

# Physics 362 Midterm II

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Name:

This is a take home exam. You have one week to complete it. You may use Maxima as needed, and you should use your homework portfolio as a reference. You may not consult with other students.

Question	Points	Score
1	40	
2	20	
3	40	
Total:	100	

1. Consider the operator  $\mathcal{A}$ , described by the relation

$$\mathcal{A}|\psi_0^{\text{SHO}}\rangle = |\psi_1^{\text{ISW}}\rangle.$$

Here  $|\psi_0^{\text{SHO}}\rangle$  is the ground state of the harmonic oscillator, and  $|\psi_1^{\text{ISW}}\rangle$  is the ground state of the infinite square well.

- (a) (12 points) Find the eigenvalues and eigenfunctions of  $\mathcal{A}$ .

- (b) (4 points) Is  $\mathcal{A}$  hermitian?

(c) (12 points) Find the matrix elements of  $\mathcal{A}$  in the SHO basis.

(d) (12 points) Find the matrix elements of  $\mathcal{A}$  in the ISW basis.

2. (a) (5 points) What is the commutator  $[T, V]$  for the harmonic oscillator?
- (b) (5 points) What is the uncertainty relation for  $T$  and  $V$  for the ground state of the harmonic oscillator?
- (c) (10 points) How do you reconcile this uncertainty with the fact that the ground state of the harmonic oscillator is a determinite state for energy?

3. The operator  $a_+ a_-$  is sometimes called the “number operator”.

(a) (10 points) Explain why this name makes sense.

(b) (10 points) What would you get if you mistakenly used  $a_- a_+$  instead?

(c) (10 points) Are  $\hat{H}$  and  $a_+ a_-$  compatible observables? What about  $\hat{x}$  and  $a_+ a_-$ ?

- (d) (10 points) Test the energy-time uncertainty principle for the wave function in problem 2.13 and the operator  $a_+ a_-$ .