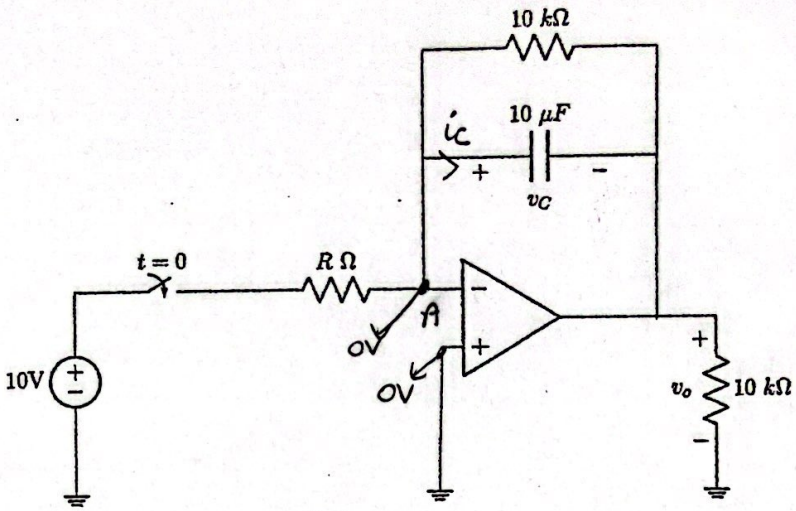
Question 3 (30 pts)

For the circuit on the right, the op-amp is ideal, has infinite gain, and operates in the linear region. The initial capacitor voltage $v_c(0^-) = 5V$. Find the resistor value R, such that $v_o(t) = -10$ V at $t = 0.5$ sec.



KCL A: $\frac{-10}{R} + \frac{-v_o}{10k} + i_c = 0$

$i_c = \frac{10}{R} + \frac{v_o}{10k} = 10^{-5}\frac{dv_c}{dt}$

$v_o = -v_c$ } const eqn.

$\Rightarrow 10^{-5}\frac{dv_c}{dt} + \frac{v_c}{10k} = \frac{10}{R} \Rightarrow \frac{dv_c}{dt} + 10v_c = \frac{10^6}{R}$

Let $\mu(t) = e^{\int 10\,dt} = e^{10t}$. Multiply the eqn by $\mu(t)$:

$e^{10t}\frac{dv_c}{dt} + e^{10t}\cdot 10v_c = e^{10t}\cdot\frac{10^6}{R} = (e^{10t}\cdot v_c)'$

$\Rightarrow e^{10t}\cdot v_c = \int \frac{10^6}{R}e^{10t}dt = \frac{10^5}{R}e^{10t} + C$

$\Rightarrow v_c(t) = \frac{10^5}{R} + Ce^{-10t}$

$v_c(0^-) = \frac{10^5}{R} + C = 5 \Rightarrow C = 5 - \frac{10^5}{R}$

$\Rightarrow v_c(t) = \frac{10^5}{R} + \left(5 - \frac{10^5}{R}\right)e^{-10t} = \frac{10^5}{R}\left(1 - e^{-10t}\right) + 5e^{-10t}$

$v_o(0.5) = -10V \Rightarrow V_c(0.5) = 10V = \frac{10^5}{R}\left(1 - e^{-5}\right) + 5e^{-5}$

$\Rightarrow \frac{10^5}{R}(1 - e^{-5}) = 10 - 5e^{-5} \Rightarrow \frac{10^5}{R} = \frac{10 - 5e^{-5}}{1 - e^{-5}}$

$\Rightarrow R = 10^5\frac{1 - e^{-5}}{10 - 5e^{-5}}\ \Omega$.