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FRA333 Robot Kinematics Class Project

SWING UP AND BALANCING OF A REACTION WHEEL INVERTED PENDULUM

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ABSTRACT:
The Reaction Wheel Inverted Pendulum (RWIP) is a system that consists of an inverted pendulum balanced by a flywheel, which is an actuated rotating reaction wheel. This system is used for research and high-level education due to its important challenges in the field of control theory, including robustness, stabilization, and nonlinearities. The objective of this system is to stabilize the inverted pendulum by actuating the reaction wheel and creating torque on the pendulum.

Keywords: Reaction Wheel Inverted Pendulum, Nonlinear Control, Linear Quadratic Regulator (LQR), Multibody Modeling, Balance Control, Swing-Up Control, Balancing Control, DC Motor, Inverted Pendulum, Control Theory, Feedback Control, PID Controller, Kalman Filter.

1. INTRODUCTION

1.1 BACKGROUND

In the realm of control theory, the reaction wheel inverted pendulum (RWIP) stands as an emblematic embodiment of a dynamic and intricate system. At its core, the RWIP amalgamates two seemingly disparate elements: an inverted pendulum and a rotating reaction wheel. The result is a mesmerizing and multifaceted system that has captivated the attention of researchers and educators alike. This introductory section embarks on a journey to uncover the essence of the RWIP, elucidating the fundamental components and objectives that make it a focal point for exploration within the realm of control theory.

The genesis of the RWIP’s significance lies in its ability to pose a multifaceted challenge that extends across both theoretical and practical domains. It epitomizes the precarious balance between stability and instability, as it seeks to stabilize an inherently unstable pendulum by means of precise actuation and control through a rotating reaction wheel. As we delve into the complexities of this system, we will unveil its remarkable capacity to address vital issues in control theory, such as robustness, stabilization, and nonlinear dynamics.

By examining the components and objectives of the RWIP, this thesis aims to provide a comprehensive understanding of the system's intricate nature. Furthermore, it underscores the significance of the RWIP as a versatile tool for both research and education, offering researchers a challenging testbed and students an invaluable educational experience. Through this exploration, we embark on a journey into the world of control theory, where the RWIP represents a captivating microcosm of the intricate dance between precision and instability, offering both challenges and opportunities that continue to inspire researchers and educators alike.

1.2 OBJECTIVE

1. STUDY THE DYNAMICS OF RWIP

Studying the dynamics of RWIP involves delving into the intricate mechanisms and behaviors that govern its operation, providing a comprehensive understanding of its functioning and impact.

2. APPLY DYNAMICS IN CONTROL SYSTEM

Applying dynamics in control systems involves using the principles of dynamic behavior and feedback to regulate and manage various processes and systems, ensuring stability, responsiveness, and desired performance outcomes.

1.3 SCOPE AND LIMITATION

Main purpose of this project is to study and apply the dynamics of RWIP in control system to understanding its functioning and impact.

1) Modelling a RWIP
2) Swing up RWIP
3) Balancing RWIP

2. LITERATURE REVIEW

2.1 MATHEMATICAL MODELING

A key component in studying control systems is mathematical modeling. Through mathematical modeling we can grasp the insight of the system dynamics and system behavior by reducing into a set of mathematical equations. Mathematical modeling usually comes in a form of differential equations derived from the fundamental law of physic such as Newton's Laws of motion, Lagrangian's equation of motion and so on. The suitable mathematical model is the one that compromise between simplicity of the model and the accuracy which model yields. We often need to neglect some of the physical properties in the model to reduce the complexity of the model. This resulted in a less accurate model but most of the time the lessen accuracy is outweighed by an easy model to analyze and design a controller with.

2.1.1 LAGRANGIAN'S EQUATION OF MOTION

The two main methods used in mathematical modeling are Newton's laws of motion and Lagrangian equation of motion for the dynamic mechanical system. Although the result from both methods is the same, the validity of using each method depend on the complexity of the system. At first, Newton's law of motion seems to be simpler than Lagrangian but in a complex system, The calculation will be tiring. Lagrangian equation of motion in general is more complex than Newton's but for complex system, Lagrangian method is far more suitable than Newton's.

First step in Lagrangian method is to define a generalized coordinate system ($q_1, q_2, q_3, \ldots, q_n$) which will determine the degree of freedom in system. qn indicated that this system has n degree of unrestrained motion with respect to reference frame. These coordinates usually are a linear or angular displacement.

After defining the coordinates, Next step is to calculate the kinetic energy and the total potential energy in terms of generalized coordinates then, Computing the total mechanical energy in the system by simply subtract the kinetic and potential energy together since energy is a scalar function. Deriving the expressions for kinetic and potential energy Lagrangian L can be found by expression below.

$$L = T - V$$

Where T is kinetic energy.

V is potential energy.

Since Lagrangian is a function of generalized coordinates and their derivatives the equation of motion will be as shown below

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = \tau_k$$

Where τ_k is generalized force or moment in the direction of q_k.

2.2 STATE SPACE MODELLING

State Space is a mathematical model that represents a physical system as inputs, outputs and states. The general for of State Space model is given by these equations:

$$\dot{X} = AX + BU$$
$$Y = CX + DU$$

Where X is the state vector.

Y is the output vector.

U is the input vector.

A is the state matrix.

B is the input matrix.

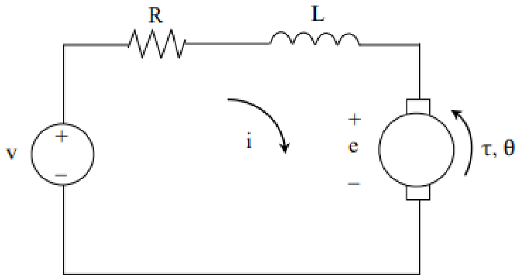
C is the output matrix.

D is the feed-forward matrix.

And the $\dot{X}$ is the first derivative of the state vector.

2.3 MOTOR MODEL

The mathematical model that represents the DC motor’s mechanical system is in terms of motor torque. In this experiment, a DC motor is used to drive the reaction wheel. In DC motors, motor’s torque is directly proportional to the motor’s current. Since we drive the motor using PWM voltage driver and not a current driver, the dynamics of the motor is required to control and approximate the torque.



Generally, the torque (τ) produced by DC motor is proportional to the current in the rotor coil which can be represented by DC motor's electrical-mechanical equation:

$\tau = k_t \cdot i;$ Where k_t is the torque constant of the motor.

The back-EMF (e) is proportional to the angular velocity of the motor's rotor which can be represented by DC motor's electrical equation:

$e = k_e \cdot \dot{\theta};$ Where k_e is the back-EMF constant of the motor.
$\dot{\theta}$ is the motor shaft angular velocity.

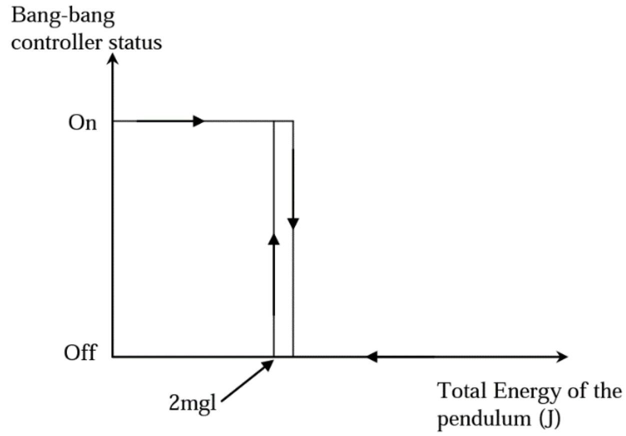
Assuming that the DC motor is 100% efficient, the torque constant k_t and back-EMF constant k_e should be equal.
Using Kirchhoff's voltage law, the motor circuit can be derived to the following expression:

$$v - Ri - L\frac{di}{dt} - k_e\dot{\theta} = 0$$

2.4 SWING UP CONTROLLER (BANG-BANG CONTROLLER)

The swing-up of this system utilized the Bang-bang controller which switched the output signal between the lower bounds and upper bounds of the input signals. With Bang-bang controller we can control the pendulum motion but not the velocity of the pendulum when it reaches the upright position which is crucial for controlling. Because if the angular velocity when the pendulum reaches the upright position is too high the reaction wheel torque might not be enough to stop the pendulum. So, another controller (LQR Controller) must be used at the moment when pendulum almost reaches the upright position to stabilize the pendulum position.

The activation of Bang-bang controller is determined by another energy-based control routine. While the Bang-bang controller increase the pendulum energy the energy-based control routine is used to detect the amount of mechanical energy in the pendulum at each movement instance. Energy controller then compare total energy of pendulum to the desired energy at each instance. If the desired energy is not reached, the Bang-bang controller will be turned on. If not, the Bang-bang controller will be turn off resulting in motor turning off.



2.5 LINEAR QUADRATIC REGULATOR (LQR) CONTROLLER

A mathematical framework for optimizing the control of linear, time-invariant system. It seeks to minimize a cost function, typically expressed as a control input over time. The controller computer a feedback control law based in the current system state to achieve stability and meet control objective.

2.6 PID CONTROLLER

Stand for “Proportional Integral Derivative Controller”. It’s the most commonly used controller in the dynamic control system. The Proportional Integral and Derivative term are summed together to achieve the control output for the system as seen in following expression

$$u(t) = K_p e(t) + K_i \int_0^{\tau} e(\tau)d\tau + K_d \frac{d}{dt} e(t)$$

Where $u(t)$ is the control input signal

K_p is the proportional gain.

K_i is the integral gain.

K_d is the derivative gain

$e(t)$ is the error.

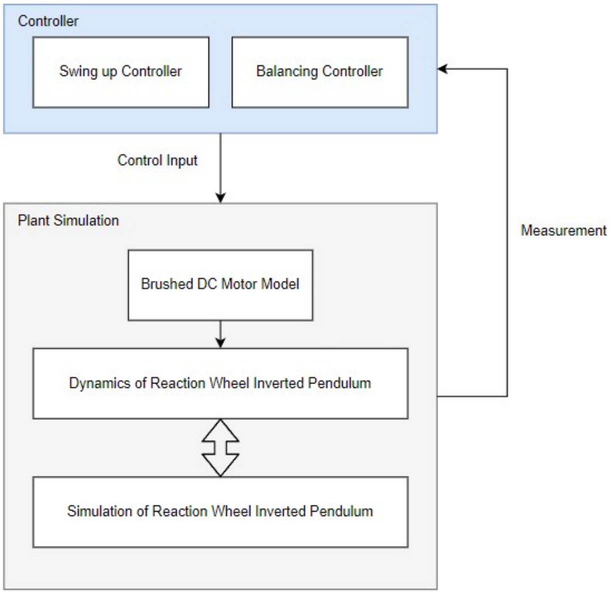
t is the instantaneous time.

τ is the integration variable.

The PID controller will be use to control the pendulum angular velocity alongside with LQR controller to see the difference between output and stability of each controller.

3. METHODOLOGY

In the framework of this project, the block diagram consists of two key components: the controller part, encompassing the essential swing-up controller and balancing controller, and the simulation part, which serves as a critical tool for measurement. The simulation segment is further divided into two elements: the Dynamics of Reaction Wheel Inverter Pendulum and the Simulation of Reaction Wheel Inverter Pendulum. These components collectively provide a comprehensive framework for the analysis and validation of the control strategies.



4. IMPLEMENTATION

5. EXPERIMENT

6. CONCLUSION

7. ACKNOWLEDGEMENT

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Appendix