COMPTON SCATTERING

OBJECTIVES
To test Compton's theory for the scattering of gamma rays by electrons and to determine whether the data are better described by the Thomson or the Klein-Nishina differential cross section.

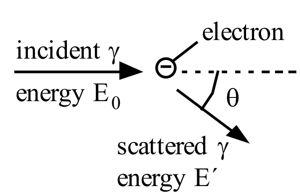
REFERENCES
Any *Modern Physics* textbook; Melissinos, Section 9.2; Preston and Dietz, *The Art of Experimental Physics*.

INTRODUCTION
In the classical theory of light, based on Maxwell's equations, scattered radiation has the same wavelength as the incident radiation. This prediction is obeyed well for radio waves and visible light. Starting about 1912 there were various reports that x-rays – used in x-ray diffraction experiments – changed wavelength when they scattered from electrons. These reports weren't taken very seriously for two reasons:

- These were difficult experiments at the time, with large uncertainties in wavelength measurements, so the experimental evidence was questionable.
- The whole idea was in disagreement with the established electromagnetic theory.

In 1923, the American physicist Arthur Compton published a paper in which he argued that if light is quantized, as Einstein had suggested, then one *should* observe that scattered x-rays have longer wavelengths than the incident x-rays. His argument was based on the idea that if photons have energy, they should also carry momentum and should undergo scattering collisions not unlike a particle. Compton used energy and momentum conservation to derive an equation for the wavelength shift as a function of scattering angle. Then, to prove his point, he carried out careful experiments that verified his claim. He received the Nobel Prize in 1927 for what we now refer to as Compton scattering.

Many physicists in 1923 were still skeptical of Einstein's theory of light quanta. Compton provided the decisive experiment that convinced everyone of the reality of photons. (Light quanta, incidentally, didn't receive the name "photon" until 1926, shortly after Compton's work.) Today, we routinely observe Compton scattering every time we use a gamma ray source in the laboratory.



Compton derived his scattering equation in terms of wavelength, but it is more useful for us – since we measure gamma energies – to describe the *energy* of a scattered gamma. If a gamma photon of energy E_0 scatters off a free electron at angle θ, the scattered photon has a lower energy E',

$$E' = \frac{E_0}{1 + (E_0/mc^2)(1 - \cos\theta)} \tag{1}$$

where mc^2 is the rest energy of the electron, 511 keV. This energy shift of the photon is the **Compton effect**. It is often useful to write this result as

$$\frac{1}{E'} = \frac{1}{E_0} + \frac{1}{mc^2}(1 - \cos\theta) \tag{2}$$

If the energy of Compton scattered photons is measured at various scattering angles, as you will do in this experiment, Eq. 2 can be used to graphically determine the incident gamma energy E_0 and also the electron rest energy mc^2. This is the first objective!

It's also interesting to inquire how the *intensity* of the scattering changes as a function of the scattering angle θ. This is a question with both a classical and a quantum mechanical answer. The number of particles dN_{sc} that reach the detector per second (the net rate) is:

$$dN_{sc} = b\left[\frac{d\sigma}{d\Omega}(\theta)\right],$$

where b is a constant and $d\sigma/d\Omega$ is a function of the scattering angle θ called the **differential cross section**, which tells you how the scattering will change as a function of the scattering angle θ. Is scattering equally likely at all angles? Or is scattering more likely at some angles, less likely at others? The differential cross section, $d\sigma/d\Omega$, will tell you.

Based on a classical derivation, the net rate dN_{sc} is

$$dN_{sc} = b\, r_0^2\left(\frac{1+\cos^2\theta}{2}\right), \tag{3}$$

where $r_0 = 2.82\times10^{-15}$ m is called the **classical electron radius**. You should go over Appendix A for a complete definition of the differential cross section and the derivation of Eq. 3. The material in the Appendix A is important!

This classical derivation is non-relativistic and it does not consider quantum effects. Eq. 3 might fail when the photon energy $E_{\text{photon}} = hf$ becomes comparable to the electron rest mass $E = mc^2$. The same problem – light scattering from an electron – can be solved in quantum electrodynamics (QED). This is the quantum, relativistic version of classical electrodynamics. The differential cross section found from QED is called the **Klein-Nishina formula**. Without derivation, here it is:

$$dN_{sc} = b\, r_0^2\left(\frac{1+\cos^2\theta}{2}\right)\times\frac{1}{\left[1+\gamma(1-\cos\theta)\right]^2}\times\left[1+\frac{\gamma^2(1-\cos\theta)^2}{(1+\cos^2\theta)\left[1+\gamma(1-\cos\theta)\right]}\right] \tag{4}$$

where $\gamma = hf/mc^2$ is the ratio of photon energy to electron rest energy. (It is not the special relativity γ.) Although this looks rather gruesome, notice that the first term is the classical result, Eq. 3, and that the terms in square brackets are each 1 when $\gamma \to 0$. Thus Eq. 4 reduces to the classical limit when $hf << mc^2$. However, Eqs. 3 and 4 do give rather different predictions for the scattered intensity as a function of angle.

You will have an opportunity to graph Eqns. 3 and 4 as part of your analysis. It's straightforward to compute and graph Eq. 4 in a spreadsheet or programming language. You will then be able to judge whether Eq. 3 or Eq. 4, or perhaps neither, best describes your experimental measurements of scattering intensity as a function of angle θ. This is your second objective! In the process, you will have to determine the value of the constant, *b*.

PROCEDURES

CAUTION — The source used in this experiment is a 2-millicurie ^{137}Cs source that emits 662 keV gammas. This is quite strong compared to our usual sources, which are typically 1 μCi. The radiation dose from an unshielded 2-millicurie ^{137}Cs source is about 0.6 milliroentgen/hr at 1 meter. Even though it is well shielded, take care to stay well away from the source except while adjusting the apparatus.

The experimental geometry (Fig. 1) is quite simple. Holes in two lead bricks collimate a beam of gamma radiation. A 1/2-inch diameter aluminum rod is placed in the beam. Nearly all the gammas pass through the aluminum (recall the gamma absorption in aluminum that you measured in Physics 340), but a small fraction scatter from electrons. A scintillation detector measures the energy spectrum of the scattered radiation. You can measure both the energy and the intensity (net counts) of the radiation at various angles.

The practical difficulty is that this is a low count rate experiment (a few counts per second) carried out very close to a strong source and in the presence of "background radiation." This goal can be accomplished, but both the source and the detector require extensive shielding.

The source – too strong for you to handle – will already be in its housing and aligned when you arrive. **CAUTION: Don't move any of the lead bricks around the source.**

Part 1

If the detector is not already at 60°, rotate it to 60°. **Set this angle very accurately.** Then lift off the top lead bricks over the detector. Note three things:

- You are lifting 20 pound bricks over a $1000 detector. Be very careful!
- The front of the detector should be 7 inches from the front edge of the aperture, as shown. In addition, the aperture should be 6 inches from the Al rod. *Check both of these*.
- The aperture width should be 1 inch. There's a spacer you can use to set this accurately.

Based on geometry, and assuming that a lead brick is thick enough to absorb most of the gammas, what determines the “effective width” of the detector: the detector itself, or the aperture? Or both equally? What is the rationale for doing it this way? After answering this question, make any necessary measurements and calculate the solid angle of the detector as seen from the rod. What is the inherent uncertainty in your angle because of your ability to measure the angle and because of the width of the subtended angle? Include this information in your report.

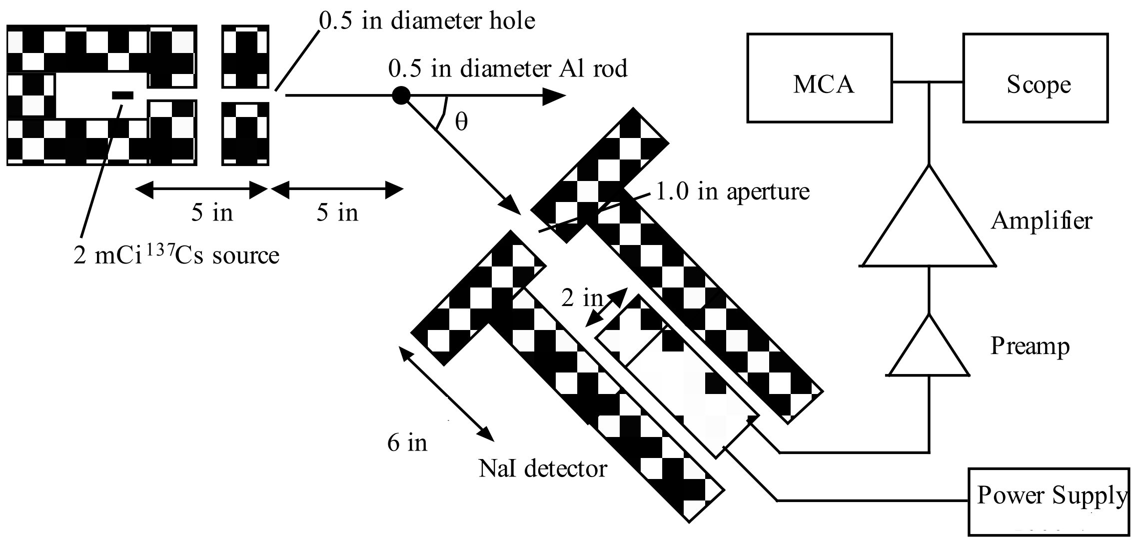


Figure 1. Experimental arrangement for Compton scattering.

Verify that the experiment is wired up as shown, then turn on the detector voltage to **600 V**. Place a weak 1 μCi ^{137}Cs calibration source right next to the detector (after removing the top lead bricks) and adjust the amplifier gain so that most pulses are 5 to 6 V. (A few will saturate the amplifier, but that's OK.)

Turn on the MCA and set the conversion gain to 256 channels (i.e., the full input range 0 - 10 V spread over 256 channels) and the display to show 256 channels. Start recording a ^{137}Cs spectrum. Adjust the amplifier gain to center the photopeak on channel 165 or 166. This is 1/4 the photon energy of 662 keV, so **your "rough calibration" is that E ≈ 4 × channel number.**

Calibration of the detector

In order to determine the energies of the Compton scattered gammas it is necessary to calibrate the detector-amplifier-MCA system very accurately. Some or all of the following will be available. Use your rough calibration to identify the various emissions.

Isotope	Half Life	Gamma energies (keV)
^{133}Ba	11 years	81.0, 302.9, 356.0
^{109}Cd	463 days	88.0
^{57}Co	272 days	122.1

^{22}Na	2.6 years	511.0
^{137}Cs	30 years	661.7
^{54}Mn	313 days	834.8

Calibration is critical. Read the following notes carefully.

- The calibration source needs to be placed along the axis of the detector, so that gammas enter the face, at least one inch away from the face. You can leave the source in the plastic box, but set the source on something (the 1-inch spacer block works well) so that it's roughly the same height as the center of the detector.
- Record a spectrum for ≈10 seconds (using the live time counter is adequate) then check how many counts you got at the center of the peak. It needs to be <1000 counts in 10 s. (A higher count rate artificially shifts the peak to lower channels because the MCA amplifier doesn't fully recover between pulses, so subsequent pulses aren't quite at full amplitude.) Weaker sources will be OK 1 inch from the detector, but stronger sources need to be moved away from the detector until you're under 1000 counts in 10 s.
- Then run long enough to get peaks with *excellent* signal-to-noise ratio. Use the MCA software "Region of Interest" tool. The region choice is critical: for a short discussion on the "Region of Interest" tool, go to Appendix B. Record the Peak centroid, the FWHM, the net rate, and the uncertainty. For this experiment you need to determine the peak positions and widths *very accurately*, to a fraction of a channel.
- The ^{57}Co source has an "extra" peak at lower energy. The identity of this peak is unknown, so you can't use it for calibration. Use only the higher-energy peak.
- ^{133}Ba has a weak gamma emission at 384 keV. It is not resolved from the larger peak at 356 keV, so you won't see it, but it is slightly distorting the peak you see by "pulling" it toward higher energies. When you later get the channel for the center of the 356 keV peak, subtract 1.0 channels to compensate for this.
- Strong peaks with channel numbers <10 are x-rays. These are not used in the calibration.
- Save your calibration spectra. Use file names such as AB_Am and AB_Co, where AB are your initials. Save in the CSV format to open later with Excel. **Do not change any gains or MCA parameters** after you've run the calibration spectrum.
- Using your software of choice (Excel, Matlab, Python) do a fit of energy versus channel number to determine the calibration equation $E = a + bN + cN^2$. Record the fit uncertainties.

Background determination

Move the calibration sources far away. Put the top bricks back over the detector. Remove the aluminum rod, then remove the lead brick from in front of the source. Now the situation is exactly as it will be when you take data except for the rod. Run a spectrum for at least 10 minutes to record your background. Make a note of any peaks or features you see. They could interfere with your data, or you might mistake a background feature for a "real" peak. Consider additional lead shielding if the background has significant peaks or structure above channel 25. Information about the background spectrum should be part of your report. Save your background spectrum.

Compton Scattering

Place the Al rod at the center of the scattering apparatus. *Record the time you start taking data*, then collect a spectrum for long enough to have a **very well-defined peak with very good signal-to-noise**. This may take 15 or 20 minutes; it's not a fast experiment! *Record the time you stop taking data.* Save this spectrum as AB_Cs60, where AB are *your* initials.

Part 1 Analysis: Do this *in lab* on Day 1.

Find the energy <u>and its uncertainty</u> of the photons scattered at 60°, then compare to the expected theoretical value. Think carefully about the error analysis. The software will give you an uncertainty for the channel of the peak center, and you also have uncertainty in both parameters of the calibration equation. How do all these uncertainties combine to give a final uncertainty for the energy of the peak? You'll probably want to consult the Taylor text.

Before proceeding, show your calibration and your analysis of the 60° scattering to your instructor. You don't want to spend 2 days collecting data only to discover that something wasn't right at this stage.

Part 2

Do as much of Part 2 on Day 1 as you can.

Your goals are:

- To test Compton's prediction for how the energy of the scattered gammas varies with scattering angle.
- If Compton's theory is correct, to use your data to determine the mass (in keV) of the electron.
- If Compton's theory is correct, to then determine whether the differential cross section for gamma scattering is better described by the Thomson cross section, the Klein-Nishina cross section, or perhaps neither.

Devise an experimental procedure and an analysis procedure that will allow you to meet these goals. Here's some information that will help your planning:

- Accuracy is important, so you need to collect scattering data long enough to have a peak with a really good signal-to-noise ratio. At small scattering angles, you can record a nice peak in 10 or 15 minutes. At the largest angles, it might take as much as 45 minutes or more. How should you best use the time you have available? **Which angles – large, small, or either – are most useful for meeting your goals? This is very important! You should consult the Melissinos text and plot the theoretical expressions for the Compton effect and the differential cross section to better select the appropriate angles for your experiment.**

- Set the angles very carefully. Even a small angle error can throw your results off. How well do you know the angle? You should include a discussion on angle uncertainty in your report.
- At very small angles (20° or less), extra lead shielding is needed to block radiation coming straight from the source and/or small-angle Compton scattering from the sides of the collimating holes in the lead bricks. Calculate the expected energy of the scattered photons and make sure your peak is in the right channel. **You'll get bad data at small angles if you're not careful!!**
- At all angles, you need to record the time spent collecting each spectrum.
- On Day 2, you'll need to do a new calibration and apply it to Day 2 data. This is because other groups have changed the amplifier since Day 1. **Simply setting the voltages and gains back to where you had them on Day 1 is not sufficient for the accuracy you need.**

ANALYSIS

Then follow the procedure of Day 1 to get the energy of each of the scattering peaks, with an energy uncertainty.

To test whether the classical Thomson cross section or the quantum Klein-Nishina cross section better describe your data, you need to correct for the efficiency of the detector. Gamma photons are detected only if they cause photoionization inside the NaI crystal. Many photons, especially as the energy increases, pass through the detector without being detected.

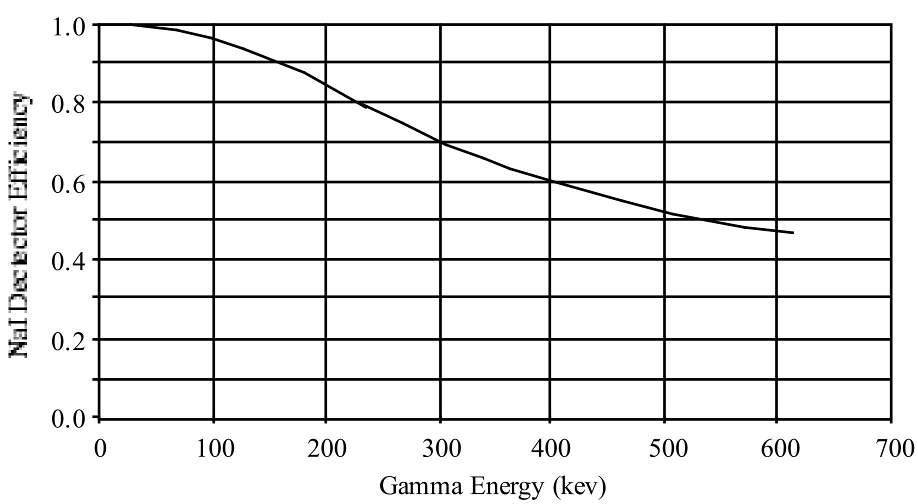


Figure 2. Detection efficiency of the NaI scintillation detector.

The detector's efficiency is defined as

$$\varepsilon = \text{probability of detection} = N_{\text{detected}}/N_{\text{incident}}.$$

Detector efficiency curves are supplied by the manufacturer. Figure 2 shows the efficiency curve for our NaI detector.

You need to compare the *intensities* of the peaks – counts per second – as a function of angle. And "counts" is not the amplitude A of the peak but the total counts associated with the peak – the net counts reported by the MCA software. Remember to correct for detector efficiency and then divide by the data collection time to get a rate (counts per second).

REPORT

Among other things asked for in this write-up, your report should:

- Derive Eq. 1 from the Compton formula for wavelength shift, as found in any modern physics textbook.
- Determine whether your data support Compton's theory, and, if so, determine a value (in keV) for the electron rest mass.
- Determine whether the Thomson or the Klein-Nishina result for the differential cross section – or perhaps neither – best describes your scattering intensity data. You can do this by comparing a graph of your scattering intensity versus scattering angle – with error bars! – to graphs of Eqns. 3 and 4. You want to compare the *shapes* of the curves, not the numerical values. Think about how to best make such a comparison.

Be careful to consider the uncertainty in the angle as well as the detector efficiency in your report!

Note: While the Background section of your report should *summarize* relevant theory, do **not** copy the long derivation from the appendix of this write-up. Copying is a waste of time and paper, and you gain no points simply for proving that you can copy.

APPENDIX A. Scattering Cross Section

Classical light scattering is analyzed in most upper division electricity and magnetism textbooks. We need to begin with the idea of a **scattering cross section**. Consider a glass box with face of area A, as shown in Fig. 3a. There are N_{dust} spherical dust particles in this box, each of radius r. As you face the box, the randomly located dust particles cover an area $A' = N_{\text{dust}}a$, where $a = \pi r^2$ is the *cross sectional area* of each dust particle. We're assuming that the dust particle density is low enough that you can see all of them, without any particles "hiding" behind others.

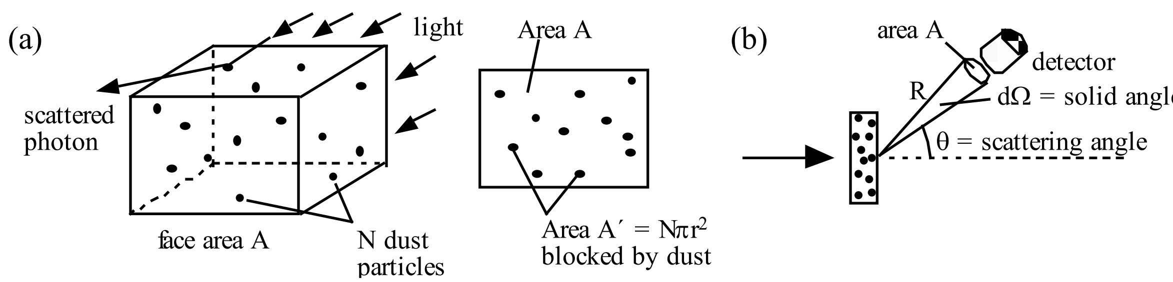


Figure 3. a) If light is incident on a box with dust particles, the probability that a photon hits a dust particle and scatters is $A'/A = N\sigma/A$, where $\sigma = \pi r^2$ is the particle's cross section. b) Experimentally, scattered particles are measured within a small solid angle $d\Omega$ at scattering angle θ. This angle-dependent scattering is described by the differential cross section $d\sigma/d\Omega$.

If there's a light on the other side of the box, the fraction of the light that is blocked by the dust particles and doesn't make it through the box is simply A'/A. Thus the **probability** that a photon will hit a dust particle and scatter to a new direction is

$$P_{\text{sc}} = \frac{A'}{A} = \frac{N_{\text{dust}}a}{A}. \tag{5}$$

Suppose the **flux** of photons (photons incident per second) entering the face of the box is N_0. The flux of scattered photons (photons scattered per second) is then

$$N_{\text{sc}} = P_{\text{sc}}N_0 = \frac{N_0}{A}N_{\text{dust}}a = I_0 N_{\text{dust}}a, \tag{6}$$

where $I_0 = N_0/A$ is the incident **intensity** (photons per second per m^2).

Dust particles are easy because a photon scatters only if it hits the particle, and a dust particle has a "sharp" well-defined edge. Suppose, however, that we shoot subatomic particles, such as photons or electrons, through a target containing other subatomic particles. Interactions between the particles will cause some to scatter. But these are long-range interactions, such as electromagnetic or nuclear forces, so the situation isn't as simple as hitting or missing a dust particle. Nonetheless, we can use the same ideas.

Suppose we have a beam of photons of intensity I_0 (photons per second per m^2) incident on a target material containing N_e electrons. If the flux of scattered photons is N_{sc} (photons scattered per second), then, in analogy with Eq. 6, we *define* the **scattering cross section** σ such that

$$N_{sc} = I_0 N_e \sigma. \tag{7}$$

That is, σ is the **effective area** of each electron for scattering photons. It is *not* the same as the geometric cross section of an electron – which, as far as we know today, is zero because electrons are true "point particles."

Notice that σ can be determined experimentally by measuring the incident intensity and the flux of scattered photons. Alternatively, you can predict the flux of scattered photons if you already know the scattering cross section or if you can calculate the cross section from theory.

Photons are scattered in all directions, but a realistic detector measures the photons scattered in one particular direction. Consider a detector of area A. The detector is placed at distance R from the target at angle θ from the incident photons, as shown in Fig. 3b. The detector will measure the flux of photons scattered at angle θ.

If A is small compared to $4\pi R^2$, then the solid angle spanned by the detector is

$$d\Omega \approx \frac{A}{R^2} \text{ steradians.} \tag{8}$$

Note: If you're not familiar with **solid angle**, now is a good time to learn. The definition $\Omega = A/R^2$ steradians is analogous to the definition $\theta = s/R$ radians for angles in a plane. The maximum value of A is the surface area of a sphere, $4\pi R^2$, so the solid angle of a full sphere is 4π steradians.

The number of particles dN_{sc} that reach the detector is found from Eq. 6:

$$dN_{sc} = I_0 N_e \left[\frac{d\sigma}{d\Omega}(\theta)\right] d\Omega, \tag{9}$$

where $d\sigma/d\Omega$, which is a function of the scattering angle θ, is called the **differential cross section**. The total cross section σ is then found by integrating over all solid angles.

What does classical electromagnetic theory predict for the scattering of photons by electrons? Figure 4a shows an electromagnetic wave with electric field $E = E_0\cos\omega t$ incident on an electron. The electron experiences a force $F = eE$ and accelerates with

$$a = F/m = eE_0\cos(\omega t)/m. \tag{10}$$

That is, the light wave's electric field causes the electron to oscillate. The oscillation axis is the same direction as the incident wave's polarization, and the oscillation frequency matches the wave's frequency.

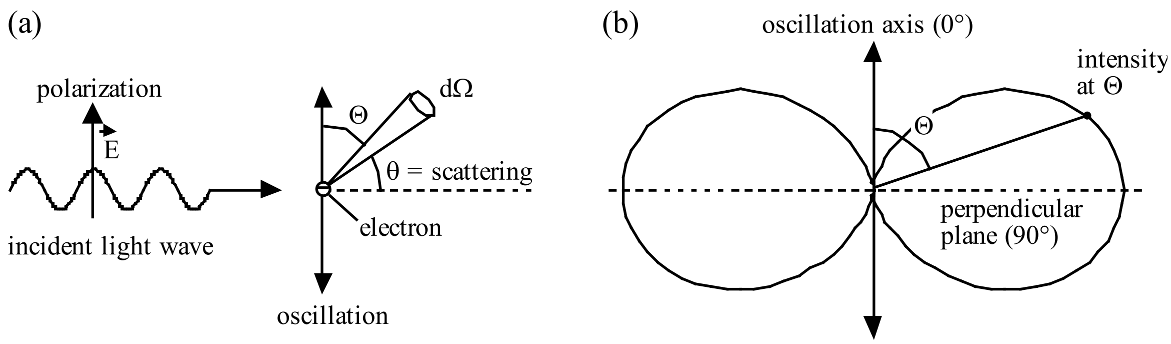


Figure 4. a) The incident light wave causes the electron to oscillate. The electron is then a small antenna and radiates at angle Θ. This is scattered radiation. b) The radiation pattern of an oscillating electron. This is a polar plot – intensity versus angle Θ. Most of the radiation is scattered in the plane perpendicular to the polarization of the incident wave.

An oscillating charge is a little dipole antenna, and if you've had Physics 408-409 you've probably learned that the radiated power of an oscillating charge e is

$$\frac{dP}{d\Omega} = \text{power radiated into } d\Omega = \frac{e^2 a^2}{(4\pi\varepsilon_0)^2 \mu_0 c^5} \sin^2\Theta, \tag{11}$$

where Θ is the angle between the line of oscillation (the same as the polarization direction of the incident wave) and the direction of radiation. The oscillating electron radiates most strongly perpendicular to the line of oscillation (Θ = 90°), and it doesn't radiate at all along the line of oscillation (Θ = 0°). That makes sense, because if you look along the line of oscillation then you don't see the electron moving. Figure 4b shows the radiation pattern of a dipole antenna.

If an electromagnetic wave is incident on a sample of electrons, those electrons start to oscillate. This extracts energy from the wave. The oscillating electrons then radiate that energy in other directions, mostly in the plane perpendicular to the oscillation. In other words, the incident light wave energy is *scattered* by the electrons. Because the electron "antennas" oscillate at the same frequency as the incident wave, the scattered radiation has the same frequency as the incident radiation. That is, classical E&M predicts *no shift* in the wavelength.

We can easily make the transition from waves to photons. The power dP is joules per second radiated into the small solid angle $d\Omega$ at angle Θ. If the light frequency is f, then $dP = dN_{sc}hf$, where dN_{sc} is the number of photons scattered (per second) into solid angle $d\Omega$ at scattering angle Θ. We can rewrite Eq. 11 as

$$dN_{sc} = \text{photons scattered into } d\Omega \text{ per s} = \left[\frac{e^2 a^2}{(4\pi\varepsilon_0)^2 \mu_0 c^5 hf} \sin^2\Theta\right] d\Omega. \tag{12}$$

Comparing Eq. 12 to Eq. 9, you can see that the term is square brackets is essentially the differential cross section for photon scattering. We'll omit the details, but the steps we still need to complete the calculation are:

- Average over 1 cycle of oscillation, since the acceleration a contains a $\cos\omega t$ term.
- Change from angle Θ, measured relative to the incident light polarization, to angle θ, measured relative to the direction of the incident light.
- Average over all possible directions of incident polarization, since we're using an unpolarized light source.

Doing so leads to the *classical* differential cross section for light scattering:

$$\frac{d\sigma}{d\Omega_{\mathrm{T}}} = r_0^2\left(\frac{1+\cos^2\theta}{2}\right), \tag{13}$$

where $r_0 = 2.82\times10^{-15}$ m is called the **classical electron radius**. Equation 13 is called the **Thomson cross section**. When integrated over all angles, the *total* scattering cross section is

$$\sigma_{\mathrm{T}} = \frac{8\pi}{3}r_0^2 = 6.66\times10^{-29}\ \mathrm{m}^2. \tag{14}$$

This is the *effective* area of an electron for scattering electromagnetic radiation. Equations 13 and 14 give very good descriptions of the scattering of radio waves and visible light waves.

APPENDIX B. “Region of Interest” Tool

The region of interest tool fits a line connecting the selected endpoints. It then subtracts the counts below this line as background. Finally, it fits a Gaussian on the remaining counts to determine the centroid and FWHM. It is therefore critical to select appropriate endpoints. Look at the example below. The blue line represents a Gaussian distribution on top of a linear background. In the first case, the endpoints are appropriately selected on the background line. Background subtraction returns the Gaussian distribution. In the second case, the endpoints are selected symmetrically around the peak. After subtracting the background, a distorted and shifted distribution is returned.

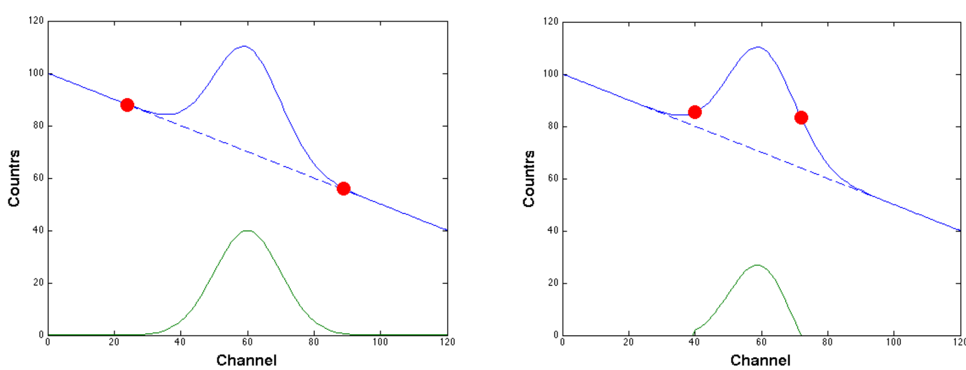


Figure 5. a) Endpoints selected on background line, b) Endpoints selected symmetrically around peak.

The software tool will also return the total and net area. The total area corresponds to the area below the blue curve, whereas the net area corresponds to the area under the green curve, after the background has been subtracted. This is the area of interest for our experiment.