Franck-Hertz

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1 Results and Analysis

The cross section of an electron-atom collision in a gas is approximately a circle of twice the radius of a gas particle. Thus, as electrons travel, they sweep out an effective volume and encounter a number of collisions depending on the number density of the gas atoms. Therefore, the mean free path, which is the average distance traveled between any two collisions, is given by

$$\lambda = \frac{1}{\pi(N/V)R^2} \tag{1}$$

where N/V is the number density of the gas and R is the radius of a gas molecule. We have omitted the $\sqrt{2}$ factor that intended to correct for the relative velocity between the electron and Mercury atoms, since an electron of 20 eV (typical for our experiment) travels with velocity $\sim 10^6$ m/s, while the Mercury gas atoms have velocity $v \sim \sqrt{k_b T/m} \sim 10^2$ m/s, so the relative velocity is only weakly dependent on the velocity of the Mercury atoms. Our runs were done for chamber temperatures 430 K and 459 K, creating a Mercury vapor pressure of 500 Pa and 1500 Pa, respectively, which amounts to $N/V \approx 10^{23}$ molecules per cubic meter and $N/V \approx 2 \cdot 10^{23}$ molecules per cubic meter, respectively. Then, with $R = 0.18$ nm as the effective radius of a Mercury atom, we find the electron's mean free path to be 100 μm at 430 K and 50 μm at 459 K, which is approximately 1% of the 1 cm distance between the cathode and the grid, meaning the 430 K electron will experience approximately 100 collisions before reaching the grid, and the 459 K electron will collide about 200 times in the same distance.

We witnessed the effects of these collisions on our apparatus as a glow of the Mercury spectrum was emitted in close proximity to the cathode grid when the filament voltage was large. In this case, a majority of the electrons emitted by the cathode quickly reach the threshold energy necessary to excite Mercury and then release it in their immediate next collision with such high probability that virtually none of the Mercury atoms become doubly excited. Low energy electrons emitted by the heated cathode are unable to promote gaseous Mercury to its first excited state, therefore, they accelerate across the cathode-grid gap, due to the grid's accelerating potential, without losing energy in collisions or scattering. Electrons that surrender their energy in collisions near the grid are rendered too weak to overcome the retarding potential of the anode and consequently they don't contribute to the current signal, causing dips in the vacuum diode IV curve, which would normally increase with the accelerating voltage in an approximately parabolic manner. These current dips are seen in Figure

1, where the first dip from the left appears when the accelerating voltage reaches the minimum value necessary to impart enough kinetic energy onto the traveling electrons such that they are capable of exciting a gaseous mercury atom right at the point where they reach the grid, then they have no kinetic energy and are repelled by the anode. Subsequent dips form at larger accelerating voltages when the electron undergoes multiple excitation collisions, for example, the second dip from the left occurs when there is an excitation collision approximately midway to the grid and a secondary excitation collision right at the grid.

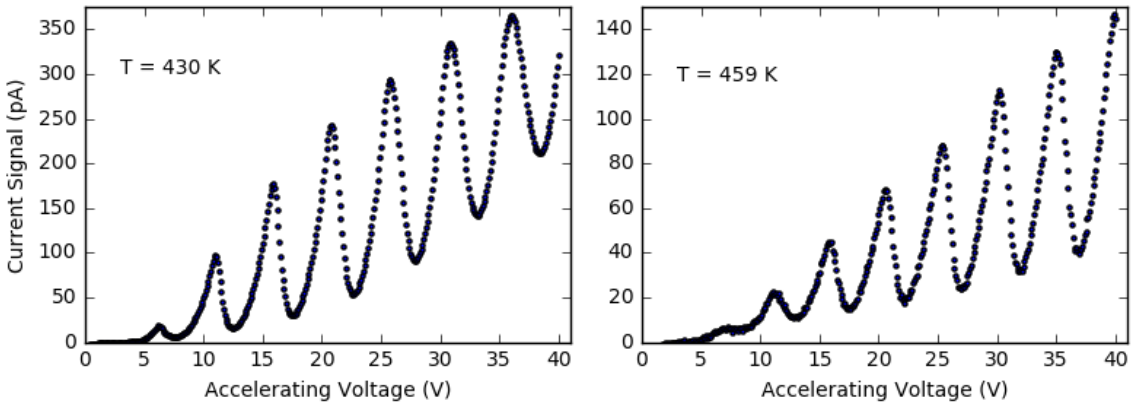

Figure 1: Current voltage curves for chamber temperatures 430 K (left) and 459 K (right), where each dip indicates an excitation collision.

In Figure 1, the current magnitude is greater and the relative depth of each dip is smaller in the case of lower chamber temperature even though the rate of electron emission from the filament is similar in both situations. This follows from the Mercury vapor pressure being 500 Pa at 430 K, in contrast with 1500 Pa at 459 K, which gives rise to fewer collision events and a higher likelihood for the electrons to reach the current collecting anode instead of being grabbed by the grid. Both figures share the presence of seven dips, which is as expected, since the electron energy gained from the accelerating voltage is independent of the chamber temperature and $k_b \Delta T \sim 1$ meV is significantly less than the excitation energy of a Mercury atom.

We found the accelerating voltage at each local minimum by fitting a second order polynomial, of the form $a + bV + cV^2$, to the I-V curve for short intervals centered around each dip. Where a, b, and c are fit parameters, V is the accelerating voltage, and the uncertainty in V_{min} is determined by the quantity $V_{min}\sqrt{(\delta b/b)^2 + (\delta c/c)^2}$, since $V_{min} = -b/2c$, where δb and δc are supplied by the fitting algorithm. An image of our fit for the leftmost dip of the 430 K scenario is seen in Figure 2.

Performing such a fit for each dip provides the total energy (in electron volts) accumulated by the electrons that underwent an excitation collision near the grid. These results are arranged according to n, which denotes the dip number when counting from the left and also, as discussed previously, the number of excited Mercury atoms due collisions, in Table 1.

The true accelerating potential experienced by the electrons is reduced due to the presence of a contact potential between the cathode and the grid. This follows from the fact that the Fermi energy of the grid is lesser than that of the cathode, so when they are connected through a circuit, electrons flow naturally into the grid until the

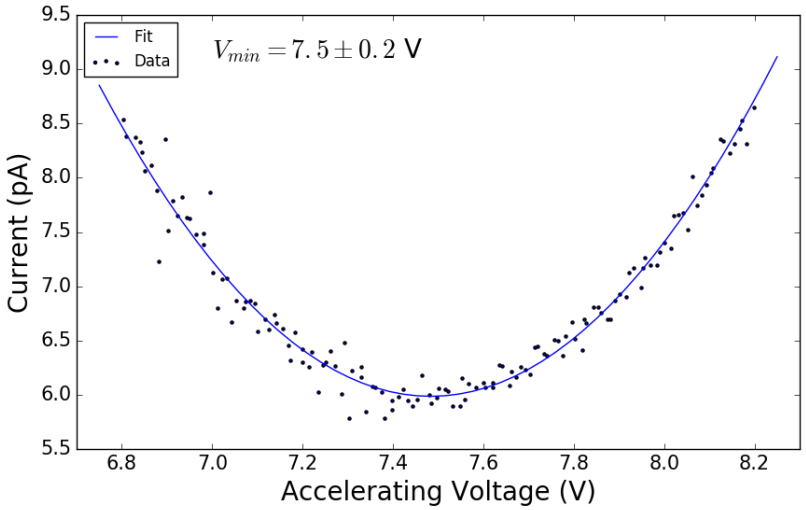

Figure 2: Quadratic best fit to determine the local minimum coordinates for the leftmost dip of the 430 K current-voltage plot.

Table 1: Total kinetic energy imparted upon the airborne electrons that underwent an excitation collision near the grid, such that they were unable to reach the anode detector and their absence lead to the current dips seen in Figure 1.

n	1	2	3	4	5	6	7
K_n (eV) at 430 K	7.5 ± 0.2	12.5 ± 0.4	17.5 ± 0.8	22.7 ± 0.6	27.9 ± 0.7	33.1 ± 0.9	38.3 ± 1.4
K_n (eV) at 459 K	7.8 ± 1.6	12.9 ± 0.3	17.5 ± 0.5	22.2 ± 0.6	27.1 ± 0.7	32.0 ± 1.0	37.1 ± 0.6

Fermi energy of both metals are equivalent. Therefore, at zero potential difference, the grid is negative compared to the cathode, since it contains "excess" electrons and the data of Figure 1 is right shifted in V by the contact potential. Then, the values of K_n from Table 1 can be expressed

$$K_n = nE_{Hg} + V_{CPD} \tag{2}$$

where E_{Hg} is energy required to promote Mercury to its first excited state and V_{CPD} is the contact potential difference. We performed a linear least squares fit to the data of Table 1 with Equation 2 to determine our measured values for E_{Hg} and V_{CPD} at each chamber temperature, which is shown in Figure 3.

At 430 K, we found $E_{Hg} = 5.14 \pm 0.02$ eV, which is on the order of the accepted value of 4.89 eV, but lies many standard deviations away, which we believe to be the result of the less than perfect quadratic fit approach for determining the minimum of each dip. Perhaps higher order fits, and/or greater measurement resolution, would remedy this issue. At 459 K, we found $E_{Hg} = 4.85 \pm 0.03$ eV, which is within two standard deviations of the accepted value, 4.89 eV. Both measurements of E_{Hg} should be independent of temperature, since the excitation energy of an atom is independent of temperature, and we expect the discrepancy between our measurements to be a consequence of assuming the dips to be only parabolic in form. At 430 K, we found $V_{CPD} = 2.21 \pm 0.09$ eV, and at 459 K, we found $V_{CPD} = 2.99 \pm 0.14$ eV, which are both reasonable numbers for the contact potential difference, since the Fermi energy of a metal such as silver is 5.5 eV, so we could expect the difference of the Fermi

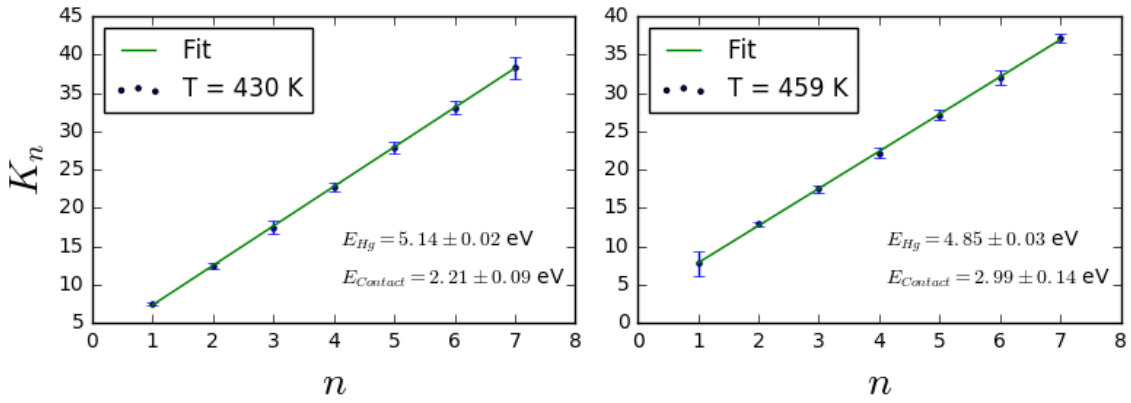

Figure 3: Linear least squares fit to the data from Table 1 to determine the contact potential and excitation energy of Mercury's first excited state.

energy between two metals to be on the order of 2 eV. The Fermi energy decreases as the temperature increases, since the density of states for electrons increases with increasing temperature, so it is not unreasonable for the contact potential difference of our two measurements to be distinct, although there are competing effects, since the cathode Fermi energy will also decrease with increasing temperature [1].

These results demonstrate that atomic energy levels (of Mercury) are quantized. If they were not quantized, Figure 1 would not display periodic dips, instead it would be indistinguishable from a typical vacuum diode I-V curve, since the energetic electrons would excite Mercury atoms with random and continuous amounts of energy.

Once an electron reaches the minimum energy required for an inelastic collision, E_{Hg}, it is reasonable to believe that it will accumulate an additional energy $eV_{acc}\lambda/L$, where L is the anode-cathode distance, since it will travel a distance of approximately λ before encountering another Mercury atom. Then, the energy gained between collisions increases when the accelerating voltage increases, and since there are many other excited states of Mercury of energy near that of its first excited state (due to weak splitting effects), the electron still loses most of its energy in inelastic collisions, thus the energy spacing between minima should increase with n according to

$$\Delta K_n = (1 + \frac{\lambda}{L}(2n - 1))E_{Hg} \tag{3}$$

where L is the anode-cathode distance [2].

In order to determine if Equation 3 follows from valid arguments, we performed a weighted linear least squares fit with the data of Table 1, which is shown in Figure 4. These results are limited by a large relative uncertainty, but at 430 K, we found $E_{Hg} = 4.92 \pm 0.02$ eV, which is within two standard deviations of the accepted value and is a more accurate measurement than our previously determined value at this temperature. At 459 K, our results gave $E_{Hg} = 4.45 \pm 0.03$ eV, which is not in agreement with the accepted value, which is likely due to the fact that the leftmost data point at this temperature was difficult to resolve from Figure 1. From these results, it is difficult to make any certain claims as to whether Equation 3 is valid, as the relative uncertainty of our quantities here is quite large, and upon inspection of Figure 4, we see that only in the 459 K scenario does ΔK_n appear to increase linearly with n.

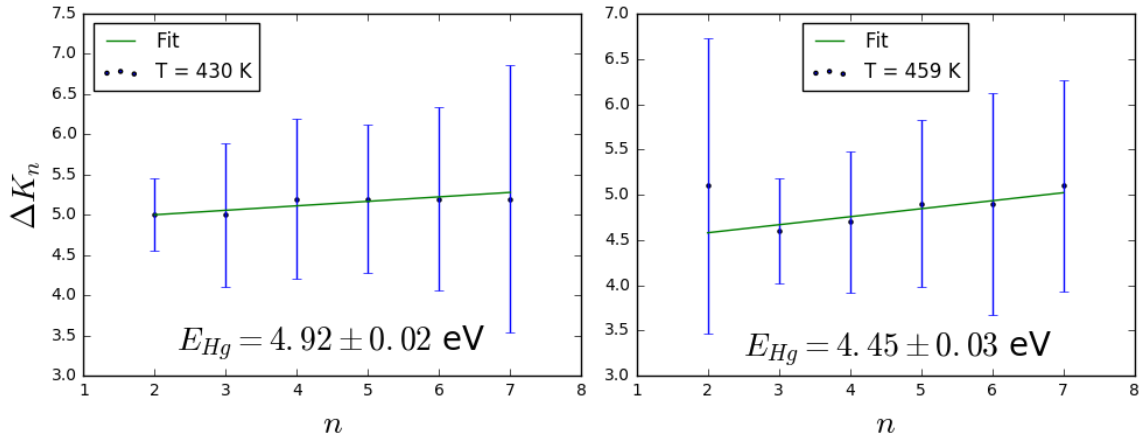

Figure 4: Weighted linear least squares fit to the values $\Delta K_n = K_n - K_{n-1}$ from Table 1 with Equation 3. The uncertainty is calculated by $\delta(\Delta K_n) = \sqrt{(\delta K_n)^2 + (\delta K_{n-1})^2}$.

2 Conclusion

We have used inelastic collisions between free electrons and gaseous Mercury to demonstrate that atomic energy levels are discrete in nature by applying a range of accelerating voltages to the airborne electrons and observing a dip in collected electron current whenever the electron energy is an integer multiple of the first excitation energy of Mercury. We performed these measurements at two different temperatures, to show that the Mercury excitation energy is independent of temperature, and found the energy necessary to excited Mercury from its ground state to its first excited state to be 5.00 ± 0.04 eV, which is within two standard deviations of the accepted value of 4.92 eV. Additionally, we were able to infer the contact potential difference between our cathode and anode, which we found to be on the order of 2.5 eV and temperature dependent.

References

[1] Ronald Brown. *Solid State Physics*. 2009.

[2] Gerald Rapior, Klaus Sengstock, and Valery Baev. New features of the Franck-Hertz experiment. *American Journal of Physics*, 74(5):423–428, 2006.