

Physics 362 Final Exam

Name:

You may use both your textbook and your homework portfolio as references with this exam.

Question	Points	Score
1	24	
2	28	
3	24	
4	24	
Total:	100	

1. **A One dimensional potential.** Consider the potential

$$V = \begin{cases} \infty & x \leq 0 \\ -\alpha/x & x > 0 \end{cases} . \quad (1)$$

(a) (8 points) Do you expect to find bound states for this potential? What about scattering states? Explain your answer

(b) (8 points) How does this potential differ from the Coulomb potential (the potential found in the hydrogen atom)?

(c) (8 points) What do you expect this difference will do to the eigenfunctions? The energies?

2. **A two state system.** The Hamiltonian for a certain two-level system is

$$\hat{H} = -|1\rangle\langle 1| + \alpha|2\rangle\langle 1| - \alpha|1\rangle\langle 2| + |2\rangle\langle 2| \quad (2)$$

(a) (10 points) Find the eigenvalues of \hat{H} .

(b) (10 points) Find the eigenfunctions of \hat{H} .

(c) (8 points) What is the matrix representing \hat{H} in this basis?

3. **3D SHO, cartesian coordinates.** In three dimensions, the harmonic oscillator potential can be written

$$V = \frac{m\omega^2}{2} (x^2 + y^2 + z^2) . \quad (3)$$

- (a) (8 points) Separate Schrödinger's equation into three ODEs for this potential.

- (b) (8 points) Use work either in your portfolio or textbook to write down the energy eigenvalues (*i.e.*, the allowed energies) and the eigenfunctions for this solution.

- (c) (8 points) Describe the degeneracy of these solutions. How many states share the same energy, and where does the degeneracy come from, physically?

4. **3D SHO, spherical coordinates.** In three dimensions, the harmonic oscillator potential can also be written

$$V = \frac{m\omega^2}{2}r^2. \quad (4)$$

- (a) (8 points) Separate Schrödinger's equation into three ODEs for this potential. (Use spherical coordinates).

- (b) (8 points) Do you expect the same degree of degeneracy in the solutions in spherical coordinates as you observed in cartesian coordinates? Why or why not?

- (c) (8 points) Give one reason you might choose spherical coordinates over cartesian coordinates for the three dimensional harmonic oscillator. Also give one reason you might choose cartesian coordinates over spherical coordinates for this problem.