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Answer 1

BASIS: $n = 1$. $6^2 - 1 = 35$ is divisible by both 5 and 7 .
IND. STEP: Assume that $6^{2k} - 1$ is divisible by both 5 and 7 . Then $6^{2k} - 1 = 35m$, where $m \in \mathbb{Z}$. Hence, $6^{2k} = 35m + 1$.
Put $k + 1$ in place of n . Then, $6^{2k+2} - 1 = 6^{2k} \times 36 - 1 = 36(35m + 1) - 1 = 1260m + 35 = 35(36m + 1)$ is divisible by 35 and hence, by both 5 and 7 .
Therefore, by induction, $6^{2n} - 1$ is divisible by both 5 and 7 , where $n \in \mathbb{N}^+$.

Answer 2

BASIS: $n = 3$. $H_3 = 8H_2 + 8H_1 + 9H_0 = 105 \leq 9^3 = 729$.
IND. STEP: Assume $H_4 \leq 9^4$, $H_5 \leq 9^5$, ... , $H_k \leq 9^k$. Then $H_{k+1} = 8H_k + 8H_{k-1} + 9H_{k-2} \leq 8 \cdot 9^k + 8 \cdot 9^{k-1} + 9 \cdot 9^{k-2} = (8/9 + 8/81 + 1/81) \cdot 9^{k+1} = 9^{k+1}$. So $H_{k+1} \leq 9^{k+1}$.

Answer 3

<u>For 0000:</u>
Assume that 0000 is a single object and is equal to 0.
00000000 $\rightarrow$ 00000 $\rightarrow \frac{5!}{5!} = 1$
00000001 $\rightarrow$ 00001 $\rightarrow \frac{5!}{4! \cdot 1!} = 5$
00000011 $\rightarrow$ 00011 $\rightarrow \frac{5!}{3! \cdot 2!} = 10$
00000111 $\rightarrow$ 00111 $\rightarrow \frac{5!}{2! \cdot 3!} = 10$
00001111 $\rightarrow$ 01111 $\rightarrow \frac{5!}{1! \cdot 4!} - 2 = 3$ (excluded 2 cases 00001111 and 11110000 since they will also occur in 1111 case.)
Total: $1 + 5 + 10 + 10 + 3 = 29$.
<u>For 1111:</u>
Similarly, 29 cases.

Result: $29 + 29 + 2 = 60$.

Answer 4

Pick 1 star in 10 distinct stars. Pick 2 habitable planets in 20 distinct habitable planets. Pick 8 nonhabitable planets in 80 distinct nonhabitable planets. Let H: habitable planet, N: nonhabitable planet. Then,

HNNNNNNNHNN $\rightarrow 6! \cdot 2! \cdot 2! \cdot 3$ (We multiply by 3 because there are 3 cases: HNNNNNNNHNN, NHNNNNNNNHN, NNHNNNNNNNH)
HNNNNNNNNHN $\rightarrow 7! \cdot 2! \cdot 1! \cdot 2$ (2 cases: HNNNNNNNNHN, NHNNNNNNNNH)
HNNNNNNNNNH $\rightarrow 8! \cdot 2!$.
Hence, the result is:
$\binom{10}{1} \times \binom{20}{2} \times \binom{80}{8} \times (6! \cdot 2! \cdot 2! \cdot 3 + 7! \cdot 2! \cdot 1! \cdot 2 + 8! \cdot 2!)$.

Answer 5

a) $a_1 = 1,\ a_2 = 2$ (11 and 2).
a_3 has 4 cases, which is $2^2 - 0$: $111, 12, 21, 3$.
a_4 has 7 cases, which is $2^3 - 1$: $1111, 112, 121, 211, 13, 31, 22$. The robot can not jump 4 cells, so a_4 does not have the 8th case.
Similarly, a_5 has 14 cases, which is $2^4 - 2$, since the robot can not jump 4 or 5 cells.
Hence, the answer is: $a_n = 2^{n-1} - (n-3) = 2^{n-1} - n + 3$.

b) $a_1 = 1,\ a_2 = 2$.

c) $a_9 = 2^8 - 9 + 3 = 250$.