Q1 (a) $\dfrac{s^2-6s-19}{(s+3)^2(s^2+2s+5)} = \dfrac{As+B}{\underset{s^2+6s+9}{(s+3)^2}} + \dfrac{Cs+D}{s^2+2s+5}$

(my name is at the end of the page.)

$$= \frac{As^3 + 2As^2 + 5As + Bs^2 + 2Bs + 5B + Cs^3 + 6Cs^2 + 9Cs + Ds^2 + 6Ds + 9D}{---}$$

$$= \frac{(A+C)s^3 + (2A+B+6C+D)s^2 + (5A+2B+9C+6D)s + (5B+9D)}{---}$$

$\Rightarrow$
$$\left.\begin{aligned} A+C &= 0 \\ 2A+B+6C+D &= 1 \\ 5A+2B+9C+6D &= -6 \\ 5B+9D &= -19 \end{aligned}\right\} \quad \begin{aligned} A &= -C \\ B+4C+D &= 1 \\ 2B+4C+6D &= -6 \\ 5B+9D &= -19 \end{aligned}$$

$\left.\begin{aligned} B+4C+D &= 1 \\ 2B+4C+6D &= -6 \end{aligned}\right\} \; 5/\; B+5D = -7$

$$\underbrace{5B+9D = -19 \qquad B+5D=-7}_{16D = -16}$$

$D = -1$

$B = -2$

$C = 1$

$A = -1$

$$\Rightarrow F(s) = \frac{-s-2}{(s+3)^2} + \frac{s-1}{\underset{(s+1)^2+4}{s^2+2s+5}}$$

$$\Rightarrow F(s) = -\frac{1}{s+3} + \frac{1}{(s+3)^2} + \frac{s+1}{(s+1)^2+2^2} + \frac{-2}{(s+1)^2+2^2}$$

$$\Rightarrow f(t) = -e^{-3t} + te^{-3t} + e^{-t}\cos 2t - e^{-t}\sin 2t \;, \quad t \geq 0$$

Batuhan Akgan / 2580181 / Section 3

Homework 1 / 18.11.2023

Q1 b $F(s) = \frac{s^2}{(s+1)^3} = \frac{A}{s+1} + \frac{B}{(s+1)^2} + \frac{C}{(s+1)^3}$

$(s+1)^2 \quad (s+1)$

$$= \frac{As^2 + (2A+B)s + (A+B+C)}{(s+1)^3}$$

$\Rightarrow A = 1,\ B = -2,\ C = 1$

$\Rightarrow F(s) = \frac{1}{s+1} + \frac{-2}{(s+1)^2} + \frac{1}{(s+1)^3}$

$\Rightarrow f(t) = e^{-t} - 2te^{-t} + \frac{t^2}{2}e^{-t}, \quad t \geq 0.$

c $F(s) = \frac{s^3}{(s^2+1)^2} = \frac{As+B}{s^2+1} + \frac{Cs+D}{(s^2+1)^2}$

(s^2+1)

$= \left(As^3 + Bs^2 + (A+C)s + (B+D)\right) / (s^2+1)^2$

$\Rightarrow A = 1,\ B = 0,\ C = -1,\ D = 0$

$\Rightarrow F(s) = \underbrace{\frac{s}{s^2+1}}_{H(s)} - \underbrace{\frac{s}{(s^2+1)^2}}_{G(s)}$

Let $\mathcal{L}^{-1}\{G(s)\} = g(t) = t \cdot g_2(t)$.

Then, $\mathcal{L}\{t\, g_2(t)\} = -\dfrac{dG_2(s)}{ds} = G(s)$.

Thus $-\int \dfrac{dG_2(s)}{ds}\, ds = -G_2 + c = \int G\, ds$

$$= \frac{-1}{2(s^2+1)} + \cancel{c}^{\,0} = -G_2 \Rightarrow G_2 = \frac{1}{2(s^2+1)}$$

$$\Rightarrow g_2(t) = \frac{1}{2}\sin t \Rightarrow g(t) = \frac{1}{2} t \sin t$$

$$\Rightarrow f(t) = h(t) - g(t) = \cos t - \frac{1}{2} t \sin t$$

Q2

$$\left.\begin{aligned} M_3 x_3'' &= -K_1 x_3 - B_1(x_3' - x_1') - B_3(x_3' - x_2'). \\ M_1 x_1'' &= -K_2 x_1 - B_2 x_1' - B_1(x_1' - x_3') - K_3(x_1 - x_2) \\ M_2 x_2'' &= -K_3(x_2 - x_1) - B_3(x_2' - x_3') + f(t) \end{aligned}\right\} \text{DE}$$

Let.

$$X = \begin{bmatrix} x_1 \\ x_1' \\ x_2 \\ x_2' \\ x_3 \\ x_3' \end{bmatrix}$$

input $= f(t)$

output $= x_3(t)$

$$x' = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ \frac{-K_2-K_3}{M_1} & \frac{-B_1-B_2}{M_1} & \frac{K_3}{M_1} & 0 & 0 & \frac{B_1}{M_1} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ \frac{K_3}{M_2} & 0 & \frac{-K_3}{M_2} & \frac{-B_3}{M_2} & 0 & \frac{B_3}{M_2} \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{B_1}{M_3} & 0 & \frac{B_3}{M_3} & \frac{-K_1}{M_3} & \frac{-B_1-B_3}{M_3} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/M_2 \\ 0 \\ 0 \end{bmatrix} f$$

$$x_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} f$$

Q3 a

$$J_m \theta_m'' = \tau_m - B(\theta_m' - \theta_L')$$

$$J_L \theta_L'' = -B(\theta_L' - \theta_m'), \quad \omega_L = \theta_L'$$

convert to laplace domain:

$$\Rightarrow \begin{cases} J_m \theta_m s^2 = \tau_m - B\theta_m s + B\theta_L s \\ J_L \theta_L s^2 = -B\theta_L s + B\theta_m s \end{cases}$$

$\theta_m s = \Omega_m$

$\theta_L s = \Omega_L$

$$\Rightarrow \theta_m = \frac{J_L \theta_L s^2 + B\theta_L s}{Bs} = \frac{J_L \theta_L s + B\theta_L}{B}$$

+

$$\Rightarrow J_m \theta_m s^2 + J_L \theta_L s^2 = T_m$$

$$\Rightarrow J_m J_L \theta_L s^3 + J_m B \theta_L s^2 + J_L \theta_L B s^2 = BT_m$$

$$\Rightarrow J_m J_L \Omega_L s^2 + J_m B \Omega_L s + J_L B \Omega_L s = BT_m$$

$$\Rightarrow \frac{\Omega_L}{T_m} = \frac{B}{J_m J_L s^2 + Bs(J_L + J_m)} = G(s)$$

(b) $i_{c1} + \dfrac{v_{c1} - v_{out}}{R} = i_{in}$ (Kirchoff's Current Law).

$i_{c2} = C_2 v'_{out}$, $i_{c1} + i_{c2} = i_{in}$

$\Rightarrow v_{c1} - v_{out} = i_{c2} \cdot R = C_2 R v'_{out}$. Differentiate both sides.

$\Rightarrow v'_{c1} = v'_{out} + C_2 R v''_{out}$

$i_{c1} = C_1 v'_{c1}$

$$\frac{i_{c1}}{C_1} = \frac{i_{in} - C_2 v'_{out}}{C_1} = v'_{out} + C_2 R v''_{out}$$

$\Rightarrow i_{in} - C_2 v'_{out} = C_1 v'_{out} + C_1 C_2 R v''_{out}$

$\Rightarrow i_{in} = (C_1 + C_2) s v_{out} + C_1 C_2 R s^2 v_{out}$

$$\Rightarrow G(s) = \frac{V_{out}}{I_{in}} = \frac{1}{(C_1 + C_2)s + C_1 C_2 R s^2}$$

(c) Both a and b are 2^{nd} order systems.

Q4 $i_a R_a + L_a i_a' + e_b = v_a$

$J_{load}\theta'' = \tau_m - B\theta' - K\theta$

$\tau_m = K i_a$, $e_b = K\omega_m$. Convert to laplace domain.

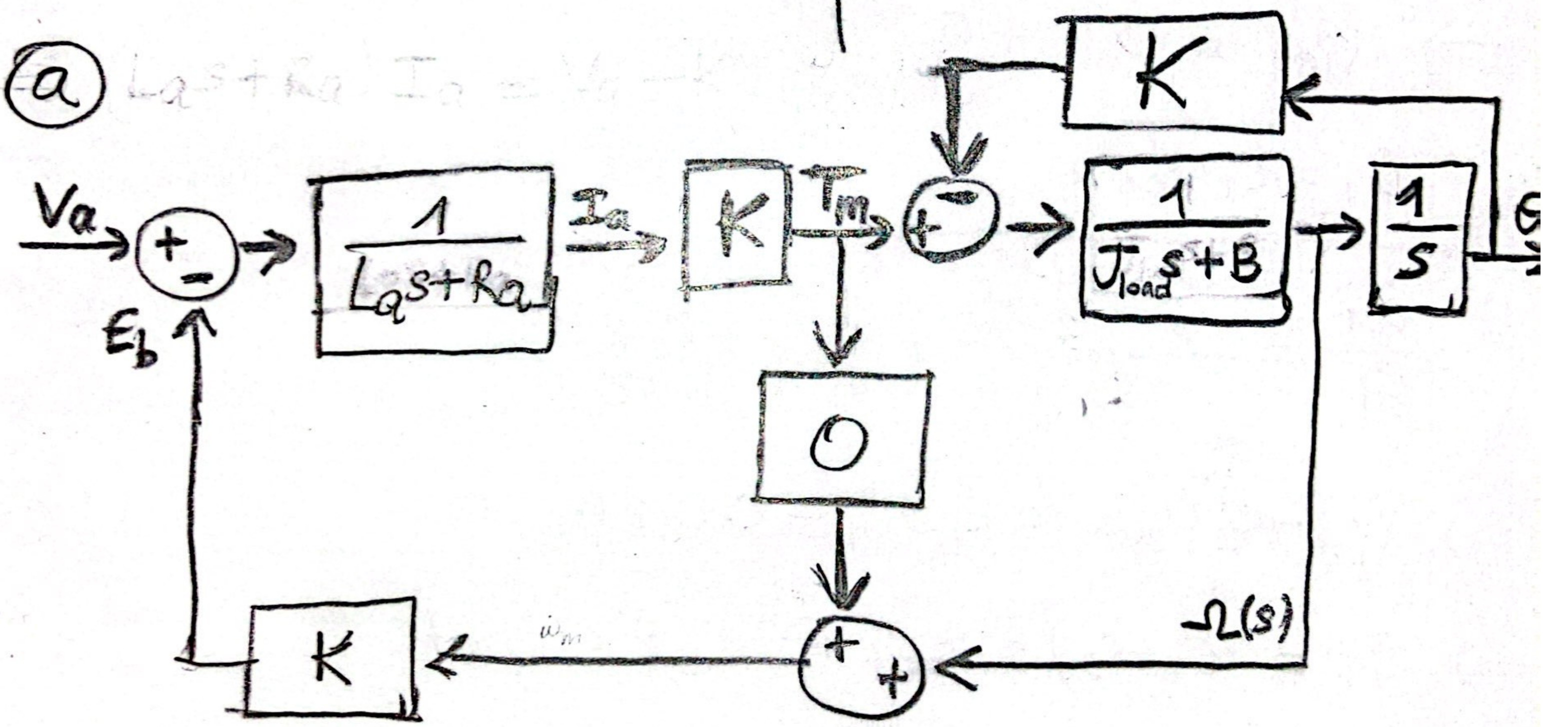
$\Rightarrow (L_a s + R_a) I_a = V_a - E_b$

$(J_{load}s^2 + Bs + K)\Theta = T_m$

$(J_{load}s + B)\Omega = T_m - K\Theta$.

$\Omega = \Theta s$

(a)



b) $i_a R_a + L_a i_a' + e_b = v_a \Rightarrow i_a' = \frac{v_a}{L_a} - \frac{e_b}{L_a} - \frac{i_a R_a}{L_a}$.

$J_{load}\,\theta'' = \tau_m - B\theta' - K\theta$, $w = \theta'$, $\tau_m = K i_a$, $e_b = K w$

$$x = \begin{bmatrix} \theta \\ w \\ i_a \end{bmatrix}.$$

$$x' = \begin{bmatrix} 0 & 1 & 0 \\ \frac{-K}{J_{load}} & \frac{-B}{J_{load}} & \frac{K}{J_{load}} \\ 0 & \frac{-K}{L_a} & \frac{-R_a}{L_a} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ 1/L_a \end{bmatrix} v_a$$

$$y = \theta = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \end{bmatrix} v_a$$