Braylee Cumro

CS 397 HW02

Problem 1

Part a:

Show the following are equivalent

- $(P \to Q) \land (P \to R) \equiv P \to (Q \land R)$

P	Q	R	$P \to Q$	$P \to R$	$(P \to Q) \land (P \to R)$	$Q \land R$	$P \to (Q \land R)$
0	0	0	1	1	1	0	1
0	0	1	1	1	1	0	1
0	1	0	1	1	1	0	1
0	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0
1	0	1	0	1	0	0	0
1	1	0	1	0	0	0	0
1	1	1	1	1	1	1	1
					☆		☆

These are equivalent b/c if we look at the truth table, their truth assignments are the same. The columns for each side of this equivalency are marked w/ a star.

- $(P \land Q) \equiv \neg(P \to \neg Q)$

P	Q	$P \land Q$	$\neg Q$	$P \to \neg Q$	$\neg(P \to \neg Q)$
0	0	0	1	1	0
0	1	0	0	1	0
1	0	0	1	1	0
1	1	1	0	0	1
		☆			☆

These are equivalent expressions. Their truth tables evaluate to the same values and are marked w/ a star on the truth table above.

Part b:

Let A be the proposition that it is sunny outside. Let B be the proposition that we are biking. Let C be the proposition that we are getting ice cream

- Write in English $A \to (B \wedge C)$

If it is sunny outside then we are biking and we are getting ice cream

b/c $\to$ = then + $\wedge$ = and

- Write in English the contrapositive of the previous statement

Contrapositive: If not B then not A

$\neg(B \wedge C) \to \neg A \equiv \neg B \vee \neg C \to \neg A$ By DeMorgan's law

If we are not biking or not getting ice cream then it is not sunny outside

Problem 2

An integer a is said to be odd if there exists an integer b such that $a = 2b+1$.

Using direct proof technique show that if a is odd then a^2 is odd.

Definitions:

odd integer: An integer x is odd if there exists an integer k such that $x = 2k+1$

Claim: The result of an odd integer squared is an odd number proof: Let a be an odd integer

Then there exists an integer m such that

$a = 2m+1$

Then

$$a^2 = (2m+1)^2$$
$$a^2 = (2m+1)(2m+1) \quad \text{[mutiply out]}$$
$$a^2 = 4m^2 + 2m + 2m + 1$$
$$a^2 = 4m^2 + 4m + 1$$
$$a^2 = 2(2m^2 + 2m) + 1$$

Let $k = 2m^2 + 2m$ then $a^2 = 2k+1$

K is an integer

Thus a^2 is odd If a is odd

□

Problem 3:

Integer a is said to be odd if there exists an integer b such that $a = 2b+1$.

Using proof by contradiction show that if n is an integer and $3n+2$ is odd, then n is odd

proof: Assume for contradiction that if n is an integer and $3n+2$ is odd then n is even

If n is even integer then by the definition of an even integer there exists an integer x such that $n = 2x$

Then

$$3n+2 = 3(2x)+2 \quad \text{[substitute 2x for n]}$$
$$= 6x+2$$
$$= 2(3x+1)$$

$3x+1$ is an integer so we can write

$3n+2 = 2(c)$ where c is an integer equal to $3x+1$

Contradiction: By our definition of an even integer, $3n+2$ an even integer. Contradicts assumption $3n+2$ is odd

In our assumption we assumed $3n+2$ to be odd, but we have found it to be even contradicting our assumption
Conclusion: If $3n+2$ is odd n must be odd

$\square$

Problem 4

Assuming that $n \in \mathbb{Z}$ $\forall n \geq 1$, show that the following statement is true using a proof by induction

$$1+3+5+\ldots+(2n-1)=n^2$$

claim: $1+3+5+\ldots+(2n-1)=n^2$. let $p(n)$ be the property that $1+3+5+\ldots+(2n-1)=n^2$

$$\sum_{i=1}^{n} 2i-1$$

$p(n)$ can also be written $\sum_{i=1}^{n} 2n-1 = n^2$

① Base case prove $P(1)$ is true

$$2n-1=n^2$$
$$2(1)-1=1^2$$
$$2-1=1$$
$$1=1$$

② Inductive assumption: Assume $P(n)$ is true for all $n \geq 1$

$$\left(\sum_{i=1}^{n} 2i-1 = n^2\right)$$

③ Inductive step: show that its true for $P(n+1)$

want to show
$\sum_{i=1}^{n+1} 2i-1 = (n+1)^2$

$(n+1)(n+1)$
$n^2+n+n+1$
n^2+2n+1

$$\sum_{i=1}^{n+1} 2i-1 = 2(n+1)-1+\sum_{i=1}^{n} 2i-1$$
$= 2(n+1)-1+n^2$ [by inductive assumption]
$= 2n+2-1+n^2$
$= 2n+1+n^2$
$= (n+1)^2$ [simplify n^2+2n+1]

④ Conclusion:

By mathematical induction $P(n)$ holds for all $n \geq 1$ b/c we showed it works for $P(1)$ and $P(n+1)$.

$\square$

Bonus Problem:

Explain how computers might be applied to proving mathematical statements. What are some limitations we might run into?

Computers might be applied to proving mathematical statements because they can run multiple tests on them quicker then by hand and offer different ways to prove these statements. One way a computer can help with proving statements is by doing proofs by exhaustion much faster than we can do by hand. If a proof has a defined/specific input that can be exhausted a computer can more quickly test all of these options and show wether the claim holds or not. We have seen that different proof techniques work on different claims better then some others. Computers offer us another technique to utilize that may make certain proofs easier. Computers offer the process of loop invariants. It may be easier in some cases to write pseudocode and find a loop invariant to prove a mathematical statement than to use another proof technique. Some limitations we might run into is that there are classes of problems that cannot be solved with a computer or in some cases could be solved but would take years for the solution to be computed. In those cases, using a computer to prove a mathematical statement may be impossible or impractical. As stated before different problems have different proof techniques that work the best, so it wouldn't be surprising to run into some problems where it is impractical to use a computer for this. Another limitation you might run into is complexity. Some algorithms while elegant if you understand them can be difficult to get a grasp on. To prove that the algorithm proves a mathematical statement you have to understand the algorithm and that may be a difficult feat to acomplish depending on your mathematical background.