CS 5/7320
Artificial Intelligence

Search with Uncertainty

AIMA Chapters 4.3-4.5

Slides by Michael Hahsler
with figures from the AIMA textbook

Types of uncertainty we consider for now*

Nondeterministic Actions:
Outcome of an action in a state is uncertain.

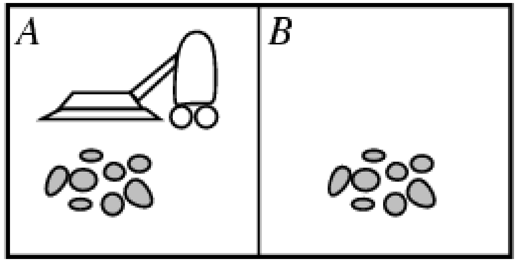
No observations:
Sensorless problem

Partially observable environments:
The agent does not know in what state the environment is.

Exploration:
Unknown environments and Online search

* we will quantify uncertainty with probabilities later.

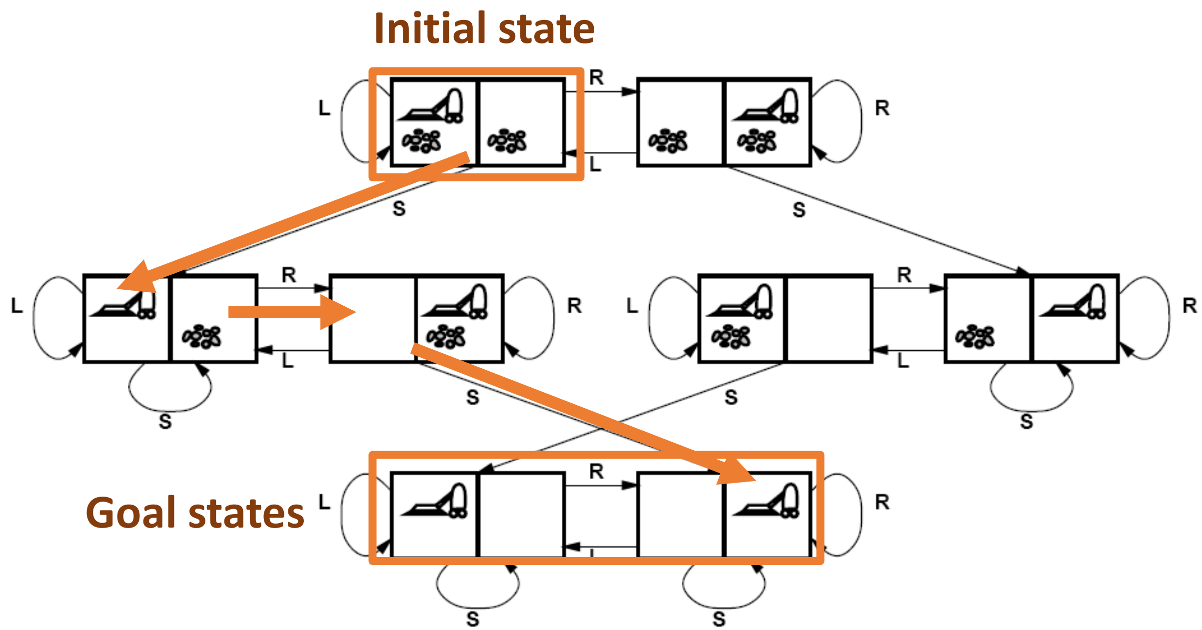
Remember: Solving search problems under Certainty



No Uncertainty

- Deterministic actions with known transition model $Result(s_1, a) = s_4$
- Full observability (we have sensors to see the whole environment)

State space: A state completely describes the environment and agent



Solution of the planning phase is a **sequence of actions** also called a **plan** that can be blindly followed: **[Suck, Right, Suck]**

Consequence of Uncertainty

Solution is typically not a fixed precomputed plan (sequence of actions), but a

conditional plan (also called strategy or policy)

that depends on percepts.

Nondeterministic Actions

Nondeterministic Actions

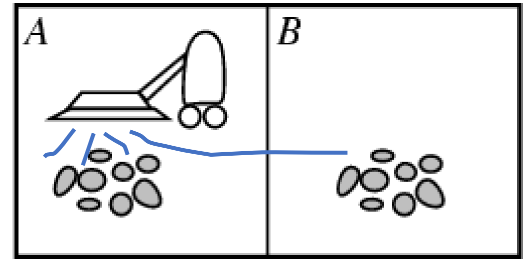
Outcome of actions in the environment is nondeterministic = **transition model need to describe uncertainty**

Example transition:

$$Results(s_1, a) = \{s_2, s_4, s_5\}$$

i.e., action a in s_1 can lead to one of several states.

Example:
Erratic Vacuum World

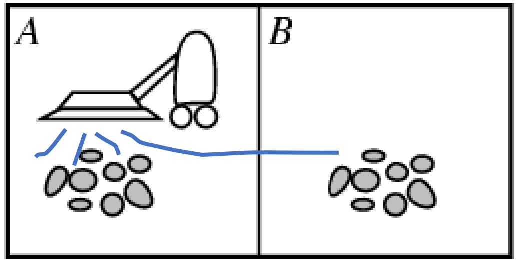


Regular fully-observable vacuum world, but the action ‘**suck**’ is more powerful and **nondeterministic**:

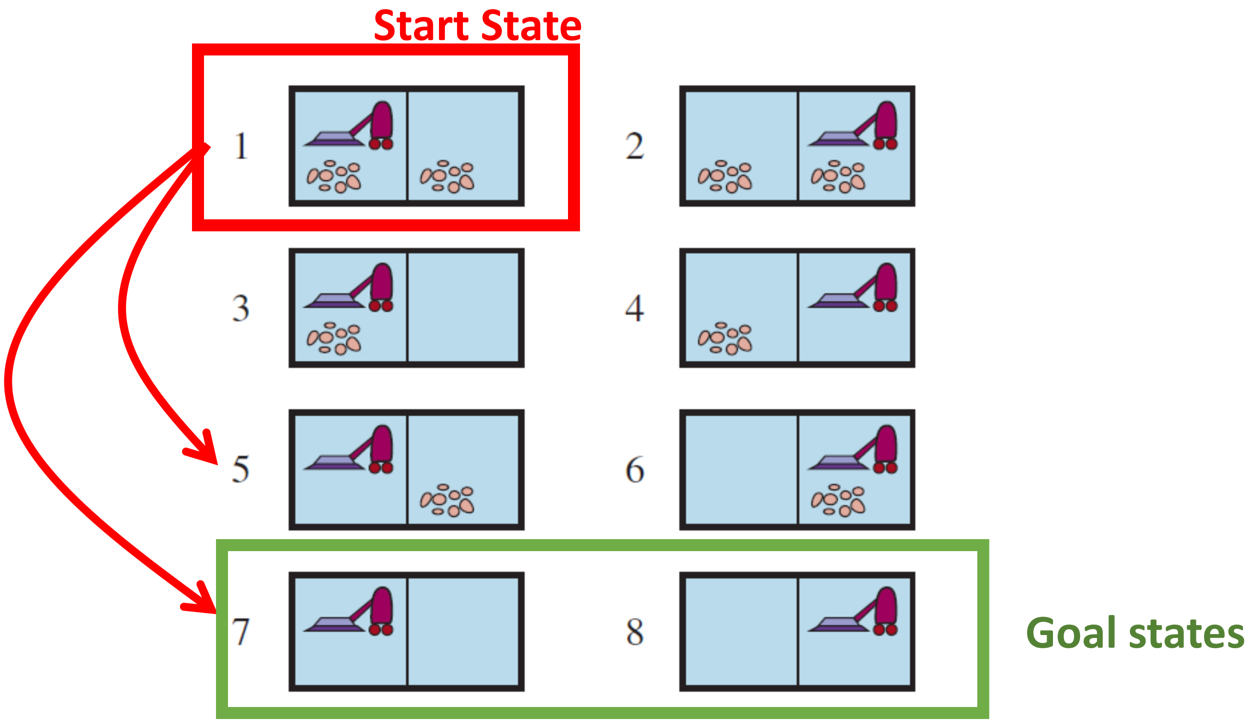
a) **On a dirty square**: cleans the square and sometimes cleans dirt on adjacent squares as well.

b) **On a clean square**: sometimes deposits some dirt on the square.

Example: Erratic Vacuum World



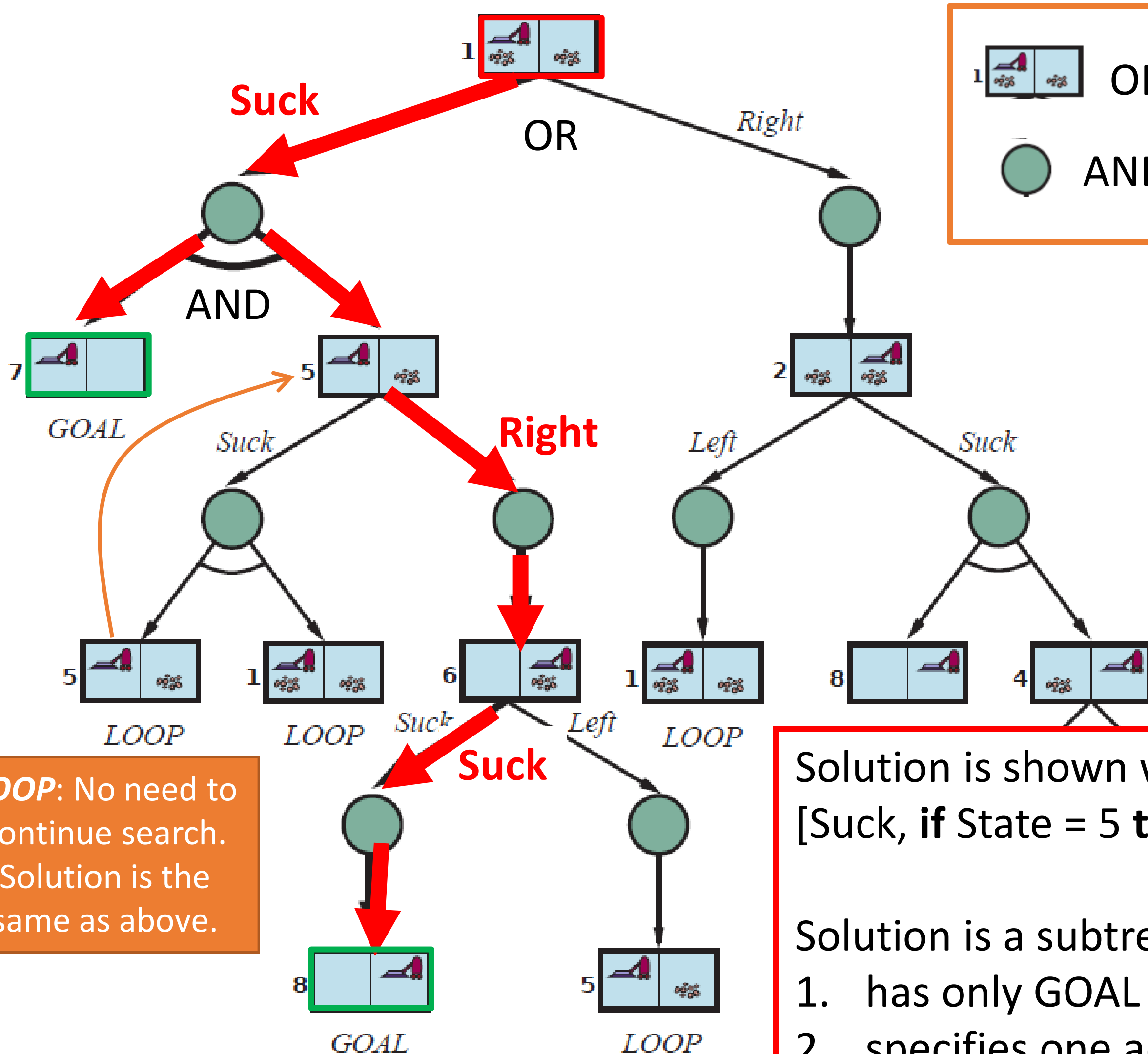
$Results(1, Suck) = \{5, 7\}$



We need a conditional plan

[Suck, **if** State = 5 **then** [Right, Suck] **else** []]

Finding a Cond. Plan: AND-OR Search Tree



OR node (choose one action)

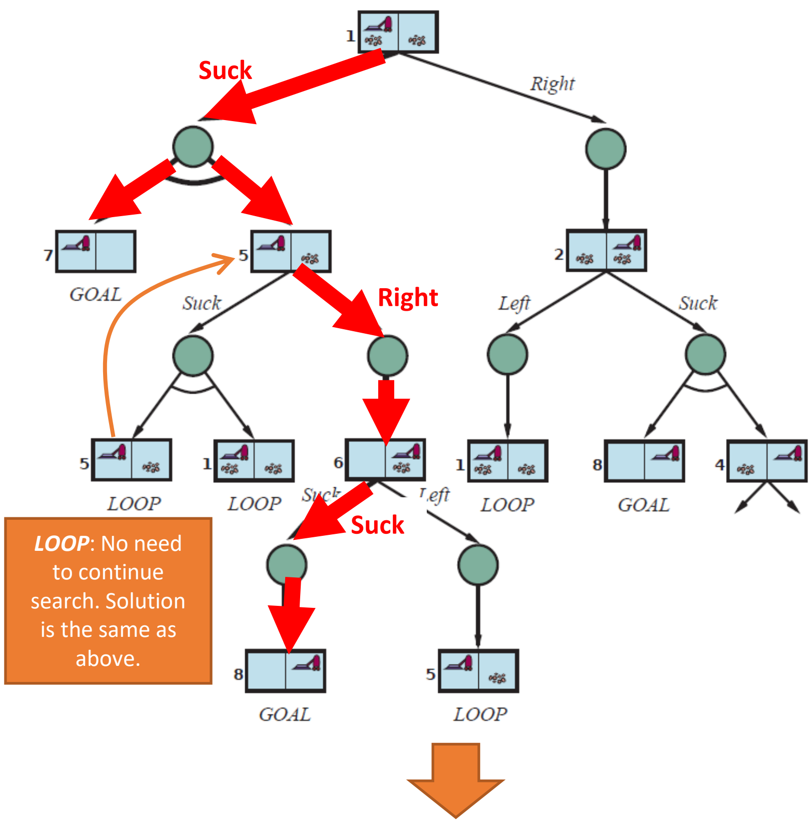
AND node (all possible outcomes)

LOOP: No need to continue search. Solution is the same as above.

Solution is shown with **bold** arrows:
[Suck, **if** State = 5 **then** [Right, Suck] **else** []]

Solution is a subtree that
1. has only GOAL leaf nodes
2. specifies one action at each OR node (state)
3. includes every outcome of AND nodes

AND-OR Tree search: Idea



[Suck, **if** State = 5 **then** [Right, Suck] **else** []]

- Descend the tree by trying an action in each OR node and considering all resulting states of the AND nodes.
- Remove branches (actions) if we cannot find a subtree below that leads to only goal nodes. (see failure in the code on the next slide).
- Stop when we find a subtree that only has goal states in all leaf nodes (loop nodes can be ignored).
- Construct the conditional plan that represents the subtree.

AND-OR Recursive DFS Algorithm

= nested If-then-else statements

path is only maintained for cycle checking!

```
function AND-OR-SEARCH(problem) returns a conditional plan, or failure
   return OR-SEARCH(problem, problem.INITIAL, [ ])

function OR-SEARCH(problem, state, path) returns a conditional plan, or failure
   if problem.IS-GOAL(state) then return the empty plan
   if IS-CYCLE(path) then return failure              // don't follow loops using path.
   for each action in problem.ACTIONS(state) do       // try all possible actions
      plan ← AND-SEARCH(problem, RESULTS(state, action), [state] + path])
      if plan ≠ failure then return [action] + plan]
   return failure                                     // fail means we found no action that leads to
                                                      // a goal-only subtree

function AND-SEARCH(problem, states, path) returns a conditional plan, or failure
   for each s_i in states do                          // try all possible outcomes, none can fail!
      plan_i ← OR-SEARCH(problem, s_i, path)          // (= belief state)
      if plan_i = failure then return failure
   return [if s_1 then plan_1 else if s_2 then plan_2 else ...if s_{n-1} then plan_{n-1} else plan_n]
```

Notes:

- The DFS search tree is implicitly created using the call stack (recursive algorithm).
- DFS is **not optimal**! BFS and A* search can be used to find better solutions (e.g., smallest subtree).

Use of Conditional Plans

- The conditional plan can be used in a **model-based reflex agent**.

Example: After the initial action "suck"

Agent

Agent's State

Step 2

Step	Program
1	[Suck,
2	**if** State = 5 **then**
3	[Right,
4	Suck]
	else
4b	[]
	]

Search with no Observations

Using Actions to
“Coerce” the World into
Known States

No Observations

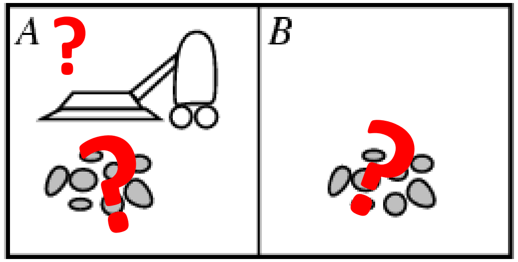
Sensorless problem = *unobservable environment* also called a *conformant problem.*

Why is this useful?

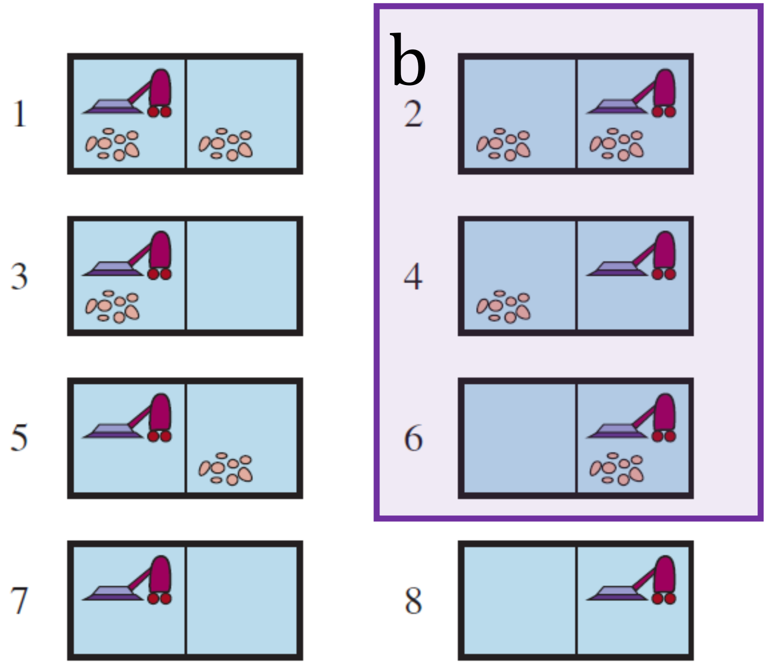
- **Example**: Doctor prescribes a broad-band antibiotic instead of performing time-consuming blood work for a more specific antibiotic. This saves time and money.

- **Basic idea**: Find a solution (a sequence of actions) that **works (reasonably well) from any state** and then just blindly execute it (open loop system).

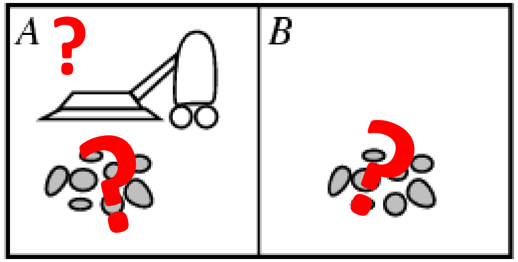
Belief State



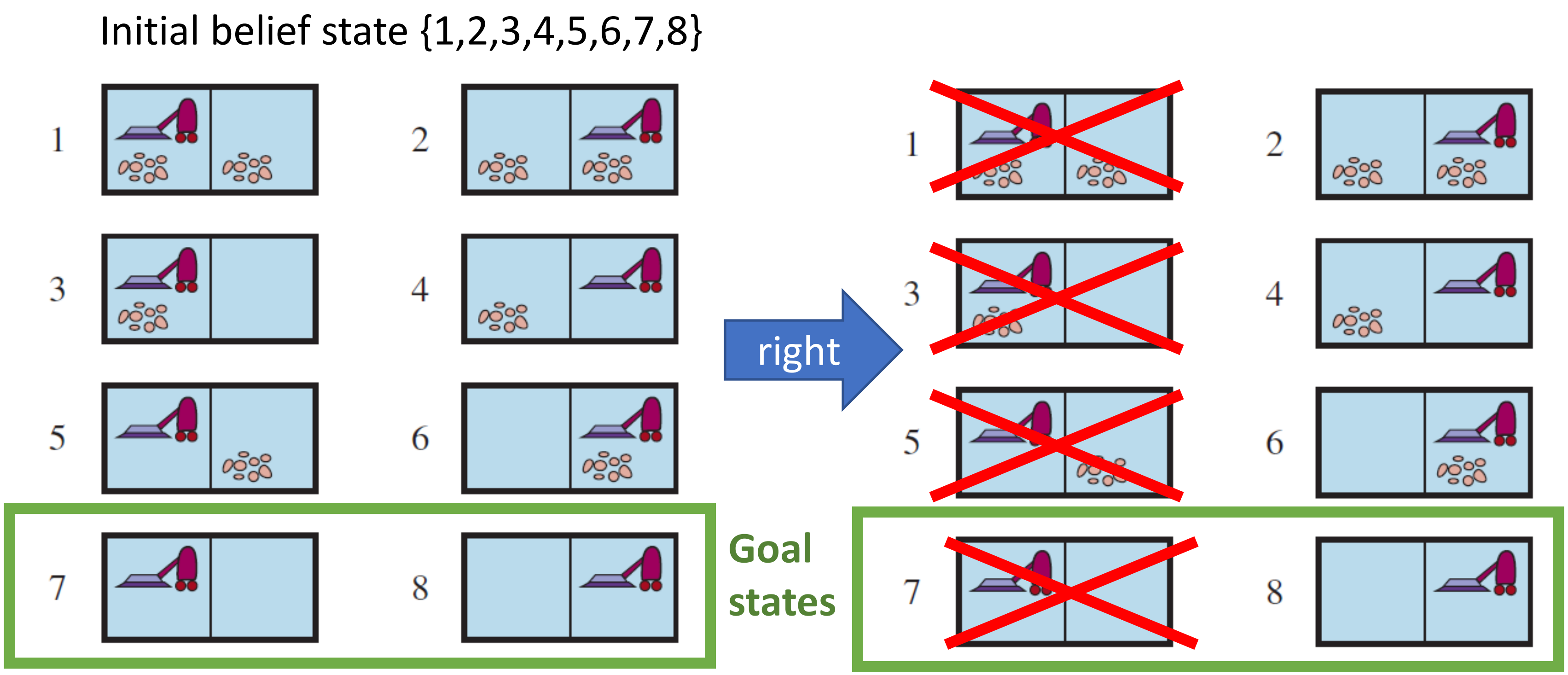
- The agent does not know in which state it is exactly in.
- However, it may know that it is in one of a set of possible states. This set is called a **belief state** of the agent.
- Example: $\mathrm{b} = \{s_2, s_4, s_6\}$



Actions to Coerce the World into States



- Actions can reduce the number of possible states.
- **Example**: Deterministic vacuum world. Agent does not know its position and the dirt distribution.



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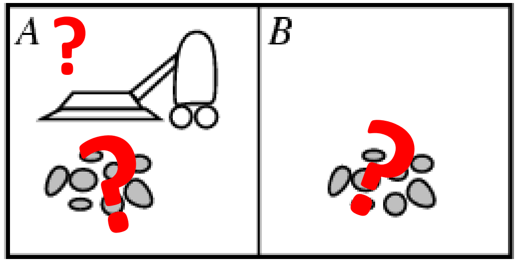

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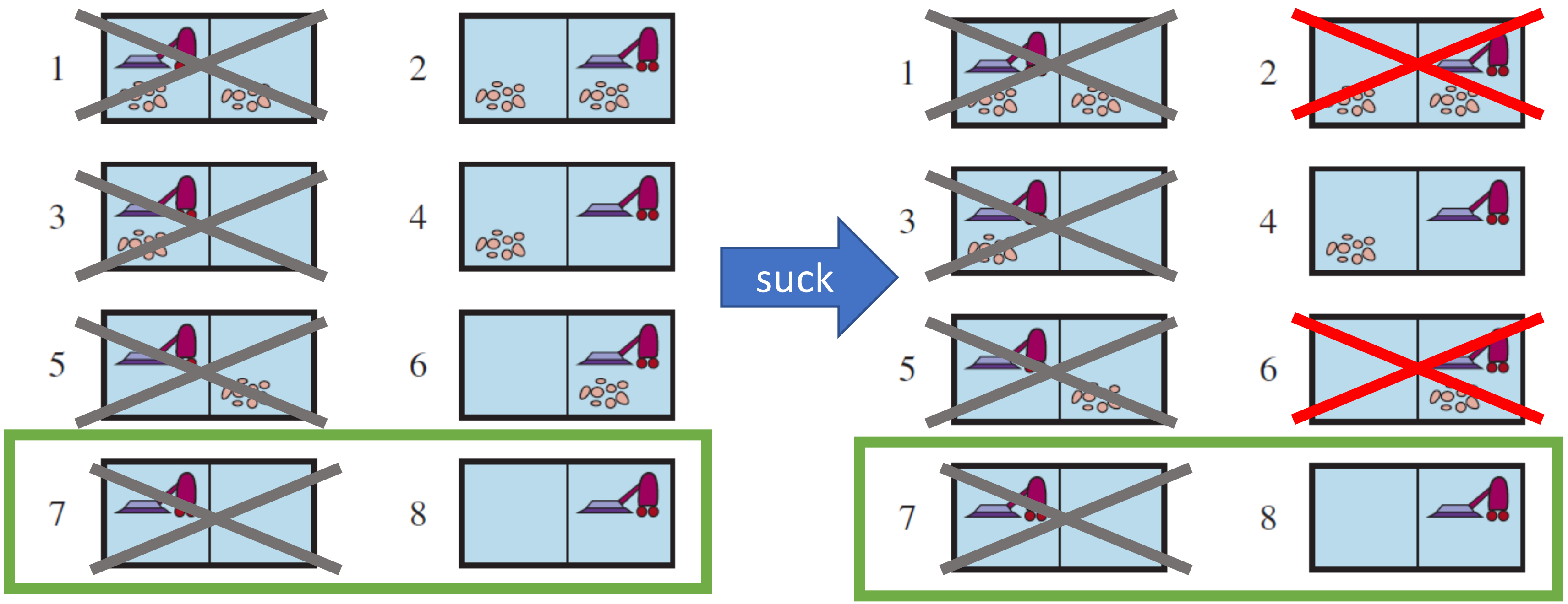

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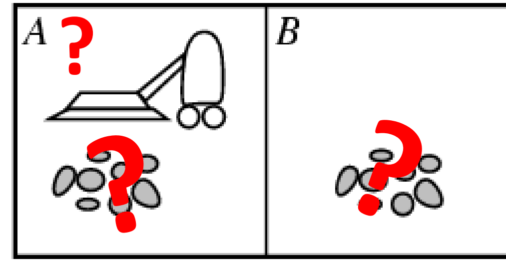
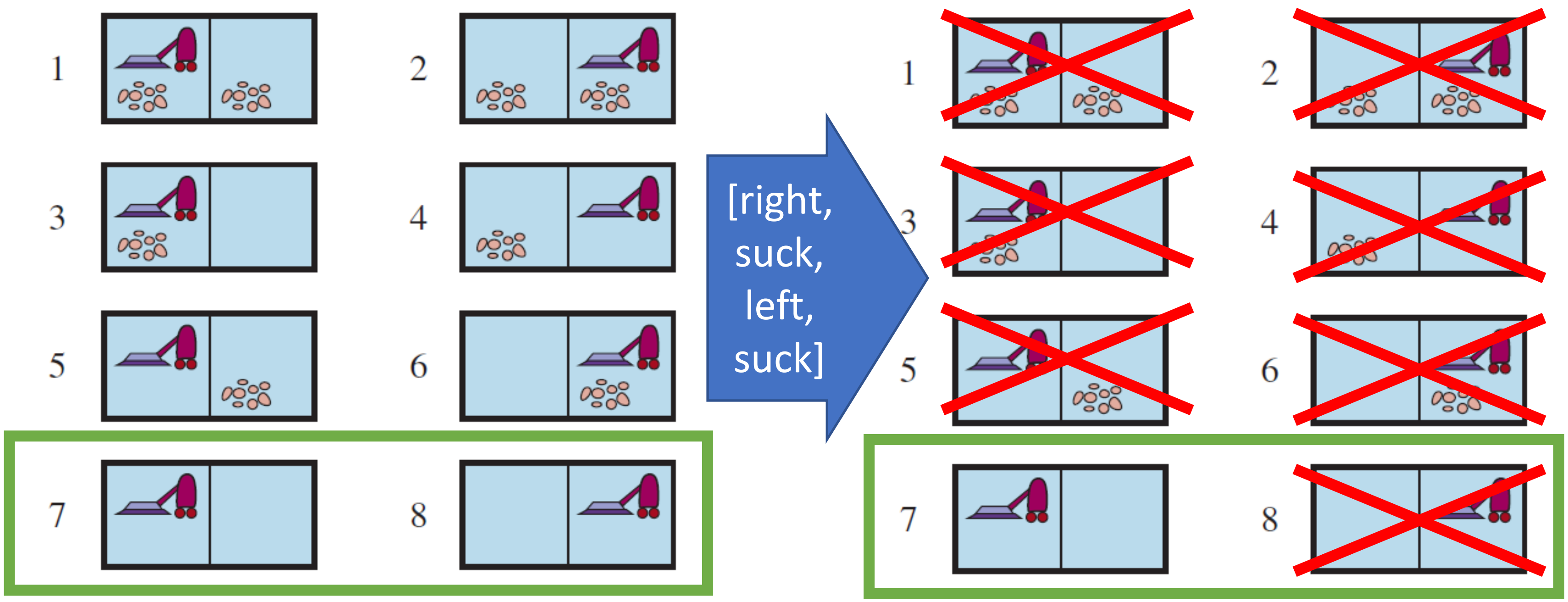
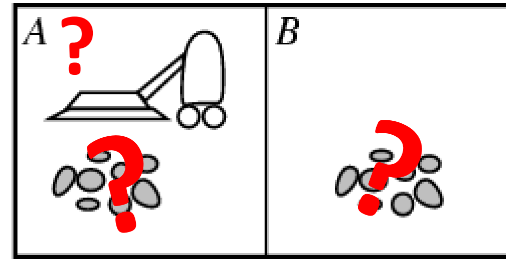


- The action sequence [right, suck, left, suck] coerces the world into the goal state 7. It works from any initial state!
- There are no observations so there is no need for a conditional plan.



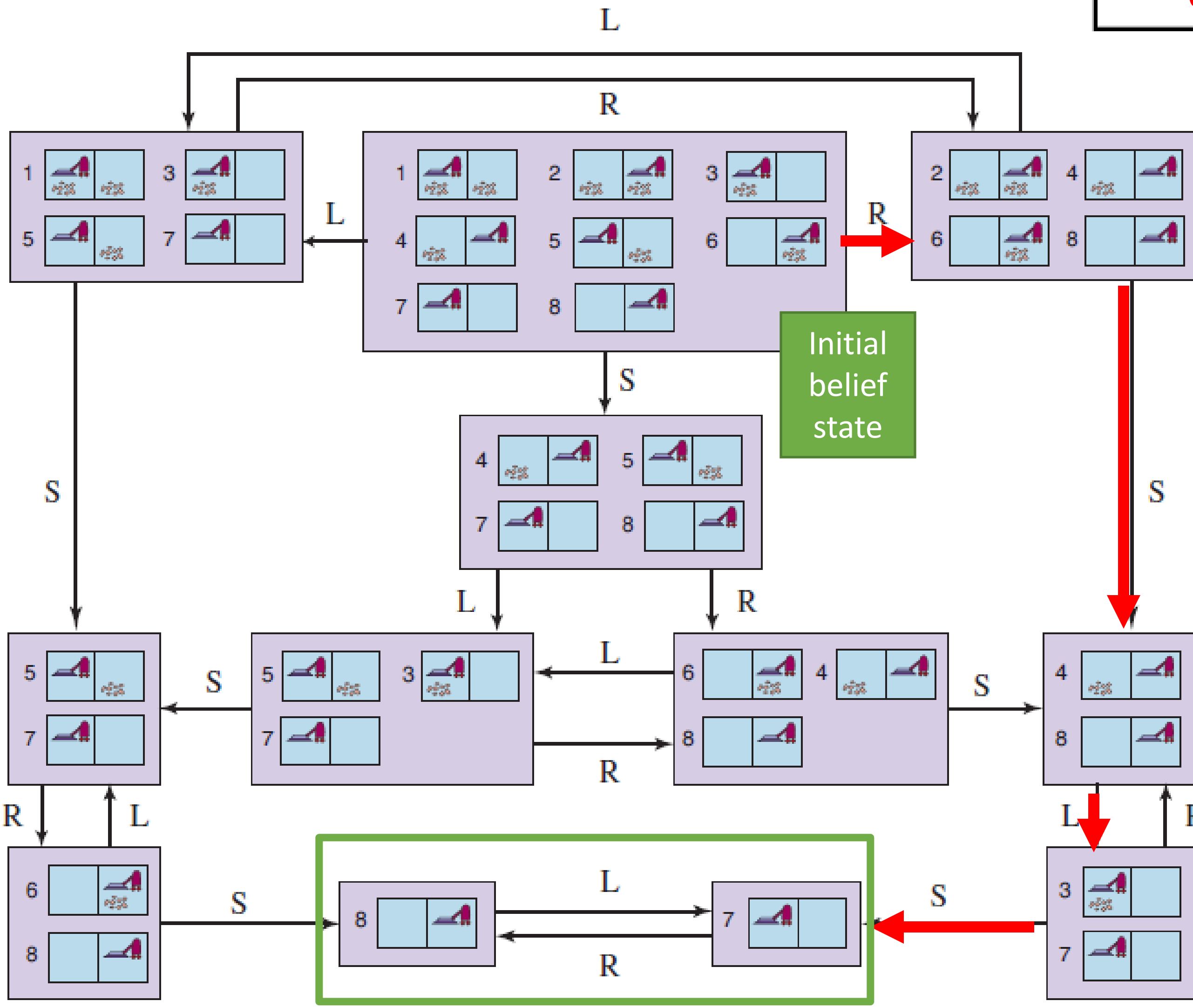
Example: The reachable belief-state space for the deterministic, sensorless vacuum world



Size of the belief state space depends on the number of states N:

$$\mathcal{P}_s = 2^N = 2^8 = 256$$

Only a small fraction (12 states) are reachable.



No observations, so we get a solution sequence from an initial belief state:
[Right, Suck, Left, Suck]

Finding a Solution Sequence

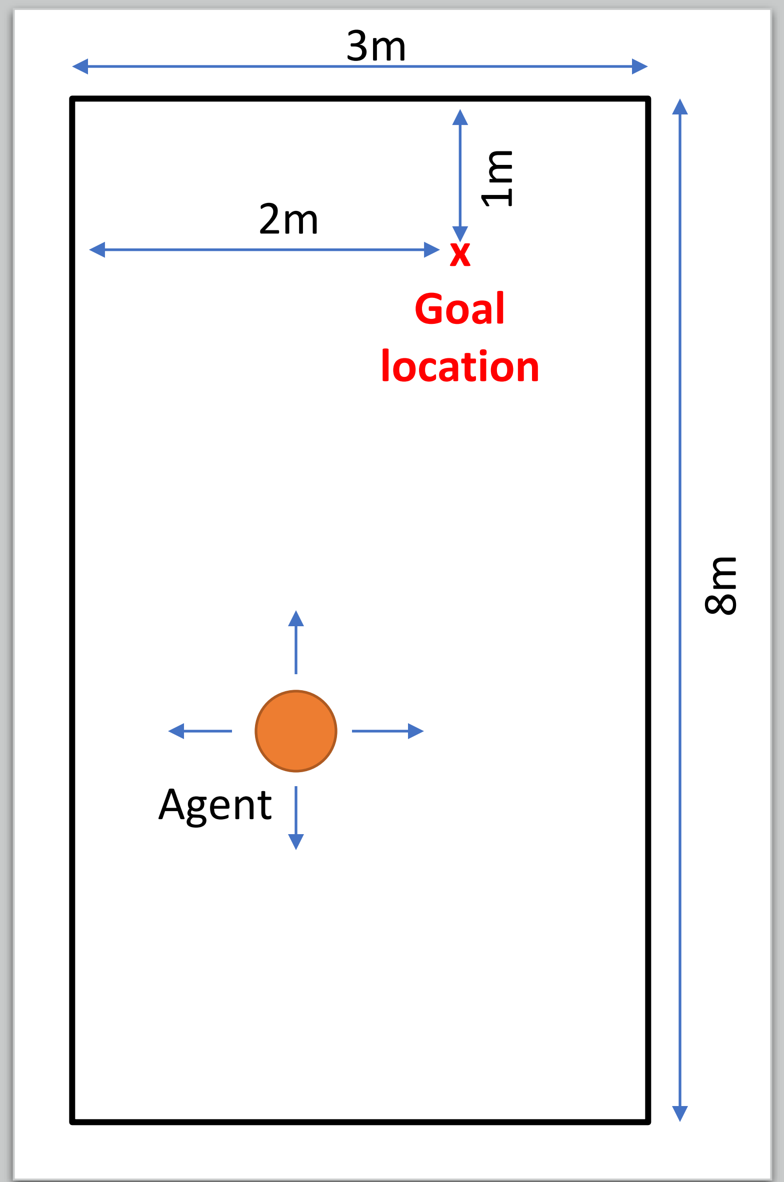
Note: State space size makes this impractical for larger problems!

Formulate as a regular search and solve with DFS, BFS or A*:

- **States**: All belief states (=powerset $\mathcal{P}_s$ of states of size 2^N for N states)
- **Initial state:** Often the belief state consisting of all states.
- **Actions:** Actions of a belief state are the union of the possible actions for all the states it contains.
- **Transition model:** $b' = Results(b, a) = \{s' : s' = Result(s, a)\ and\ s \in b\}$
- **Goal test:** Are all states in the belief state goal states?
- **Simplifying property:** If a belief state (e.g., $b_1 = \{1,2,3,4,5\}$) is solvable (i.e., there is a sequence of actions that coerce all states to only goal states), then belief states that are subsets (e.g., $b_2 = \{2,5\}$) are also solved using the same action sequence. Used to prune the search tree.

Other approach:

- **Incremental belief-state search**. Generate a solution that works for one state and check if it also works for all other states. If it does not, then modify the solution slightly. This is similar to local search.



Case Study

The agent can move up, down right, left.

The agent has no sensors and does not know its current location.

1. Can you navigate to the goal location? How?

2. What would you need to know about the environment?

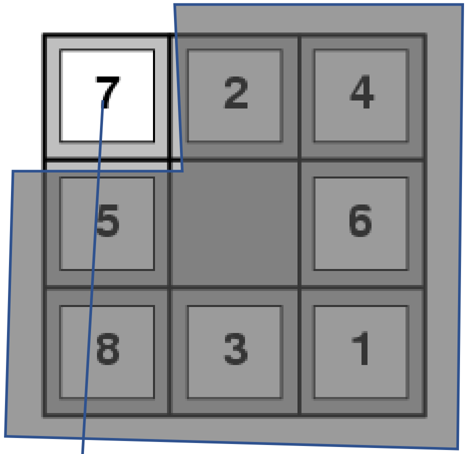
3. What type of agent can do this?

Partially Observable Environments

Using Observations to
Learn About the State

Percepts and Observability

- Many problems cannot be solved efficiently without sensing (e.g., 8-puzzle).
- We need to see at least one square.



$Percept(s) = Tile7$

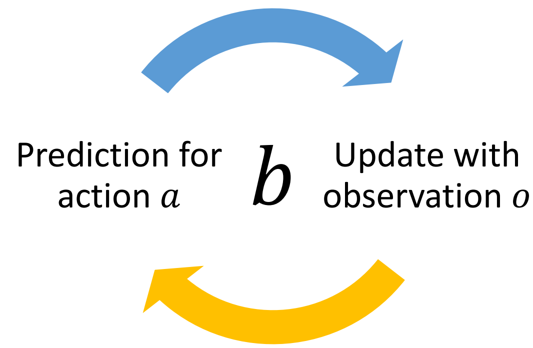
Problem: Many states (different order of the hidden tiles) can produce the same percept!

Percept function: $Percept(s)$

s is the state

- **Fully observable**: $Percept(s) = s$
- **Sensorless**: $Percept(s) = null$
- **Partially observable**: $Percept(s) = o$
  o is called an observation and tells us something about s

Use Observations to Learn About the State



Agents choose an action and then receive an observation.
Idea: Observations can be used to learn about the agent's state.

Assume we have a current belief state b (i.e., the set of states we could be in).

Prediction for action: Choose an action a and compute a new belief state that results from the action.

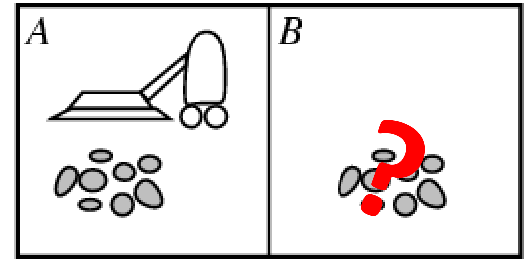
$$\hat{b} = Predict(b, a) = \bigcup_{s \in b} Predict(s, a)$$

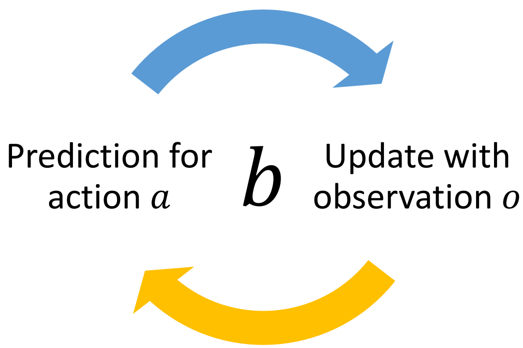
Update with observation: You receive an observation o and only keep states that are consistent with the new observation. The belief after observing o is:

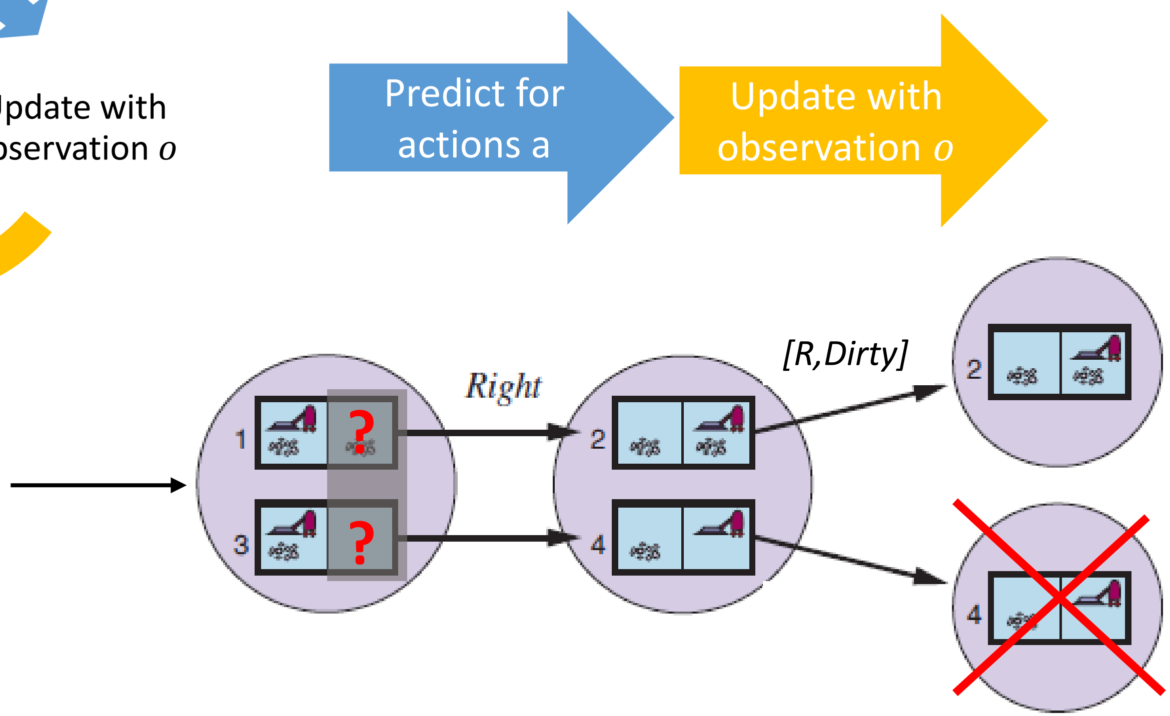
$$b_o = Update(\hat{b}, o) = \{s : s \in \hat{b} \wedge Percept(s) = o\}$$

Both steps in one: $b \leftarrow Update(Predict(b, a), o)$

Example: Deterministic local sensing vacuum world





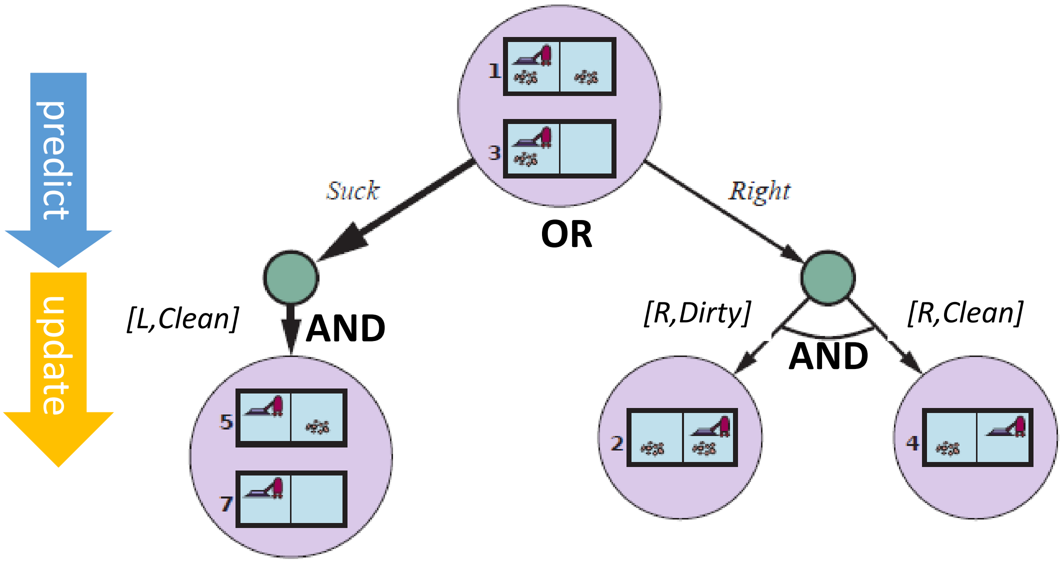


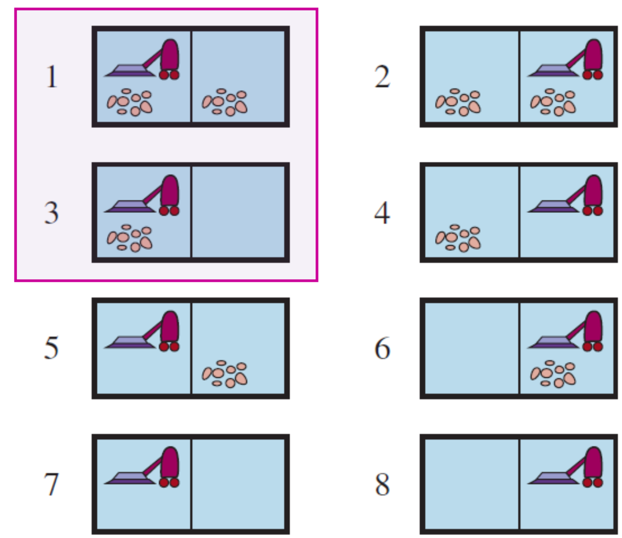
$$b \leftarrow Update(Predict(\ b\ ,\ a\ ),\ o\ )$$

$$Update(Predict(\{1{,}3\}, Right), [R, Dirty]) = \{2\}$$

Solving Partially Observable Problems

Use an AND-OR tree of belief states to create a conditional plan

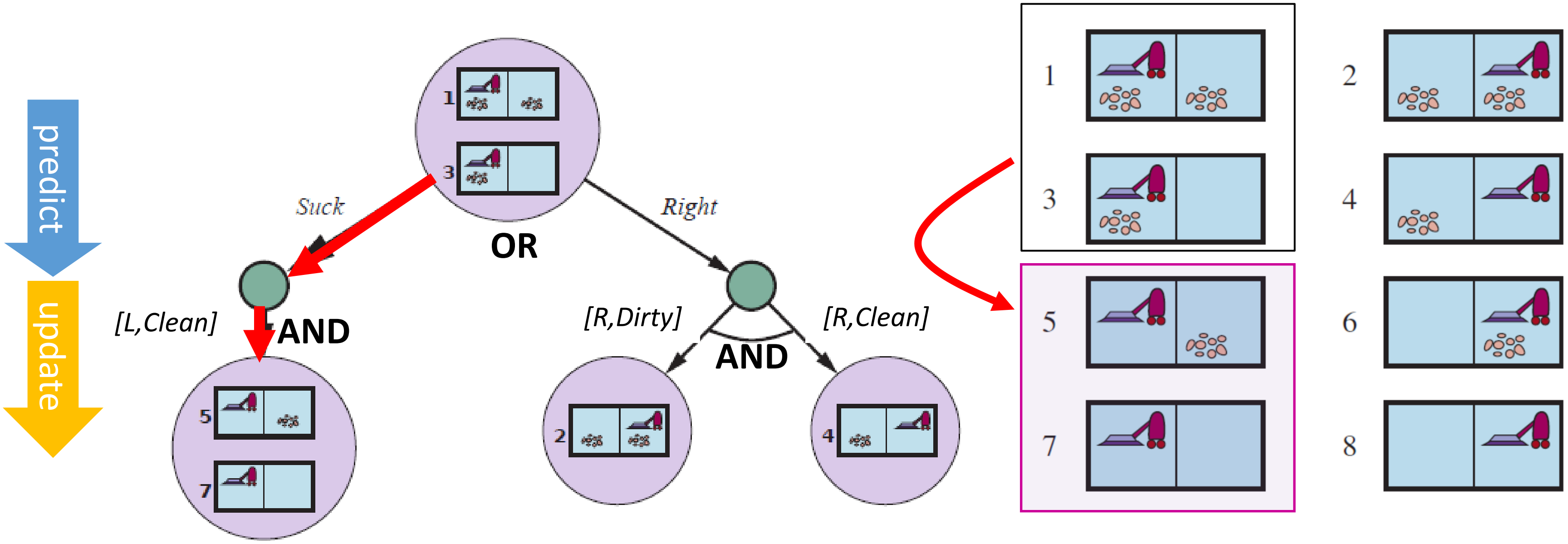




Solution: *[Suck, Right,* **if** *b = {6}* **then** *Suck* **else** *[]]*

Solving Partially Observable Problems

Use an AND-OR tree to create a conditional plan



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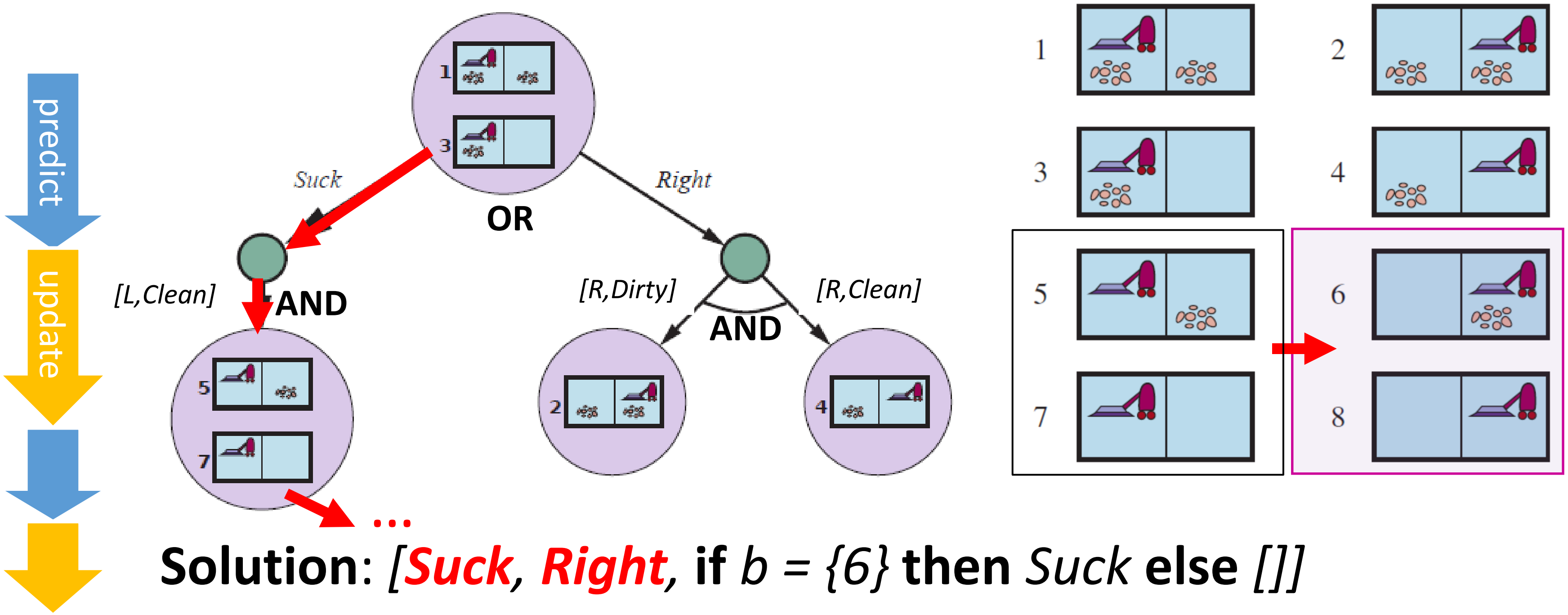

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Solving Partially Observable Problems

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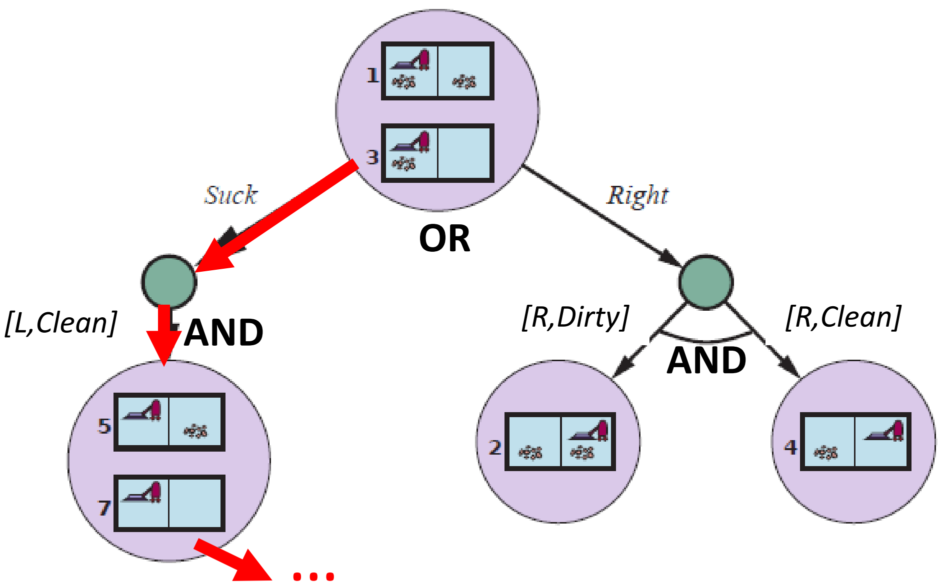
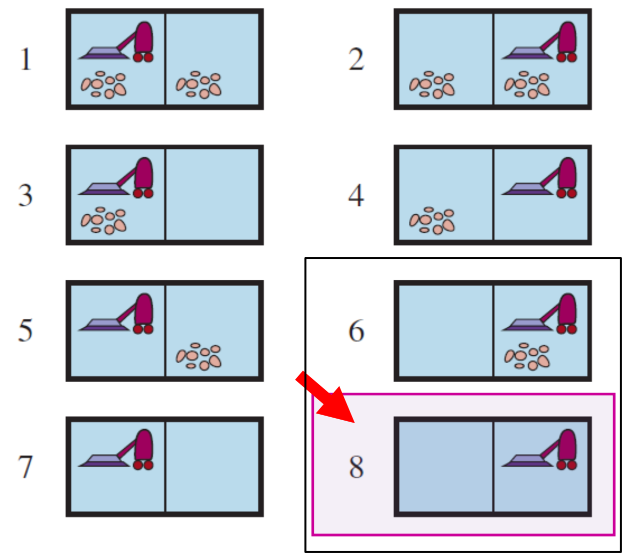
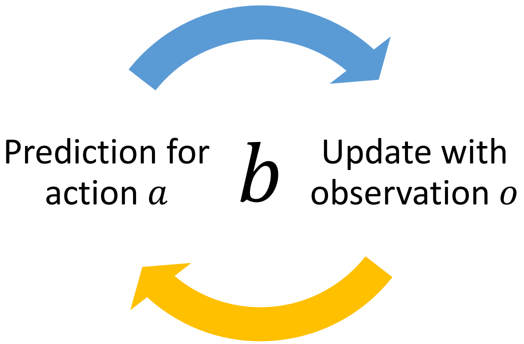




Solution: *[Suck, Right,* **if** *b = {6}* **then** *Suck* **else** *[]]*

State Estimation and Approximate Belief States

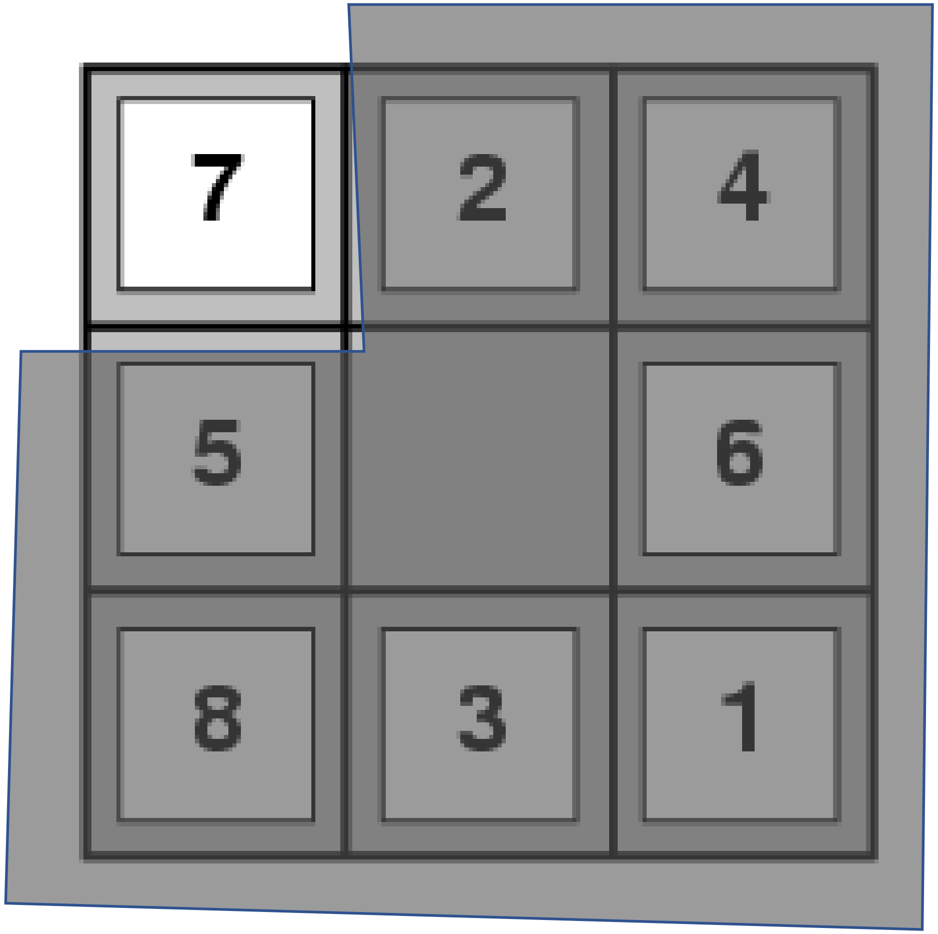


- Agents choose an **action** and then receive an **observation** from the environment.
- The agent keep track of its belief state using the following update:

$$b \leftarrow Update(Predict(b, a), o)$$

- This process is often called
    - **monitoring**,
    - **filtering**, or
    - **state estimation**.

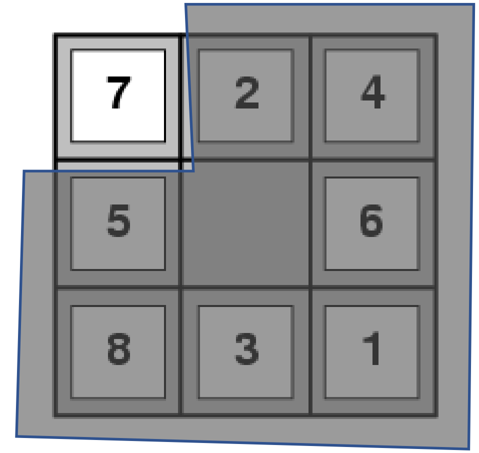
- The agent needs to be able to update its belief state following observations in **real time**! For many practical application, there is only time to compute an **approximate belief state!** These approximate methods are used in control theory and reinforcement learning.



Case Study

Partially
Observable
8-Puzzle

Partially Observable 8-Puzzle



1. Give a problem description for each step.
   - States:
   - Initial state:
   - Actions:
   - Transition model:
   - Goal test:
   - Percept function:

2. The problem can be solved using an AND-OR Tree, but is there an easier solution?

   a. What type of agent do we use?

   b. What algorithms can be used?

Exploration

Unknown Environment and Online Search

Online Search

- **Recall offline search**: Create a plan using the state space as a model before taking any action. The **plan** can be a sequence of actions or a **conditional plan** to account for uncertainty.

- **Online search** explores the real world one action at a time. Prediction is replaced by "act" and update by "observe."

- Useful for
  - **Real-time problems**: When offline computation takes too long and there is a penalty for sitting around and thinking.
  - **Nondeterministic domain**: Only focus on what actually happens instead of planning for everything!
  - **Unknown environment**: The agent has no complete model of how the environment works. It needs to explore an unknown state space and/or what actions do. I.e., it needs to **learn the transition model** $f : S \times A \rightarrow S$

Design Considerations for Online Search

- **Knowledge**: What does the agent already know about the outcome of actions? E.g., does go north and then south lead to the same location?
- **Safely explorable state space/world**: There are **no irreversible actions** (e.g., traps, cliffs). At least the agent needs to be able to avoid these actions.
- **Exploration order**: Expanding nodes in **local order** is more efficient if you have to execute the actions to get observations: Depth-first search with backtracking instead of BFS or A* Search.

Online Search: Model-based Agent Program for Unknown Transition model

Environment is deterministic but

- only partially observable ($percept(s) = current\ location$, state space may be unknown)
- unknown transition model (function $result$).

Approach: The algorithm builds the map $result(s, a) \rightarrow s'$ by trying all actions and backtracks when all actions in a state have been explored.

function ONLINE-DFS-AGENT($problem$, s') **returns** an action
    persistent: s, a, the previous state and action, initially null
        $result$, a table mapping (s, a) to s', initially empty
        $untried$, a table mapping s to a list of untried actions
        $unbacktracked$, a table mapping s to a list of states never backtracked to

    if $problem$.IS-GOAL(s') **then return** $stop$
    if s' is a new state (not in $untried$) **then** $untried[s'] \leftarrow problem$.ACTIONS($s'$)
    if s is not null **then**
        $result[s, a] \leftarrow s'$
        add s to the front of $unbacktracked[s']$
    if $untried[s']$ is empty **then**
        if $unbacktracked[s']$ is empty **then return** $stop$
        else $a \leftarrow$ an action b such that $result[s', b] =$ POP($unbacktracked[s']$)
    else $a \leftarrow$ POP($untried[s']$)
    $s \leftarrow s'$
    return a

Learns results function (= transition function)

Untried is the “frontier”

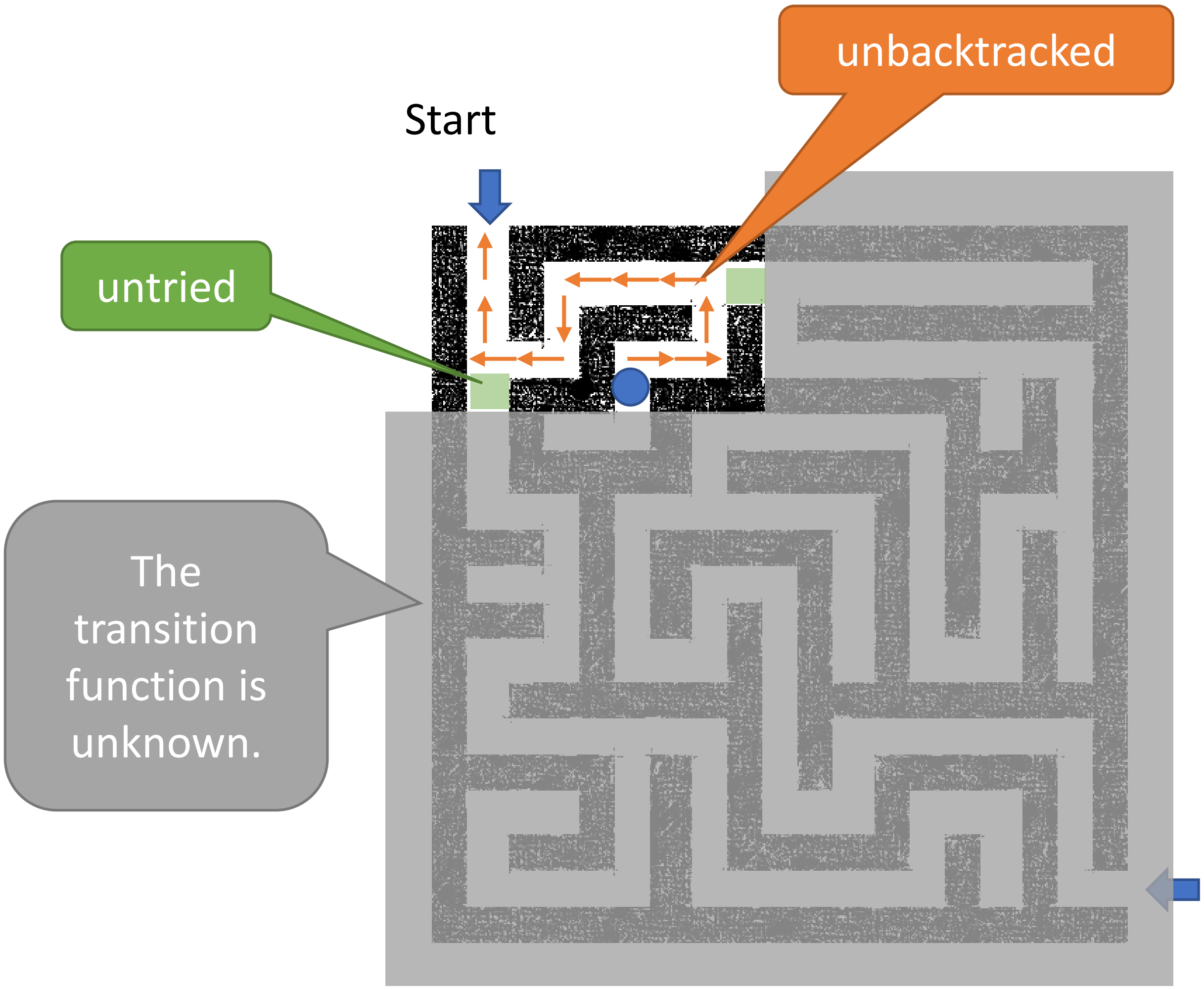
unbacktracked store the current path

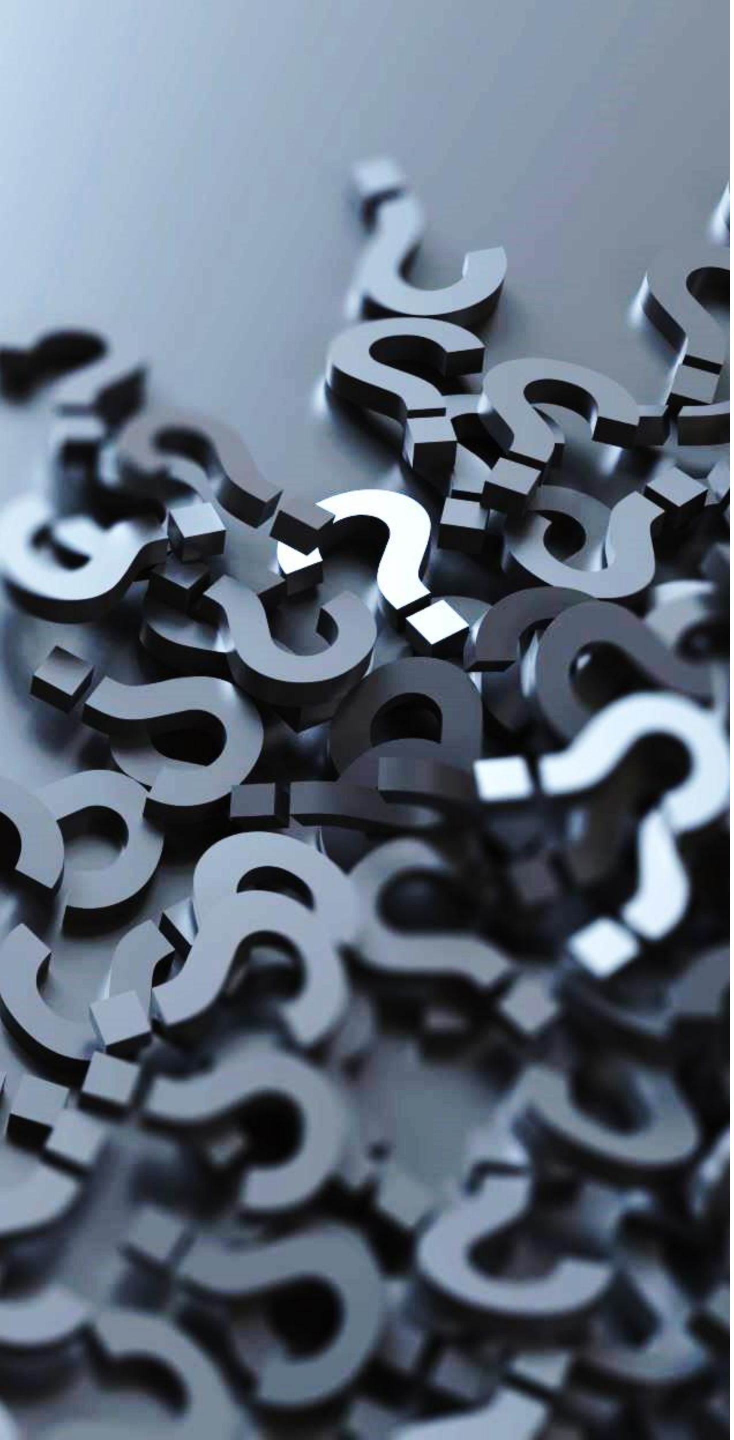
Record the found transition

Keep breadcrumbs to go back

Case Study: DFS with Backtracking for a Maze

- We can only see adjacent squares and don't know where the goal is!
- We cannot plan but we have to explore!
- We know what we have already explored.





Important concepts that you should be able to explain and use now…

- Difference between solution types:
  - a fixed actions sequence, and
  - a conditional plan (also called a strategy or policy).
- What are belief states?
- How actions can be used to coerce the world into states.
- How observations can be used to learn about the state: State estimation with repeated predict and update steps.
- The use of AND-OR trees.