THE FRANCK-HERTZ EXPERIMENT

OBJECTIVES

To obtain evidence for quantized atomic energy levels, to measure the energy of the first excited state in the mercury atom, to determine the mean-free path of electrons in mercury vapor, and to measure the contact potential difference between the cathode/anode in the mercury tube.

REFERENCES

Almost any *Modern Physics* text; Preston, *The Art of Experimental Physics*; Melissinos, Chapter 1 Section 3.

INTRODUCTION

This experiment was performed in 1914 by Franck and Hertz, 1 year after Bohr introduced his model of the atom and 12 years before Schrödinger invented quantum mechanics. The Franck-Hertz experiment provided important evidence that atomic energies are quantized. They were awarded the Nobel Prize in 1925.

In a vacuum, electrons emitted by a heated cathode can accelerate across a gap to an anode that is positive relative to the cathode. The current increases more rapidly than linear as the voltage increases. Figure 1 shows a typical current-voltage curve of a vacuum diode (which is what this device is) when the vacuum inside is good (electron mean free path >> cathode-anode spacing). If the electrons start from rest, then from energy conservation their kinetic energy as they reach the anode is

$$K = eV_{\text{anode}}. \tag{1}$$

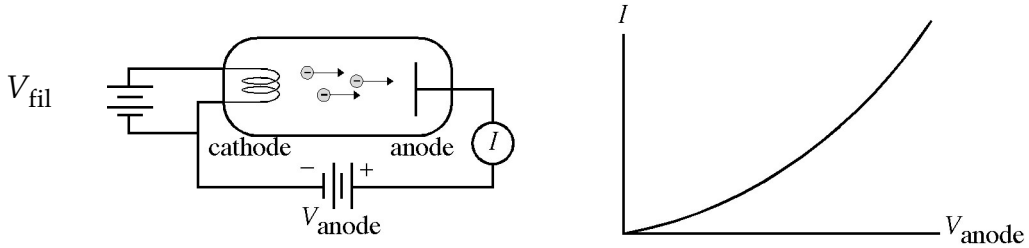

Figure 1. In a good vacuum, the current I increases rapidly as the anode voltage V_{anode} is increased. V_{fil} is the power supply that heats the electron gun filament.

Now suppose the tube contains mercury vapor at a low pressure. The mean free path of an electron in the gas is given by simple kinetic theory as

$$\lambda = \frac{1}{\sqrt{2}\pi (N/V) R^2}, \tag{2}$$

where N/V is the number density of the gas (atoms per m^3) and R is the radius of a mercury atom. If the electrons are moving slowly (small anode voltage), then they undergo *elastic* collisions in which kinetic energy is conserved. If the mean free path is less than the cathode-anode spacing,

then the electrons zig-zag their way to the anode as they repeatedly bounce off mercury atoms. This doesn't significantly change the current-voltage curve. And although their route is now more circuitous, the electrons still reach the anode with $K = eV_{\text{anode}}$.

But when the electrons reach kinetic energies higher than the atomic excitation energies, the collisions can excite the atoms from their ground state to excited states. This is *collisional excitation* of the atoms, an inelastic collision. That is, the electrons lose kinetic energy, the energy being transferred to the atomic electrons.

To observe an inelastic collision we need a *triode* rather than a diode. A triode has a wire-mesh *grid* that accelerates the electrons. The grid's *accelerating voltage* V_{accel} controls the current, just like the anode in a diode, because the accelerating voltage establishes the electric field that accelerates the electrons. Electrons that pass through the grid and reach the anode are measured as current I.

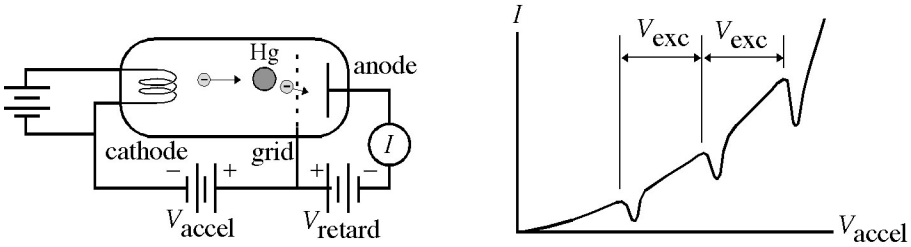

Figure 2. Electrons collide with mercury atoms. If an electron has an inelastic collision near the grid, it doesn't have enough kinetic energy to move uphill against the retarding voltage. These electrons don't reach the anode, and the current drops.

The electron kinetic energy increases as the electrons move toward the grid. As the accelerating voltage is increased, the current I begins to increase just like in a diode. But when the accelerating voltage matches the excitation energy of the first excited state in mercury [V_{accel} (in volts) = E_{exc} (in eV)], the electrons have enough energy to collisionally excite this state *just as they reach the grid* – but not before. An inelastic collision in which the electron gives most or all of its kinetic energy to the mercury atom leaves the electron essentially at rest.

If we now apply a small *retarding voltage* $V_{\text{retard}} \approx 1$ V between the grid and the anode, the electrons, which have just given up their kinetic energy in collisions very near the grid, will not have sufficient kinetic energy to move "uphill" and reach the anode. Consequently, the current collected at the anode will suddenly drop. This is shown in Figure 2.

The *maxima* in the current represent the point where excitation of mercury atoms is just beginning, with excitation from the fastest electrons occurring right at the grid. Increasing the accelerating voltage slightly causes the current to *fall* as the region of excitation begins to expand outward from the grid, allowing more electrons to lose energy in inelastic collisions. Because the electrons have a distribution of speeds, not all the same speed, the *minima* in the curves represents mercury excitation by electrons having the most probable speed – i.e., most electrons

have a speed very near this. *You'll measure the minima* since they most represent "average" electrons, and thus we can then apply statistical ideas about mean free path.

Eventually a voltage is reached at which an electron can have a collision, recoil with $K \approx 0$, and still have enough distance before the grid to re-gain sufficient kinetic energy to make it up the slight potential hill (against the retarding voltage) to the anode. Thus the current "recovers" as V_{accel} is increased further. The small retarding voltage stops only those electrons that have an inelastic collision very near the grid.

The current drops again when the accelerating voltage reaches twice the mercury excitation energy [V_{accel} (in volts) = $2\times E_{\text{exc}}$ (in eV)]. In this case, the electron has enough energy to undergo an inelastic collision when it is halfway to the grid. It starts over, but now can accelerate to where, just as it reaches the grid, it has a second inelastic collision. This second collision, very near the grid, leaves the electron unable to overcome the retarding voltage to reach the anode, so the current drops again.

In fact, you should be able to see that the current will drop every time the accelerating voltage is an integer multiple of the excitation energy of the mercury atoms. When V_{accel} is n times the excitation energy, an electron has n-1 inelastic collisions as it moves toward the grid and an nth collision right at the grid. The electron then cannot make it up the potential hill to the anode, and the current drops. Consequently, the current-voltage curve has an overall rise like the vacuum diode, but it reaches a maximum and then dips every time

$$V_{\text{accel}}\,(\text{in volts}) = n\times E_{\text{exc}}\,(\text{in eV}). \qquad (3)$$

However, the accelerating voltage that you measure with the voltmeter is not the same as the potential difference the electrons experience because of the **contact potential difference** between the grid and cathode, which are made of different materials. Contact potential difference is a subtle but important idea. Consider a cathode with a small work function W_{C} and a grid with a larger work function W_{G}. (Cathodes have small work functions so that it's relatively easy to make them electron emitters.) The work function is the energy difference between the Fermi energy of electrons inside the metal and free electrons outside the metal. If you have not yet done the Photoelectric Effect experiment, go read the work function discussion on pages 1–2 and Figure 1 of Photoelectric Effect.

Figure 3a shows an *isolated* cathode and grid – that is, no electrical contact between them. Because there is no electric field between them, their surfaces must be at the same potential. The energy of an electron just outside the surface of either is zero. Because $W_{\text{C}} < W_{\text{G}}$, electrons near the Fermi level *inside* the cathode have more energy than electrons near the Fermi level inside the grid.

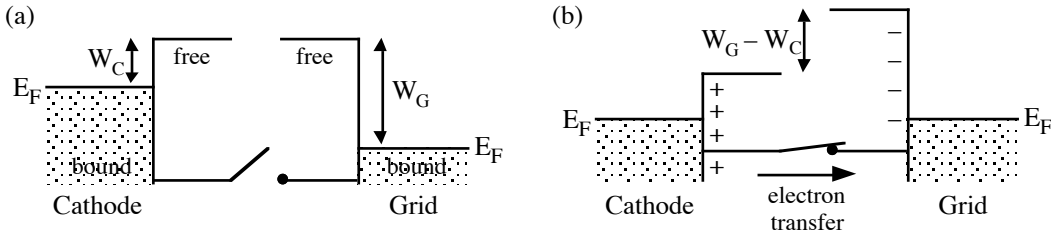

Figure 3. a) Isolated cathode and grid. No electric field means no potential difference between their surfaces. b) When connected, electrons flow from the cathode to the grid until their Fermi energies match. Now the grid is negative relative to the cathode.

Now suppose we connect the cathode and grid with a wire, as in Figure 3b. The more energetic electrons in the cathode will move to the grid, like water seeking a common level between two containers. The electron transfer does two things:

- The cathode potential shifts downward and the grid potential upward until their Fermi levels match. This is equilibrium, with no electrons going either direction.
- The grid, which received electrons, is now negative with respect to the cathode. The potential difference between the *surfaces* of the metals is

$$V_{\text{cpd}} = (W_{\text{G}} - W_{\text{C}})/e \tag{4}$$

This is called the *contact potential difference*. V_{cpd} is defined as a positive number, but the grid is *negative* with respect to the cathode. Be sure you understand why.

The contact potential difference does not affect electric circuits, where the electrons move inside the conductors, but it does affect experiments where the electrons move through free space from one metal to another. Even with no applied voltage between the cathode and grid, an electron wanting to go from the cathode to the grid would have to move "uphill" against the retarding contact potential difference. If an external voltage $V_{\text{accel applied}}$ is applied between the grid and the cathode (this is the voltage you measure with the voltmeter), the *actual* accelerating potential experienced by an electron is

$$V_{\text{accel actual}} = V_{\text{accel applied}} - V_{\text{cpd}}. \tag{5}$$

If the mercury excitation energy is 5 eV and the contact potential difference is 1 V, then the first minimum in Figure 2 will occur not at $V_{\text{accel applied}} = 5$ V, as you might expect, but at $V_{\text{accel applied}} = 6$ V. In other words, **the entire pattern of periodic minima in Figure 2 is shifted rightward by V_{cpd}.**

There's one last piece of the puzzle. Mercury has many excited states. You might expect to see an irregular series of maxima in the current, corresponding to each of the excited states. Instead, you'll find a very periodic series of maxima corresponding only to the first excited state of mercury. The reason is that an electron would need to travel further, to gain more kinetic energy, before it could excite a higher-energy state. The mean free path in the mercury doesn't allow this extra travel distance. Once an electron has sufficient energy to excite the first state, it will very quickly do so and lose its energy. The probability is extremely small that an electron could travel the extra distance without undergoing a collision, so we don't see the excitation of higher energy states.

EQUIPMENT

A triode vacuum tube contains a drop of Hg. By heating the tube in an oven, a relatively high vapor pressure of Hg is produced. The tube has three electrodes: an indirectly heated *cathode*, a perforated *grid* through which the accelerated electrons pass, and an *anode* that collects the electrons. A variable acceleration voltage V_{accel} is applied between the cathode and grid and a small retarding potential V_{retard} is applied between the grid and the anode. Figure 4 shows the electrical arrangement. Our tube, unlike Figure 2, has an indirectly heated cathode rather than a filament.

Note that the voltage you supply and measure is $V_{\text{accel applied}}$ in Equation (5). The potential difference experienced by the electrons is lower by V_{cpd}.

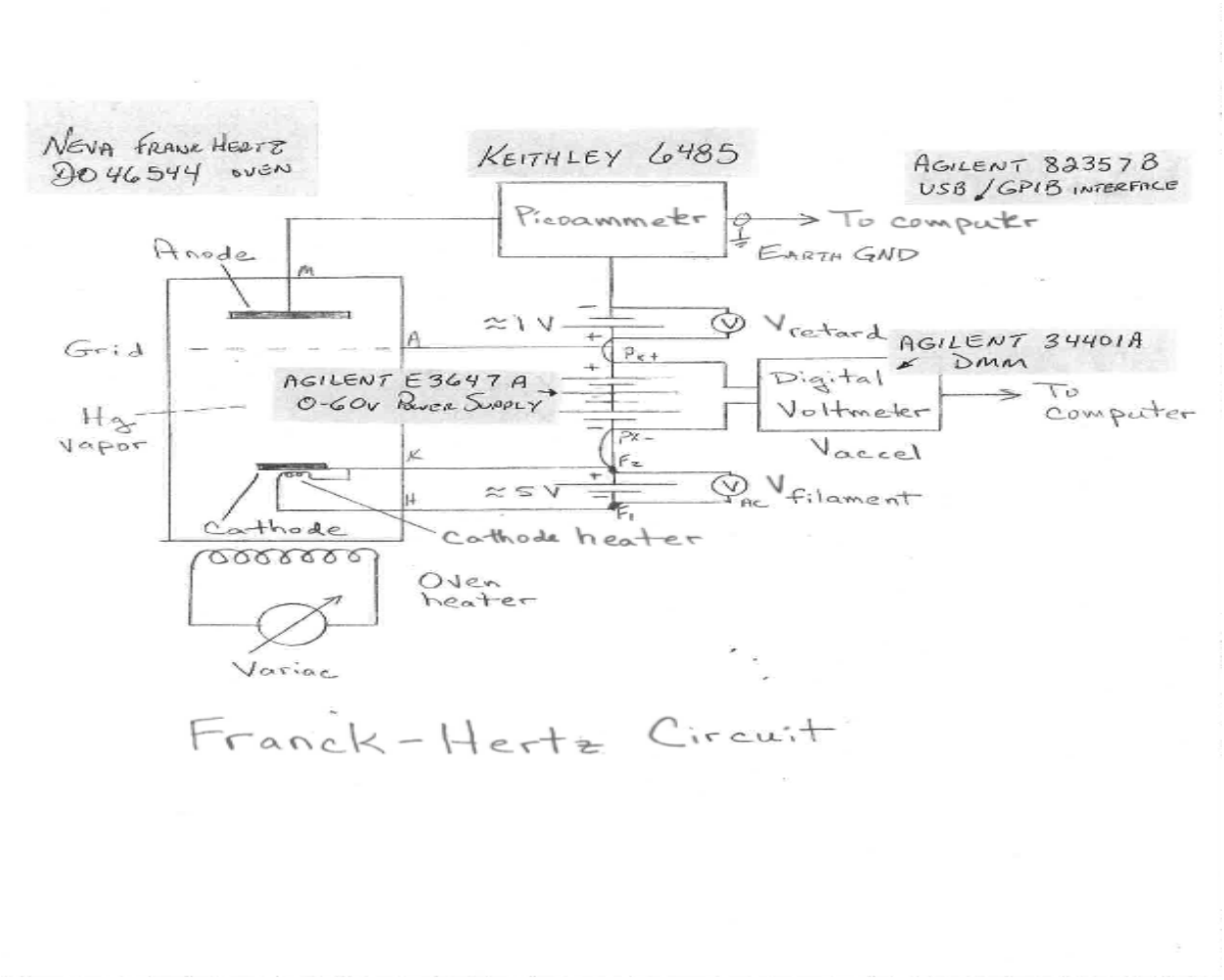

Figure 4. Schematic of the Frank-Hertz experiment.

PROCEDURE

1. Make sure the filament voltage knob, the retarding voltage knob, the voltage knobs on the two power supplies, and the Variac knob are all at zero (full ccw), then turn things on. Set the range switch on the picoammeter (electrometer) to 2 nA.

2. Look through the side window and position the thermometer bulb right at the center of the vacuum tube. Turn on the oven by turning the Variac up to 110 V. **THEN PAY ATTENTION TO THE THERMOMETER!** When the thermometer gets to ≈140°C, turn the Variac down to ≈65 V and then monitor the temperature closely. Adjust the Variac so that the temperature "coasts" up to a temperature between 155°C and 160°C.

3. When the tube reaches 150°C, almost to full temperature:

- Turn on the retarding voltage and set $V_{\text{retard}} = 1.0$ V.
- Turn up the power supply to an accelerating voltage of 36.0 V.
- *Slowly* (over many seconds) increase the filament voltage (this heats the cathode until it begins to emit electrons) until the anode current as read on the picoammeter is ≈ 0.4 nA = 4×10^{-10} A. The exact value isn't critical, just get close. You'll need to make fine adjustments of the filament voltage as the tube settles at its final temperature.
- Wait until current reading is stable and unchanging at ≈0.4 nA (fluctuating no more than ±0.01 nA) and the temperature is stable (changing no more than ±1°C) over a 3 minute period. It may take awhile to stabilize to this degree. While you're waiting:
    - Look through the side window and identify the cathode, the grid, and the anode. If you turn the room lights off, you should barely be able to see the cathode glowing.
    - As best as possible, estimate the cathode-to-grid distance.

4. Now you're ready for a scan.

- Turn the accelerating voltage to zero.
- Start LabView in the 341 folder on the PC. Instructions are at the top of the screen.
- Enter the starting voltage (0 V), the stopping voltage (40 V) and the step size (0.1 V). Click START to start a scan.
- STOP LabView data acquisition.
- Save the data, which can later be opened in Excel. If you want to rerun without saving, click Cancel and then tell LabView to Continue. Press START to start another run. (You can get a picture/screenshot by right clicking in the graph and using "Copy Data." At bottom you can export to Excel.)

5. Your scan should clearly demonstrate the Franck-Hertz effect. Run a scan over a 3 V wide window around each minimum point with a step size of 0.01 V. Every time you adjust the start/stop voltage values, click the tick mark ("Enter text") that appears on the top left of your LabView window before starting the scan.

Load the data in Matlab and plot them. Using the data cursor, you should now be able to determine the minimum points with an uncertainty of around 0.05 V.
Do you see a light emission in the tube when the current dips? Where and why?

6. Return the accelerating voltage to 36.0 V so that you can monitor current, then increase the temperature to 185–190°C (remember to return the picoammeter and digital multimeter to "local" control). The current will drop as the mercury density increases. Adjust the filament voltage to try keeping the current at 0.050 nA, although you may reach maximum filament voltage first. When temperature and current are stable, repeat steps 4 and 5. Then turn all knobs to zero and turn things off.

ANALYSIS AND REPORT

Part I:

- Convert all currents to pA, then make graphs of current versus accelerating voltage. These are for display, not for measurement.
- The total energy gained *in eV* by the electron in moving from cathode to grid is numerically the same as the acceleration voltage V in volts. That is, V = 15 V means the electrons gain kinetic energy K = 15 eV. Using your voltages of the minima, make a table of the energy gains K_n (in eV) of the electrons at these minima, starting from n = 1 for the first minimum at ≈7.5 V.
- For each of the two high temperatures, use *all* of the energies in your data table to determine (a) the energy of the first excited state in mercury and (b) the contact potential difference between the cathode and grid. Using a graph would be good. Compare your results to each other (should the results change with temperature?) and with the known mercury energy level of 4.89 eV.
- Estimate the mercury vapor pressure at the temperatures of your two runs and also at room temperature (20°C) using $P = 8.7 \cdot 10^{(9-(3110/T))}$ in Pascals, where T is in Kelvin. Use the ideal gas law to determine the number density of mercury atoms, then use Eq. (2) of this write-up to calculate the mean free paths of electrons through the mercury vapor. Use R = 0.18 nm as the radius of mercury atoms. Compare the mean free paths to each other and to the cathode-grid distance in the tube. Also estimate the number of collisions an electron has while moving from cathode to grid.
- In your two curves, what does and what does not change with temperature? Why?
- Explain how your experimental results lead to the conclusion that atomic energy levels are quantized. In particular, how would the experimental results have been different if atoms did not have quantized energy levels – i.e., the non-quantized Rutherford model of the atom. Why do we only see the first excited state?

Part II:

- There are reasons to think that the spacing between minima should not be constant, as our simple theory predicts, but should increase with n. Consider an electron that excites a mercury atom. The energy of the electron before the inelastic collision is *at least*, but not

necessarily equal to E_{exc}. An electron gains energy $\Delta E = eV_{acc}\lambda / L$ between collisions, where L is the anode-cathode distance. Therefore, as the accelerating voltage increases, the energy gained between collisions increases as well. This idea is presented in more detail in "New features of the Franck-Hertz experiment", G. Rapior, K. Sengstock, V. Baeva, Am. J. Phys. 74, 423 (2006). In this paper, available on PolyLearn, an expression for the spacing between two minima is derived:

$$\Delta E(n) = \left[1 + \frac{\lambda}{L}(2n-1)\right] E_{exc}\,, \tag{6}$$

- Test the validity of this expression using your data at each temperature. Fit your data for each temperature setting to the function above. How does the value for E_{exc} from your fit compare to the accepted value?