Question 3 (25 pts)

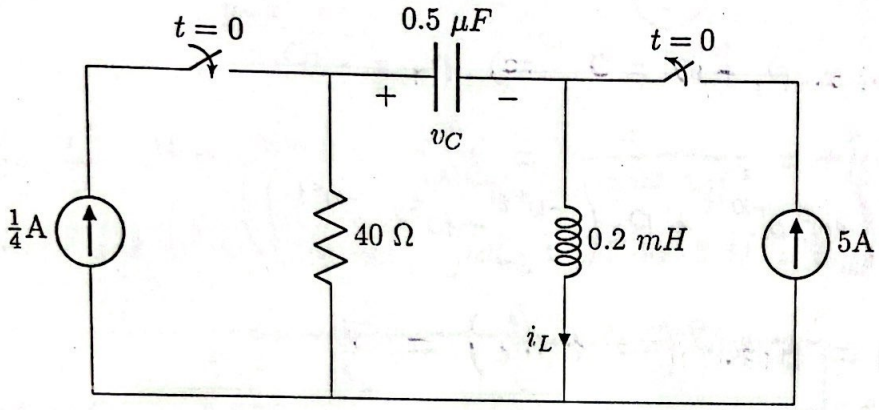
In the circuit shown below, the switch on the left has been kept open for a long time before closing at $t = 0$. Similary, the switch on the right has been kept closed for a long time before opening at $t = 0$.

a) Find the initial values of the capacitor voltage, $v_C(0^-)$, and the inductor current, $i_L(0^-)$.

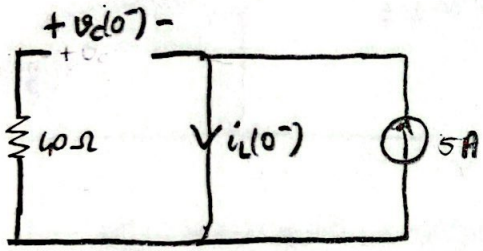
b) For $t > 0$, derive the differential equation for the circuit and write the characteristic equation for the capacitor voltage, $v_C(t)$. Find the roots of the characteristic equation.

c) Find $v_C(t)$ and $i_L(t)$ for $t \geq 0$.

d) Is the capacitor voltage response overdamped, underdamped or critically damped? Explain.



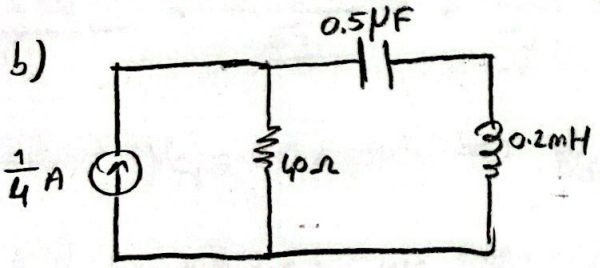
a)

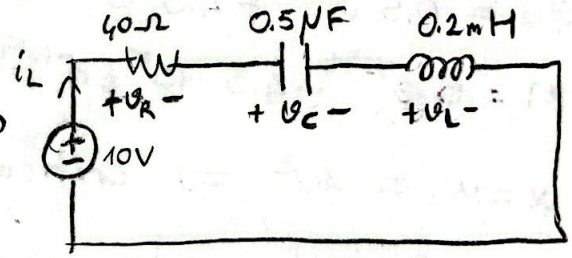


$i_L(0^-) = 5A$.

$v_C(0^-) = 0\,V$.

b)





KVL: $v_R + v_C + v_L = 10 \Rightarrow 40 i_L + v_C + 0.2 \cdot 10^{-3} \frac{di_L}{dt} = 10$

$i_L = 0.5 \cdot 10^{-6} \cdot \frac{dv_C}{dt}$; $\frac{di_L}{dt} = 5 \cdot 10^{-7} \frac{d^2 v_C}{dt^2}$

$\Rightarrow 2 \cdot 10^{-5} \frac{dv_C}{dt} + v_C + 10^{-10} \frac{d^2 v_C}{dt^2} = 10$

$\Rightarrow \frac{d^2 v_C}{dt^2} + 2 \cdot 10^5 \frac{dv_C}{dt} + 10^{10} v_C = 10^{11}$

$\alpha = \frac{R}{2L} = \frac{20}{0.2 \cdot 10^{-3}} = 10^5$, $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-1} \cdot 10^{-6} \cdot 10^{-3}}} = 10^5$

$\alpha = \omega_0 \Rightarrow$ critically damped.