

If there are only even numbers in the set then all subsets sum to even number. There are  $2^N$  subset overall including empty subset. Thus, with empty subset excluded we have

$$X = 2^N - 1$$

If there are odd numbers in the set they must be considered as pairs, fours, etc.: only sums of even number of odd numbers is even. Let the number of even numbers in the set be  $N_e$  and the number of odd numbers be  $N_o$ :  $N_e + N_o = N$ .

We can choose  $C_{N_o}^2$  pairs from  $N_o$  numbers,  $C_{N_o}^4$  fours,  $C_{N_o}^6$  sixes, etc. Overall we can choose

$$\sum_{k=2, k \bmod 2=0}^{N_o} C_{N_o}^k$$

even counts of odd numbers. Thus total number of even subsets is:

$$X = (2^{N_e} - 1) + (2^{N_e} - 1) \cdot \sum_{k=2, k \bmod 2=0}^{N_o} C_{N_o}^k + \sum_{k=2, k \bmod 2=0}^{N_o} C_{N_o}^k$$

Because

$$\sum_{k=0}^N C_N^k = 2^N$$

we have

$$\sum_{k=2, k \bmod 2=0}^{N_o} C_{N_o}^k = \frac{2^{N_o}}{2} - C_{N_o}^0 = 2^{N_o-1} - 1$$

(we must get rid of half of choose terms and  $C_{N_o}^0$ ).

Then

$$\begin{aligned} X &= (2^{N_e} - 1) + (2^{N_e} - 1) \cdot (2^{N_o-1} - 1) + (2^{N_o-1} - 1) = (2^{N_e} - 1) \cdot (1 + 2^{N_o-1} - 1) + 2^{N_o-1} - 1 = \\ &= (2^{N_e} - 1) \cdot 2^{N_o-1} + 2^{N_o-1} - 1 = 2^{N_o-1} \cdot (2^{N_e} - 1 + 1) - 1 = 2^{N_o-1} \cdot 2^{N_e} - 1 = 2^{N_o+N_e-1} - 1 = 2^{N-1} - 1 \end{aligned}$$

As a result we have

$$X = 2^{N-1} - 1.$$