Sphere Surface Area Explanation

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It can be shown that the total surface area of a sphere can be calculated by:

$$\int_0^{2\pi} d\phi \int_0^{\pi} sin\theta d\theta = 4\pi$$

For some arbitrary portion surface area we can calculate the surface area by:

$$\int_a^b d\phi \int_c^d sin\theta d\theta = Area$$

Simplifying this becomes:

$$\int_a^b d\phi \int_c^d sin\theta d\theta = (b-a)(-cos(d)+cos(c)) = (b-a)(cos(c)-cos(d))$$

Additionally, we know that:

$$b - a = d - c = \pi/r = constant$$

Where r denotes the resolution that is user-defined. This comes from the fact that the for-loops are defined as the following:

```
for(i = 0; i < ANGULAR_RES; i++){
    sph.theta = double(PI)/double(ANGULAR_RES) * double(i);
    for(j = 0; j < ANGULAR_RES*2; j++){
        sph.phi = double(PI*2.0)/double(ANGULAR_RES*2.0) * double(j);
```

Therefore, we can rearrange the above equation into an equation only dependent on d.

$$(b-a)(cos(c)-cos(d)) = \frac{\pi}{r}\left(cos\left(d-\frac{\pi}{r}\right)-cos(d)\right) = Area$$

Where d is the variable that we are looping over in the θ for-loop. So, in the code, I continually add this area until we have integrated over the whole sphere. I implement an "area-counter" variable, that successively adds these areas for only the angles that fulfill the various criterion for a pair match.