{n=2}^{\infty} a_{n-2}x^n = 3x\sum_{n=2}^{\infty} a_{n-1}x^{n-1} + 4x^2\sum_{n=2}^{\infty} a_{n-2}x^{n-2}$. Hence,
$A(x) - a_0 - a_1 x = 3x(A(x) - a_0) + 4x^2 A(x) \leftrightarrow (4x^2 + 3x - 1)A(x) = (3x-1)a_0 - xa_1 = 2x - 1$
$\leftrightarrow A(x) = \frac{2x-1}{(4x-1)(x+1)} = \frac{A}{4x-1} + \frac{B}{x+1} \leftrightarrow 2x - 1 = A(x+1) + B(4x-1) \rightarrow A + 4B = 2,\ A - B = -1 \rightarrow A = -2/5,\ B = 3/5 \rightarrow A(x) = \frac{-2/5}{4x-1} + \frac{3/5}{x+1} = \frac{2}{5}\frac{1}{1-4x} + \frac{3}{5}\frac{1}{1+x}$

$\frac{1}{1-4x} \leftrightarrow < 1, 4, 16, 64, ..., 4^n, ... >$

$\frac{1}{1+x} \leftrightarrow < 1, -1, 1, -1, ..., (-1)^n, ... >$

So, $\frac{2}{5}\frac{1}{1-4x} \leftrightarrow < 2/5, 8/5, 32/5, 128/5, ..., 4^n \cdot 2/5, ... >$

$\frac{3}{5}\frac{1}{1+x} \leftrightarrow < 3/5, -3/5, 3/5, -3/5, ..., (-1)^n \cdot 3/5, ... >$

Hence, $a_n = \frac{2}{5} \cdot 4^n + \frac{3}{5} \cdot (-1)^n$.

Answer 2

a)

$< 1, 1, 1, 1, 1, 1, ... >= \frac{1}{1-x}$

$< 1, 3, 9, 27, 81, 243, ... >= \frac{1}{1-3x}$

$< 0, 3, 9, 27, 81, 243, ... >= \frac{1}{1-3x} - 1 = \frac{3x}{1-3x}$

$< 1, 4, 10, 28, 82, 244, ... >= \frac{3x}{1-3x} + \frac{1}{1-x} = \frac{1-3x^2}{(1-x)\cdot(1-3x)}$

$< 2, 5, 11, 29, 83, 245, ... >= \frac{1-3x^2}{(1-x)\cdot(1-3x)} + \frac{1}{1-x} = \frac{2-3x-3x^2}{(1-x)\cdot(1-3x)} = F(x)$ is the generating function of the sequence.

b)

$G(x) = \frac{7-9x}{(2x-1)(x-1)} = \frac{A}{2x-1} + \frac{B}{x-1} \leftrightarrow 7 - 9x = A(x-1) + B(2x-1) \rightarrow A + 2B = -9,\ -A - B = 7 \rightarrow A = -5,\ B = -2 \rightarrow G(x) = \frac{-5}{2x-1} + \frac{-2}{x-1} = \frac{5}{1-2x} + \frac{2}{1-x}$

$\frac{1}{1-2x} \leftrightarrow < 1, 2, 4, 8, ..., 2^n, ... >$

$\frac{1}{1-x} \leftrightarrow < 1, 1, 1, 1, ..., 1, ... >$

Hence, $\frac{5}{1-2x} \leftrightarrow <5, 10, 20, 40, ..., 5 \cdot 2^n, ... >$

$\frac{2}{1-x} \leftrightarrow <2, 2, 2, 2, ..., 2, ... >$

So, $a_n = 5 \cdot 2^n + 2.$

Answer 3

a)

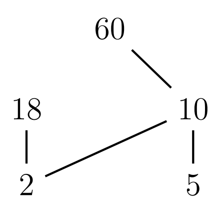
$R = \{(3,4), (3,5), (4,5), (5,12), (5,13), (12,13), (7,24), (24,25), (7,25), ...\}$
R is not reflexive since, for example, it does not contain $(3,3)$. So R is not an equivalence relation.

b)

$R = \{((1,1),(1,1)), ((1,2),(1,2)), ((1,2),(2,0)), ...\}$
R is reflexive since $\forall x_1, x_2 \in \mathbb{R} \quad (x_1, x_2)R(x_1, x_2)$.
R is symmetric since $\forall x_1, x_2, y_1, y_2 \in \mathbb{R} \quad ((x_1, x_2)R(y_1, y_2) \rightarrow (y_1, y_2)R(x_1, x_2))$.
R is transitive since $\forall x_1, x_2, y_1, y_2, z_1, z_2 \in \mathbb{R} \quad ((x_1, x_2)R(y_1, y_2) \wedge (y_1, y_2)R(z_1, z_2) \rightarrow (x_1, x_2)R(z_1, z_2))$.
Thus, R is an equivalence relation.
$[(1,-2)] = \{(x,y) \mid 2x+y = 0\}$ is the equivalence class of $(1,-2)$. It represents the line $y = -2x$ on the Cartesian coordinate system.

Answer 4

a)



b)

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c)

$R_s = R \cup R^{-1} = \{(2,2),(2,10),(2,18),(2,60),(5,5),(5,10),(5,60),(10,2),(10,5),$
$(10,10),(10,60),(18,2),(18,18),(60,2),(60,5),(60,10),(60,60)\}$

$$M_{R_s} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$S = \{(10,2),(18,2),(60,2),(10,5),(60,5),(60,10)\}$ is the set of all pairs (x,y) such that $(x,y) \in R_s \wedge (x,y) \notin R$.

d)

2 does not divide 5. Also, 10 does not divide 18. Also, 18 does not divide 60. Therefore, it is impossible to create a total ordering by changing only one element.
If we remove 5 and 18, and add 30; then R will become a total order relation on A.