MIDDLE EAST TECHNICAL UNIVERSITY
DEPARTMENT OF COMPUTER ENGINEERING

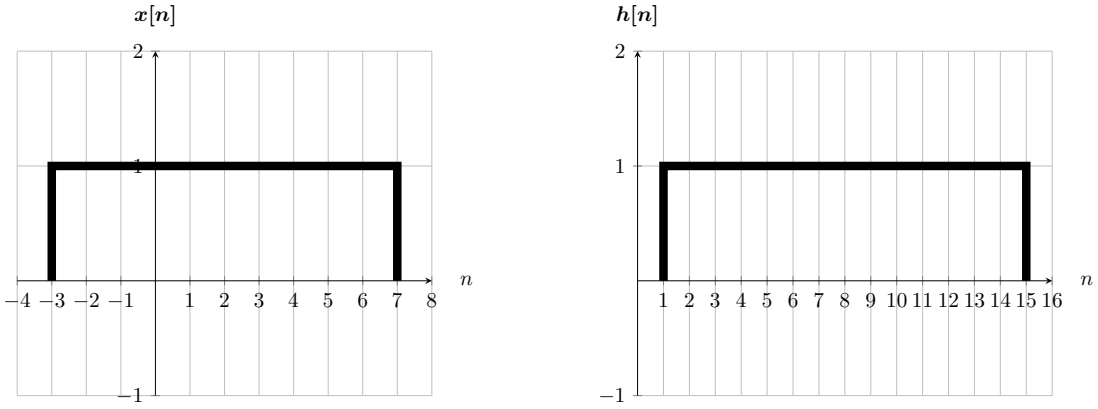
CENG 384

Spring 2024
Homework 2

Regulations:

- **Grouping:** You are strongly encouraged to work in pairs.
- **Drawing Plots:** Clearly label the coordinate axes and make sure that your plots are not open to different interpretations.
- **Submission:** You need to submit a pdf file named 'hw2.pdf' to the odtuclass page of the course. You need to use the given template 'hw2.tex' to generate your pdf files. Otherwise you will receive zero.
- **Deadline:** 23:55, April 02, 2024 (Tuesday).
- **Late Submission:** Not allowed.

1. (10 pts) Find the output, $y[n]$, of the system for the following input and impulse response plots:



(a) n vs. $x[n]$.

(b) n vs. $h[n]$.

Figure 1: $x[n]$ and $h[n]$.

2. (15 pts) Find and plot the outputs of the discrete time LTI systems given below for the input $x[n] = \delta[n] + 2\delta[n-2] - 3\delta[n-4]$ and the impulse response $h[n] = 2\delta[n+2] + \delta[n-2]$:
   (a) (5 pts) $y_1[n] = x[n] * h[n]$
   (b) (5 pts) $y_2[n] = x[n+2] * h[n]$
   (c) (5 pts) $y_3[n] = x[n+2] * h[n-2]$

3. (20 pts) Consider a discrete time casual LTI system, represented by the following difference equation:

$$y[n] = \frac{1}{5}x[n-1] + x[n].$$

   (a) (4 pts) Find the impulse response, $h[n]$, of the system.
   (b) (4 pts) Find the output, $y[n]$, for the input $x[n] = \delta[n-2]$.
   (c) (4 pts) Is this system BIBO stable? Explain.
   (d) (4 pts) Does this system have memory? Explain.
   (e) (4 pts) Is this system invertible? If yes, find its inverse.

4. (15 pts) The transfer function of a continuous time LTI system is given as follows:

$$H(\lambda) = \frac{2\lambda}{\lambda^2 - 2\lambda + 1},$$

   where the system is initially at rest.
   (a) (5 pts) Find the differential equation which represents this system.
   (b) (5 pts) Find the output this system for $x(t) = 0$.
   (c) (5 pts) Find the output this system for $x(t) = (2t+1)u(t)$.

5. (15 pts) A discrete time LTI system, which is initially at rest, is represented by the following difference equation:

$$y[n] = \frac{1}{5}y[n-1] + 2x[n-2].$$

   (a) (5 pts) Find the impulse response of this system.

   (b) (5 pts) Find the transfer function of this system.

   (c) (5 pts) Find a block diagram representation of this system using the adders and unit delay operators.

6. (10 pts) A continuous time casual LTI system is represented by the following differential equation:

$$y(t) = -\frac{1}{2}y'(t) + 4x(t).$$

   (a) (5 pts) Find a block diagram representation of this system using integrators and adders.

   (b) (5 pts) Find a block diagram representation of this system using differentiators and adders.

7. (15 pts) Programming.

   Write a Python program that calculates and plots the output of a causal Linear Time-Invariant (LTI) system defined by a given difference equation when $x[n] = \delta[n-1]$:

$$y[n] = \frac{1}{4}y[n-1] + x[n],$$

   without using built-in convolution functions (e.g., numpy.convolve()). Plot the first 5 samples of $y[n]$ using **matplotlib.pyplot**. You should write your code in `Python 3`. You are not allowed to use any library other than **matplotlib.pyplot**.