MSc Thesis

Transporting an overproduction of locally generated renewable energy in an island context

SET 3901: Graduation Project

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Preface

This report was written by me, a Sustainable Energy Technology student at Delft University of Technology, as part of my Thesis Project.

For my final project in this program, I chose to tackle a complex topic about commodity transport. With a strong focus on renewable energy throughout my studies, the challenge of energy system modelling, particularly in addressing a real-world issue, offered a fascinating area to investigate. Moreover, the project's emphasis on Linear and Mixed-Integer Programming, combined with the use of Python, aligned perfectly with my interests in both mathematics and programming.

Projects such as these are of immense significance as the world confronts the pressing need for clean energy solutions and the looming impacts of climate change. Managing the overproduction of renewable energy is becoming increasingly critical and finding efficient ways to transport surplus energy is essential to achieving a sustainable energy future.

Finally, I would like to thank my supervisors Stefan Pfenninger and Bryn Pickering for their help and support during my Thesis Project.

Berat Kaya
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Abstract

This study examines the impact of realistic transport constraints on the supply of aluminium from Iceland to the Netherlands using the Calliope energy system model. Two scenarios are developed: an idealized baseline with continuous, lossless transmission of aluminium (analogous to unlimited pipeline flow), and a discrete shipping scenario featuring a single 20,000-ton capacity ship with a 72-hour one-way transit and fuel consumption requirements. Both scenarios are run over a one-month period (January 2005, hourly resolution) to meet a constant demand in the Netherlands. The baseline scenario demonstrates the best-case theoretical supply. In contrast, the discrete shipping scenario yields a more complex operational profile: aluminium is delivered in batches roughly every 6–9 days and has a short initial supply shortfall (≈7,200 tons unmet during the first 72 hours). After this initial lag, the single vessel is able to continuously satisfy demand by cycling shipments. Fuel consumption for transport in the discrete scenario was 3,360 units (costing €70), highlighting a minor cost and emissions factor absent in the baseline. Comparing the scenarios reveals that ignoring shipping constraints (as in the baseline) overestimates supply reliability and understates operational requirements, whereas the constrained model captures the timing, capacity limits, and fuel costs that characterize real-world commodity transport. Overall, this report demonstrates the importance of incorporating discrete transport modelling in energy systems analysis to inform more practical and robust infrastructure planning.

Contents

1 Introduction

The urgent global challenge of climate change has led to a sweeping movement across countries and industries to drastically reduce greenhouse gas emissions and transition towards sustainable energy systems. In Europe, the European Union (EU) has set ambitious targets to achieve a climate-neutral economy by 2050, with energy system decarbonization at the forefront of these goals. Transitioning to low-carbon energy sources, however, is a complex task, involving significant trade-offs and operational challenges (Pickering et al., 2022). For instance, the intermittency of renewable energy sources, such as wind and solar power, creates variability in energy supply that complicates efforts to meet consistent demand. This variability is especially challenging in industries and applications that require stable, reliable energy supplies, such as the transport of energy-intensive resources across long distances.

As such, energy system optimization is imperative, with a key area in need of this being transport. To effectively address such a transport problem, advanced energy system modeling tools are required. One tool capable of building energy models is the Calliope framework, which also provides spatial and temporal optimization (Verrascina, 2022). However, while it is possible to use continuous power transmissions for modeling within the framework, it needs to be improved to handle more complex cases. One such case is the discrete transportation of energy carriers, within the maritime industry for example.

To effectively setup a test model within Calliope to apply these improvements to, a case study will be built focusing on Iceland. This country is a world leader in renewable energy, producing a majority of its total primary energy supply locally through energy-intensive processes powered primarily by hydropower and geothermal energy, which then gets exported in the form of aluminium. Although this makes Iceland one of the greenest producers of aluminium, transporting the aluminium to its primary markets in Europe requires significant energy. Maritime transport, which consumes fossil fuels and contributes to carbon emissions, is currently the standard method for moving aluminium from Iceland to Europe. As such, optimizing the aluminium transport process between Iceland and the Netherlands to minimize emissions and fuel costs is a pressing priority for sustainable supply chain management. The aim of this report is to use the Calliope framework to optimize the transport of renewable energy from Iceland to The Netherlands using ships and investigate potential trade-offs compared to other methods, such as power transmission.

The remainder of this report is organized as follows:

- Chapter 2: Literature Review – This chapter reviews the relevant literature on European energy system decarbonization, energy modeling frameworks, maritime transport optimization, and scheduling of mobile energy resources. Particular emphasis is placed on the methodologies used to model intermittent energy sources and discrete transport events.
- Chapter 3: Methodology – This chapter details the model setup and mathematical formulation. It describes the system components (nodes, transport ship, fuel consumption) and explains the constraints and optimization objectives used in the MILP model within the Calliope framework.
- Chapter 4: Results – This chapter presents the model results, analyzing the effectiveness of various transport schedules and fuel consumption strategies. The impact of renewable energy variability on fuel costs, emissions, and scheduling is explored in depth.
- Chapter 5: Discussion – This chapter interprets the findings and discusses the implications for sustainable maritime transport in the context of climate-neutral energy systems. Potential trade-offs between cost and sustainability are evaluated, and recommendations are made for future transport optimization.
- Chapter 6: Conclusion – The final chapter summarizes the key contributions of this thesis and outlines potential areas for further research, including possible extensions to multi-node systems and other energy-intensive supply chains.

2 Literature Review

This chapter presents a structured review of the existing literature to establish the theoretical and methodological foundation for modelling discrete, delayed transport of renewable energy in island contexts. First, the chapter provides a justification for improving transport modelling by highlighting significant limitations of conventional energy system optimization models, especially their inability to accurately represent discrete transport events and associated time delays, such as maritime shipment of energy-intensive products. Second, the chapter systematically examines recent methodological advances in energy system modelling and maritime transport optimization. Emphasis is placed on the suitability of Mixed-Integer Linear Programming (MILP) and the Calliope framework for addressing these modelling challenges. Collectively, the literature reviewed here outlines the current knowledge gaps and methodological requirements that justify and guide the development of the modelling approach used in this report.

2.1. Limitations of existing transport modelling in energy systems

Isolated renewable-rich regions like Iceland face unique challenges in exporting surplus energy. With no grid connection to continental markets, Iceland effectively embodies its renewable electricity in energy-intensive products (aluminium) for export. However, shipping these products introduces discrete transport events and time delays that traditional energy system models struggle to represent. Most optimization frameworks assume continuous, instantaneous flows between regions, an approach suitable for pipelines or power lines but not for discrete shipments by sea. For example, natural gas transport is typically modelled at a coarse time resolution in energy models, implicitly treating fuel delivery as continuous. This fails to capture the reality of maritime transport, where a ship must be loaded, sail for days, then unload, a process that cannot be approximated by a steady hourly flow. Existing energy system models rarely incorporate the timing of shipments or transit delays, often necessitating assumptions or separate scheduling simulations. This gap means that critical dynamics, such as scheduling a voyage when renewable-derived fuel is abundant, or stockpiling product until a ship returns, are overlooked. In short, conventional models lack in the case of island energy exports, justifying improved transport modelling that can handle discrete shipments and their delays.

Another limitation is the treatment of non-continuous decisions and binary events. Standard linear programming models avoid integer variables for tractability, but this forces oversimplifications like fractional shipping or instantaneous delivery. Delayed delivery can cause supply–demand mismatches if not modelled, and ignoring it can underestimate storage needs or fuel timing issues. Past studies have noted that integrating diverse temporal scales (production vs. transport schedules) in a single model is challenging and often omitted. Even in pipeline gas modelling, researchers have had to introduce MILP formulations to approximate non-linear or time-coupled constraints. Representing a cargo ship's departure and arrival inherently requires binary and time-linking constraints (e.g. a ship either sails on a given day or not, and arrives after a known transit time). Without such discrete modelling, an energy export scenario could be inaccurately represented as a continuous flow of energy, rather than a batched, delayed process. To address these shortcomings, a modelling approach that treats transport as a scheduled discrete event is crucial to accurately simulate the exports of renewable energy from an island via ship.

2.2. Approaches to optimizing energy transport in literature

2.2.1. Energy system decarbonization and transport integration

Recent energy system studies underline the importance of integrating transport options into decarbonization strategies. For example, Pickering et al. (2022) examine pathways for a carbon-neutral European energy system that eliminates fossil fuel imports, using a high resolution optimization model. Their work explores the diversity of configurations needed to meet demand with renewables under spatial and technical constraints, highlighting trade-offs in network expansion, storage, and resource allocation. Notably, while Europe's case assumes grid interconnections, the study's attention to trade-offs and timing is relevant to an island context. It suggests that when and how energy is transported can significantly affect costs and feasibility. The methodologies in such continental models, often built in flexible frameworks like Calliope, demonstrate how high spatial and temporal resolution can capture complex supply chains. By customizing these frameworks, one can evaluate unconventional transport modes. In fact, the European scenarios indirectly support this approach: they show that eliminating fossil fuels requires not just local renewable deployment but also smart allocation of energy across regions and time. In an Iceland-to-Europe setting, this translates to intelligently scheduling aluminium shipments (as “energy carriers”) to align with renewable availability and demand. The literature thus motivates treating maritime transport as an integral part of the energy system, rather than an external logistics issue.

In another region-specific analysis, Verrascina (2022) uses Calliope to model the energy transition of Sardinia as it phases out coal and integrates renewables. This study demonstrates the flexibility of the Calliope framework in regional scenarios, exploring pathways for Sardinia’s transition while accounting for renewable resource variability. The successful application of Calliope for Sardinia is relevant for this report, which uses the same framework to model aluminium transport and fuel use between Iceland and the Netherlands. This case shows Calliope’s effectiveness in integrating localized renewable sources into an island’s energy system, aligning with this report’s goal of representing Iceland’s renewable energy surplus and its impact on fuel requirements for maritime transport. Importantly, when direct grid interconnections are not feasible, recent research emphasizes converting surplus renewable electricity into transportable fuels like hydrogen as a strategic alternative for isolated systems. Non-interconnected areas (islands with no mainland grid link) can leverage such carriers to “export” energy, underscoring the rationale for Iceland’s indirect energy export (e.g. via aluminium or hydrogen). These insights reinforce the justification for modelling an island’s surplus energy transport, guiding the development of a system that balances local renewable integration with export-oriented use of that energy.

2.2.2. Maritime transport optimization and fuel efficiency

The field of maritime transport optimization provides tools and insights applicable to energy cargo scheduling. A body of work has focused on reducing shipping costs and emissions through optimal routing, scheduling, and vessel deployment. For example, Wang & Wang (2021) address liner shipping operations by developing a MILP model to schedule heterogeneous vessels on a route, optimizing their departure times and speeds. This kind of operational research, while centered on container ships, demonstrates the value of discrete scheduling for cost minimization, achieving measurable savings (their model cut total costs by 5% in a case study). The principles carry over to energy transports: just as container shipments benefit from optimized timing and vessel choice, aluminium exports can be scheduled to minimize fuel use and waiting time. Moreover, maritime decarbonization studies emphasize fuel efficiency techniques (like slow steaming and routing) to cut emissions. These require modelling the interplay between fuel consumption and schedules – a linkage very relevant when the fuel itself is produced from renewables. If ships are to use climate-neutral fuels (e.g. hydrogen or ammonia made in Iceland), the timing of fuel production and refuelling becomes a critical optimization aspect. Researchers have begun to explicitly consider such couplings. For instance, a recent open-source model was developed to assess the costs of shipping hydrogen-based fuels over long distances, reflecting growing interest in transporting renewable energy in chemical form. Although that model was primarily a cost calculator, it reinforces the idea that shipping routes, fuel technology, and energy availability must be evaluated together in system planning. These studies collectively underscore that optimizing maritime transport, whether for goods or energy commodities, hinges on careful scheduling and integration with energy supply dynamics.

Groppi et al. (2022) explore various renewable fuel solutions for a small island’s maritime transport sector and evaluate these within their EPLANoptMAC model. Focusing on the island of Favignana, their work provides insights into decarbonizing ship-based transport in limited geographic areas with local renewable fuel production. This study used a heuristic linear programming approach to optimize the island’s transportation energy mix, identifying cost-effective solutions while meeting operational constraints. The challenges of modelling discrete transport events (ship voyages) and fuel consumption align well with Groppi’s methodology, emphasizing the need for flexible energy models capable of evaluating both fuel demand and renewable integration in transport-reliant regions. Işıklı et al. (2020) further contribute to this area by developing statistical models to predict ship fuel consumption based on operational factors such as speed and distance. Their work underscores the importance of accurate fuel consumption estimation, which is critical for representing fuel demand in maritime aluminium transport. The methodology presented by Işıklı and colleagues aids this report’s modelling of fuel use by providing insights into the operational variables affecting fuel efficiency. By incorporating such empirical models, the optimization could ensure realistic fuel use estimates, supporting a cost-effective and low-carbon transport solution.

Beyond improvements in operational efficiency, recent literature highlights the adoption of carbon-free fuels in shipping as a key decarbonization strategy. The International Maritime Organization's initial strategy targets a 50% reduction in maritime greenhouse gas emissions by 2050 (relative to 2008 levels), pressuring the sector to explore alternatives to conventional bunker fuel. Integrating alternative energy carriers such as hydrogen, ammonia, and battery systems into shipping is increasingly seen as a viable pathway to reduce the industry's environmental impact. These alternative fuels are carbon-free at the point of use and could drastically cut emissions if produced from renewable energy. However, their deployment requires careful consideration of energy density, storage, and safety. For example, ammonia's toxicity and hydrogen's low volumetric density pose challenges. Notably, pilot projects have begun to demonstrate the feasibility of transporting renewable-based fuels by ship. In 2022, the Suiso Frontier – the world's first liquid hydrogen carrier – successfully delivered 2.6 tonnes of liquefied hydrogen from Australia to Japan. This Hydrogen Energy Supply Chain (HESC) project proved that maritime transport of liquid hydrogen is technically achievable, marking an important milestone in energy export logistics. Such developments highlight both the opportunities and modelling challenges for island contexts: if an island like Iceland produces green hydrogen or ammonia from surplus power, shipping these fuels abroad becomes a potential export strategy. Energy system models must therefore be capable of evaluating scenarios in which locally generated renewables are converted to transportable forms and shipped overseas. The feasibility of these options will depend on factors like fuel production costs, ship scheduling, and infrastructure, all of which need to be captured in a comprehensive modelling framework. In summary, there is a clear need to optimize how islands can ferry out excess renewable energy in a sustainable and economically viable way.

2.2.3. Mixed-integer optimization in energy system models

The turn to Mixed-Integer Linear Programming (MILP) is a natural evolution as we introduce binary variables for shipments and other on/off decisions. MILP is widely used in energy systems research whenever discrete choices are involved, from unit commitment in power systems to infrastructure placement and supply chain design. For instance, Kim et al. (2019) formulate a MILP model for optimal design of a future hydrogen supply network, including production and transportation infrastructure. Similarly, multi-period MILP models have been applied to green hydrogen supply chains to decide facility locations, transportation modes, and storage, all while minimizing cost or emissions. These examples demonstrate that MILP optimization can effectively handle the combination of investment decisions and operational scheduling in an energy context. The trade-off is computational complexity, but modern solvers and careful model formulation often make it tractable for moderate scales. In the case of transporting Iceland's energy via ships, the decision to send a shipment in a given time period is binary, as is the decision to use (or not use) a particular fuel tank at a given time. Using MILP allows the model to rigorously choose optimal shipping schedules (an inherently integer problem) alongside the continuous decisions of energy production and consumption. This integrated approach is supported by recent studies: Wang et al. (2022) integrated vehicle routing with power system reconfiguration in a MILP, and others have noted that combining operational scheduling with planning models is achievable through integer programming at the cost of greater solution times. Given the moderate scale of a two-node Iceland–Netherlands system, a MILP approach is feasible and warranted for the accuracy gains. By capturing the on/off nature of shipments and the timing of fuel use, the model could explore strategies like skipping shipments during low renewable periods or sending extra shipments when excess power would otherwise be curtailed. No purely linear model could capture these dynamics, hence the need for MILP in the transport optimization.

To incorporate discrete transport events in energy models, some researchers have proposed novel scheduling frameworks. A particularly relevant concept is the "separable" scheduling of mobile energy resources introduced by Wang et al. (2022). In their study on distribution grid resilience, Wang and colleagues propose a Separable Mobile Energy Storage System (SMESS) approach. "Separable" denotes that the mobile carrier (e.g. a truck or ship) and the energy payload (battery modules, or by analogy, shipments of aluminium) are treated as independent components. This allows a single carrier to transport multiple batches in sequence and deploy them optimally, rather than assuming a fixed continuous flow. Crucially, they formulate the joint routing and scheduling of the carrier and modules as a MILP problem. The SMESS framework explicitly models each trip as a discrete event and includes time delays for moving resources, closely mirroring the challenges of batch transport in our island scenario. By separating the vehicle's movement from the resource itself, the model can decide, for example, to delay a trip until enough fuel is available or to reroute a tanker to where it's most needed. This idea directly informs the present thesis: we can treat the aluminium shipments (energy carriers) and the fuel supply for ships as interdependent but distinct elements, scheduled through binary decision variables. In practice, that means the model can decide to send a ship only when certain conditions are met (adequate aluminium stock, lower fuel price due to high renewable output, etc.), rather than enforcing a regular continuous delivery. Their MILP scheduling responded to emergencies by rapidly redeploying mobile units, thus highlighting system resilience and flexibility. In a non-emergency context like Iceland's exports, the same emphasis on flexibility translates to economic and environmental gains: the model can defer shipments during dips in renewable generation and accelerate them when surplus power makes synthetic fuel cheap. This ensures that transport is synergistic with the energy system's state, a strategy supported by the separable scheduling literature. The takeaway is that discrete-event MILP models have been successfully used to coordinate mobile energy carriers with supply conditions, validating the use of this approach. By adopting similar MILP scheduling, the aim is to handle the complexity of tracking ship location, departure/arrival timing, and fuel consumption within a unified optimization framework.

Zhang et al. (2014) present a multi-objective optimization framework that balances economic and environmental objectives within supply chain management. Their focus on minimizing both cost and carbon emissions offers a useful basis for multi-criteria models, which seeks to balance sustainability goals with operational fuel costs in a discrete energy transport system. The methods for evaluating trade-offs in Zhang et al.'s work provide guidance for structuring the model's objectives and constraints. In particular, their approach of simultaneous cost-carbon minimization supports the aim to integrate sustainable practices into an energy-intensive supply chain (the shipping of aluminium), ensuring that financial costs and emissions are jointly optimized.

Cutore et al. (2024) develop a framework for designing island hydrogen supply chains using MILP with a multi-period approach. Applied to Corsica's energy system, their model optimizes the deployment of hydrogen production, storage, and distribution infrastructure to support sustainable mobility on the island. This work demonstrates how MILP can capture the temporal dynamics of fuel production and use in an island context, and it exemplifies the kind of methodological precedent that underpins our approach to Iceland's energy export problem. The MILP approach allows modelling of discrete decisions (e.g. when to produce and transport hydrogen) over a planning horizon, highlighting the importance of timing in utilizing renewable overproduction.

Similarly, Florez et al. (2024) use a MILP optimization to co-design renewable ammonia and hydrogen production with maritime export logistics in an "island" energy system in Saudi Arabia. Their study proposes a flexible production and transportation strategy for green hydrogen/ammonia, leveraging coastal wind and solar resources to supply export markets. By optimizing across the full supply chain – from electrolyzers and synthesis plants to shipping schedules – the model identifies cost-optimal ways to export surplus renewable energy as fuels. The results illustrate how excess intermittent generation can be converted into export commodities, and how shipping frequency and tanker size are chosen in the optimal solution. This example reinforces the value of MILP in handling complex, time-coupled decisions: it can simultaneously decide investment in conversion capacity and the scheduling of fuel shipments.

Another methodological advance is the explicit modelling of time delays and mobile resources in energy system optimization. Li et al. (2024) present an optimal scheduling model for an integrated power-hydrogen system that incorporates vessel-based hydrogen transport. In their approach, ships act as moving storage, ferrying hydrogen between nodes; crucially, the transit and berthing times of the hydrogen vessels are modelled to ensure energy is delivered when and where needed. This kind of formulation requires integer variables to represent the presence or absence of a vessel at a given time and location, thus falling naturally into an MILP framework. By considering the vessel's scheduling (departure, travel, arrival) within the optimization, Li et al. show that significant flexibility can be achieved in balancing a distributed grid – effectively treating ships as part of the energy network. The inclusion of similar time-delay constraints in our Calliope model (via custom delay and state transition constraints) is directly inspired by such work, ensuring that the discrete nature of shipping is respected in the optimization. It highlights a key challenge in modelling renewable transport: synchronizing energy production with the availability of a transport asset, a problem well-suited to MILP techniques that can handle binary timing decisions.

Open-source energy modelling frameworks have kept pace with these advancements, allowing researchers to integrate transport modalities into broader energy systems. State-of-the-art tools like Calliope and PyPSA are capable of high spatial and temporal resolution modelling and can be extended with custom constraints for novel technologies. Calliope, in particular, has been applied to diverse contexts from city-level analyses to continental grids, demonstrating the versatility needed for an island energy export study. Its flexible structure enables defining separate "nodes" (e.g. Iceland and the Netherlands) and linking them with user-defined shipping routes and delays. Indeed, Calliope's developers have demonstrated its use in complex multi-node scenarios, such as fully decarbonized multi-island power systems. This provides methodological confidence that a custom MILP formulation for a mobile energy system can be implemented within Calliope's framework.

In our case, the ability to enforce a transit delay for shipments and a mutually exclusive location constraint for the single ship (so it cannot be in two places at once) draws on the precedent of incorporating integer decision variables in an energy model – a practice that tools like Calliope (and PyPSA via add-ons) support.

In summary, recent literature strongly justifies the chosen methodology. Studies on island energy systems and renewable fuel transport have proven that MILP-based models can capture the discrete, time-dependent nature of shipping energy off-grid. From hydrogen supply chains in Corsica and Fiji to global green ammonia trade networks, researchers are employing optimization models to find least-cost pathways for moving energy in space and time. These works provide both a justification and a template for this thesis. They show that with careful formulation, an island's surplus renewable energy can be optimally allocated to various export options (be it via material proxies like aluminium or energy-dense carriers like H□/NH□), and that the logistics of transportation can be endogenously decided by the model. By building on this body of knowledge, the present study's MILP approach in Calliope is well-grounded in methodological precedent, and it extends this line of research to the novel case of exporting Iceland's renewable energy through maritime aluminium shipments. The added focus on integrating a mobile element (a ship) within an energy systems model addresses a noted gap in standard modelling tools, pushing the boundary of what open-source frameworks like Calliope can represent in pursuit of sustainable energy transport solutions.

2.2.4. Selection of a MILP-enabled calliope framework

To implement the above ideas, the Calliope energy system modelling framework with MILP capabilities is used. Calliope has flexibility in representing custom technologies and constraints, as well as support for multi-scale analysis. It is an open-source framework designed for arbitrarily high spatial and temporal resolution in energy modelling, which is essential for capturing the hourly renewable variability and multi-day shipping cycles in this case. Importantly, Calliope's solver interface can handle both linear and mixed-integer formulations. In fact, Calliope explicitly allows users to define binary decision variables for capacities or operations, and it can leverage commercial MILP solvers to tackle the resulting problem. This capability has been demonstrated in prior studies – for example, Calliope has been utilized for national-scale models and even extended with piecewise linearization for complex technology behaviours. The framework cleanly separates the data from the mathematical formulation, making it straightforward to add our new transport constraints. Moreover, using an existing framework like Calliope ensures that the model aligns with best practices in energy systems modelling, and results can be compared or extended in the future. In summary, Calliope's multi-node, multi-period optimization approach, augmented with MILP features, provides an ideal platform for this subject. The reviewed literature supports this choice, thus insights from high-resolution energy modelling, maritime optimization, and MILP scheduling of energy transports are combined into a coherent modelling strategy. This literature foundation gives confidence that a MILP-driven Calliope model can capture the complex interaction between Iceland's renewable energy overproduction, the timing of fuel and aluminium production, and the logistics of maritime transport, thus filling the gap identified in current energy system models.

3 Methodology

This report uses a case study focused on Iceland and the Netherlands to explore the optimisation of aluminium transport within a climate-neutral energy framework. These two countries were selected based on Iceland's unique energy profile and the established trade relationship between Iceland as a major aluminium exporter and the Netherlands as a primary importer. The aim is to develop an optimization model that minimizes fuel costs and carbon emissions associated with transporting aluminium between Iceland and the Netherlands. By modelling a two-node system where aluminium is produced in Iceland using renewable energy and transported by ship to meet demand in the Netherlands, this report explores the trade-offs involved in transport scheduling, fuel consumption, and renewable energy availability. The model is constructed using the Calliope framework, with fuel production costs in Iceland dynamically linked to renewable energy output, creating a flexible and climate-neutral energy system.

3.1. Case study rationale

Iceland is uniquely positioned as an ideal location for examining renewable energy-driven production due to its abundant natural resources and overproduction of renewable energy. Unlike many countries in Europe, Iceland derives the majority of its total primary energy supply locally from renewable sources, particularly geothermal and hydropower. Iceland produces approximately ten times as much electricity per resident as the European Union average, making it one of the most energy-rich nations per capita. However, this overproduction poses the challenge of effectively utilising the surplus energy, as direct export of electricity from Iceland to Europe is technically and economically challenging due to its geographic isolation and the high costs associated with long-distance electricity transmission infrastructure.

One solution Iceland has adopted is to export this surplus renewable energy indirectly by refining aluminium. Aluminium refining is an energy-intensive process, and by using its excess electricity for this purpose, Iceland has created a sustainable method of "exporting" energy in the form of aluminium. This aluminium is then shipped internationally, with the Netherlands as one of the primary importers. This report's case study thus provides an opportunity to investigate the efficiency and sustainability of aluminium as an energy export medium and evaluate its trade-offs against other possible methods, such as direct electricity transmission.

The Netherlands was chosen as the import destination for Iceland's aluminium due to its role as one of Iceland's key aluminium trading partners. As a major industrial and logistics hub within Europe, the Netherlands is strategically positioned to import and distribute aluminium throughout the continent. This established trade relationship between Iceland and the Netherlands provides a practical basis for modelling aluminium transport and allows for a realistic assessment of the trade-offs involved in using aluminium as a carrier of Iceland's renewable energy exports.

By focusing on Iceland and the Netherlands, this case study can provide valuable insights into the economic and environmental trade-offs of using aluminium as a renewable energy export method. Following the modelling, the study will assess the efficiency of this approach compared to alternatives such as submarine power cables for direct electricity transmission. The high capital and technical requirements of submarine cables, as well as transmission losses over long distances, make direct power transmission challenging.

3.2. Model structure and assumptions

The model is structured around a two-node system. Node 1 represents Iceland, where aluminium is produced using renewable energy sources (hydropower and geothermal energy). Fuel for the transport ship is produced in Iceland based on renewable energy availability, and fuel costs are variable, responding dynamically to energy production levels. Node 2 represents the Netherlands, where there is a continuous demand for aluminium. This node serves as the destination for aluminium shipments from Iceland.

The transport of aluminium between these nodes is modelled as a discrete event where each shipment consumes fuel. Additionally, the model incorporates a time delay to reflect the transit period between Iceland and The Netherlands, ensuring that fuel costs are accurately represented as variable and responsive to Iceland's renewable energy conditions.

A key assumption is that renewable energy production in Iceland is variable, impacting fuel production costs over time. Fuel cost is not constant; instead, it fluctuates with Iceland's renewable energy output, reflecting a climate-neutral approach. Another assumption is the model accounting for other demands on Iceland's renewable energy resources beyond fuel production. Renewable energy allocation to fuel production competes with other essential electricity needs, ensuring a realistic balance between energy demands. The final assumption is that each aluminium shipment represents an independent event with variable timing and fuel costs determined by the renewable energy availability in Iceland.

3.3. Model mathematics

The mathematical model is based on Mixed-Integer Linear Programming (MILP), adapted from the separable scheduling approach outlined by Wang et al. (2022). The MILP framework enables discrete event scheduling and optimizes the timing and fuel usage of aluminium shipments, incorporating dynamic fuel costs and energy availability.

Sets

- T: Set of time spans (e.g., $T = \{1, 2, \ldots, D\}$), where D is the length of the scheduling horizon.
- $N = \{1, 2\}$: Set of nodes (e.g., port in Iceland and port in the Netherlands).
- S: Set of ships.
- F: Set of fuel depots.

Parameters

- c_s: Carrying capacity of ship s.
- η_s: Fuel consumption rate of ship s.
- σ_s: Fuel capacity of ship s.
- α_i: Initial aluminium stock at node i.
- β_i: Aluminium demand at node i.

Variables

- $x_{s,i,t}$: Binary variable indicating if ship s is at node i during time span t.
- $y_{s,i,t}$: Binary variable indicating if ship s is travelling to node i during time span t.
- $a_{s,i,t}$: Amount of aluminium transported by ship s to node i during time span t.
- $f_{s,i,t}$: Amount of fuel consumed by ship s at node i during time span t.
- $\phi_{i,t}$: Amount of aluminium stock at node i during time span t.
- $\theta_{f,t}$: Amount of fuel at depot f during time span t.

Constraints

The following constraints define the behaviour and limitations of the system for optimising aluminium transport under renewable energy constraints.

1. State Transition of Ships

The state transition constraints ensure that a ship can only be in one state at any given time: either stationary at a node or travelling between nodes.

$$x_{s,i,t+1} \geq x_{s,i,t} + y_{s,i,t} - 1, \quad \forall s \in S, \forall i \in N, \forall t \in T \tag{3.1}$$
$$x_{s,i,t+1} \leq x_{s,i,t} + y_{s,i,t}, \quad \forall s \in S, \forall i \in N, \forall t \in T \tag{3.2}$$

- **Variables:**
  - $x_{s,i,t}$: A binary variable indicating if ship s is at node i during time span t (1 if at the node, 0 otherwise).
  - $y_{s,i,t}$: A binary variable indicating if ship s is travelling from node i during time span t (1 if travelling, 0 otherwise).
- **First Constraint** $x_{s,i,t+1} \geq x_{s,i,t} + y_{s,i,t} - 1$:
  - Ensures that if a ship was at node i in the previous time step ($x_{s,i,t} = 1$) and started travelling ($y_{s,i,t} = 1$), it will not remain at the node in the next time step ($x_{s,i,t+1} = 0$).
- **Second Constraint** $x_{s,i,t+1} \leq x_{s,i,t} + y_{s,i,t}$:
  - Prevents the ship from being at the node and in transit simultaneously.
  - Ensures logical movement by enforcing that if the ship is travelling, it will not be counted as ”at the node” in the subsequent time step.

2. Fuel Consumption and Restock Constraint (Refuel/Restock only when not travelling)

This constraint limits fuel consumption or restocking to times when the ship is stationary at a node.

$$f_{s,i,t} \leq \sigma_s \cdot (1 - y_{s,i,t}), \quad \forall s \in S, \forall i \in N, \forall t \in T \tag{3.3}$$

- **Variables:**
  - $f_{s,i,t}$: The amount of fuel consumed or restocked by ship s at node i during time span t.
  - $y_{s,i,t}$: Binary variable indicating if ship s is travelling from node i during time span t.
  - σ_s: Fuel capacity of ship s.
- The term $(1 - y_{s,i,t})$ acts as an ”enabler” for refueling:
  - When $y_{s,i,t} = 0$ (stationary), $f_{s,i,t} \leq \sigma_s$ allows refueling up to capacity.
  - When $y_{s,i,t} = 1$ (travelling), $f_{s,i,t} = 0$, preventing refueling while in transit.

3. Ship Capacity Constraint

This constraint limits the amount of aluminium a ship can carry to its maximum capacity.

$$a_{s,i,t} \leq c_s \cdot (1 - y_{s,i,t}), \quad \forall s \in S, \forall i \in N, \forall t \in T \tag{3.4}$$

- **Variables:**
  - $a_{s,i,t}$: Amount of aluminium transported by ship s to node i during time span t.
  - $y_{s,i,t}$: Binary variable indicating if the ship is moving during time step t.
  - c_s: Carrying capacity of ship s.
- The term $(1 - y_{s,i,t})$ again enables aluminium transport only when stationary.
  - When $y_{s,i,t} = 0$, $a_{s,i,t} \leq c_s$ allows loading up to the ship’s capacity.
  - When $y_{s,i,t} = 1$, $a_{s,i,t} = 0$, preventing loading while in transit.

4. Aluminium Stock Balance

This constraint maintains the aluminium stock balance at each node over time, accounting for aluminium transported and local demand.

$$\phi_{i,t+1} = \phi_{i,t} + \sum_{s \in S} a_{s,i,t} - \beta_i, \quad \forall i \in N, \forall t \in T \tag{3.5}$$

- **Variables:**
  - $\phi_{i,t}$: The amount of aluminium stock at node i during time step t.
  - $a_{s,i,t}$: Amount of aluminium transported by ship s to node i during time span t.
  - β_i: Aluminium demand at node i.
- Updates stock at node i for the next time step based on:
  - Previous stock, incoming aluminium from ships, and demand.

5. Fuel Depot Balance

This constraint tracks the fuel available at each depot over time.

$$\theta_{f,t+1} = \theta_{f,t} - \sum_{s \in S} f_{s,i,t}, \quad \forall f \in F, \forall t \in T \tag{3.6}$$

- **Variables:**
  - $\theta_{f,t}$: The amount of fuel at depot f during time span t.
  - $f_{s,i,t}$: Amount of fuel consumed by ship s at node i during time span t.
- Updates fuel stock at depot f for the next time step by deducting fuel consumed by all ships.

3.4. Model Implementation in Calliope

The model is implemented in the Calliope framework, which supports the configuration of custom constraints and dynamic energy systems. Calliope enables the integration of renewable energy variability and fuel production constraints, ensuring that the fuel costs and aluminium transport schedules are optimised for sustainability. The model structure, along with the transport and energy constraints, is specified across multiple `.yaml` files, each of which defines different aspects of the system. These files establish the nodes, technologies, and custom constraints required for the aluminium transport model and are as follows:

- `model.yaml`: This file contains the overall configuration for the model, specifying the model structure and key settings, including the optimisation objective, time horizon, and overarching parameters that guide the simulation of aluminium transport between Iceland and the Netherlands.
- `scenarios.yml`: This file specifies the start and end date of the model, as well as the timestepresolution.
- `nodes.yaml`: This file defines the nodes in the model. Specifically, Iceland as the aluminium production and fuel supply location and the Netherlands as the aluminium demand destination. The nodes are set up with relevant parameters for renewable energy production, aluminium stock, and demand constraints.
- `techs.yaml`: This file outlines the technologies used in the model, including aluminium production, storage, transport technologies, and fuel supply. Each technology is specified with its operational constraints, capacity limits, and associated costs.
- `custom_constraints_delay.yml`: This file defines the time delay constraints in the model, ensuring that aluminium sent from Iceland at time t only arrives in the Netherlands after a fixed delay. This constraint captures the transit time and enforces realistic timing for aluminium shipments.

- `custom_constraints_state.yml`: This file implements the state transition constraints for the transport ship. It ensures that the ship's location and state (either stationary at a node or travelling) are updated sequentially, allowing for realistic movement between nodes and prohibiting simultaneous presence at multiple locations.
- `custom_constraints_fuel.yml`: This file links the fuel usage to the transported aluminium.

The model is built and executed in Calliope, with the results exported for further analysis. Data processing and visualisation are performed in a Jupyter Notebook, where the model outputs are analysed to interpret transport schedules, fuel consumption patterns, and overall system efficiency. The Jupyter Notebook allows for the application of data processing techniques to extract insights, generate visual representations (such as graphs and charts), and explore the impact of different scenarios on fuel costs and aluminium transport timing. Additional visualisation is performed with the Calligraph tool.

By structuring the model through these configuration files and leveraging Jupyter Notebook and Calligraph for post-processing, the methodology provides a comprehensive, flexible framework for evaluating the aluminium transport model under varying renewable energy availability and demand conditions.

4 Results

In the baseline scenario with direct transmission, aluminium flows smoothly and continuously from Iceland to the Netherlands. The model delivers a steady 20,000 tons per day throughout January, meeting the constant demand in the Netherlands.

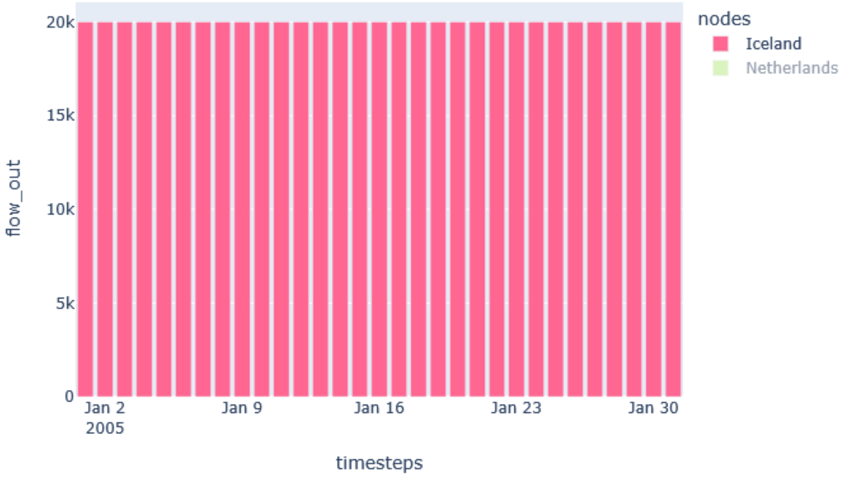

Figure 4.1: Baseline flow.

In the discrete case, there is no aluminium delivered during the initial 72 hours of the month due to the transit delay. This leads to an initial drop in inventory to zero and an inability to meet demand until the first shipment arrives. When the first ship reaches the Netherlands (after 72 hours), it delivers a large batch of aluminium (up to 20,000 tons in a single arrival). This causes a sudden jump in available aluminium inventory.

Under the discrete scenario assumptions, the model scheduled four shipments within the month of January. In practical terms, this corresponds to roughly one ship arrival every 6–9 days. The first three shipments occurred at approximately weekly intervals (every 6 days, aligned with the vessel's minimum 144-hour round-trip cycle time). The fourth shipment was delayed longer (approximately 13 days after the third) because the earlier deliveries had created a buffer of aluminium that allowed a longer gap. This flexible timing illustrates that the model does not simply dispatch the ship at the maximum possible frequency, but rather optimizes the schedule to balance supply and demand. By the end of January, all required aluminium has been delivered with only four trips instead of five, because the final trip carried a partial load (about 7,200 tons instead of the full 20,000 tons capacity).

An explicit output of the model's discrete transport scenario is the fuel consumption associated with shipping. In the baseline scenario, there is no fuel usage attributed to aluminium transport because the model idealizes the link as an efficient, lossless transmission (or it can be seen as having an infinite-efficiency link with no associated fuel). In reality, transporting aluminium via ship incurs fuel costs, which the discrete model captures. Each voyage of the single ship consumes fuel proportional to the mass of aluminium carried. The assumed rate is 0.05 units of fuel per ton of aluminium transported (i.e. 5% of the cargo mass in fuel units, or 1 fuel unit per 20 tons). With a cost of €0.02 per fuel unit, each full 20,000-ton shipment uses about 1,000 fuel units, costing approximately €20.

Over the month, the discrete scenario's optimized schedule of four trips resulted in a total fuel consumption of roughly 3,360 fuel units. In monetary terms this is about €67.2 spent on fuel for January. Broken down by trip: the first three full shipments each used approximately 1,000 fuel units (€20 each), and the final partial shipment (approximately 7,200 tons) used around 360 fuel units (approximately €7.20). These operational costs are extremely small relative to the scale of the commodity being transported (on the order of €0.001 per ton of aluminium). They are almost negligible in the overall system cost, meaning that in this scenario the fuel constraint serves more as a physical realism check (and a potential emissions proxy) than a significant economic burden. The baseline scenario, having no such fuel requirement, would not incur these costs at all. However, it also does not account for any energy or emissions required for transport, which is a limitation that the discrete model addresses.

5 Discussion

Introducing a finite transit time (72 hours) fundamentally changes the system's behaviour compared to the idealized baseline. The baseline scenario, with instantaneous transmission, effectively assumes zero transport delay, which is that aluminium produced in Iceland can appear in the Netherlands at the same moment it's needed. This assumption glosses over any logistical challenges. In the discrete shipping scenario, however, the 72-hour shipping delay means the Netherlands must operate for three days without resupply at the start. The discrete scenario imposed a single-ship constraint (only one vessel available, with a fixed capacity of 20,000 tons per trip). This constraint introduces a hard limit on how much aluminium can be on the move at any given time and how frequently shipments can occur.

If demand were to increase, or if additional constraints were present (for example, limited loading/unloading rates, or required downtime for the ship), a single ship might no longer suffice. The baseline scenario, by contrast, implicitly has unbounded capacity, as it can supply any amount instantaneously. By comparing the two, you can see that the baseline might mask potential bottlenecks.

Another observation from the results is how the single-ship constraint affected scheduling. With only one vessel, the timing of each trip is critical. The vessel cannot be in two places at once, so it must complete a full round trip (72 h outbound, unload, 72 h return) before it can ship again. This enforces a natural minimum interval between deliveries (approximately 6 days). The discussion of shipping frequency noted that the model initially dispatched ships at the minimum interval, but later allowed a longer gap.

Comparing the baseline and discrete scenarios provides insight into the importance of modelling realistic constraints in commodity transport. The baseline scenario, with continuous and unconstrained transport, represents an ideal that maximizes service reliability and simplicity: no matter when aluminium is needed, it is delivered, and there are no losses or delays. In terms of system cost, the baseline might appear optimal since it incurs no transport fuel expense and never fails to meet demand. However, this comes at the expense of realism. The baseline effectively assumes an infrastructure that does not exist (for example, a hypothetical direct aluminium pipeline or unlimited fleet), and thus its insights might be misleading if used for planning. It does not reveal the operational dynamics of shipping such as inventory requirements, shipment scheduling, or potential vulnerabilities (like what happens during a 3-day storm when ships cannot sail).

6 Conclusion

In this report, the idea was to model the transport of aluminium from Iceland to the Netherlands using the Calliope energy system framework, comparing an idealized continuous transport scenario with a realistic discrete shipping scenario. The Results demonstrate clear differences between the two approaches. In the baseline (continuous) scenario, aluminium flow was modelled as an uninterrupted, lossless transmission, with no delays, no need for storage, and no fuel consumption. It effectively treated commodity transport as if it were akin to transmitting electricity over a wire, which is an assumption that, while simplifying the optimization, ignores the realities of physical shipping.

The discrete shipping scenario, on the other hand, introduced key real-world constraints: a single ship with finite capacity, a 72-hour one-way voyage time (resulting in a 6-day round-trip), and fuel requirements for each ton transported. Incorporating these constraints yielded a more complex but more faithful representation of the system. It was found that the Netherlands could still be supplied with the required aluminium, but the pattern of supply was pulsed rather than continuous. Approximately four shipments per month were needed to satisfy the demand, and the timing of these shipments was critical. Notably, the discrete scenario experienced a 3-day supply gap at the start (since the first batch was en route), during which a portion of demand went unmet.

While the model successfully captured the high-level behavior of a maritime aluminium transport system, there are several limitations and opportunities for further refinement. First, the scenario was limited to a one-month period (January 2005) with constant demand. This simplification helped clarify the system's response under steady conditions, but real demand could be variable or growing over time. Future work could involve running the model over a full year (to capture seasonal variations or long-term steady-state behavior) and exploring how the shipping schedule adapts to fluctuating demand (for instance, higher demand in certain weeks or a trend increase).

The single-ship assumption itself is a limitation that could be varied. The number of ships was fixed to one, in order to test the viability of a minimal transport fleet. However, in reality one might consider deploying multiple ships or different ship sizes. A logical next step is to model scenarios with two or more ships, or a mix of ship sizes, to see how that improves supply reliability and what the cost trade-offs are. The model could also be extended to consider ship availability and scheduling in more detail. For instance, adding constraints for layover times, loading/unloading durations, or limiting departures to certain days (perhaps due to port schedules or convoy planning). Such details would make the model more complex but would increase its fidelity for operational planning.

In conclusion, modelling the aluminium transport from Iceland to the Netherlands with Calliope has highlighted the importance of including temporal and capacity constraints in system analysis. The Results and Discussion chapters show that while an idealized model can serve as a theoretical benchmark, a realistic model is essential for practical planning.

References

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A Appendix

A.1. model.yaml

Listing A.1: model.yaml configuration file

```yaml
import:
  - "model_config/techs.yaml"
  - "model_config/locations.yaml"
  - "scenarios.yaml"

config:
  init:
    name: main model
    calliope_version: 0.7.0
    time_subset: ["2005-01-01", "2005-01-31"]
    broadcast_param_data: true

  build:
    ensure_feasibility: true
    mode: plan
    add_math: [custom_constraints_state.yml, custom_constraints_delay.yml]

  solve:
    solver: gurobi
    zero_threshold: 1e-10

parameters:
  objective_cost_weights:
    data: 1
    index: [monetary]
    dims: costs
  bigM: 1e6

data_tables:
  time_varying_parameters:
    data: data_tables/supply_and_demand.csv
    rows: timesteps
    columns: [comment, nodes, techs, parameters]
    drop: comment
  cost_parameters:
    data: data_tables/costs.csv
    rows: techs
    columns: [parameters, comment]
    drop: comment
    add_dims:
      costs: monetary
```

A.2. scenarios.yaml

Listing A.2: custom_constraints_delay.yml configuration file

```
scenarios:
  default:
    timeseries:
      time:
        coordinates:
          - time
        data:
          time:
            interval: 1H  # Time steps are hourly
            start: 2005-01-01  # Start date
            end: 2005-01-31  # End date
```

A.3. locations.yaml

Listing A.3: nodes.yaml configuration file

```
##
# nodes
##

nodes:
  Iceland:
    latitude: 64.1355
    longitude: -21.8954
    techs:
      fuel_supply:
        carrier: fuel
      aluminium_supply:
        carrier: aluminium

  Netherlands:
    latitude: 52.3676
    longitude: 4.9041
    techs:
      aluminium_demand:
        carrier: aluminium
```

A.4. techs.yaml

Listing A.4: nodes.yaml configuration file

```yaml
##
# TECHNOLOGY DEFINITIONS
##

techs:

  ##
  # Supply
  ##

  fuel_supply:
    name: "Fuel Supply"
    color: "#E37A72"
    base_tech: supply
    carrier_out: fuel
    flow_cap_max: .inf

  aluminium_supply:
    name: "Aluminium Supply"
    color: "#F9CF22"
    base_tech: supply
    carrier_out: aluminium
    flow_cap_max: .inf

  ##
  # Demand
  ##

  aluminium_demand:
    name: "Aluminium Demand"
    color: "#072486"
    base_tech: demand
    carrier_in: aluminium

  ##
  # Transmission
  ##

  aluminium_transport_tech:
    link_from: Iceland
    link_to: Netherlands
    name: "Aluminium Transport"
    color: "#8465A9"
    base_tech: transmission
    carrier_in: aluminium
    carrier_out: aluminium
    flow_cap_max: 20000 # Maximum capacity for aluminium transmission
    flow_out_eff: 1  # Transmission efficiency
```

A.5. custom_constraints_fuel.yml

Listing A.5: custom_constraints_delay.yml configuration file

```
constraints:
  fuel_usage_for_transport:
    description: "Link fuel usage to aluminium transport: fuel_in = 0.05 *
        aluminium_transported"
    foreach: [techs, timesteps]
    where: techs=aluminium_transport_tech AND fuel_supply
    equations:
      - expression: >
          flow_out[nodes=Iceland, carriers=fuel] ==
          0.05 * flow_out[nodes=Iceland, carriers=aluminium]
```

A.6. custom_constraints_delay.yml

Listing A.6: custom_constraints_delay.yml configuration file

```
constraints:
  aluminium_transport_delay:
    description: >
      Enforce a 72-hour delay for aluminium transport from Iceland to Netherlands.
      The amount arriving at NL at time t equals the amount sent from Iceland at time t
          - 72.
    foreach: [techs, timesteps]
    where: techs=aluminium_transport_tech
    equations:
      - expression: >
          flow_in[nodes=Netherlands, carriers=aluminium]
          == default_if_empty(
               roll(flow_out[nodes=Iceland, carriers=aluminium], timesteps=72),
               default=0
             )
```

A.7. custom_constraints_state.yml

Listing A.7: custom_constraints_state.yml configuration file

```
variables:
  ship_depart:
    description: "1 if a ship departs with an aluminium shipment from Iceland in this timestep, else 0"
    foreach: [techs, timesteps]
    where: techs=aluminium_transport_tech AND nodes=Iceland
    domain: integer # bound to 0-1 to make it binary
    bounds:
      min: 0
      max: 1

  ship_available:
    description: "Number of ships available at origin (Iceland) at a given timestep"
    foreach: [techs, timesteps]
    where: techs=aluminium_transport_tech AND nodes=Iceland
    domain: integer # non-negative integer
    bounds:
      min: 0
      max: 1 # number of ships, 1 in this case

constraints:
  shipping_capacity_link:
    description: "Allow aluminium flow out of Iceland only when a ship departs (enforce discrete ship usage)"
    foreach: [nodes, techs, timesteps]
    where: techs=aluminium_transport_tech
    equations:
      - expression: flow_out[nodes=Iceland, carriers=aluminium] <= ship_depart * 20000 # Use the max flow of the tech as capacity, 20000 in this case

  ship_roundtrip_cycle:
    description: >
      Ensures the ship returns to origin before another departure can occur.
      ship_available(t) = ship_available(t-1) + (returning ship at t) - (departing ship at t).
    foreach: [techs, timesteps]
    where: techs=aluminium_transport_tech AND nodes=Iceland
    equations:
      - expression: ship_available == $prev_available + $ship_returned - ship_depart
    sub_expressions:
      prev_available:
        # For t=0, set initial number of ships; for t>0, use previous timestep's availability
        - where: timesteps = get_val_at_index(timesteps=0) # first timestep
          expression: "1" # number of ships
        - where: NOT timesteps = get_val_at_index(timesteps=0)
          expression: roll(ship_available, timesteps=1) # ship_available at t-1
      ship_returned:
        # ship_returned at time t equals ship_depart at time t - 72 (a ship that left 72 hours ago returns now)
        - expression: default_if_empty(roll(ship_depart, timesteps=144), default=0)
        # use 2*72 = 144 for the return (round-trip). The roll shifts departures forward by 144h.
```