THE FUNDAMENTAL UNIT OF CHARGE (Millikan's oil drop)

Objectives:

In this experiment, small, charged oil drops are levitated in an electric field. You can determine the value of the fundamental unit of charge by measuring the speeds of the drops as they fall at terminal velocity in free fall and in electric fields in different directions.

Background:

1) Review the Millikan oil drop experiment in a Modern Physics text and Melissinos pages 1-10.
2) Error propagation/mean/standard deviation from Chapters 3 and 4 of Taylor.

Theory and Apparatus:

J. J. Thomson measured the electron e/m in 1897. He *assumed* that the electron mass is much smaller than the mass of atoms – and hence that the electron is a **subatomic particle** – but he had no direct evidence. An obvious challenge was to measure either e or m directly. Thomson's students attempted to do this with water droplets, which were known to often have a small charge, but it was the American physicist Robert Millikan, a decade later, who developed a technique for measuring the charge with great precision.

It may seem "obvious" to us today that all electrons have the same charge, but that was not at all clear in 1900. It was thought by many that electrons would have a *range* of charges, with e being simply an average. One of the most important aspects of Millikan's experiment was the discovery of a *fundamental unit of charge*. That is, charge is **quantized**, and all objects have a charge that is an integer multiple of *e*.

The electron charge e is called a fundamental constant of nature, along with other constants such as c, h, m_{elec}, and others. An important task of experimental physics during the last century has been to determine the values of the fundamental constants with the highest possible accuracy. Most are now know to 6 or more significant figures. The currently accepted value of *e* is

$$e = (1.6021773 \pm 0.0000005) \times 10^{-19} \text{ C.}$$

This experiment will give you a chance to measure a fundamental constant. With care, you can achieve an accuracy of close to ±1%, or about ±0.02×10⁻¹⁹ C. *It is important*, to reach this level of

accuracy, to do this experiment slowly and carefully. It will take a bit of practice to learn how to generate and then measure the motion of the small drops. If you rush to start measuring the first drops you see, your results probably won't be very good. But if you spend some time studying the apparatus and practicing the procedures, then you'll be able to generate quality data and obtain a respectable result.

When a lightweight oil is sprayed through a very small nozzle, some of the oil drops become slightly charged due to friction. Both positive and negative charges are observed, and you'll find that most drops have $|q|$ = few $\times e$, where "few" may range from 2 or 3 to 10 or more. Both types of drops are very small, with diameters (≈ 1 μm) too small to measure accurately even with a high quality microscope.
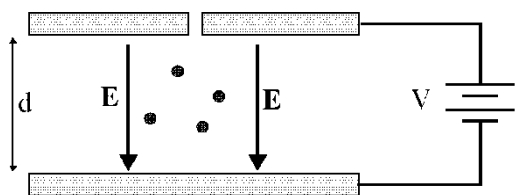
The figure shows a few drops between two electrodes with separation d and voltage V. To begin, assume $V = 0$. Thus the electric field $E = 0$. The drop falls, and a drop this small reaches its terminal velocity within a few milliseconds. It then falls with a *constant* fall velocity v_f. For slow velocities, it is well established that the aerodynamic drag force is proportional to v_f: $F_{drag} = kv_f$. In dynamic equilibrium,

$$F_{net} = kv_f - mg = 0 \tag{1}$$

where m is the mass of the *drop*, not the electron mass.

Now turn on a voltage V with polarity such that it exerts an upward force on the drop (positive V for negative drops and vice versa). The drop now moves at *constant* upward speed v_u, and the drag force is now exerted downward. In this case,

$$F_{net} = qE - kv_u - mg = 0 \tag{2}$$

where q is the drop's charge. We've written the equations so that really $q = |q|$ and the v's are speeds, not velocities. So all quantities are positive. It's not hard to eliminate the constant k from Eqs. (1) and (2) to get

$$q_u = \frac{mg}{E}\left(\frac{v_u}{v_f} + 1\right) \tag{3}$$
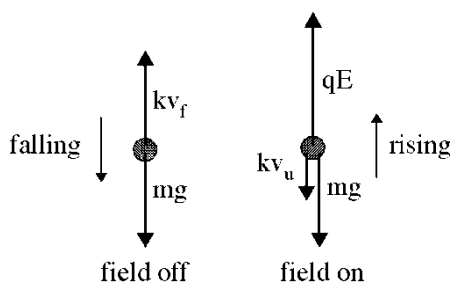
where the subscript q_u indicates a measurement when the electric field drives the drop *up*.

Alternatively, a reversed V can drive the drop downward even faster than free-fall, at the *down* speed v_d. You'll later show that this measurement leads to

$$q_d = \frac{mg}{E}\left(\frac{v_d}{v_f} - 1\right) \tag{4}$$

Note that these expressions do not require a knowledge of the electron mass. Neither do they make any assumption about the nature of q. We may find q to be either discrete or continuous, and that different drops have either the same or different amounts of charge. We'll let the experiment tell us. The experimental challenge is to determine m, E, and various velocities with high precision.

The electric field is $E = V/d$. We'll measure both V and d to better than 1% accuracy, so the uncertainty δE will be insignificant. The local value of g can be taken as 9.80 m/s^2, with negligible uncertainty.

We'll measure velocities by using a stopwatch to time how long it takes the drop to move a known distance h, then calculating $v = h/t$ with $h = 0.50$ mm. Although the stopwatch is good to 0.01 s, there are two sources of possible uncertainty. First, your reaction time in clicking the stopwatch will vary from drop to drop. However, we'll assume that this pretty well averages out over several repeated measurements. More significant is an *inherent* variation in the time t due to the Brownian motion of such a tiny drop. You'll find that a single drop can vary by as much as 1 s, or even a bit more, in a 10 s fall. Consequently, it takes many measurements to obtain a good average and a careful use of error analysis to track the uncertainties.

The mass of a drop can be obtained from its radius r and its density ρ by
$$m = \tfrac{4}{3}\pi r^3 \rho . \tag{5}$$
But r is much too small to measure through the telescope. Instead, we'll use Stoke's law, which tells us that the terminal velocity of a slowly moving drop is
$$v_f = \frac{2g\rho r^2}{9\eta},$$
where η is the viscosity of the medium. By measuring v_f, we can indirectly find the drop radius
$$r = \sqrt{\frac{9\eta v_f}{2g\rho}} \tag{6}$$
The viscosity of air depends on the temperature, but it is known and tabulated and can be well approximated by the empirical formula
$$\eta = \left[1.800 + \frac{T(\text{in }^\circ\text{C}) - 15.0^\circ\text{C}}{209}\right] \times 10^{-5}\,\frac{\text{Ns}}{\text{m}^2} . \tag{7}$$

You'll need to calculate η to three decimal places.

The drop's density is a given quantity. Strictly speaking, the air has a small buoyancy (Archimede's principle), and every ρ really should be $\rho_{\text{drop}} - \rho_{\text{air}}$. The air density depends on both temperature

and pressure. However, the air density is only $\approx 0.1\%$ of the drop's density, so an adequate approximation is to simply decrease ρ_{drop} by 1 kg/m³. This has already been done, so you don't have to. With this slight correction

$$\rho_{oil} = 885 \text{ kg/m}^3$$

There's still one more issue. Millikan found that the value of e seemed to have a slight dependence on the drop size. Smaller drops yielded a slightly larger value of e. Millikan eventually realized that he was seeing a breakdown of Stoke's law for the drag force. Stoke's law assumes that the particle moves in a perfectly continuous medium. But when the drop's diameter is $< 10^{-6}$ m, the drop size is approaching the mean free path of molecules in the air ($\lambda \approx 10^{-7}$ m at STP). At these distances the air appears granular rather than continuous. By clever measurements, Millikan was able to show that Stoke's law could be modified by a <u>correction factor</u>. The value of q given in Eq. (3) or (4) has to be multiplied by the correction factor

$$\gamma = \left(\frac{1}{1+b/r}\right)^{3/2} \tag{8}$$

where b is a constant and r is the drop radius. Although b depends on pressure and temperature, the value

$$b = 0.82 \times 10^{-7} \text{ m}$$

will be sufficiently accurate for our experiments at $p = 1$ atm and near $T \approx 22°C$.

The correction factor of Eq. (8) fails if r gets to be much less than $\approx 10\lambda$. In that case, the correction factor would need higher order terms. Under our conditions, we need to restrict our measurements to drops with $r > 5 \times 10^{-7}$ m, and preferably $> 7 \times 10^{-7}$ m.

Thus the basic procedure will be to measure v_f and either v_u or v_d. From v_f, Eq. (6) can be used to compute the drop radius. Then Eq. (5) can be used to compute the mass. Then the corrected versions of Eqs. (3) and (4) give the charge as

$$q_u = \gamma \cdot \frac{mg}{E}\left(\frac{v_u}{v_f}+1\right) \tag{9}$$

or

$$q_d = \gamma \cdot \frac{mg}{E}\left(\frac{v_d}{v_f}-1\right) \tag{10}$$

where γ is the correction factor of Eq. (8). (It is *not* the special relativity γ.)

There are several steps in the process, so the issue of propagation of errors becomes important. You will later show that the uncertainties in q_u and q_d are given by

$$\delta q_{\mathrm{u}} = q_{\mathrm{u}} \cdot \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta v_{\mathrm{u}}}{v_{\mathrm{u}} + v_{\mathrm{f}}}\right)^2 + \left(\frac{v_{\mathrm{u}}}{v_{\mathrm{f}}} \cdot \frac{\delta v_{\mathrm{f}}}{v_{\mathrm{u}} + v_{\mathrm{f}}}\right)^2}$$

$$\delta q_{\mathrm{d}} = q_{\mathrm{d}} \cdot \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta v_{\mathrm{d}}}{v_{\mathrm{d}} - v_{\mathrm{f}}}\right)^2 + \left(\frac{v_{\mathrm{d}}}{v_{\mathrm{f}}} \cdot \frac{\delta v_{\mathrm{f}}}{v_{\mathrm{d}} - v_{\mathrm{f}}}\right)^2}.$$

(11)

At the end, your result is $q_{\mathrm{u}} \pm \delta q_{\mathrm{u}}$ or $q_{\mathrm{d}} \pm \delta q_{\mathrm{d}}$. If you make both measurements on the same drops, the error bars for q_{u} and q_{d} should overlap about two-thirds of the time.

Procedure:
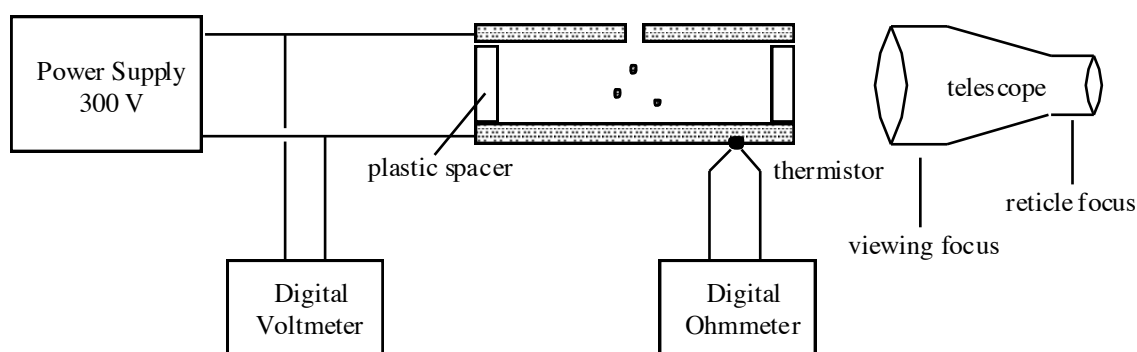
1. Make sure the power supply is turned off. Remove the plastic lid and the droplet hole cover, which is the black piece sitting on the upper electrode. CAREFULLY remove the cylindrical shield by pulling it straight up. Lift off the upper electrode and place it UPSIDE DOWN (experimental side up), out of the way. This is a polished surface, and it's very important not to scratch it. Lift off the plastic spacer, and use micrometers to measure its thickness at 3 or 4 points. Be sure to measure the main part of the spacer, not the slightly raised rim. Make each measurement to ±0.01 mm, then average and record your results. This is d. There should be almost no variation in your measurements, and your average will be so good compared to other measurements that its uncertainty is irrelevant.

While the spacer is off, notice the two things sticking through the lower plate. One is a spring-loaded wire that brings voltage to the upper electrode when it's replaced. The other is a very weak alpha emitter (thorium) that will be used to change the charge on the drops. Move the ionization source lever back and forth, noticing that the source points toward the center of the electrodes when the label says ON and is turned away when the label says OFF.

Replace the plastic spacer, making sure it's seated level. Place the upper electrode back on, giving it a bit of a wiggle to make sure it is also level and resting firmly against the spacer. Push the shield back down on the pins, but DO NOT replace the droplet hole cover.

Finally, adjust the feet of the apparatus by screwing them in and out until the bubble in the bubble level is exactly centered. The oil drops will drift sideways out of your field of view if the apparatus isn't perfectly level.

2. Just to the right of the cylindrical shield is a black knob that looks like the droplet hole cover. This is a focusing wire. Unscrew it *carefully*, then *carefully* insert the wire through the hole in the top electrode, where the droplet hole cover normally goes.
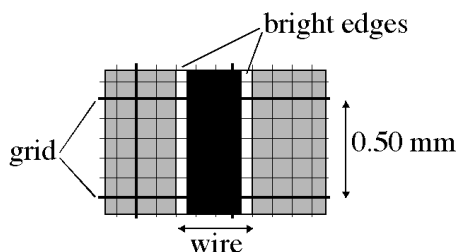
Turn on the illumination lamp, then look through the eyepiece. You should see two parallel vertical stripes of light and a grid of lines. The stripes of light are the two edges of the focusing wire, being lit from behind. First, turn the focusing ring on the *eyepiece* of the telescope to get the best possible focus for the grid. These lines are 0.10 mm apart, and you'll notice that they're grouped into 0.50 mm "divisions." Next, turn the focusing ring near the input end of the telescope to get the best focus for the wire.

You'll see two screws on the lamp housing. The screw sticking out by the leveling bubble moves the light source horizontally. Make small adjustments until the right side of the wire is slightly brighter than the left and you have the best contrast between the lighted edges and the dark center. The screw on the top moves the light source vertically. Make small adjustments until the brightest part of the wire is in the center of the grid.

Remove the focusing needle, carefully screw it back into its holder, replace the droplet hole cover, and then replace the plastic lid.

3. Turn on the power supply and the two digital meters. Turn the 100-volt knob up to 300 V and read the voltmeter. It may differ just slightly from 300 V, this is OK but be sure to record the actual voltage. Use this voltage for all measurements.

The other meter is set up to read resistance and is connected to a "thermistor" in the bottom electrode. A thermistor is a device whose resistance varies in a known way with temperature and that can be used as a thermometer. You'll see a chart on the top of the apparatus (also at end of this

write up) that converts resistance measurements to temperature. The temperature inside will be slightly warmer than the room temperature due to heating of the enclosed air by the light. And the temperature may rise during the experiment, so check the temperature for every drop you measure.

4. Pick up the oil sprayer and make a couple of practice squeezes over your hand. You want a quick but not too powerful squeeze that produces a fine oil mist. Don't oversqueeze, which will release way too much oil.

The procedure for loading oil drops is as follows.
*   Make sure the power supply is at 300 V. Put the plate charging switch in its middle position. This internally disconnects the power supply and shorts the electrodes together to give V = 0 and no field.
*   Turn the ionization source lever to its middle position. This opens a vent hole so that air can flow through the chamber.
*   Place the nozzle in the hole in the plastic lid. Give the bulb a quick squeeze. This fills the upper chamber, above the electrodes, with an oil mist.
*   Now, without removing the sprayer, give the bulb a <u>slow</u> steady squeeze. This will overpressurize the upper chamber and force some of the drops through a small hole in the droplet hole cover. Sight through the telescope during or immediately after this squeeze. You should see a cloud of drops appear. Some will be bright, others not well focused, and all will be slowly falling. An ideal situation would be 20 or 30 drops, but somewhat more is OK. The situation to avoid is so many drops that you can't focus your attention on just one.
*   Turn the ionization source lever to the OFF position. This closes the vent hole to prevent drafts.
*   Begin searching for a drop that
    i) falls one full division of 5 lines (0.5 mm) in less than 15 s,
    ii) you can drive back up by turning the plate charging switch to either + or –, and
    iii) whose rise time for 0.5 mm is >3 s.
    Fall times over 15 s indicate a drop too small for the correction factor of Eq. (8) to work. And a rise time <3 s indicates that the drop is too highly charged to obtain useful results. It doesn't matter whether you use + or – voltage to drive the drop back up since we're not going to worry about the sign of the charge.
*   If need be, turn the focusing ring on the telescope (at the input end, not the eyepiece) to get a better focus on your chosen drop.

You'll probably have to examine quite a few drops to find one that fits these criteria. All the drops may drift away before you find one. One way to bring new drops into view is to just slightly lift and drop the plastic lid on top, causing a sudden air current. If need be, lift and drop the lid several times

to clear the old drops out and spray a new batch in. Finding a "good" drop requires some patience, but once you have it you'll keep it through a whole series of measurements – long enough to begin developing a close personal relationship with it.

As you begin making measurements, you'll find this is a two-person operation. One has to manipulate the drop and the timer while the other records the direction of motion and the times. The viewer can't look up to record measurements without losing track of "your" drop. Lab partners should switch roles so that both get to be viewers for at least one full set of measurements.

5. Once you've selected a "good" drop, check your voltage, record the temperature, then begin a series of measurements. Record the time t_f for the drop to fall 0.50 mm with the field off. Then switch the field on and record the time t_u for the drop to rise 0.50 mm. Repeat this over and over until you have a minimum of 8 and preferably 10 or more values of t_f. You'll want to put these values in columns in your lab notebook and later in your report.

After you have a sufficient number of t_f values, you can change the procedure a bit. Alternate pushing the drop up and measuring t_u with using the field to <u>push</u> the drop down and measure t_d.

   Note: You may find that the value of t_u suddenly changes between one measurement and the next. That's OK, and even useful if it isn't changing too often. Either cosmic rays passing through the container or collisions between the drop and water molecules in the air can change the drop's charge. It will be interesting, during the analysis, to learn the *amount* by which the charge changes during these interactions.

It sometimes happens that a charge-changing interaction neutralizes the drop. It then falls out the field of view, and there's nothing you can do. Other drops may disappear from too much sideways motion (although check the leveling of the apparatus if this happens repeatedly). Sometimes you simply have to abandon a drop and start over.

Obtain adequate measurements on <u>at least</u> 3 separate drops. Measurements on 4 or 5 drops will greatly improve your final results.

Analysis:

1. For each drop, compute the average and the standard deviation σ_t of <u>all</u> the measured values of t_f.

Sometimes you may have one value that seems totally out of line with all the others, and you're tempted to throw it out. But keep in mind that with ≈10 measurements, you expect 1 of those to differ from the mean by 1.6σ, and there's a 5% chance (half a measurement) of a 2σ deviation. So here's a reasonable "rule of thumb" for whether you can throw out a point. Compute the mean and standard deviation with all data. If one point differs from the mean by >2.5σ (a 1% chance), that seems pretty unlikely in a data set of ≈10 points. You then have reasonable cause to reject that point. Throw it out, then recompute the mean and standard deviation without it.

The standard deviation σ_t describes the spread of the data. But the uncertainty δt_f specifies how well you can determine the mean. Taking more measurements doesn't change the spread, but it does improve the reliability of the average. So what you want to report as the uncertainty in the fall time is <u>not</u> the standard deviation σ_t of the data, but the *standard deviation of the mean*, namely

$$\delta t_f = \frac{\sigma_t}{\sqrt{N}} \tag{12}$$

where N is the number of data points. So report your result for the fall time as

$$t_{f,\,best} = \overline{t_f} \pm \delta t_f \tag{13}$$

Now compute the fall velocity v_f (in m/s) and use propagation of error (<u>show your work</u>) to determine the uncertainty δv_f. You can use $h = 0.50$ mm with $\delta h = 0$.

2. Use your measured values of V and d to compute the electric field $E = V/d$. Make sure you have the right SI units! The uncertainty δE can be neglected compared to other uncertainties in this experiment.

3. Use the temperature for each drop and Eq. (7) to compute the air viscosity η. Keep 3 decimal places. Then use Eq. (6) to compute the drop radius and Eq. (5) to compute the drop mass m. Use propagation of error (<u>show your work</u>) to determine the uncertainty δm. You can use $\rho_{oil} = 885$ kg/m³, which includes a slight correction for the density of air.

Then compute the correction factor γ from Eq. (8) for each drop.

4. **Read and follow this step carefully**. First, look at all your t_u and t_d values for each drop and group together those values that seem to be the same, within statistical fluctuations. You may have two or more groups if the drop's charge changed during the measurements. Any groups with less than 3 values simply don't have enough data to be useful. For each group with 3 or more values, proceed as in Step 1 to find the average time and the mean of the standard deviation. Then compute $v_u \pm \delta v_u$ and $v_d \pm \delta v_d$. You may end up with several values for each drop, so you'll want to organize your data carefully.

Note: Grouping the data is not a license to ignore data points. All data until a clear charge-change must be grouped together. There's inherently a fair amount of variation in the data, so one point that seems a bit out of line is OK. What you're looking for are any "step function" changes to a different average time.

For each value of v_u and v_d for each drop, use Eqs. (9) and (10) to calculate the drop's charge and Eq. (11) to calculate the uncertainty. **Report results as (x.xx ± 0.xx)×10^{-19} C.**

For each drop that was exposed to the ionization source, calculate the charge before and after exposure.

5. You could divide each value of q by the known e to see if it's an integer multiple – but that's cheating since you're assuming the answer in your analysis. Instead, you want to find the largest common divisor of each charge value that you've determined. This is your experimental value of e. Do this by dividing each q by $n = 1, 2, 3, ...$ and then looking for the largest value that seems to be common to all your measurements. For example, if your measured $q = 4.4, 6.2,$ and 9.1 for three drops, you would find that $4.4/3 = 1.47$, $6.2/4 = 1.55$, and $9.1/6 = 1.52$ are all very nearly the same.

Make a table to show all your results, with columns labeled: drop number, q, δq, n, $e=q/n$, $\delta e=\delta q/n$.

6. Determine your experimental value for the fundamental unit of charge and your experimental uncertainty: $e \pm \delta e$. Show your error analysis. Compare the value of e determined from your data with the accepted value. Remember that "to compare" means "Does the accepted value fall within your uncertainty δe. If not, by how many standard deviations does it miss? Is this acceptable?"

Where the charge of a dropped changed, is the change an integer multiple of your e?

7. Combine this result with your value from Charge to Mass Ratio for e/m, the electron charge-to-mass ratio, to deduce an experimental value for the electron mass: $m \pm \delta m$. Show explicitly how you determine δm. Compare to the accepted value. So with two experiments, you've measured two fundamental constants of nature.

8. IF you've finished experiment Atomic Spectra, proceed with this part. If not, record your values of e and m in your lab book, and you'll do this calculation then.

From quantum mechanics, E_R – in joules! – is defined as

$$E_R = \frac{me^4}{(4\pi\varepsilon_0)^2 \cdot 2\hbar^2}$$

where m is the electron mass. In experiment Atomic Spectra you determined experimental values for E_R. Combine this with your experimental values of e and δe and of m and δm to find an experimental value for Planck's constant h and its uncertainty δh (<u>show your error analysis</u>). Since you didn't find an explicit result for δE_R, you can use your percentage error in E_R as a reasonable estimate for δE_R. Compare your value for h with the accepted value.

Additional Questions:

1. Derive Eq. (4) for a drop that is driven downward by the electric field.

2. Prove Eqs. (11) for the uncertainty in q_u and q_d. You can assume that the uncertainty in γ, E, and g are negligible compared to the uncertainties in m and the speeds.

Resistance/Temperature Table for Pasco Oil Drop Apparatus

Millikan Oil Drop Apparatus Thermistor Resistance at Various Temperatures

THERMISTOR RESISTANCE TABLE

°C	X 10^6 Ω	°C	X 10^6 Ω	°C	X 10^6 Ω
10	3.239	20	2.300	30	1.774
11	3.118	21	2.233	31	1.736
12	3.004	22	2.169	32	1.700
13	2.897	23	2.110	33	1.666
14	2.795	24	2.053	34	1.634
15	2.700	25	2.000	35	1.603
16	2.610	26	1.950	36	1.574
17	2.526	27	1.902	37	1.547
18	2.446	28	1.857	38	1.521
19	2.371	29	1.815	39	1.496