MXB324 Group Project
Groundwater Modelling

See the *Project Assessment Guide* (available on Blackboard under Assessment) for a full description of assessment tasks, due dates, weightings, etc.

Contents

1 Background

Groundwater models are used to inform impacts of proposed developments. Depending upon the identified impacts, mitigation measures may be required, or project approval may not be granted. Your group has been requested to prepare a groundwater model to simulate changes to the water resources in a given region of Queensland. You will also study the impact of the evapotranspiration effects arising due to surface evaporation and transpiration from the nearby crop plantation. To assist with the study, a two-dimensional groundwater model will be developed and implemented in MATLAB to simulate groundwater levels. Boundary conditions will be based upon rainfall, groundwater discharge to the river, evapotranspiration and water extraction via bores for irrigation and town usage.

There is an ongoing concern about the state of groundwater aquifers in this region and your group has been asked to provide advice on a water management plan for the confined and unconfined aquifers. The concerns relate to the extraction rates of groundwater for irrigation and town water purposes and also the impact of the evapotranspiration effects arising due to the crop grown in this region. Fresh water is extracted from the aquifer at a variable rate all year round. However, the rate at which the aquifer is recharged by fresh water depends on the local rainfall and potential interaction with a river located to the west. There are longterm weather predictions from the Bureau of Meteorology that indicate potential drought seasons are imminent and management are concerned that in these dry seasons the water table may be dropping dangerously low, which could lead to problems in the future with salinity.

Managing the water table is a key aspect to groundwater management in an irrigation area. If the water table is too low the water supply can be reduced which could be detrimental to the ecosystem health and flow in the river; if the water table is too high there is a chance that the crop will become water logged. By using a simple groundwater model we can assess the impacts to both the river and water table given climate data available from the Bureau of Meteorology website http://www.bom.gov.au/climate/averages/tables/ca_qld_names.shtml and hydrogeological characteristics of the site (refer lectures 1–2 from GHD Principal Environmental Engineer Mr James Dowdeswell).

The site under consideration is based upon the region with irrigation and town water extraction that occurs in a sandstone (semi-confined) aquifer and overlaying (unconfined) alluvial aquifer. This sequence (refer Figure 1) is typical of several locations in Queensland.

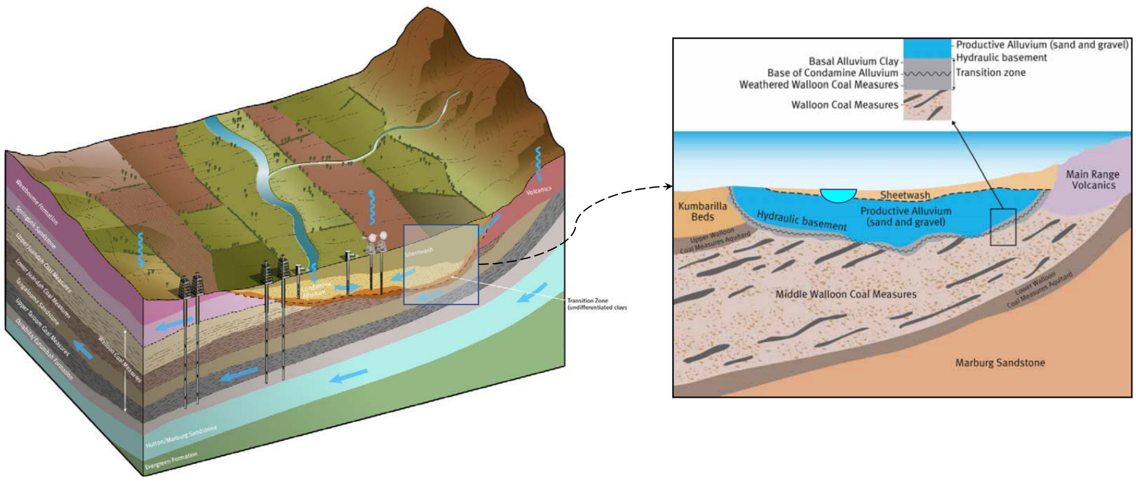


Figure 1: Regional Conceptual Modelling Framework.

Your team is competing for a lucrative consulting contract based on this pilot study and you have been instructed to put together a preliminary model and analysis. By thoroughly addressing the questions posed in this study, you aim to demonstrate that you are capable of undertaking a fully detailed study in the future.

2 Model

2.1 Richards' Equation

In a porous medium such as soil, there exist gaps between the solid grains that form pores. In a perfectly dry soil, these pores are filled with air. However, as water is allowed to infiltrate the soil, the pores begin to fill with water. The flow of groundwater is described by Richards' equation:

$$\frac{\partial \psi}{\partial t} + \boldsymbol{\nabla} \cdot \mathbf{q} = Q\,, \tag{1}$$

where ψ is the volumetric water content, Q is the flow rate, and the effective velocity $\mathbf{q}$ is given by Darcy's law:

$$\mathbf{q} = -k\mathbf{K}\boldsymbol{\nabla} H\,, \tag{2}$$

where the total hydraulic head is given by $H = h + z$, h is the pressure head (m), $\mathbf{K}$ is the hydraulic conductivity, k is the relative permeability, z is the vertical direction (m) and x the horizontal direction (m).

It is known that the location has two distinct aquifers that are separated by a confining layer (refer Figure 2):

1. *Unconfined aquifer* in the alluvium zone from $0\text{m} \le x \le 50\text{m}, 30\text{m} \le z \le 80\text{m}$ and $50\text{m} < x \le 350\text{m}, 40\text{m} < z \le 80\text{m}$ containing a soil-type with well defined geological and hydraulic material properties.
2. *Semi-confined aquifer* in the sandstone from $0\text{m} \le x \le 500\text{m}, 0\text{m} \le z \le 30\text{m}$ and $350\text{m} < x \le 500\text{m}, 30\text{m} \le z \le 80\text{m}$ containing sandstone with defined geological and hydraulic material properties.
3. *Confining layer* from $50\text{m} < x \le 350\text{m}$ and $30\text{m} \le z \le 40\text{m}$ with well defined geological and hydraulic material properties.

To account for this variation, it is assumed that the underlying porous structure can be characterised in each zone by a *representative unit cell.* Under these conditions, the flux term is written as:

$$\mathbf{q} = -k\mathbf{K}_{\text{eff}}^{(r)}\boldsymbol{\nabla} H\,, \tag{3}$$

where the spatially-varying hydraulic conductivity $\mathbf{K}$ has been replaced by a constant "effective" tensor $\mathbf{K}_{\text{eff}}^{(r)} = \begin{pmatrix} K_{\text{xx}}^{(r)} & 0 \\ 0 & K_{\text{zz}}^{(r)} \end{pmatrix}$, where $r = a$ for the alluvium aquifer, $r = c$ for the confining layer and $r = s$ for the sandstone.

We make the following assumptions (which may not all be entirely reasonable, but are used for this assignment so that a two-dimensional model is appropriate):

- the land area in question is large, and effects in the y–direction are negligible
- the aquifer extends to a uniform depth over the area in question
- permeability in the horizontal directions is greater than in the vertical direction
- soil properties are homogeneous within each layer over the depth of the aquifer

These assumptions reduce the problem to the following two-dimensional nonlinear partial differential equation (PDE) on the domain $\Omega = \{(x, z) \,|\, 0 \le x \le L_1, 0 \le z \le L_2\}$ and $t > 0$:

$$\frac{\partial \psi(h)}{\partial t} + \frac{\partial}{\partial x}\left[-k(h) K_{\mathrm{xx}}^{(r)} \frac{\partial h}{\partial x}\right] + \frac{\partial}{\partial z}\left[-k(h) K_{\mathrm{zz}}^{(r)} \left(\frac{\partial h}{\partial z} + 1\right)\right] = Q(t)\,, \tag{4}$$

where $z = 0$ is the model base and $z = L_2$ is the land surface, which for the purposes of this study is taken as $L_2 = 80\text{m}$. The width of the aquifer is $L_1 = 500\text{m}$.

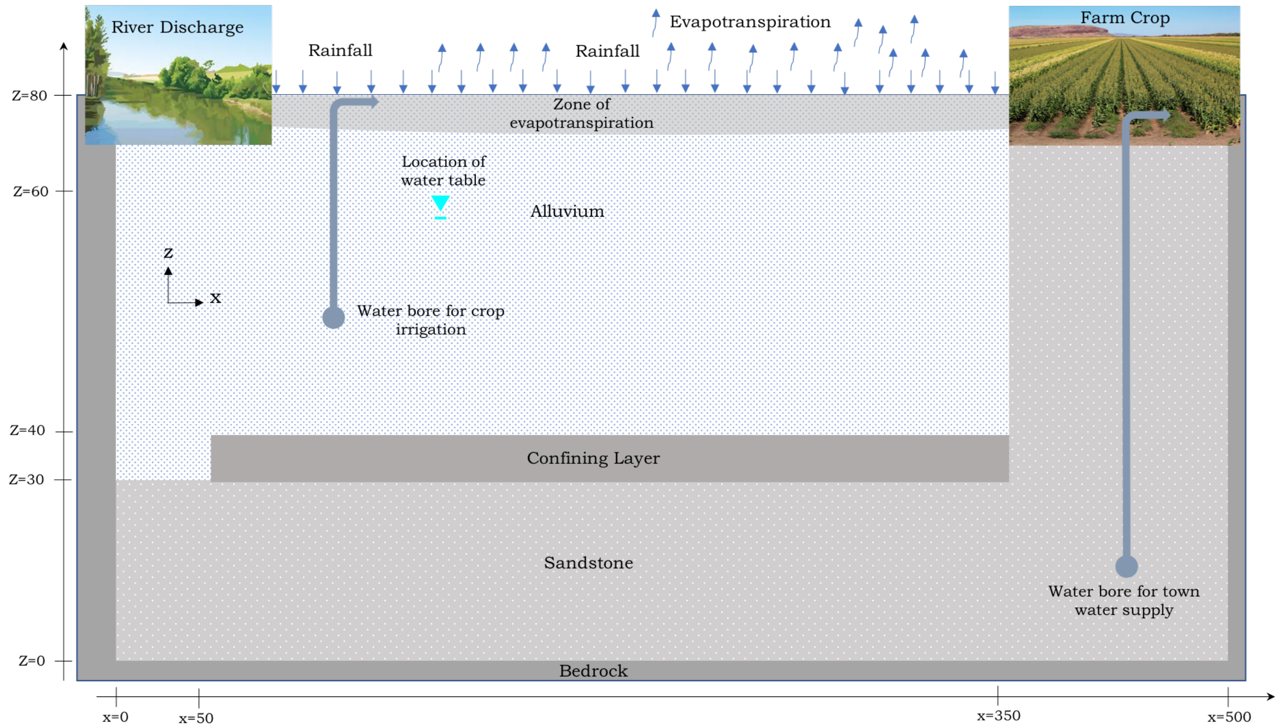


Figure 2: Schematic diagram of the aquifer.

2.2 Closure Conditions

The van Genuchten model provides the empirical form for the water content $\psi(h)$ and relative permeability $k(h)$:

$$\psi(h) = \begin{cases} \psi_{\text{res}} + S(h)(\psi_{\text{sat}} - \psi_{\text{res}}), & h < 0 \\ \psi_{\text{sat}}, & h \ge 0 \end{cases} \tag{5}$$

and

$$k(h) = \begin{cases} \sqrt{S(h)}\left(1 - (1 - S(h)^{1/m})^m\right)^2, & h < 0 \\ 1, & h \ge 0\,, \end{cases} \tag{6}$$

where

$$S(h) = \begin{cases} (1 + (-\alpha h)^n)^{-m}, & h < 0 \\ 1, & h \ge 0 \end{cases} \tag{7}$$

is the relative saturation. The geological and hydrodynamical material parameters are listed in the following table:

	K_{xx} (m/d)	K_{zz} (m/d)	ψ_{res}	ψ_{sat}	α (1/m)	n
Alluvium	2.6	0.91	0.01	0.33	1.43	1.51
Confining Layer	0.08	0.0159	0.106	0.4686	1.04	1.3954
Sandstone	3.9	1.17	0.0286	0.3658	2.8	2.239

and $m = 1 - 1/n$.

A schematic illustrating the heterogeneity of the porous medium is depicted in Figure 3 for the four elements in the mesh located around node (x_P, z_P) each having different material properties $\alpha_k, n_k, m_k, \psi_{\text{res}_k}, \psi_{\text{sat}_k}, k = 1, \ldots, 4$. To evaluate the water content $\psi_P(h_P)$ at node (x_P, z_P) we take an average over the four sub-control volumes ΔV_{scv_k} as follows:

$$\psi_P(h_P) = \frac{\sum_{k=1}^{4} \Delta V_{\text{scv}_k} \, \psi^{(k)}(h_P)}{\sum_{k=1}^{4} \Delta V_{\text{scv}_k}}.$$

The quantities used in the average are the water contents $\psi^{(k)}(h_P)$ evaluated at $h_P = h(x_P, z_P)$ in each of the four sub-control volumes surrounding (x_P, z_P) using the material properties of the k^{th} element.

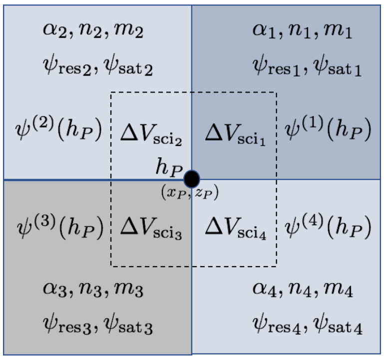


Figure 3: Treating heterogeneity in control volumes.

Evaluation of the relative permeability $k_P(h_P)$ is carried out in a similar manner.

2.3 River Flow Boundary Considerations

Discharge from/to the river can be approximated by specifying the flux

$$\mathbf{q} \cdot \mathbf{n} = K_{\text{R}} \, \Delta H \, , \; 60 \leq z \leq 80 \tag{8}$$

where K_{R} is the river hydraulic conductivity (m/d) in the Alluvium zone and $\mathbf{n}$ is the unit normal vector to the boundary. The river bottom is located 20m from the surface. When the total head $h(0, z, t) + z \leq 65$m, the hydraulic gradient (m/m) is given by $\Delta H = (H - (h(0, z, t) + z)) / \Delta x$, whereas when the total head is lower than the river bottom, $h(0, z, t) + z < 60$m, the hydraulic gradient is taken as $\Delta H = (H - 60)/\Delta x$. We assume that the river thickness is 50m so that $\Delta x = 50$m with head 15m below ground level ($H = 65$m). You should also investigate periods where discharge from the aquifer to the river is possible, i.e., given suitable water table conditions throughout the year it is possible that if the total head 65m $< h(0, z, t) + z \leq$ 80m, the hydraulic gradient results in flow from the aquifer to the river. For your simulations investigate the impact of using different values of $0.1 \leq K_R \leq 0.5 \; m/d$.

2.4 Initial and other Boundary Conditions

Initially the head profile is specified with linear variation

$$h(x, z, 0) = h_{\text{bot}} + (h_{\text{top}} - h_{\text{bot}})z/L_2 \text{ m}, \quad (x, z) \in \Omega.$$

Use for testing purposes $h_{\text{bot}} = -5$ and $h_{\text{top}} = -10$. The boundary conditions on $\partial\Omega$ at $x = L_1$ and $z = 0$ are no-flow, i.e. $\mathbf{q} \cdot \mathbf{n} = 0$, corresponding to the bedrock layer upon which the aquifer sits. Apart from the flow conditions discussed above for the river located at $x = 0, 60 \leq z \leq 80$, the remaining boundary at $x = 0$ is also taken as no flow. The boundary condition at $z = L_2$ depends on the local rainfall. As a very simplified approach, we will suppose the annual rainfall can be modelled by the function

$$q_{\text{rain}} = \mathrm{r}_f + \mathrm{r}_f \cos(2\pi t/365) \text{ mm/day}, \quad t \text{ in days},$$

where according to the 2017 Bureau of Meteorology Climate statistics for Australian sites at Dalby Airport (Site number 41522) http://www.bom.gov.au/jsp/ncc/cdio/weatherData/av?p_nccObsCode=136&p_display_type=dailyDataFile&p_startYear=&p_c=&p_stn_num=041522 it can be determined that $\mathrm{r}_f = 1.71$ mm/day.

If the aquifer is not fully saturated, then the boundary condition at $z = L_2$ is $\mathbf{q} \cdot \mathbf{n} = q_{\text{rain}}$, i.e. the rain soaks into the soil. If the aquifer is fully saturated, the rain runs off the surface rather than soaking into the soil.

To obtain the initial state of the aquifer you should run the model without pumping until a pseudo steady state is reached. After this point in time, the water bore extraction is introduced and the model should be run for an additional 10 years to study the evolution of the water table.

2.5 Evapotranspiration

Evapotranspiration is the process by which water is transferred from the land to the atmosphere by evaporation from the soil and by transpiration from the nearby crop plantation. Here, we use a simplistic model for the evaporation zone $0 \leq x \leq 350\text{m}, 78\text{m} \leq z \leq 80\text{m}$ and for the transpiration zone for the crop $350\text{m} < x \leq 500\text{m}, 76\text{m} \leq z \leq 80\text{m}$ via the use of the following flow term:

$$Q(z) = \begin{cases} \frac{-R(z-L_2+\ell)^2}{\ell^2} & L_2 - \ell \leq z \leq L_2 \\ 0 & 0 \leq z \leq L_2 - \ell \end{cases} \tag{9}$$

which is assumed to be active only if the moisture content at a given node satisfies $\psi(h^n)/\psi_{\text{sat}} > 0.5$, where ψ_{sat} would be the average saturated water content computed by taking into account the heterogeneity surrounding the node point. The constant R is taken in the evaporation zone as 2.5% of the rainfall value at the surface of the aquifer and $\ell = 2\text{m}$. The root system of the crop plantation is quite deep and we therefore take $\ell = 4\text{m}$ with the constant $R = 3.5\%$ of the rainfall value at the surface of the aquifer.

2.6 Pumping Rates

Presently the farmers are pumping from the very bottom of the unconfined aquifer. Legislation requires that the pumping rate Q is such that the total water extracted over one year is less than or equal to a quarter of the total (predicted) annual rainfall. Town water is pumped from the semi-confined aquifer at a constant rate such that the total water extracted over one year is less than or equal to a half of the total (predicted) annual rainfall. The irrigation and town water bores are currently located at the coordinates of $(100, 50)$ and $(450, 10)$ respectively.

3 Solution Methodology

3.1 2D Finite Volume Code

Develop MATLAB code that solves (4) using the *vertex-centred* Finite Volume Method (FVM) subject to the appropriate initial and boundary conditions. Your code should account for non-uniform grid spacing, different temporal discretisation schemes $\left(\theta = 1 \text{ or } \frac{1}{2}\right)$ and completely general boundary conditions at the boundaries of the domain $\Omega = [0, L_1] \times [0, L_2]$ located along the edges of the domain $\partial\Omega$ at $x = 0$ and $x = L_1$ for $0 \leq z \leq L_2$ and $z = 0$ and $z = L_2$ for $0 \leq x \leq L_1$. You should implement and investigate the following spatial weighting strategies for the terms $k(h)K_{\mathrm{xx}}^{(r)}$ and $k(h)K_{\mathrm{zz}}^{(r)}$ of the flux in (4) at the control volume faces: (i) *arithmetic average* of the values at the upstream and downstream nodes; and (ii) the use of *upwinding*, where the value at the control volume face is approximated using the value at the upstream node.

The finite volume method when applied to equation (4) produces a discrete analogue of the model in the form

$$\mathbf{F}(\mathbf{u}^{(n+1)}) = \mathbf{0}\,, \tag{10}$$

where $\mathbf{u}^{(n+1)}$ is a vector containing the unknown nodal pressure head values at time level t_{n+1}. This nonlinear system must be solved at each time step to advance the solution in time. You will investigate a range of different solvers based on Newton's method for solving (10).

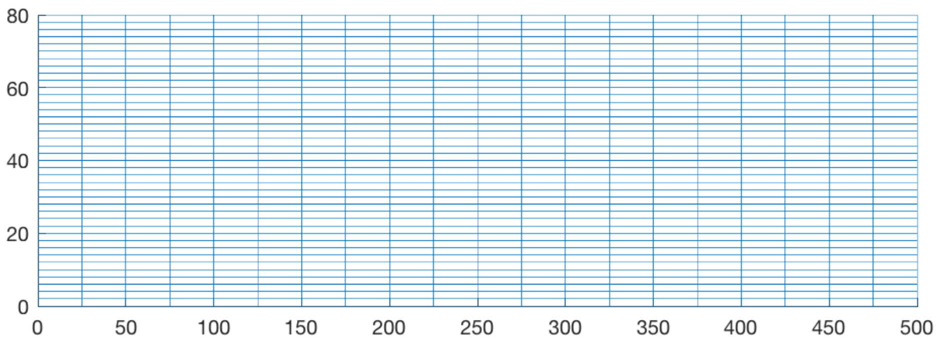
To solve the above nonlinear system you are to use an inexact Globally-Convergent Newton-Krylov method. You should implement and investigate different variations of the Newton method (full Newton, Chord, Shamanskii), the Krylov subspace method GMRES with different right preconditioning strategies and the simple line searching method discussed in lectures. The Jacobian matrix should be approximated using suitable finite difference approximations that exploit the banded structure of the matrix. You should reordered your nodes in the mesh using the reverse Cuthill-Mckee algorithm via the `symrcm` command in MATLAB. You should also investigate the use of the Jacobian-free Newton-Krylov strategy discussed in lectures. Your code should use the full Jacobian matrix as a preconditioner ($\mathbf{M} = \mathbf{J}$) using either a complete LU decomposition or an incomplete LU decomposition using the MATLAB commands `lu` or `ilu`. Heuristics based on the convergence behaviour of both the Krylov and Newton iterations should be used to determine when the preconditioner is updated (refreshed): if either the Newton or Krylov iterations fail to converge, or if convergence is achieved but exceeds a user-defined number of iterations, the preconditioner should be refreshed. Your group will also develop a simple adaptive time stepping strategy for the FVM solver that is also based on heuristics related to the convergence behaviour of Newton's method and GMRES.

You should also investigate the use of the modified Eisenstat-Walker formula[1] for determining the forcing term η_k (with suitable choices for the parameters η_{max}, γ and α) given below to terminate the Krylov solver at each iteration of the inexact Newton-Krylov method:

$$\eta_k^R = \gamma \left(\frac{\|\mathbf{F}^{(k)}\|_\infty}{\|\mathbf{F}^{(k-1)}\|_\infty} \right)^\alpha,$$

$$\eta_k^S = \begin{cases} \eta_{max}, & k = 0 \\ \min(\eta_{max}, \eta_k^R), & k > 0, \gamma\eta_{k-1}^\alpha \le 0.1, \\ \min(\eta_{max}, \max(\eta_k^R, \gamma\eta_{k-1}^\alpha)), & k > 0, \gamma\eta_{k-1}^\alpha > 0.1, \end{cases}$$

$$\eta_k = \min\left(\eta_{max}, \max\left(\eta_k^S, \frac{0.5(\tau_a + \tau_r\|\mathbf{F}^{(0)}\|_\infty)}{\|\mathbf{F}^{(k)}\|_\infty} \right) \right).$$



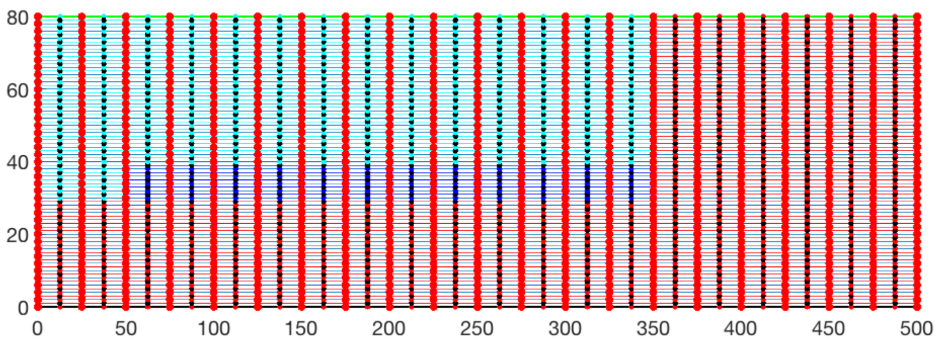


Figure 4: Computational mesh exhibiting varying zones having different material properties.

You should compare runtimes and appropriate iteration statistics for convergence of the different solution strategies in tabular format. Important assessable components of this code are implementation of the time-stepping strategy, nonlinear solver, preconditioned Krylov solver and treatment of the flux term. You are also encouraged to use visualisation methods to depict the solution fields graphically. You should experiment with different mesh sizes and

[1]S. C. Eisenstat and H. F. Walker (1996), Choosing the forcing terms in an inexact Newton Method, SIAM J. Sci. Comput, 17(1), pp. 16-32.

time step combinations and tabulate the total accumulated function calls for your simulations. Explain which solution strategy you would recommend and give reasons to support your claims in your report.

To test your code, start by considering a uniform mesh of size of dimension 21×41 for Ω that accommodates the different zones of the aquifer. A typical mesh is exhibited in Figure 4. Then, for the case of constant rainfall across the upper boundary (at $z = 80$) over the period of simulation you should confirm that the rate of increase of the average water content in the aquifer is linear. By performing an integration of the model (4) over Ω and applying the appropriate boundary conditions on $\partial\Omega$ you will arrive at a linear differential equation for the average water content

$$\bar{\psi} = \frac{1}{L_1 L_2} \iint_\Omega \psi(x, z, t)\, dx dz\,,$$

that can be solved to allow you to identify the precise form of the linear function in time. Use this function to check your code is working correctly.

4 Analysis

You have been asked to advise on the following aspects.

- What is the profile of the water content, pressure head and water table in the aquifer over the course of the simulation?
- How does the location of the water table change over the course of the simulation? In particular, how many days each year is it below the river head of $z = 65$? What differences do you observe (if any) between the pre and post pumping conditions?
- Devise a comparison strategy to analyse the impact of pre and post pumping conditions. In particular, focus on the interaction between the river and the aquifer and the periods (if any) of no flow to the river.
- Investigate the impact of dry periods over the course of the simulation during which there will be no drawdown from the river. More information will be provided on this climate data at a later stage.
- Repeat the analysis using more realistic annual rainfall data (suitably smoothed) taken from the Bureau of Meteorology. Investigate what happens to the river discharge during periods of drought (again assuming there will be no drawdown from the river during these periods) for the pre and post development conditions.