$$h_i(t) \begin{cases} h_0(t) = 0 & 0 \leq t \leq T \\ h_1(t) & 0 \leq t \leq T \end{cases}$$

- $g_r(t)$: la réponse impulsionnelle du filtre de réception.
- $G_r(f)$ la réponse en fréquence.
- échantillonneur $t = t_0$

$x(t) \rightarrow \boxed{G_r(f)} \rightarrow y(t)$

$$x(t) = h_i(t) + b(t)$$

$b(t)$ AWGN $\mu_b = 0$
$\sigma_b^2 = \frac{N}{2}$

1- Exprimer en fonction $G_r(f)$ et $H_0(f)$ ou $H_1(f)$, en $a_0(t), a_1(t)$

$$a_0(t) = h_0(t) * g_r(t)$$

$$a_0(t) = \int_{-\infty}^{+\infty} H_0(f) \cdot G_r(f) \cdot e^{j2\pi f t} \, df$$

$$\boxed{a_0(t_0) = \int_{-\infty}^{+\infty} H_0(f) \, G_r(f) \, e^{j2\pi f t_0} \, df}$$

$$a_1(t) = h_1(t) * g_r(t)$$

$$a_1(t) = \int_{-\infty}^{+\infty} H_1(f) \cdot G(f) \cdot e^{j2\pi f t} \, df$$

$$a_1(t_0) = \int_{-\infty}^{+\infty} H(f) \, G_r(f) \cdot e^{j2\pi f t_0} \, df$$

2- Déterminer en fonction de N_0, de $G_r(f)$ la valeur $\sigma_{b_0}^2$ du

$$\sigma_{b_0}^2 = E[b_0^2] - \cancel{E[b_0]}^{2}$$

$R_{bb}(\tau) \rightarrow \boxed{G_r(f)} \rightarrow R_{b_0 b_0}(\tau)$

$$\sigma_{b_0}^2 = E[b_0^2] \quad \dots \quad (1)$$

$$\sigma_{b_0}^2 = R_{b_0 b_0}(0)$$

$$S_{b_0 b_0}(f) = |G_r(f)|^2 \cdot S_{bb}(f)$$

$$R_{b_0 b_0}(\tau) = \int_{-\infty}^{+\infty} |G_r(f)|^2 \cdot S_{bb}(\tau) \cdot e^{j2\pi f \tau} \, df$$

$$R_{b_0 b_0}(0) = \int_{-\infty}^{+\infty} |G_r(f)|^2 \cdot S_{bb}(f)$$

on a $S_{bb}(f) = N/2$

$$\boxed{R_{b_0 b_0}(0) = \sigma_{b_0}^2 = \int_{-\infty}^{+\infty} |G_r(f)|^2 \cdot N/2} \quad (1)$$

$$R_{bb}(\tau) = N_0\,\delta(\tau)$$

$$S_{bb}(f) = TF[R_{bb}(\tau)] = N_0 = \eta/2$$

$$\boxed{R_{b_0 b_0} = a_{b_0}^2 = N_0 \int_{-\infty}^{+\infty} Gr(f)^2}$$

4- $Pe = Q\left(\dfrac{a_0 - a_2}{2 a_{b_0}}\right)$ [démonstration dans TD ou bien cours.]

5- Déduire l'expression de la probabilité d'erreur Pe en fonction de Ed, η

$$Pe = Q\left(\frac{(a_0 - a_1)}{2 a_{b_0}}\right)$$

on a :

$\left(\frac{S}{N}\right)_0$ la sortie du filtre est égal :

$$\left(\frac{S}{N_0}\right) = \frac{a_i^2}{a_{b_0}^2} = \frac{\left|\int_{-\infty}^{+\infty} Gr(f) \cdot Hi(f)\, e^{j2\pi f_0 t_0}\, df\right|}{\cancel{\int_{-\infty}^{+\infty} |H(f)|^2 df} \int_{-\infty}^{+\infty} Gr^2(f)\, df}$$

d'après 3ami shwartz :

$$\left(\frac{S}{N_0}\right) \leq \frac{\left|\int_{-\infty}^{+\infty} Hi(f)\right|^2 df \cdot \cancel{\int_{-\infty}^{+\infty} Gr^2(f)\, df}}{\eta/2 \cdot \cancel{\int_{-\infty}^{+\infty} Gr^2(f)}}$$

$$\boxed{\left(\frac{S}{N}\right)_0 \leq \frac{\int_{-\infty}^{+\infty} |Hi(f)|^2}{\eta/2}}$$

2

d'après perseval $\int^{T} hi^2(t) = \int_{-\infty}^{+\infty} |Hi(f)|$ [l'energie ou la puissance dans domaine temporelle ...

$h_1(t)$ $\quad 0 \le t \le T$

$$\left(\frac{S}{N_0}\right) \le \frac{2E}{\eta}$$

E : l'énergie du signal $h(t)$

$$\left(\frac{S}{N_0}\right)_{max} = \frac{a_i^2}{a_{b_0}^2} = \frac{2E}{\eta}.$$

$$Q\left(\frac{a_0 - a_1}{2a_{b_0}}\right)$$

pour un signal différentielle

$a_0 - a_1$

$$\left(\frac{S}{N_0}\right) = \frac{(a_0 - a_1)^2}{a_{b_0}^2} = \frac{2\int_0^T (h_0(t) - h_1(t))^2 dt}{\eta}$$

$$E_d = \int_0^T (h_0(t) - h_1(t))^2 dt$$

$$\left(\frac{S}{N_0}\right) \Rightarrow \frac{(a_0 - a_1)^2}{a_{b_0}^2} = \frac{2E_d}{\eta}$$

$$\Rightarrow \left(\frac{(a_0 - a_1)^2}{2a_{b_0}^2} = \frac{E_d}{\eta}\right) \times \frac{1}{2}$$

$$\Rightarrow \frac{(a_0 - a_1)^2}{4a_{b_0}^2} = \frac{E_d}{2\eta}$$

$$\boxed{\frac{a_0 - a_1}{2a_{b_0}} = \sqrt{\frac{E_d}{2\eta}}}$$

$$P_e = Q\left(\frac{a_0 - a_1}{2a_{b_0}}\right) = Q\left(\sqrt{\frac{E_d}{2\eta}}\right)$$

$$P_e = Q\left(\frac{a_0 - a_1}{2\,a_{b_0}}\right) = Q\left(\sqrt{\frac{E_d}{2\eta}}\right)$$

$$E_d = \int_0^T \left(h_1(t) - h_2(t)\right)^2$$

$$E_d = \int_0^T \left(A\cos(w_p t) + A\cos(w_p t)\right)^2 dt$$

$$E_d = \int_0^T 4A^2 \cos^2(W_p t)\, dt$$

$$E_d = 4A^2 \left[\int_0^T \frac{1}{2}\, dt + \int_0^T \cos(2W_p t)\, dt \right.$$

$$= \frac{4A^2 T}{2} + \frac{4A^2}{2W_p} \sin(2W_p T)$$

$$= 2A^2 T + \frac{2A^2}{w_p} \sin(2w_p T)$$

car T est multiple de T_p donc $\sin(2W_p T) = 0$

$$\boxed{E_d = 2A^2 T}$$

$$P_e = Q\left(\sqrt{\frac{E_d}{2\eta}}\right) = Q\left(\sqrt{\frac{2A^2T}{2\eta}}\right) = Q\left(\sqrt{\frac{A^2T}{\eta}}\right)$$

$$\boxed{E_d = Q\left(\sqrt{\frac{A^2T}{\eta}}\right)}$$

6- ona: $P_e = Q\left(\sqrt{\frac{E_d}{2\eta}}\right)$

$$\begin{cases} h_1(t) = A\cos(w_{p_1} t) & 0 \le t \le T \\ h_2(t) = A\cos(w_{p_2} t) & 0 \le t \le T \end{cases}$$

$w_{p_1} T \gg 1$;

$w_{p_2} T \gg 1$;

$(w_{p_1} - w_{p_2}) T \gg 1$

$$E_d = \int_0^T (h_1(t) - h_2(t))^2 dt$$

$$= \int_0^T (A\cos(w_{p_1} t) - A\cos(w_{p_2} t))^2 dt$$

$$= \underbrace{\int_0^T A^2\cos^2(w_{p_1} t)\,dt}_{1} + \underbrace{\int_0^T A^2\cos^2(w_{p_2} t)\,dt}_{2} - \underbrace{2A^2\int_0^T \cos(w_{p_1} t)\cos(w_{p_2} t)\,dt}_{3}$$

$=$

$$1 = \int_0^T A^2\cos^2(w_{p_1} t)\,dt = \int_0^T \frac{A^2}{2} dt + \int_0^T \frac{A^2}{2}\cos(2w_{p_1} t)\,dt$$

$$1 = \frac{A^2 T}{2} + \cancel{\frac{A^2}{4w_{p_1}}\sin(2w_{p_1} T)}$$

0

$$2 = \frac{A^2}{2} T + \cancel{\frac{A^2}{4w_{p_2}}\sin(2w_{p_2} T)}$$

$$3 = 2A^2\int_0^T \cos(t(w_{p_1}+w_{p_2}))\,dt + 2A^2\int_0^T \cos(t(w_{p_1}-w_{p_2}))\,dt$$

0 0

$$= \frac{2A^2}{w_{p_1}+w_{p_2}}\cancel{\sin(T(w_{p_1}+w_{p_2}))} + \cancel{\frac{2A^2}{w_{p_1}-w_{p_2}}}\sin(T(w_{p_1}-w_{p_2}))$$

$$E_d = \frac{A^2 T}{2} + \frac{A^2 T}{2} = A^2 T \qquad \boxed{E_d = A^2 T}$$

5

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$+ \quad 1 + \sin(4\pi(f_1 \cdot t))$

$E_d = A^2 T$

calculer E_d

$$E_d = \int_0^T R(t)^2 \, dt$$

$$= \int_0^T A^2 \cos^2(w_{p1}) \, dt$$

$$= \int_0^T \frac{A^2}{2} \, dt + \cancel{\int_0^T \frac{A^2}{4w_p} s}$$

$$= \int_0^T \frac{A^2}{2} \, dt + \int_0^T \frac{A^2}{2} \cos(2w_{p1}t) \, dt$$

$$= \frac{A^2 T}{2} + \frac{A^2}{4w_{p1}} \cancelto{0}{\sin(2w_{p1}T)} \qquad w_{p1} >> 1$$

$$\boxed{E_d = \frac{A^2 T}{2}}$$

$$\boxed{P_e = Q\left(\sqrt{\frac{A^2 T}{2\eta}}\right) = Q\left(\sqrt{\frac{E_b}{\eta}}\right)}$$

$$CS \begin{cases} T & \text{si } |f| \leq \frac{1-\alpha}{2T} \\ \frac{T}{2}\left[1+\sin\left(\frac{\pi T}{\alpha}\left(\frac{1}{2T}-|f|\right)\right)\right] & \frac{1-\alpha}{2T} \leq |f| \leq \frac{1+\alpha}{2T} \\ 0 & \text{ailleurs} \end{cases}$$

$$R(f) \begin{cases} T & \text{si } |f| \leq \frac{3}{8T} \\ T\left(\frac{1}{2} - \frac{1}{2}\sin(4\pi|f|\cdot T)\right) & \frac{3}{8T} \leq f \leq \frac{5}{8T} \\ 0 & \text{ailleurs} \end{cases}$$

$$\frac{1-\alpha}{2T} = \frac{3}{8T} \longrightarrow \boxed{\alpha = \frac{1}{4}}$$

remplace $\alpha = \frac{1}{4}$ sur CS

$$\begin{cases} T & \text{si } |f| \leq \frac{1-1/4}{2T} = \frac{3}{8T} \\ \underbrace{\frac{T}{2}\left[1+\sin\left(\frac{\pi T}{1/4}\left(\frac{1}{2T}-|f|\right)\right)\right]}_{A} & \frac{3}{8T} \leq |f| \leq \frac{5}{8T} \end{cases}$$

$$A = \frac{T}{2}\left[1+\sin\left(\frac{\pi T}{1/4}\left(\frac{1}{2T}-|f|\right)\right)\right]$$

$$= \frac{T}{2}\left[1-\sin\left(4\pi\cancel{T}\left[\frac{1-2T|f|}{2\cancel{T}}\right]\right)\right]$$

$$= \frac{T}{2}\left[1-\sin\left(2\pi\left[1-2T|f|\right]\right)\right] = \frac{T}{2}\left[1-\sin\left(2\pi - 4\pi|f|T\right)\right]$$

7

$\frac{\gt}{8T}$

$$A = \frac{T}{2}\left[1 - \sin\left(-4\pi (f) T\right)\right]$$

$$= T\left[\frac{1 + \sin(4\pi |f| \cdot T)}{2}\right]$$

R(f) vérifie le critère de Nyquist

9 :

TD كميا