CENG 384 - Signals and Systems for Computer Engineers
20232
Written Assignment 4 Solutions

May 22, 2024

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1. (a)

$$\begin{aligned}
\int_{-\infty}^{t} x(\tau) - \int_{-\infty}^{t} 6y(\tau) + 4x(t) - 5y(t) &= y'(t) \\
x(t) - 6y(t) + 4x'(t) - 5y'(t) &= y''(t) \\
4x'(t) + x(t) &= y''(t) + 5y'(t) + 6y(t)
\end{aligned}$$

(b)

$$\begin{aligned}
4j\omega X(j\omega) + X(j\omega) &= (j\omega)^2 Y(j\omega) + 5j\omega Y(j\omega) + 6Y(j\omega) \\
(4j\omega + 1)X(j\omega) &= ((j\omega)^2 + 5j\omega + 6)Y(j\omega) \\
H(j\omega) &= \frac{4j\omega + 1}{(j\omega)^2 + 5j\omega + 6}
\end{aligned}$$

(c)

$$\begin{aligned}
\frac{4j\omega + 1}{(j\omega)^2 + 5j\omega + 6} &= \frac{B}{j\omega + 3} + \frac{A}{j\omega + 2} \\
Aj\omega + 3A + Bj\omega + 2B &= 4j\omega + 1 \\
A + B = 4 &\qquad 3A + 2B = 1 \\
A = -7 &\qquad B = 11 \\
H(j\omega) &= \frac{11}{j\omega + 3} - \frac{7}{j\omega + 2} \\
h(t) &= (11e^{-3t} - 7e^{-2t})u(t)
\end{aligned}$$

(d)

$$\begin{aligned}
Y(j\omega) &= X(j\omega) \cdot H(j\omega) \\
&= \frac{1}{4} \cdot \frac{1}{\frac{1}{4} + j\omega} \cdot \frac{4j\omega + 1}{(j\omega)^2 + 5j\omega + 6} \\
&= \frac{1}{(j\omega + 2)(j\omega + 3)} = \frac{A}{j\omega + 2} + \frac{B}{j\omega + 3}
\end{aligned}$$

$Aj\omega + 3A + Bj\omega + 2B = 1$
$A + B = 0 \quad 3A + 2B = 1 \quad \Rightarrow \quad A = 1 \quad B = -1$

$$\begin{aligned}
Y(j\omega) &= \frac{1}{j\omega + 2} - \frac{1}{j\omega + 3} \\
y(t) &= (e^{-2t} - e^{-3t})u(t)
\end{aligned}$$

2. (a)

$$\begin{aligned}
H(j\omega) &= \frac{j\omega + 4}{-\omega^2 + 5j\omega + 6} \\
\frac{Y(j\omega)}{X(j\omega)} &= \frac{j\omega + 4}{(j\omega)^2 + 5j\omega + 6}
\end{aligned}$$

$$\begin{aligned}
((j\omega)^2 + 5j\omega + 6)Y(j\omega) &= (j\omega + 4)X(j\omega) \\
y''(t) + 5y'(t) + 6y(t) &= x'(t) + 4x(t)
\end{aligned}$$

(b)

$$\begin{aligned} H(j\omega) &= \frac{j\omega+4}{(j\omega)^2+5j\omega+6} \\ &= \frac{A}{j\omega+2}+\frac{B}{j\omega+3} \end{aligned}$$

$Aj\omega+3A+Bj\omega+2B = j\omega+4$
$A+B=1 \quad 3A+2B=4 \quad \Rightarrow \quad A=2 \quad B=-1$

$$\begin{aligned} H(j\omega) &= \frac{2}{j\omega+2}-\frac{1}{j\omega+3} \\ h(t) &= (2e^{-2t}-e^{-3t})u(t) \end{aligned}$$

(c)

$$X(j\omega) = \frac{1}{j\omega+4}-\frac{1}{(j\omega+4)^2}$$

$$\begin{aligned} Y(j\omega) &= X(j\omega)\cdot H(j\omega) \\ &= \frac{1}{(j\omega+2)(j\omega+4)} \end{aligned}$$

(d)

$$Y(j\omega) = \frac{A}{j\omega+2}+\frac{B}{j\omega+4}$$

$Aj\omega+4A+Bj\omega+2B=1$
$A+B=0 \quad 4A+2B=1 \quad \Rightarrow \quad A=\frac{1}{2} \quad B=-\frac{1}{2}$

$$\begin{aligned} Y(j\omega) &= \frac{1/2}{j\omega+2}-\frac{1/2}{j\omega+4} \\ y(t) &= \frac{1}{2}(e^{-2t}-e^{-4t})u(t) \end{aligned}$$

3. (a)

$$x[n] = \left(\frac{2}{3}\right)^n u[n] \longleftrightarrow X(e^{j\omega}) = \frac{1}{1-\frac{2}{3}e^{-j\omega}}$$

$$y[n] = n\left(\frac{2}{3}\right)^{n+1} u[n] = \left(\frac{2}{3}(n+1)\left(\frac{2}{3}\right)^n - \frac{2}{3}\left(\frac{2}{3}\right)^n\right)u[n]$$

$$Y(e^{j\omega}) = \frac{2/3}{(1-\frac{2}{3}e^{-j\omega})^2}-\frac{2/3}{1-\frac{2}{3}e^{-j\omega}}$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2/3}{1-\frac{2}{3}e^{-j\omega}}-\frac{2}{3}$$

(b) Take IFT

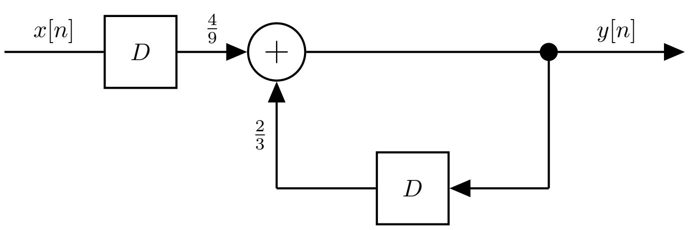
$$h[n] = \left(\frac{2}{3}\right)^{n+1}u[n]-\frac{2}{3}\delta[n]$$

(c)

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{4e^{-j\omega}}{9-6e^{-j\omega}}$$

$$9y[n]-6y[n-1] = 4x[n-1]$$

(d) The block diagram:



4. (a)

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = 2x[n]$$

(b)

$$Y(e^{j\omega})\left(1 - \frac{3}{4}e^{-j\omega} + \frac{1}{8}e^{-2j\omega}\right) = 2X(e^{j\omega})$$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{2}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})}$$

(c) By partial fraction we get

$$H(e^{j\omega}) = \frac{4}{1-\frac{1}{2}e^{-j\omega}} - \frac{2}{1-\frac{1}{4}e^{-j\omega}}$$

Take IFT and get

$$h[n] = 4\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{4}\right)^n u[n]$$

(d)

$$x[n] = \left(\frac{1}{4}\right)^n u[n] \overset{\text{FT}}{\longleftrightarrow} X(e^{j\omega}) = \frac{1}{1-\frac{1}{4}e^{-j\omega}}$$

$$Y(e^{j\omega}) = X(e^{j\omega})\cdot H(e^{j\omega}) = \frac{2}{(1-\frac{1}{2}e^{-j\omega})(1-\frac{1}{4}e^{-j\omega})^2}$$

By partial fraction we get

$$Y(e^{j\omega}) = \frac{-4}{1-\frac{1}{4}e^{-j\omega}} - \frac{2}{(1-\frac{1}{4}e^{-j\omega})^2} + \frac{8}{1-\frac{1}{2}e^{-j\omega}}$$

Take IFT and get

$$y[n] = -4\left(\frac{1}{4}\right)^n u[n] - 2(n+1)\left(\frac{1}{4}\right)^n u[n] + 8\left(\frac{1}{2}\right)^n u[n]$$

5.

$$H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega})$$

$$h_1[n] = \left(\frac{1}{3}\right)^n u[n] \overset{\text{FT}}{\longleftrightarrow} H_1(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$\begin{aligned} H_2(e^{j\omega}) &= H(e^{j\omega}) - H_1(e^{j\omega}) \\ &= \frac{-8}{4-e^{-j\omega}} = \frac{-2}{1-\frac{1}{4}e^{-j\omega}} \end{aligned}$$

Take IFT

$$h_2[n] = -2\left(\frac{1}{4}\right)^n u[n]$$

6.

```
import numpy as np
import matplotlib.pyplot as plt

# Parameters
a = 0.5
max_limit = 10
n = np.arange(-max_limit + 1, max_limit)
x = a ** np.abs(n)

# Discrete-time Fourier Transform
Wmax = 3 * np.pi
K = 4
k = np.linspace(0, K, 1001)
W = k * Wmax / K
XW = np.dot(x, np.exp(-1j * np.outer(n, W)))
XW_Mag = np.real(XW)

# Mirror the frequency and magnitude arrays to show negative frequencies
W = np.concatenate((-np.flip(W[1:]), W))
XW_Mag = np.concatenate((np.flip(XW_Mag[1:]), XW_Mag))

# Plotting
plt.figure(figsize=(10, 8))

# Plot for the discrete-time sequence x[n]
plt.subplot(2, 1, 1)
plt.stem(n, x, 'g', basefmt=" ")
plt.title('Discrete Time Sequence x[n] for a>0')
```

```
    plt.xlabel('n')
    plt.ylabel('x[n]')
    plt.axhline(0, color='black',linewidth=0.5)
    plt.axvline(0, color='black',linewidth=0.5)

    # Plot for the Discrete Time Fourier Transform X(exp(jW))
    plt.subplot(2, 1, 2)
    plt.plot(W, XW_Mag, color='b')
    plt.title('Discrete Time Fourier Transform X(exp(jW))')
    plt.xlabel('Frequency (rad/sample)')
    plt.ylabel('|X(exp(jW))|')
    plt.axhline(0, color='black',linewidth=0.5)
    plt.axvline(0, color='black',linewidth=0.5)

    plt.tight_layout()
    plt.show()
```

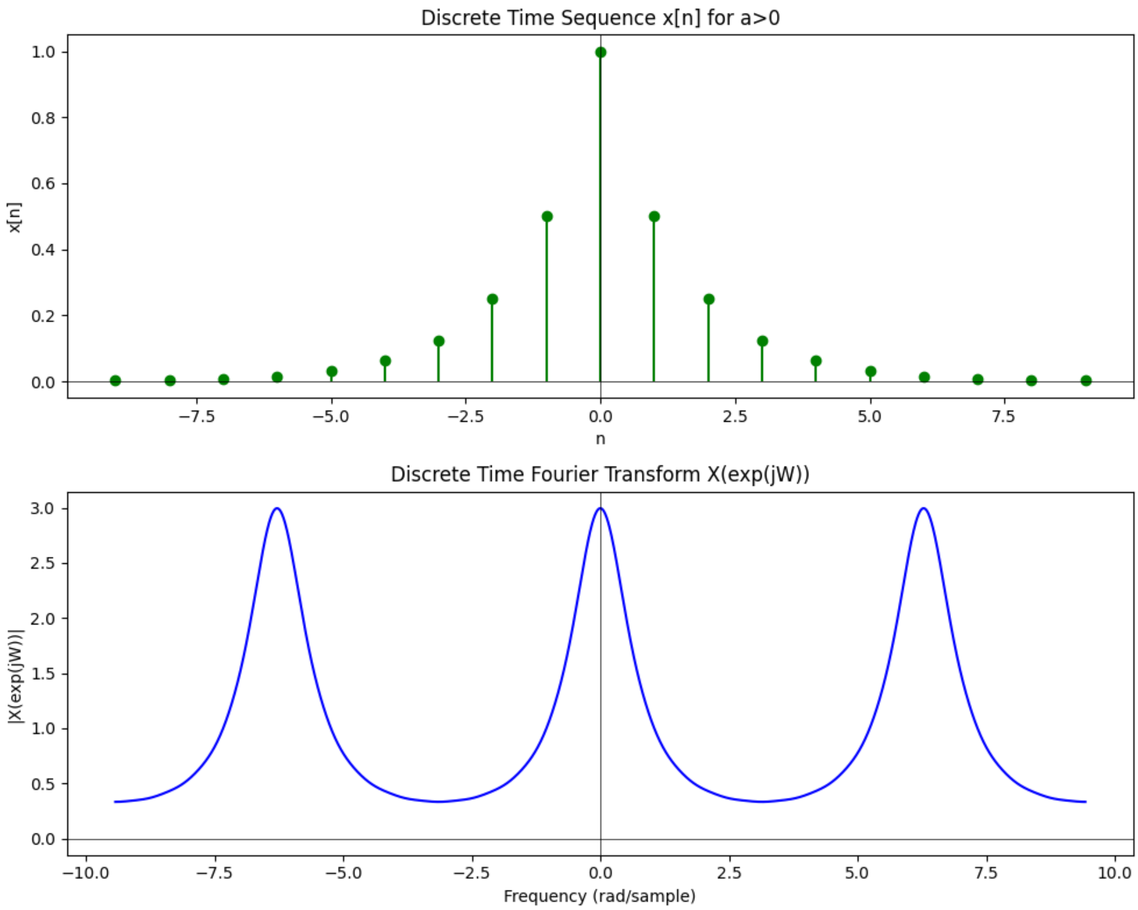


Figure 1: Discrete Time Sequence and Fourier Transform