









## Code

```
from pylab import *
close('all')
# function to calculate numerical derivatives
def numericalDerivative(u):
    dudy = zeros(N)
    for i in range (0,N):
        if i > 0 and i < N-1:
           \mathtt{dym} \; = \; \mathtt{yp} \, [\, \mathtt{i} \! + \! 1] \! - \! \mathtt{yp} \, [\, \mathtt{i} \, ]
           dyp = yp[i]-yp[i-1]
           dudy[i] = u[i+1]*dym/((dym+dyp)*dyp) \setminus
                  + u[i]*(dyp-dym)/(dyp*dym) \
                  - \ \mathtt{u} \, [\, \mathtt{i} \, -1\, ] * \, \mathtt{dyp} \, / \, (\, (\, \mathtt{dym} + \mathtt{dyp} \,) * \, \mathtt{dym} \, )
        elif i > 0:
           \begin{array}{l} {\rm dy} \, = \, {\rm yp} \, [\, {\rm i}\, ] - {\rm yp} \, [\, {\rm i}\, -1] \\ {\rm dudy} \, [\, {\rm i}\, ] \, = \, \left(\, {\rm u} \, [\, {\rm i}\, ] - {\rm u} \, [\, {\rm i}\, -1]\, \right) / {\rm dy} \end{array}
        elif i < N-1:
           dy = yp[i+1]-yp[i]
           dudy[i] = (u[i+1]-u[i])/dy
    return dudy
# Simulation parameters
Re = 2003
nu = 1/0.485e+5
\mathtt{u\_tau} \ = \ 0.41302030 \, \mathtt{e}{-1}
delta = Re*nu/u_tau
kappa = 0.4
prof = loadtxt('profiles/Re2000.prof', comments='%')
bal = loadtxt('balances/Re2000.bal.kbal', comments='%')
yt = prof[:,0]
yp = prof[:,1]
Up = prof[:,2]
\mathtt{up} \, = \, \mathtt{prof} \, [\, \colon \, , 3 \, ]
\mathtt{vp} \, = \, \mathtt{prof} \, [\, \colon , 4\, ]
\mathtt{wp} = \mathtt{prof} \left[:, 5\right]
\mathtt{uvp} = \mathtt{prof} \left[:, 10\right]
uwp = prof[:,11]
vwp = prof[:,12]
\mathtt{epsilon} \, = -\mathtt{bal} \, [\, \colon , 2\, ]
production = bal[:,3]
p_diff = bal[:,5]
t_diff = bal[:,6]
\mathtt{k} \ = \ 0.5 * (\,\mathtt{up} * \mathtt{up} + \mathtt{vp} * \mathtt{vp} + \mathtt{wp} * \mathtt{wp} \,)
N = size(yp)
# Check log profile
figure()
\label{logarithmic'} \begin{array}{ll} \texttt{plot}(\texttt{yp}\,[1:]\,,\; \texttt{log}(\texttt{yp}\,[1:])\,/\texttt{kappa} + 5,\; \texttt{label} = \texttt{'logarithmic'}) \\ \texttt{plot}(\texttt{yp}\,,\; \texttt{Up}\,,\; \texttt{label} = \texttt{'exact'}) \end{array}
legend(loc='best')
xlabel(r'$y^+$')
ylabel(r'$U^+$')
# Check mixing length validity
dUdy = numericalDerivative(Up)
dkdy = numericalDerivative(k)
```

```
l_m = kappa*delta*yp
{\tt l\_exact} \, = \, {\tt sqrt} \, (\, {\tt abs} \, (\, {\tt uvp/dUdy} \, {**2}) \, )
figure()
plot(yp, l_m, label='Prandtl mixing length model')
plot(yp, l_exact, label='exact mixing length')
legend(loc='best')
xlabel(r'\$y^+\$')
ylabel(r'$1_m$')
# Compare ke and exact Reynolds stress predictions
\mathtt{uu\_exact} \, = \, \mathtt{up}\!*\!\mathtt{up}\!-\!2/3\!*\!\mathtt{k}
vv_exact = vp*vp-2/3*k
\mathtt{ww\_exact} \ = \ \mathtt{wp*wp} - 2/3 \mathtt{*k}
uv_exact = uvp
uw_exact = uwp
vw_exact = vwp
nu_t_ke = 0.09*k**2/epsilon
uu_ke = zeros(N)
vv_ke = zeros(N)
ww_ke = zeros(N)
\mathtt{uv\_ke} \, = \, -\mathtt{nu\_t\_ke*dUdy}
uw_ke = zeros(N)
vw_ke = zeros(N)
figure()
plot(yp, uu_exact, label='uu', color='b')
plot(yp, vv_exact, label='vv', color='r')
plot(yp, vv_exact, label='vv', color='g')
plot(yp, uv_exact, label='uv', color='g')
plot(yp, uv_exact, label='uv', color='m')
plot(yp, uw_exact, label='uw', color='c')
plot(yp, vw_exact, label='vw', color='y')
plot(yp, uu_ke, linestyle='-', label='uu', color='b')
plot(yp, vv_ke, linestyle='-', label='vv', color='r')
plot(yp, ww_ke, linestyle='-', label='ww', color='g')
plot(yp, uv_ke, linestyle='-', label='uv', color='m')
plot(yp, uw_ke, linestyle='-', label='uw', color='c')
plot(yp, vw_ke, linestyle='-', label='vw', color='c')
plot(yp, vw_ke, linestyle='-', label='vw', color='y')
legend(loc='best')
xlabel(r'$y^+$')
ylabel(r'\$\ overline\{u_iu_j\}-\ frac\{2\}\{3\}k\ delta_\{ij\}\$')
# ke estimate of the production term
{\tt production\_ke} \ = \ {\tt nu\_t\_ke*dUdy**2}
figure()
{\tt plot}\,(\,{\tt yp}\,,\ {\tt production}\,,\ {\tt label='\,exact'}\,)
{\tt plot}\,(\,{\tt yp}\,,\ {\tt production\_ke}\,,\ {\tt label='\,ke'}\,)
legend(loc='best')
xlabel(r'\$y^+\$')
ylabel(r'turbulent production')
figure()
transport_exact = t_diff + p_diff
transport_ke = numericalDerivative(nu_t_ke*dkdy)
plot(yp, transport_exact, label='exact')
plot(yp, transport_ke, label='ke')
legend(loc='best')
xlabel(r'\$y^+\$')
ylabel(r'turbulent transport')
show()
```