

# Student Information

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## Answer 1

a)

$p$	$q$	$p \wedge q$	$\neg p \vee \neg q$	$(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$
$T$	$T$	$T$	$F$	$F$
$T$	$F$	$F$	$T$	$F$
$F$	$T$	$F$	$T$	$F$
$F$	$F$	$F$	$T$	$F$

The statement  $(p \wedge q) \leftrightarrow (\neg p \vee \neg q)$  is false under every interpretation. Hence, the statement is a contradiction.

b)

$$\begin{aligned} & p \rightarrow ((q \vee \neg q) \rightarrow (p \wedge q)) && \text{given.} \\ & p \rightarrow (T \rightarrow (p \wedge q)) && \text{TABLE 6, Negation laws, line 1.} \\ & p \rightarrow (F \vee (p \wedge q)) && \text{TABLE 7, line 1.} \\ & p \rightarrow (p \wedge q) && \text{TABLE 6, Identity laws, line 2.} \\ & \neg p \vee (p \wedge q) && \text{TABLE 7, line 1.} \\ & (\neg p \vee p) \wedge (\neg p \vee q) && \text{TABLE 6, Distributive laws, line 1.} \\ & T \wedge (\neg p \vee q) && \text{TABLE 6, Negation laws, line 1.} \\ & \neg p \vee q && \text{TABLE 6, Identity laws, line 1.} \end{aligned}$$

## Answer 2

- a)  $\forall x \exists y W(x, y)$
- b)  $\exists x \exists y (\neg F(x, y))$
- c)  $\forall x (W(x, P) \rightarrow A(\text{Ali}, x))$
- d)  $\exists y (W(\text{Büşra}, y) \wedge F(\text{TUBITAK}, y))$
- e)  $\exists x \exists y \exists z (S(x, y) \wedge S(x, z) \wedge y \neq z)$
- f)  $\forall x \forall y \exists p (\neg (W(x, p) \wedge W(y, p) \wedge x \neq y))$
- g)  $\exists x \exists y \exists p (W(x, p) \wedge W(y, p) \wedge \forall z (W(z, p) \rightarrow x = z \oplus y = z))$

### Answer 3

1		$p \rightarrow q$	
2		$(q \wedge \neg r) \rightarrow s$	
3		$\neg s$	
4			
		$q \wedge \neg r$	
5		$s$	$\Rightarrow$ E, 2, 4
6		$\perp$	$\neg$ E, 3, 5
7		$\neg(q \wedge \neg r)$	$\neg$ I, 4–6
8			
		$\neg(\neg q \vee r)$	
9			
		$\neg q$	
10		$\neg q \vee r$	$\vee$ I, 9
11		$\perp$	$\neg$ E, 8, 10
12		$\neg\neg q$	$\neg$ I, 9–11
13		$q$	$\neg\neg$ E, 12
14			
		$r$	
15		$\neg q \vee r$	$\vee$ I, 14
16		$\perp$	$\neg$ E, 8, 15
17		$\neg r$	$\neg$ I, 14–16
18		$q \wedge \neg r$	$\wedge$ I, 13, 17
19		$\perp$	$\neg$ E, 7, 18
20		$\neg q \vee r$	$\neg$ I, 8–19
21			
		$q$	
22			
		$\neg q$	
23		$\perp$	$\neg$ E, 21, 22
24			
		$\neg r$	
25		$\perp$	R, 23
26		$\neg\neg r$	$\neg$ I, 24–25
27		$r$	$\neg\neg$ E, 26

28				$r$	
29				$r$	R, 28
30				$r$	$\vee$ E, 20, 22–27, 28–29
31				$q \rightarrow r$	$\Rightarrow$ I, 21–30
32				$p$	
33				$q$	$\Rightarrow$ E, 1
34				$r$	$\Rightarrow$ E, 31
35				$p \rightarrow r$	$\Rightarrow$ I, 32–34

## Answer 4

1		$p$	
2		$p \rightarrow (q \wedge r)$	
3		$r \rightarrow s$	
4		$q \wedge r$	$\Rightarrow$ E, 1, 2
5		$q$	$\wedge$ E, 4
6		$r$	$\wedge$ E, 4
7		$s$	$\Rightarrow$ E, 3, 6
8		$s \wedge q$	$\wedge$ I, 5, 7
9			
10			
11			
12			

Line 12 is the negation of what Barış said. Thus, Barış is lying.

## Answer 5

1	$\forall x(P(x) \rightarrow (Q(x) \rightarrow R(x)))$	
2	$\exists x(P(x))$	
3	$\forall x(\neg R(x))$	
4	$P(a) \rightarrow (Q(a) \rightarrow R(a))$	$\forall E, 1$
5	$P(a)$	
6	$Q(a) \rightarrow R(a)$	$\Rightarrow E, 4, 5$
7	$Q(a)$	
8	$R(a)$	$\Rightarrow E, 6, 7$
9	$\neg R(a)$	$\forall E, 3$
10	$\perp$	$\neg E, 8, 9$
11	$\neg Q(a)$	$\neg I, 7-10$
12	$\exists x(\neg Q(x))$	$\exists I, 11$
13	$\exists x(\neg Q(x))$	$\exists E, 2, 5-12$