MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF COMPUTER ENGINEERING

CENG 384

Spring 2024
Homework 1

Regulations:

- **Grouping:** You are strongly encouraged to work in pairs.
- **Drawing Plots:** Clearly label the coordinate axes and make sure that your plots are not open to different interpretations.
- **Submission:** You need to submit a pdf file named 'hw1.pdf' to the odtuclass page of the course. You need to use the given template 'hw1.tex' to generate your pdf files. Otherwise you will receive zero.
- **Deadline:** 23:55, March 17, 2024 (Sunday).
- **Late Submission:** Not allowed.

1. (10 pts) Consider the following complex number:

$$z = \frac{\sqrt{2} + \sqrt{2}j}{2 + 2\sqrt{3}j}.$$

   (a) (5 pts) Find the real and imaginary parts of z.

   (b) (5 pts) Find the magnitude and phase of z.

2. (8 pts) Given the signal $x(t)$ in Figure 1, draw the signal $y(t) = x(\frac{1}{2}t - 2)$.

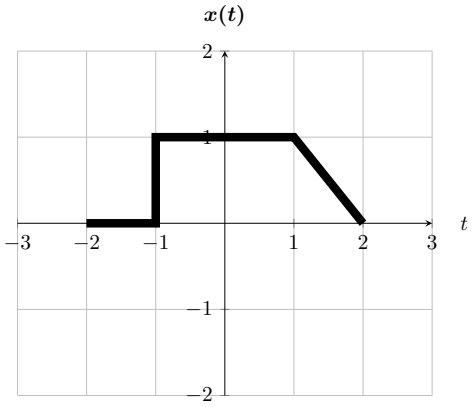


Figure 1: t vs. $x(t)$.

3. (17 pts) Given the signal $x[n]$ in Figure 2,

   (a) (5 pts) Find an analytical expression of $x[n]$ using shifted impulse functions.

   (b) (7 pts) Find and plot $y[n] = x[2n + 2] + x[1 - n]$.

   (c) (5 pts) Express $y[n] = x[2n + 2] + x[1 - n]$ in terms of the shifted impulse functions.

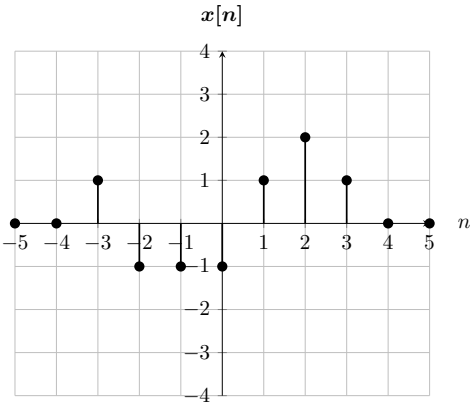
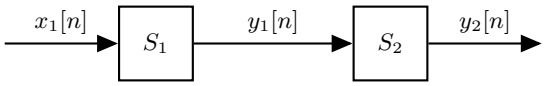


Figure 2: n vs. $x[n]$.

4. (15 pts) Determine whether the following signals are periodic and if periodic, find the fundamental period.
   (a) (5 pts) $x_1[n] = \cos(\frac{5\pi}{2}n)$,
   (b) (5 pts) $x_2[n] = \sin(5n)$,
   (c) (5 pts) $x_3(t) = 5\sin(4t + \frac{\pi}{3})$.

5. (10 pts) Show that,
$$\delta(at) = \frac{1}{a}\delta(t).$$

6. (20 pts) Consider two discrete time subsystems $S1$ and $S2$, defined by the following difference equations;
$$S1 : y_1[n] = 4x_1[n] + 2x_1[n-1],$$
$$S2 : y_2[n] = y_1[n-2].$$
   Suppose that $S1$ and $S2$ are connected in series to form an overall system, S, as shown below:



   (a) (5 pts) Find the difference equation for the overall system, which relates the input $x[n] = x_1[n]$ and output $y[n] = y_2[n]$.
   (b) (5 pts) Would the system equation you obtain in part a be different if the order of the series connection of $S1$ and $S2$ is reversed? In other words, is the series connection of sub systems commutative? Verify your answer.
   (c) (5 pts) Is the overall system S linear? Show if the superposition property holds or not.
   (d) (5 pts) Is the overall system S time invariant? Verify your answer.

7. (20 pts) Programming.
   Construct a Python program to evaluate the linearity of two different discrete-time systems using SymPy. With SymPy, you should define symbolic variables for the time index and the input signals, create symbolic functions to represent the systems, and use coefficients for linear combinations of inputs. Upon classification, the program should output **The given system is a Linear system** if the system adheres to linear properties, or **The given system is a Non-Linear system** if it does not. Write your solution in Python, utilizing SymPy for symbolic mathematics operations. The systems to be tested are:
   (a) (10 pts) $y[n] = n.x[n]$,
   (b) (10 pts) $y[n] = x^2[n]$.