EE381 – Systems and Control	METU EEE – Fall 2023

HOMEWORK ASSIGNMENT – II
Due @23:59 26/12/2023

Q1. Consider a unity feedback system with open loop transfer function given as

$$G(s) = \frac{K(s+4)}{s(s+2)}.$$

a. Draw the root locus with respect to $K \geq 0$ with all the details (break-away/in points, imaginary axis crossings).

b. Mark the point on the root locus for which the closed-loop system has minimum damping ratio.

c. Analytically express the damping ratio of the closed loop system in terms of K and find the value of K which achieves minimum damping ratio.

d. Find the value of K for which minimum settling time (5%) is achieved when the closed loop system has the damping ratio $\xi = \sqrt{3}/2$.

Q2. Consider a unity feedback system with open loop transfer function

$$G(s) = \frac{K(s^2 - 2s + 2)}{s(s+1)(s+2)(s+4)}.$$

a. Draw the root-locus with respect to $K \geq 0$ with all the details (break-away/in points, imaginary axis crossings).

b. Find the location of all closed-loop poles for the $K \geq 0$ value(s) at which there are closed-loop poles on the imaginary axis.

c. What is the range of $K \geq 0$ for the stability of the closed-loop system.

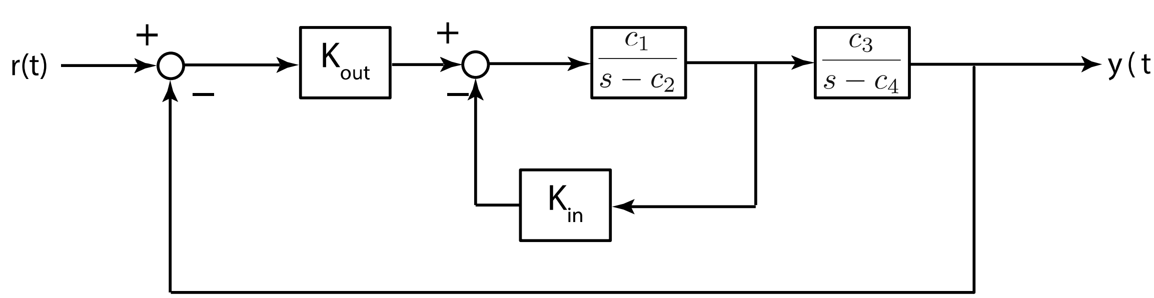
d. What is the range of values of $K \geq 0$ for which no oscillations are observed in the step response of the closed-loop system?

e. Calculate analytically the value of $K \geq 0$ at which there are two complex closed-loop poles with real part -4. What is(are) the frequency(frequencies) of oscillations observed in the step response of the system at this K value?

f. Describe how you can analytically find the value of $K \geq 0$ at which the step response of the closed-loop system has oscillations with frequency 3 rad/sec (No actual calculations are necessary). Show the related point on the root locus.

Note: In this question, feel free to use any computer tool for calculating the roots of high-order polynomials.

Q3. In this question you are going to analyze the following feedback-control system topology.



where $\boldsymbol{c_1}, \boldsymbol{c_2}, \boldsymbol{c_3}, \& \boldsymbol{c_4}$ are the last **4 non-zero digits** of your **student ID number,** e.g., if your student id no is 123405-6, then $c_1 = 3, c_2 = 4, c_3 = 5, c_4 = 6$.

a. Find the range of $\mathbf{K_{out}}$ and $\mathbf{K_{in}}$ values that makes the makes the closed-loop system stable.

b. Based on your result in **part a,** find a $\mathbf{K_{in}}$ value such that it is possible to tune $\mathbf{K_{out}}$ to obtain a stable closed-loop system. After that, draw the detailed root locus diagram of the system with respect to $\mathbf{K_{out}}$**.** **(**In the root-locus diagram you are also supposed to compute following details provided that they are applicable: *centroid of the asymptotes, break-in and break-away points (and associated gains), the point(s) and corresponding gains when the root-locus intersects the imaginary axis*).

c. Based on your result in p**art a,** find a $\mathbf{K_{out}}$ value such that it is possible to tune $\mathbf{K_{in}}$ to obtain a stable closed-loop system. After that, draw the detailed root locus diagram of the system with respect to $\mathbf{K_{in}}$**.** **(**In the root-locus diagram you are also supposed to compute following details provided that they are applicable: *centroid of the asymptotes, break-in and break-away points (and associated gains), the point(s) and corresponding gains when the root-locus intersects the imaginary axis*).