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hw01 CS397

Problem 1:

- Define the set of even integers b/t 2 and 50 using set-builder notation

$S = \{x \in \mathbb{Z} \mid 2 \text{ divides } x \text{ and } 2 < x < 50\}$

where S is a set

1b.

Page 26 Problem 0.3 parts a, b, c, d, e and f

Let A be the set $\{x, y, z\}$ and B be the set $\{x, y\}$

a. Is A a subset of B?

No

Not all elements of A are also elements of b. Z is an element of A not present in set B

b. Is B a subset of A?

Yes

all elements of set B are also elements of set A

c. What is $A \cup B$?

$A \cup B = \{x, y, z\}$

d. What is $A \cap B$?

$A \cap B = \{x, y\}$

e. What is $A \times B$?

$A \times B = \{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\}$

f. What is the powerset of B?

$P(B) = \{\{\}, \{x\}, \{y\}, \{x, y\}\}$

empty set can also be $\emptyset$

Problem 2

2a. On page 26, Problem 0.6 parts a, b, c, d and e

$X = \{1, 2, 3, 4, 5\}$ $Y = \{6, 7, 8, 9, 10\}$

unary function $f: X \to Y$ and binary function $g: X \times Y \to Y$

described in tables

n	f(n)
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

a. what is the value of $f(2)$?

$f(2) = 7$

b. range + domain of f?

range: outputs domain: input

- The domain will be the values of set X. X is mapped to Y + the input in set X which is $\{1, 2, 3, 4, 5\}$ ∴ domain: X
- The codomain is Y
- the range is 6 and 7. The range are the values actually mapped to / the target space.

c. what is the value of $g(2,10)$?

$g(2,10) = 6$

d. what are the range + domain of g?

range: outputs domain: input

- The domain is the cross product of sets X + Y. $(X \times Y)$
- The range is the values in set Y: 6, 7, 8, 9, 10

e. What is the value of $g(4, f(4))$?

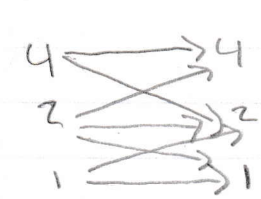
$f(4) = 7$

$g(4,7) = 8$

2b

Page 26, Problem 0.7 Parts a, b, c

For each part give a relation that satisfies the condition



a. Reflexive and symmetric but not transitive

set= $\{4,2,1\}$

Reflexive: if for every x, xRx

Symmetric: if for every x + y, xRy implies yRx

$R = \{(4,4), (4,2), (2,4), (2,2), (1,1), (1,2), (2,1)\}$

Reflexive: $(4,4) \in R$ $(2,2) \in R$ $(1,1) \in R$

- no element x is not paired w/ itself at some point in R

Symmetric: $(4,2)$ + $(2,4) \in R$ $(1,2)$ + $(2,1) \in R$

- when elements x,y do not match the inverse is also in R

NOT Transitive: $(4,2) \in R$ + $(2,1) \in R$ but $(4,1) \notin R$

- In other words $4 = 2$ + $2 = 1$ but $4 \neq 1$ so its not transitive.

b. Reflexive + transitive but not symmetric

set= $\{5,2,7\}$

Transitive: if for every x, y + z, xRy + yRz implies xRz

$R = \{(5,5), (5,7), (5,2), (2,2), (7,7), (2,7)\}$

Reflexive: For every $a \in \{5,2,7\}$ $(a,a) \in R$

Is reflexive b/c $(5,5)$ + $(2,2)$ + $(7,7) \in R$

Transitive: if $(a,b) \in R$ + $(b,c) \in R$ then $(a,c) \in R$

It is transitive b/c $(5,2)$ $(2,7)$ + $(5,7) \in R$

Symetric: if $(a,b) \in R$ then $(b,a) \in R$

It is NOT symmetric b/c $(5,7) \in R$ but $(7,5) \notin R$

+ $(2,7) \in R$ but $(7,2) \notin R$

2b continued

c. Symmetric + transitive but not Reflexive

Set = $\{5, 9\}$

$R = \{(5,5), (5,9), (9,5)\}$

Reflexive: For every $a \in \{5,9\}$ $(a,a) \in R$

- NOT Reflexive: $(9,9) \notin R$

Symmetric: if $(a,b) \in R$ then $(b,a) \in R$

- It is symmetric $(5,9) \in R$ + $(9,5) \in R$

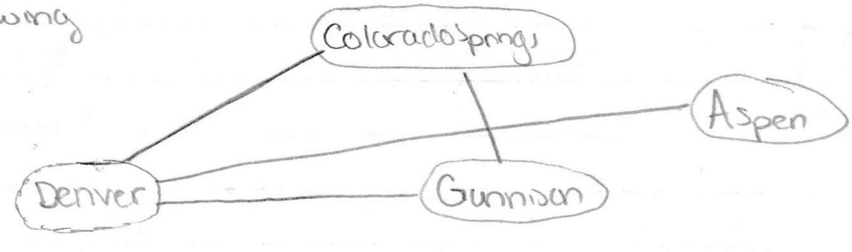
Transitive: if $(a,b) \in R$ + $(b,c) \in R$ then $(a,c) \in R$

- It is Transitive $(5,9) \in R$ + $(9,5) \in R$ + $(5,5) \in R$

Problem 3

Undirected graph G = (V = {Denver, Colorado Springs, Gunnison, Aspen}, E = {(Denver, Colorado Springs), (Gunnison, Denver), (Gunnison, Colorado Springs), (Aspen, Denver)})

1. Drawing



2. Degree of vertex Denver = 3

3. Simple path from Gunnison to Aspen?

Simple path = no repeated vertices

Simple path: Gunnison, Denver, Aspen

4. Does the graph have any cycles? Yes

- A path is a cycle if it starts + ends at the same node
- the cycle here is Denver, Colorado Springs, Gunnison, Denver

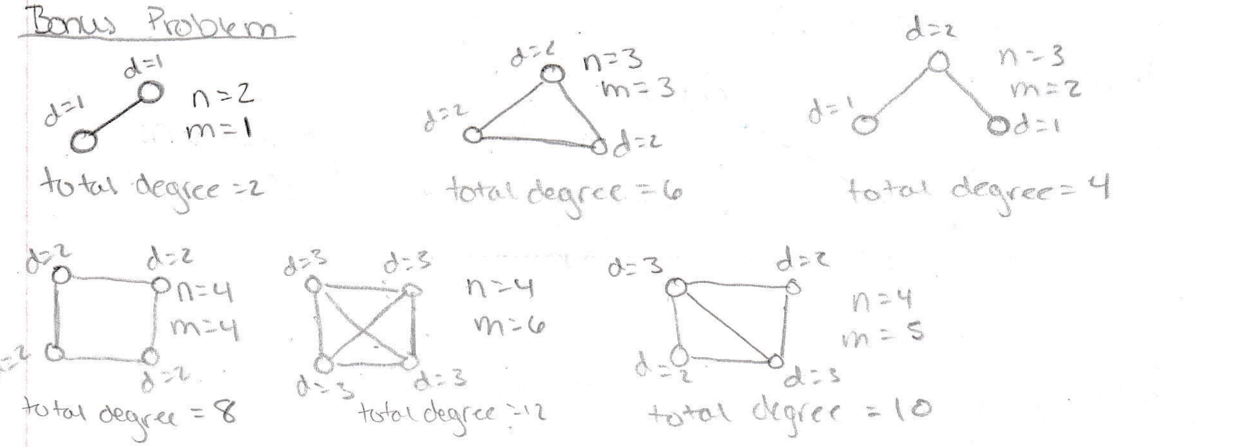
5. Is the graph G' = (V = {Denver, Gunnison}, E = {(Gunnison, Denver)}) a subgraph of G

- Yes
- vertices of G' are subset of vertices in G (G' vertices are in G)
- edges in G' = to edges of G w/ corresponding vertices → G has a edge (Gunnison, Denver) as well

6. Is G a tree?

G is not a tree b/c it has a cycle in it

Bonus Problem



The total degree seems to be equal to 2m. This makes sense intuitively b/c when you add 1 edge you increase the total degree by 2 b/c it increases the degree of 2 vertices by 1. This is easiest to see in image 1. This changes if self loops are allowed.

Problem 4

(a) Lexicographical - standard ordering

- aaaa
- aaaab
- aab
- abc
- azcbc
- b
- cab
- zz

(b) shortlex order - shorter strings before longer

- b
- zz
- aab
- abc
- cab
- aaaa
- aaaab
- azcbc