$$\sum_{i=1}^{n}\sum_{j=1}^{m} lcm(i,j)$$
$$= \sum_{d=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{m}[gcd(i,j)==d]\frac{ij}{d}$$
$$= \sum_{d=1}^{n}\sum_{i=1}^{n/d}\sum_{j=1}^{m/d}[gcd(i,j)==1]ijd$$
$$= \sum_{d=1}^{n} d\sum_{i=1}^{n/d}\sum_{j=1}^{m/d} i*j \sum_{d'|i,d'|j}\mu(d')$$
$$= \sum_{d=1}^{n}\sum_{d'=1}^{n/d}\sum_{i=1}^{n/dd'}\sum_{j=1}^{m/dd'} dijd'^2\mu(d')$$

令 $D=dd'$ $s(x,y)=\frac{xy(x+1)(y+1)}{4}$ 则进一步可得

$$= \sum_{D=1}^{n} s(\tfrac{n}{D},\tfrac{m}{D})D\sum_{d'|D} d'\mu(d')$$