# CS 5/7320
# Artificial Intelligence

# Learning from Examples

AIMA Chapter 19

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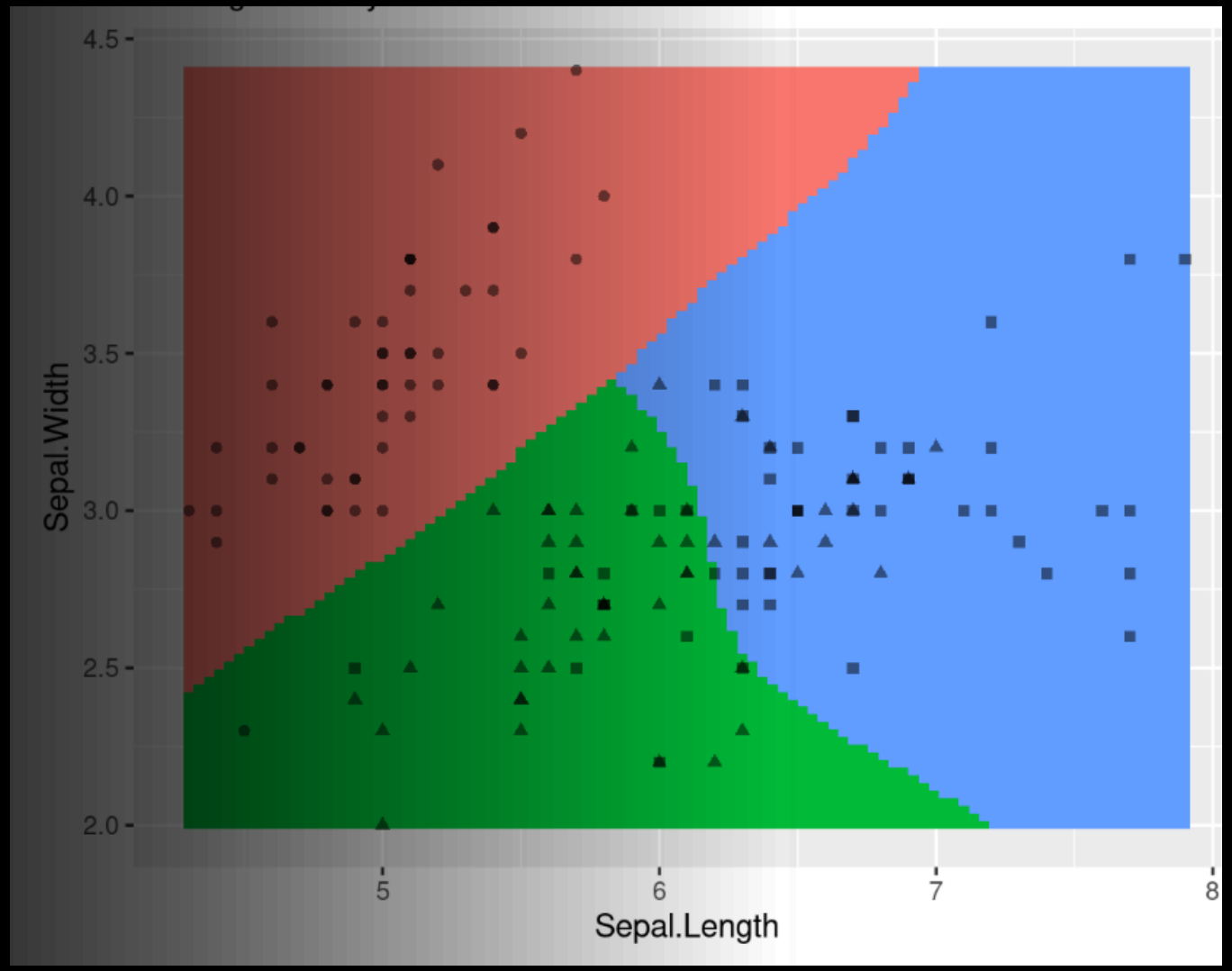
Based on slides by Dan Klein, Pieter Abbeel, Sergey Levine and A. Farhadi. All CS188 materials are at http://ai.berkeley.edu with figures from the AIMA textbook.



# Learning from Examples: Machine Learning

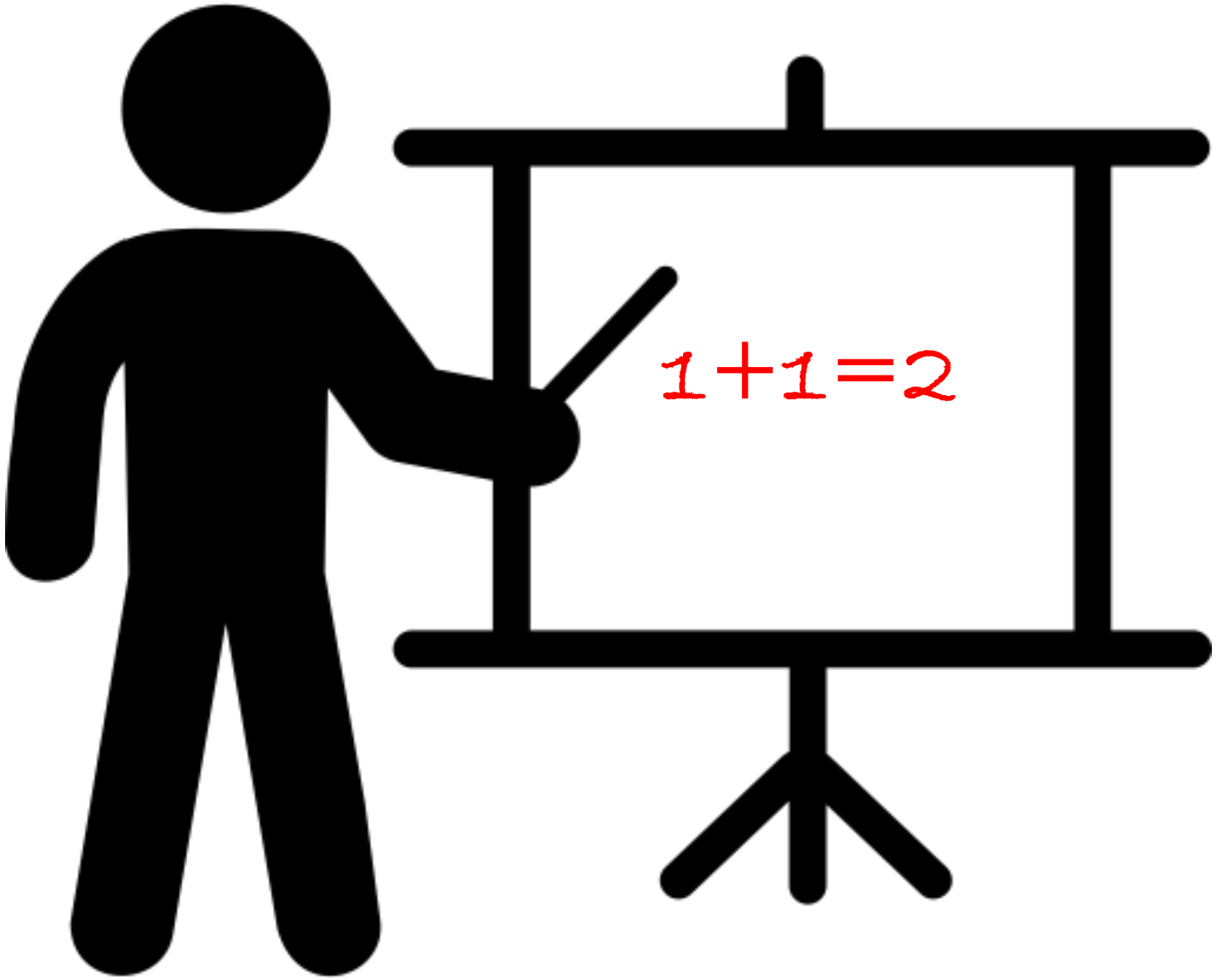
- **Up until now:** hand-craft algorithms to make rational/optimal or at least good decisions. Examples: Search strategies, heuristics.

- **Learning**: Improve performance after making observations about the world. That is, learn what works and what doesn't.

- **Machine learning:** how to build a model from data/experience
  - **Supervised Learning**: Learn a function to map input to output. Examples:
    - Use a naïve Bayesian classifier to distinguish between spam/no spam
    - Learn a playout policy to simulate games (current board -> good move)
  - Unsupervised Learning: Organize data (e.g., clustering, embedding)
  - Reinforcement Learning: Learn from rewards/punishment (e.g., winning a game).

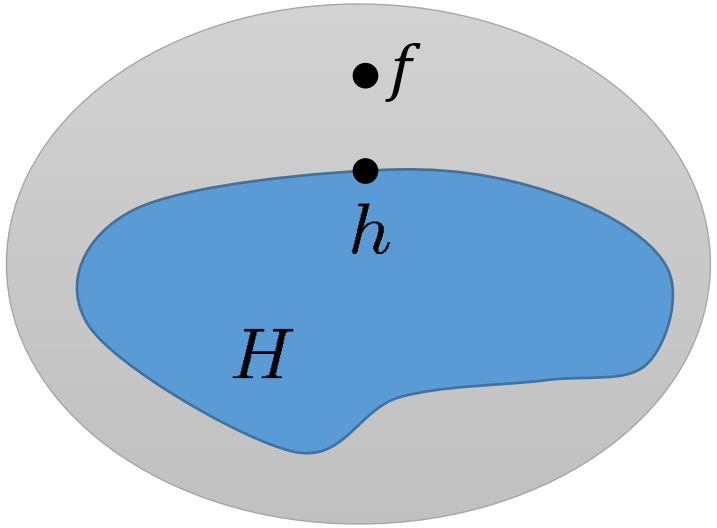
- **Learning vs. hard coding the agent function**
  - Designer cannot anticipate all possible future situations.
  - Designer may have examples but does not know how to program a solution.

# Supervised Learning



# Supervised Learning

- Examples
  - Input-output pairs: $\mathrm{E} = (x_1, y_1), \ldots, (x_i, y_i), \ldots, (x_N, y_N)$, where $x$ is a vectors called the feature vector.
  - We assume that the examples are produced iid (with noise and errors) from a target function $y = f(x)$.

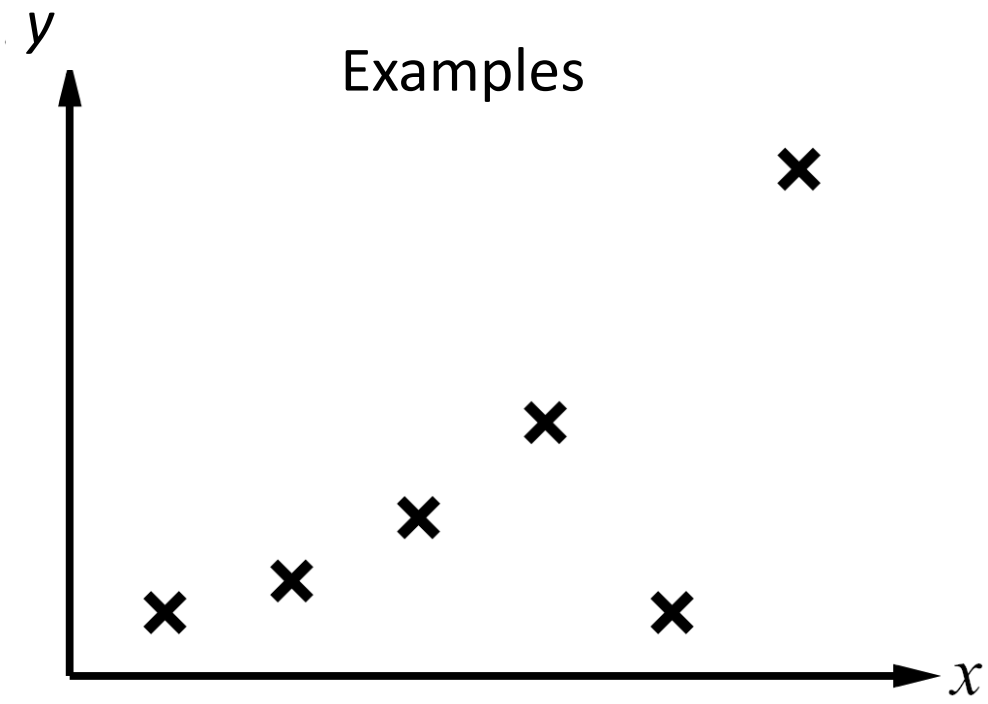


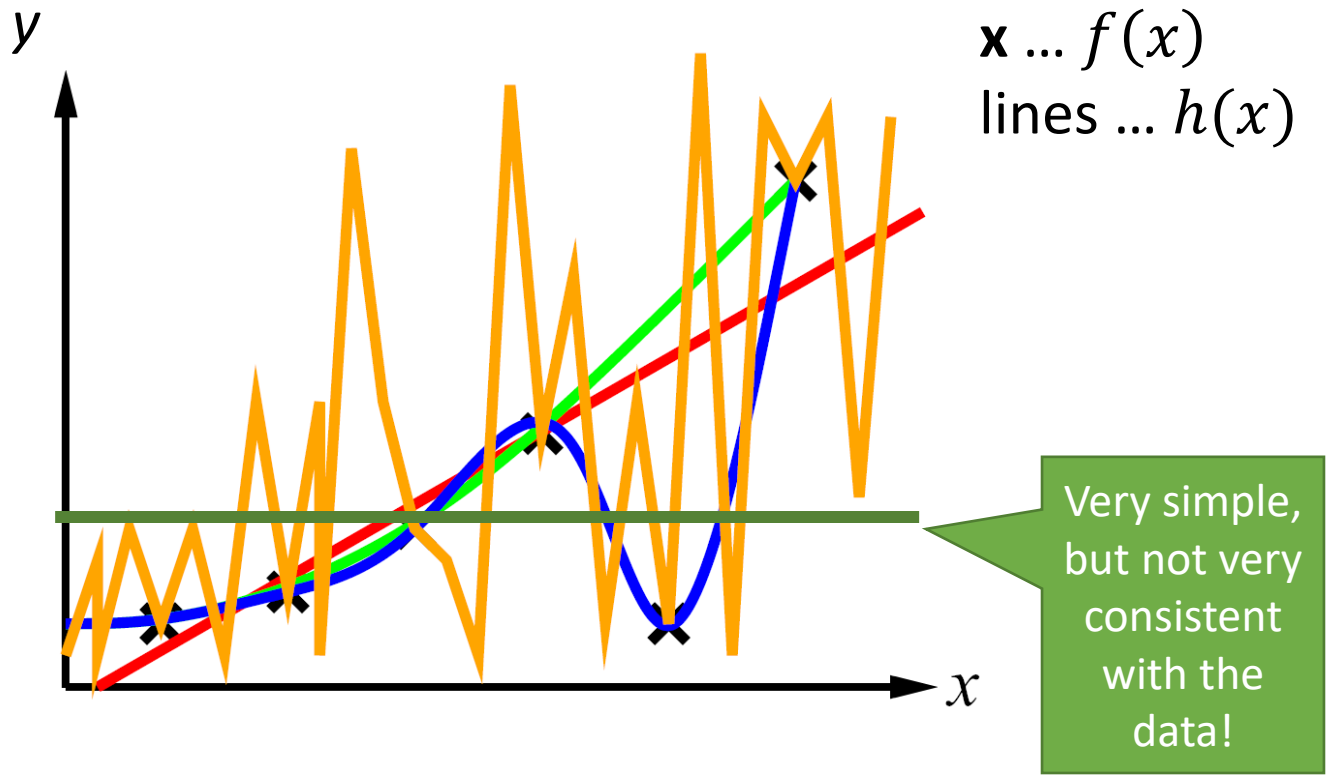

- Learning problem
  - Given a hypothesis space $H$ of possible models.
  - Find a hypothesis $h \in H$ such that $\hat{y}_i = h(x_i) \approx y_i$
  - That is, we want to approximate $f$ by $h$ using $\mathrm{E}$.

- Includes
  - Classification (outputs = class labels). E.g. $x$ is an email and $f(x)$ is spam / ham
  - Regression (outputs = real numbers). E.g. $x$ is a house and $f(x)$ is its selling price

# Consistency vs. Simplicity

Example: Univariate curve fitting (regression, function approximation)




- **Consistency:** $h(x_i) \approx y_i$
- **Simplicity**

# Measuring Consistency using Loss

**Goal of learning**: Find a hypothesis that makes good predictions that are consistent with the examples $\mathrm{E} = (x_1, y_1), \ldots, (x_i, y_i), \ldots, (x_N, y_N)$.

That is, $\hat{y} = h(x) \approx y$.

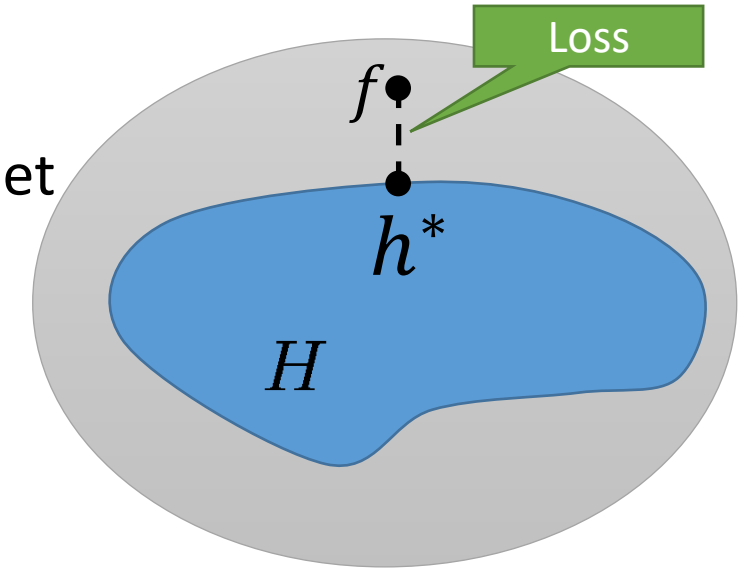
- **Measure mistakes:** Loss function $L(y, \hat{y})$
  - Absolute-value loss $L_1(y, \hat{y}) = |y - \hat{y}|$
  - Squared-error loss $L_2(y, \hat{y}) = (y - \hat{y})^2$
  - 0/1 loss $L_{0/1}(y, \hat{y}) = 0 \text{ if } y = \hat{y}, \text{else } 1$
  - Log loss and others…

For Regression

For Classification

- **Empirical loss:** average loss over the N examples in the dataset

$$EmpLoss_{L,E}(h) = \frac{1}{|E|} \sum_{(x,y) \in E} L(y, h(x))$$



# Learning Consistent $h$ by Minimizing the Loss

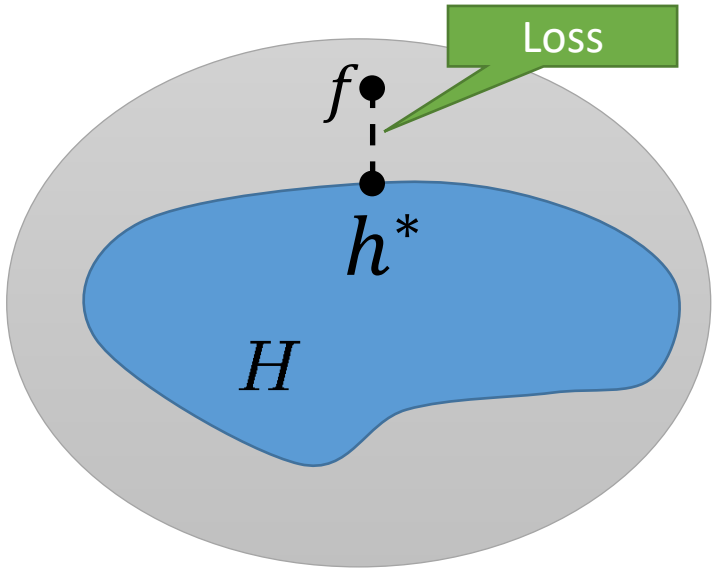
- Empirical loss

$$EmpLoss_{L,E}(h) = \frac{1}{|E|} \sum_{(x,y)\in E} L(y, h(x))$$

- Learning the best hypothesis (approximation)

$$h^* = \underset{h\in H}{\operatorname{argmin}}\, EmpLoss_{L,E}(h)$$

- Reasons for $h^* \neq f$
  a) Realizability: $f \notin H$
  b) $f$ is nondeterministic or examples are noisy
  c) Computationally intractable to search all $H$



# The Bayes Classifier

**For 0/1 loss**, the empirical loss is minimized by the model that predicts for each $x$ the most likely class $y$ using MAP (Maximum a posteriori) estimates. This is called the Bayes classifier.

$$h(x)^* = \operatorname*{argmax}_y P(Y = y \mid X = x) = \operatorname*{argmax}_y \frac{P(x \mid y)\, P(y)}{P(x)} = \operatorname*{argmax}_y P(x \mid y)\, P(y)$$

**Optimality**: The **Bayes classifier is optimal.** It is the most consistent classifier possible with the lowest possible error called the **Bayes error rate**. No better classifier is possible!

**Issue**: The classifier is not simple with $P(x \mid y)\, P(y) = P(x, y)$

- It **needs the complete joint probability** which requires in the general case a probability table with one entry for each possible value for the feature vector $x$.
- This is impractical and most classifiers try to approximate the Bayes classifier using fewer parameters.

# Simplicity

**Ease of use**

- Simpler hypotheses have fewer parameters to store.
- Simpler hypotheses are easier to estimate.

**Generalization**: How well does the hypothesis perform on new data?

- We do not want the model to be too specific to the training examples (an issue called **overfitting**).
- Simpler models typically generalize better to new examples.

**How to achieve simplicity?**

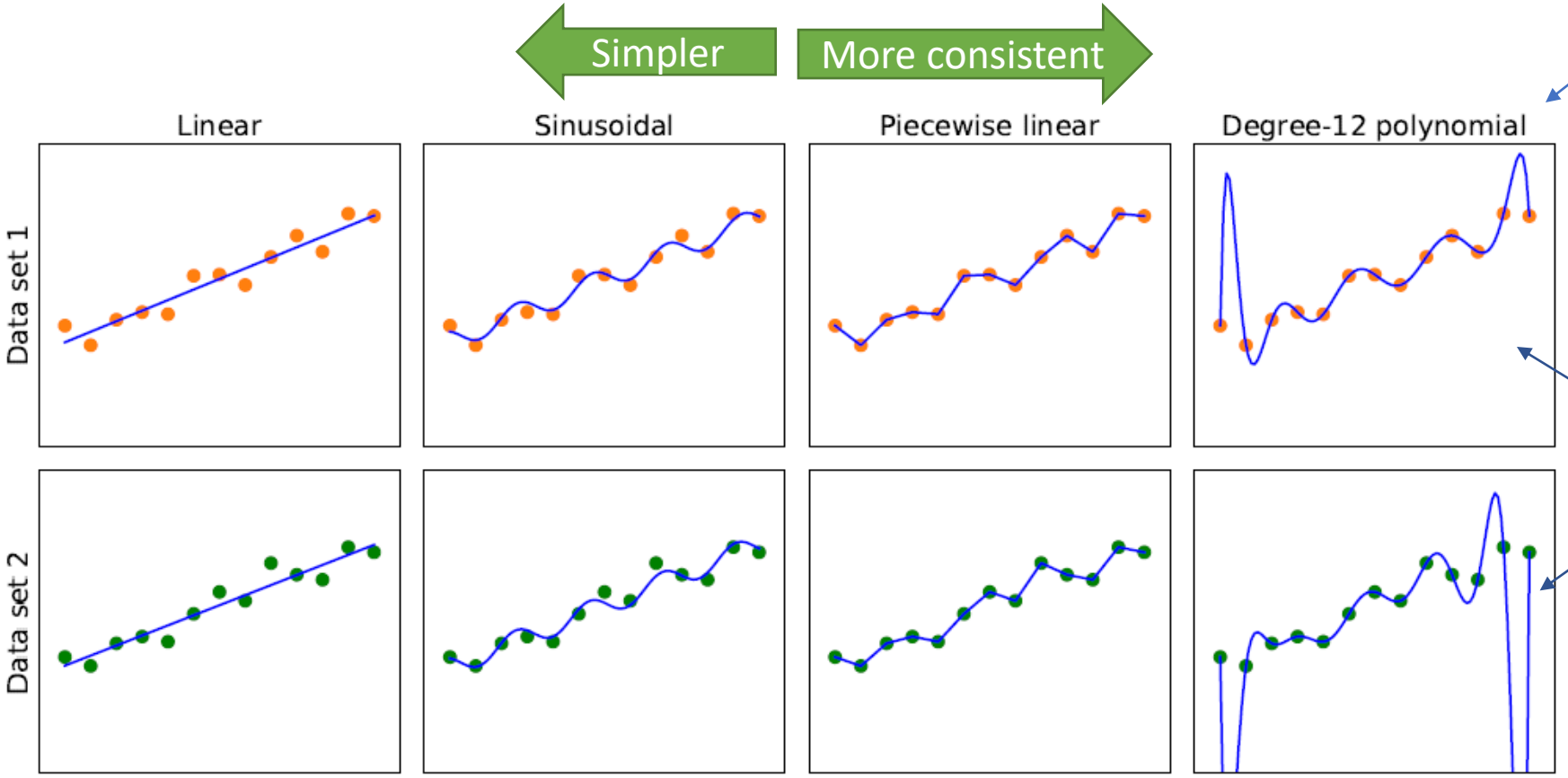
a) Restrict H to simpler models (e.g., assumptions like independence, linear models are called **model bias**)

b) Feature selection (use fewer variables from the feature vector $x$)

**c) Regularization** (penalize for complexity)

$$h^* = \underset{h \in H}{\operatorname{argmin}} \left[ EmpLoss_{L,E}(h) + \underbrace{\lambda \, Complexity(h)}_{\text{Penalty term}} \right]$$

# Model Selection: Bias vs. Variance




Overfitting

Points: Two samples from the same function $f$

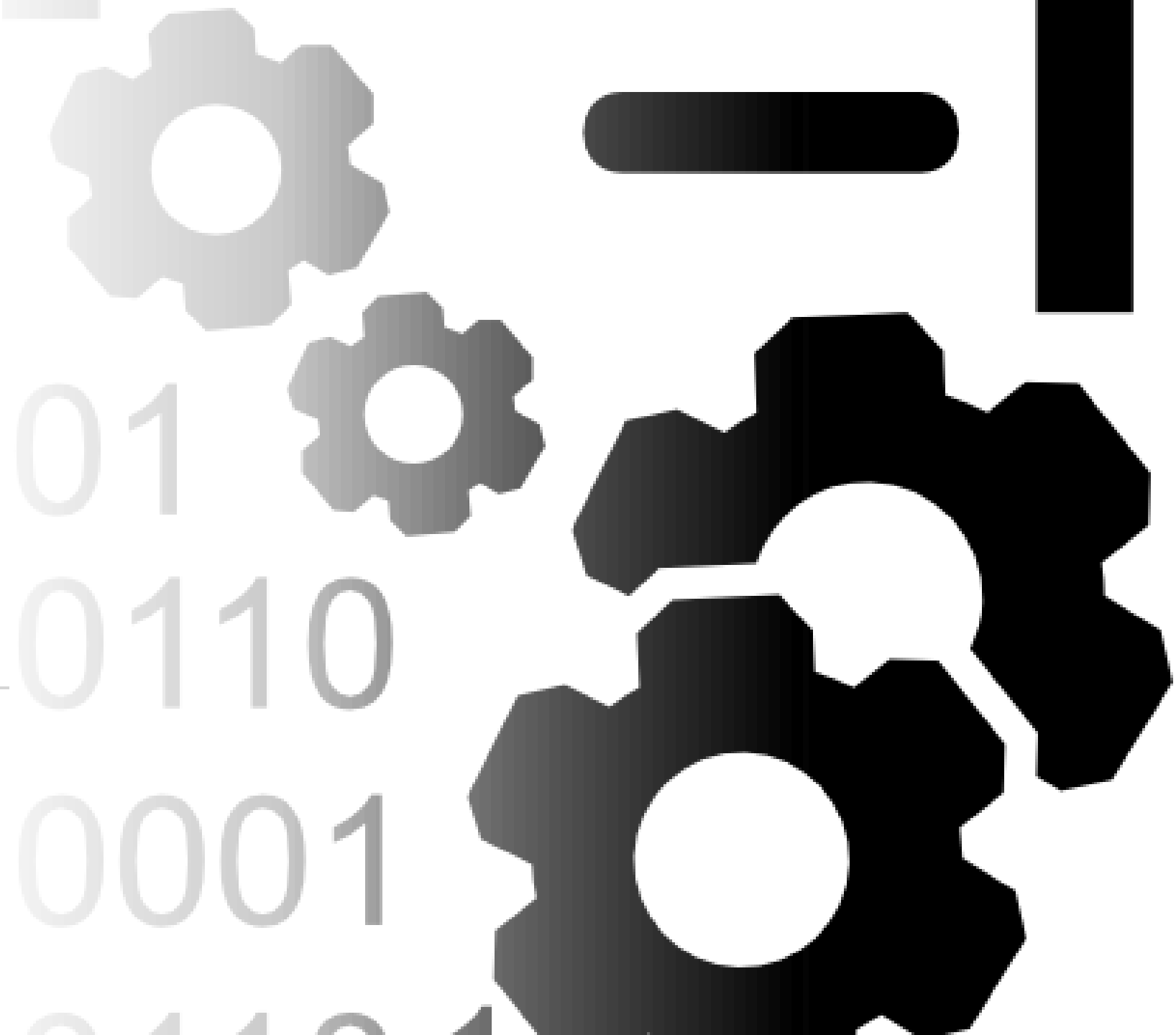
Lines: the learned function $h$.

High **Bias**: restrictions by the model class Low

Low **Variance**: difference in the model due to slightly different data. high

This is a tradeoff

# Data



# The Dataset

Features
Variables
Attributes $x$

Class
Label $y$

Examples
Instances
Observations

| Example | Input Attributes | | | | | | | | | | Output |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Alt* | *Bar* | *Fri* | *Hun* | *Pat* | *Price* | *Rain* | *Res* | *Type* | *Est* | *WillWait* |
| $\mathbf{x}_1$ | *Yes* | *No* | *No* | *Yes* | *Some* | *$$$* | *No* | *Yes* | *French* | *0–10* | $y_1 = Yes$ |
| $\mathbf{x}_2$ | *Yes* | *No* | *No* | *Yes* | *Full* | *$* | *No* | *No* | *Thai* | *30–60* | $y_2 = No$ |
| $\mathbf{x}_3$ | *No* | *Yes* | *No* | *No* | *Some* | *$* | *No* | *No* | *Burger* | *0–10* | $y_3 = Yes$ |
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| $\mathbf{x}_6$ | *No* | *Yes* | *No* | *Yes* | *Some* | *$$* | *Yes* | *Yes* | *Italian* | *0–10* | $y_6 = Yes$ |
| $\mathbf{x}_7$ | *No* | *Yes* | *No* | *No* | *None* | *$* | *Yes* | *No* | *Burger* | *0–10* | $y_7 = No$ |
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| $\mathbf{x}_{11}$ | *No* | *No* | *No* | *No* | *None* | *$* | *No* | *No* | *Thai* | *0–10* | $y_{11} = No$ |
| $\mathbf{x}_{12}$ | *Yes* | *Yes* | *Yes* | *Yes* | *Full* | *$* | *No* | *No* | *Burger* | *30–60* | $y_{12} = Yes$ |

Alternative
Hungry
Patrons
Reservation
Wait time

Find a hypothesis (called “model”) to predict the class given the features.

# Feature Engineering

- Add information sources as new variables to the model.
- Add derived features that help the classifier (e.g., $x^2$).

- Example for Spam detection: In addition to words
  - Have you emailed the sender before?
  - Have 1000+ other people just gotten the same email?
  - Is the header information consistent?
  - Is the email in ALL CAPS?
  - Do inline URLs point where they say they point?
  - Does the email address you by (your) name?

- **Feature Selection**: Which features should be used in the model is a model selection problem (choose between models with different features).

# Training and Testing

# Model Evaluation (Testing)

The model was trained on the training examples $E$. We want to test how well the model will perform on new examples $T$ (i.e., how well it **generalizes to new data**).

- **Testing loss**: Calculate the empirical loss for predictions on a testing data set $T$ that is different from the data used for training.

$$EmpLoss_{L,T}(h) = \frac{1}{|T|} \sum_{(x,y) \in T} L(y, h(x))$$

- For classification we often use the **accuracy** measure, the proportion of correctly classified test examples.

$$accuracy(h, T) = \frac{1}{|T|} \sum_{(x,y) \in T} [h(x) = y] = 1 - EmpLoss_{L_{0/1},T}(h)$$

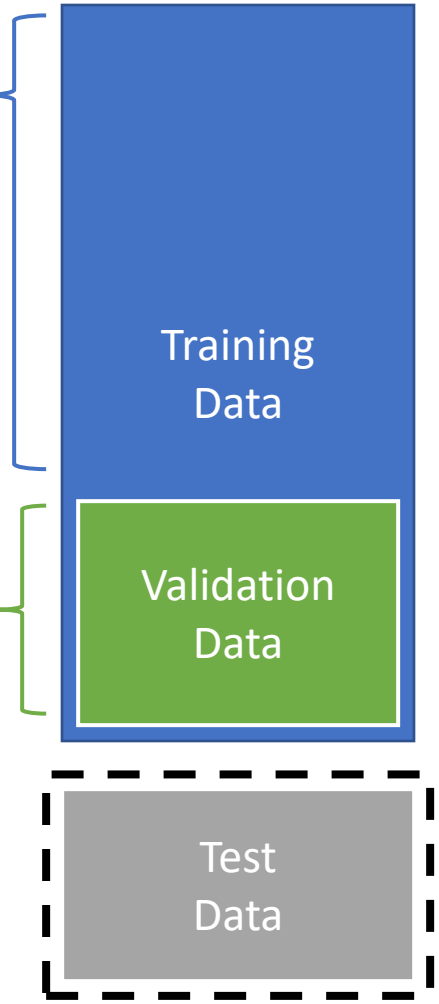
$[c]$ is an indicator function returning 1 if $c = True$ and otherwise 0

# Training a Model

- Models are “trained” (learned) on **the training data.** This involved estimating the model parameters (the model): E.g., probabilities, weights, factors.
- **Hyperparameters**: Many learning algorithms have choices for. E.g., learning rate, regularization $\lambda$, maximal decision tree depth, selected features. The algorithm tries to optimizes the model parameters given user-specified hyperparameters.
- We need to tune the hyperparameters!

# Hyperparameter Tuning/Model Selection

1. Hold a validation data set back from the training data.
2. **Learn models** using the training set with different hyperparameters. Often a grid of possible hyperparameter combinations or some greedy search is used.
3. **Evaluate the models** using the validation data and choose the model with the best accuracy. Selecting the right type of model, hyperparameters and features is called **model selection**.
4. Learn the final model with the chosen hyperparameters using all training (including validation data).




- Notes:
  - The validation set was not used for training, so we get generalization accuracy for the different hyperparameter setings.
  - If no model selection is necessary, then no validation set is used.

# Testing a Model

- After the model is selected, the final model is evaluated against the test set to **estimate the final model accuracy**.
- Very important: never “peek” at the test set during learning!

# How to Split the Dataset

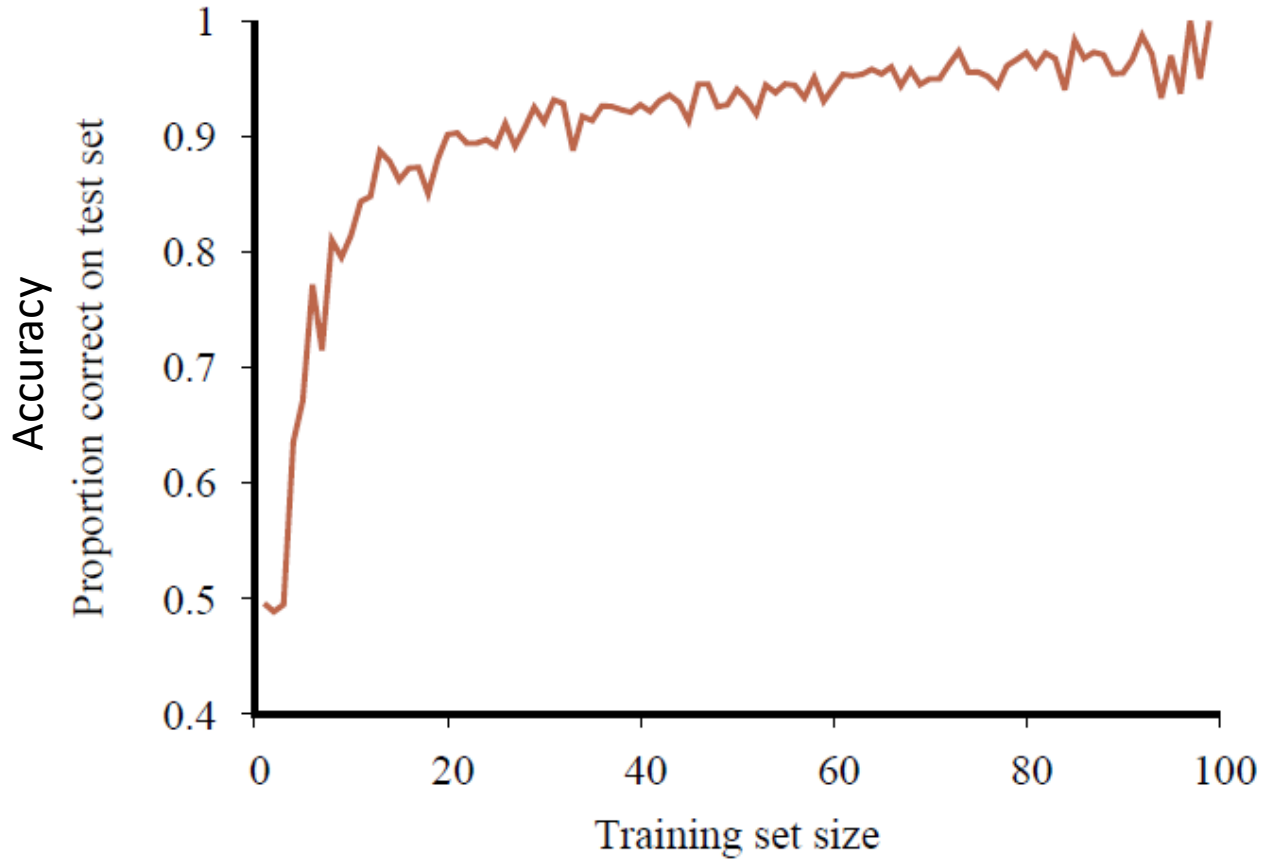
- **Random splits:** Split the data randomly in, e.g., 60% training, 20% validation, and 20% testing.

- **Stratified splits:** Like random splits, but balance classes and other properties of the examples.

- **k-fold cross validation:** Use training & validation data better
  - split the training & validation data randomly into k folds.
  - For k rounds hold 1 fold back for testing and use the remaining $k - 1$ folds for training.
  - Use the average error/accuracy as a better estimate.
  - Some algorithms/tools do this internally.

- **LOOCV** (leave-one-out cross validation): $k = n$ used if very little data is available.

# Learning Curve:
# The Effect the Training Data Size




Accuracy of a classifier when the amount of available training data increases.

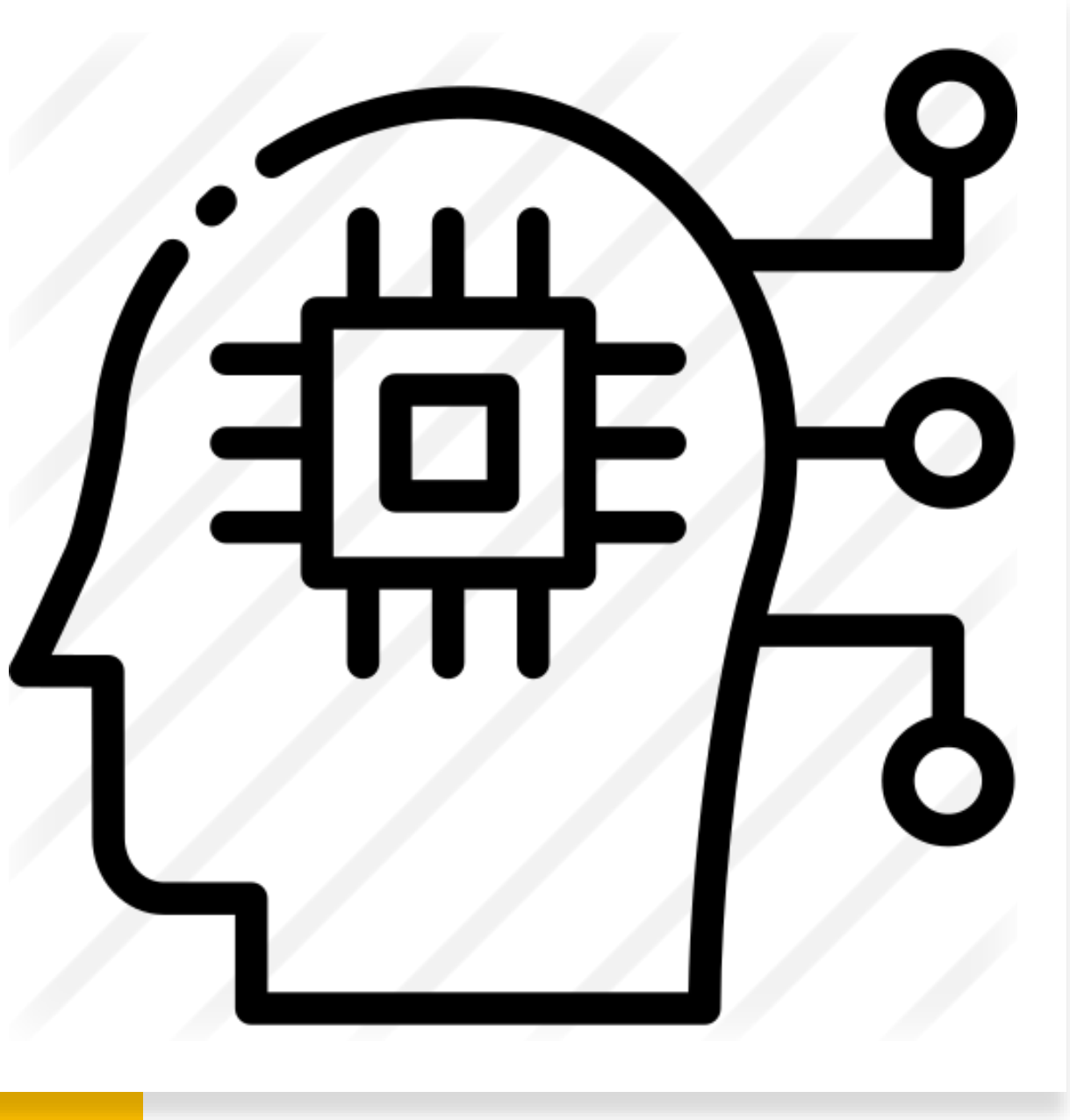
**More data is better!**

# Comparing to a Baselines

- First step: get a **baseline**
  - Baselines are very simple straw man model.
  - Helps to determine how hard the task is.
  - Helps to find out what a good accuracy is.

- **Weak baseline**: The most frequent label classifier
  - Gives all test instances whatever label was most common in the training set.
    - Example: For spam filtering, give every message the label “ham.”
  - Accuracy might be very high if the problem is skewed (called class imbalance).
    - Example: If calling everything “ham” gets already 66% right, so a classifier that gets 70% isn’t very good…

- **Strong baseline**: For research, we typically compare to previous work as a baseline.



# Types of Models

Regression: Predict a number

Classification: Predict a label

# Regression: Linear Regression

Model: $h_{\boldsymbol{w}}(\boldsymbol{x}_j) = w_o + w_1 x_{j,1} + \cdots + w_n x_{j,n} = \sum_i w_i x_{j,i} = \boldsymbol{w}^T \boldsymbol{x}_j$

Empirical Loss: $L(\boldsymbol{w}) = \|\boldsymbol{X}\boldsymbol{w} - \boldsymbol{y}\|^2$

Squared error loss over the whole data matrix $\boldsymbol{X}$

Gradient: $\nabla L(\boldsymbol{w}) = 2\boldsymbol{X}^T(\boldsymbol{X}\boldsymbol{w} - \boldsymbol{y})$

The gradient is a vector of partial derivatives

$$\nabla L(\boldsymbol{w}) = \left[\frac{\partial L}{\partial w_1}(\boldsymbol{w}), \frac{\partial L}{\partial w_2}(\boldsymbol{w}), \ldots, \frac{\partial L}{\partial w_n}(\boldsymbol{w})\right]^T$$

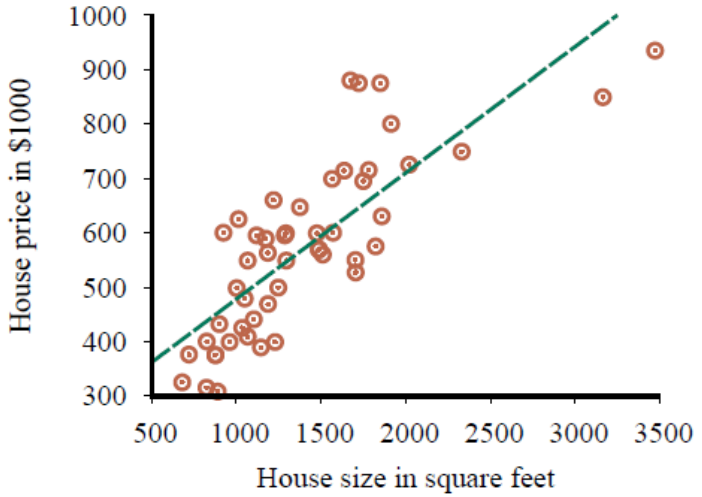
Find: $\nabla L(\boldsymbol{w}) = 0$

Gradient descend:

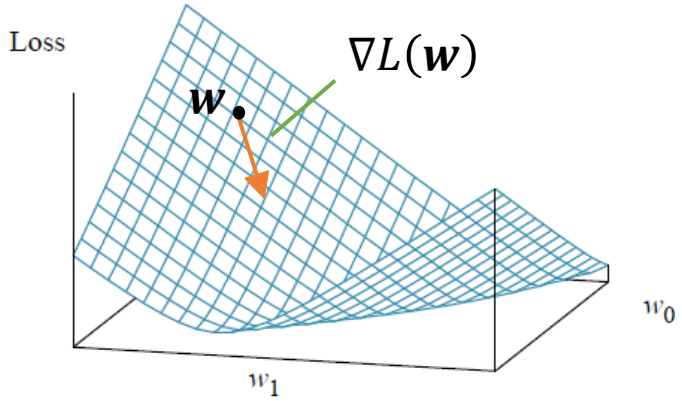
$$\boldsymbol{w} = \boldsymbol{w} - \alpha \nabla L(\boldsymbol{w})$$

Analytical solution:

$$\boldsymbol{w}^* = \underbrace{(\boldsymbol{X}^T\boldsymbol{X})^{-1}\boldsymbol{X}^T}_{\text{Pseudo inverse}}\boldsymbol{y}$$




(a)




(b)

# Naïve Bayes Classifier

- Approximates a Bayes classifier with the **naïve independence assumption** that all $n$ features are conditional independent given the class.

$$h(x) = \underset{y}{\operatorname{argmax}} P(y) \prod_{i=1}^{n} P(x_i \mid y)$$

- We have only used discrete features so far, but it can be extended to continuous features. Gaussian Naïve Bayes Classifier assumes that continuous features have:

$$P(x_i \mid y) \sim N(\mu_y, \sigma_y)$$

The parameters for $N(\mu_y, \sigma_y)$ are estimated from data.

# Decision Trees

| Example | Input Attributes | | | | | | | | | | Output |
|---|---|---|---|---|---|---|---|---|---|---|---|
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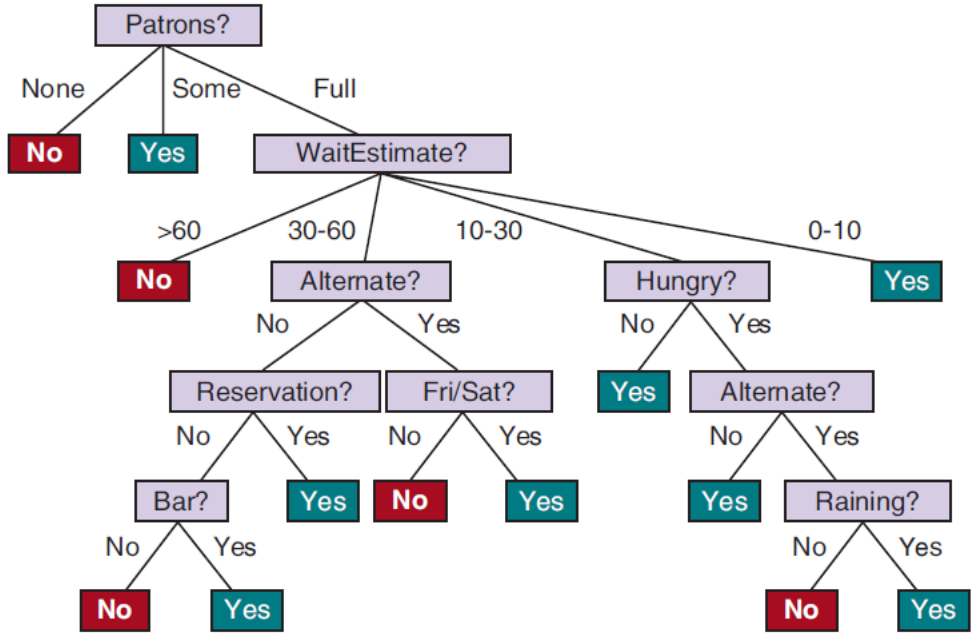
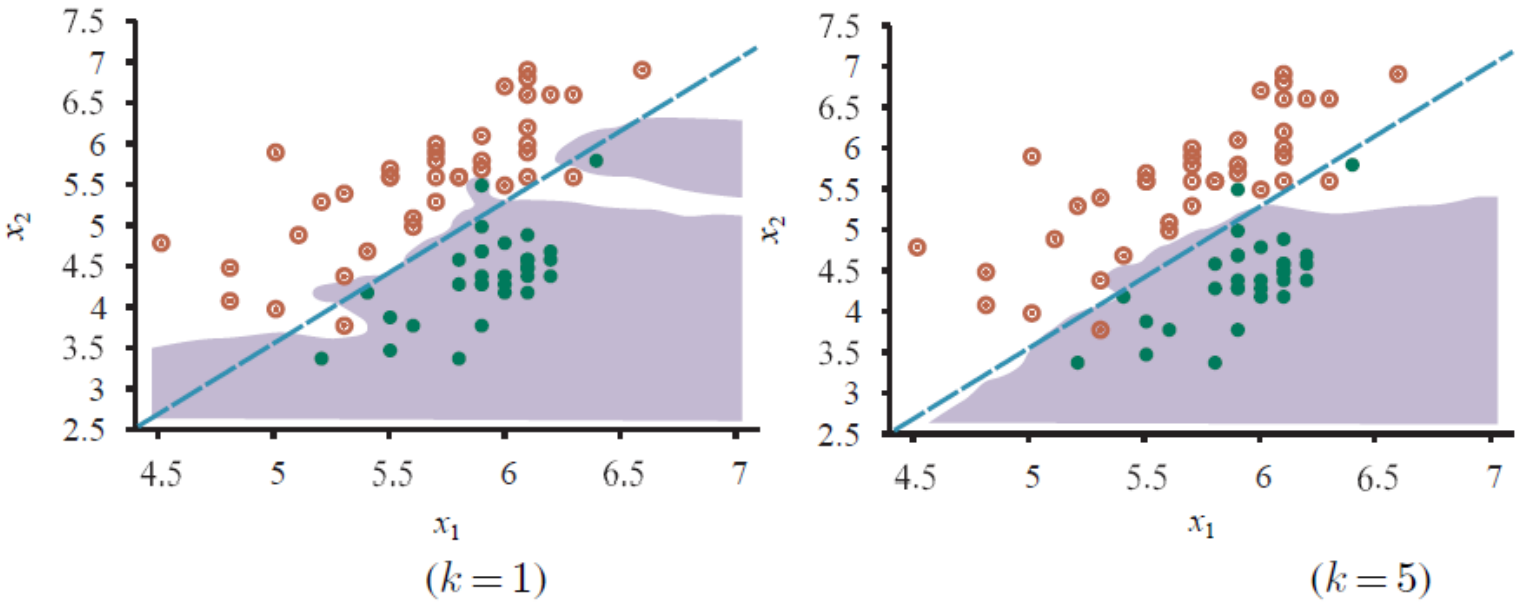


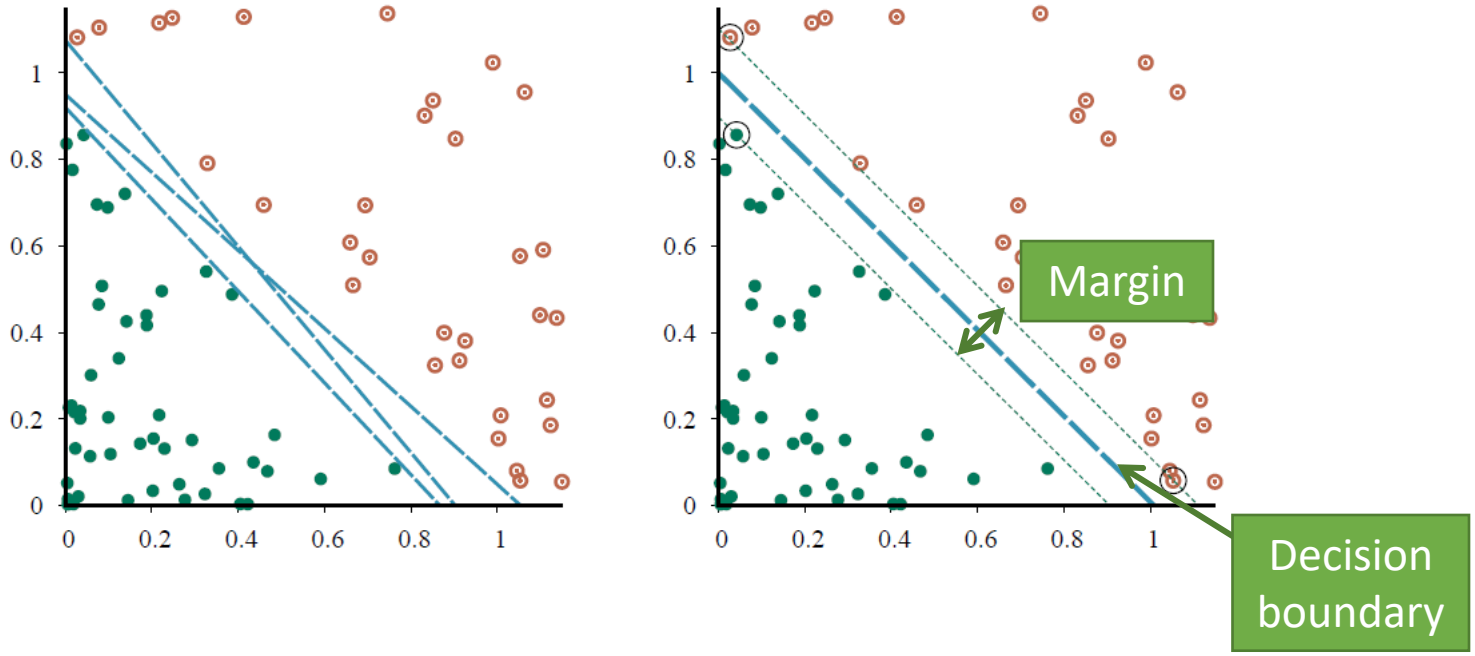

- A **sequence of decisions** represented as a tree.
- Many implementations that differ by
  - How to select features to split?
  - When to stop splitting?
  - Is the tree pruned?
- Approximates a Bayesian classifier by $h(x) = \underset{y}{\operatorname{argmax}} P(Y = y \mid \text{leafNodeMatching}(x))$

# K-Nearest Neighbors Classifier



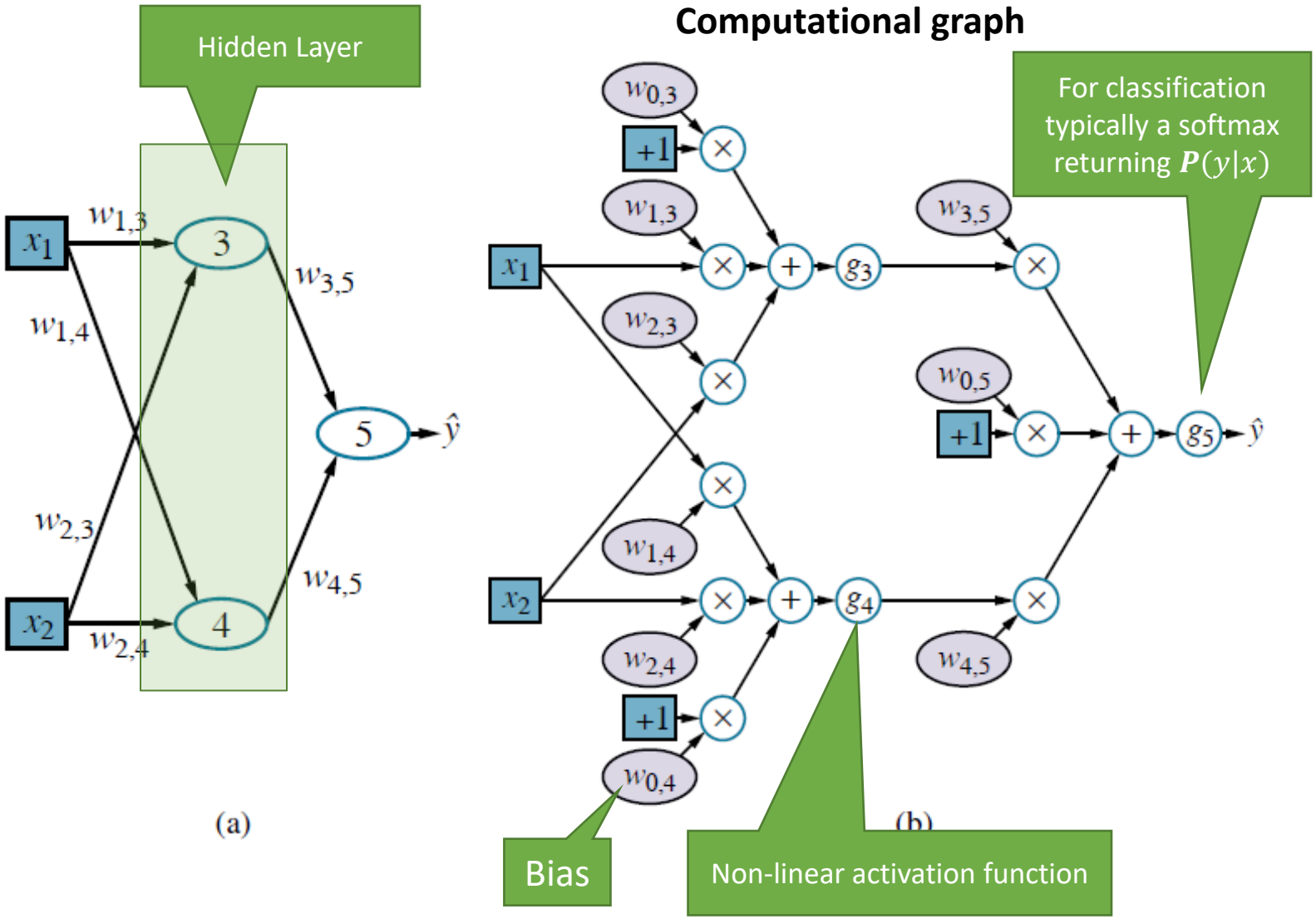

- Class is predicted by looking at the majority in the set of the k nearest **neighbors**. k is a hyperparameter. Larger k smooth the decision boundary.
- Neighbors are found using a distance measure (e.g., Euclidean distance between points).
- Approximates a Bayesian classifier by $h(x) = \underset{y}{\operatorname{argmax}} P(Y = y \mid \text{neighborhood}(x))$

# Support Vector Machine (SVM)




- Linear classifier that finds **the maximum margin separator** using only the points that are “support vectors” and quadratic optimization.
- The kernel trick can be used to learn non-linear decision boundaries.

# Artificial Neural Networks/Deep Learning




- Represent $\hat{y} = h(x)$ as a network of weighted sums with non-linear activation functions (e.g., logistic, ReLU).
- Learn weights **w** from examples using **backpropagation** of prediction errors $\mathrm{L}(\hat{y}, y)$ (gradient descend).
- ANNs are **universal approximators**. Large networks can approximate any function (no bias). **Regularization** is typically used to avoid overfitting.
- **Deep learning** adds more hidden layers and layer types (e.g., convolution layers) for better learning.

# Other Popular Models and Methods

## Many other models exist

- **Generalized linear model (GLM):** This important model family includes **linear regression** and the classification method **logistic regression.**

## Often used methods

- **Regularization:** enforce simplicity by using a penalty for complexity.
- **Kernel trick:** Let a linear classifier learn non-linear decision boundaries ( = a linear boundary in a high dimensional space).
- **Ensemble Learning:** Use many models and combine the results (e.g., random forest, boosting).
- **Embedding and Dimensionality Reduction:** Learn how to represent data in a simpler way.

# Example Use of ML for Intelligent Agents

## Learn Actions

- Directly learn the best action from examples.

$$action = h(state)$$

- This model can also be used as a **playout policy** for Monte Carlo tree search with data from self-play.

## Learn Heuristics

- Learn evaluation functions for states.

$$eval = h(state)$$

- Can learn a **heuristic** for minimax search from examples.

## Perception

- **Natural language processing:** Use deep learning / word embeddings / language models to understand concepts, translate between languages, or generate text.
- Speech recognition: Identify the most likely sequence of words.
- Vision: Object recognition in images/videos. Generate images/video.