RADIOACTIVE DECAY

Objectives:

To measure the half lives of radioactive isotopes produced by neutron activation in a) indium and b) silver.

Background:

1) Review radioactive decay and half lives in a modern physics book.
2) Chapter 8 in Taylor on least-squares fitting. Give special attention to pages 194-196 on exponential functions.
3) Melissinos pages 354-365.

Theory and Apparatus:

A.   Radioactive Decay

Excited states of quantum systems, be they atoms or nuclei, decay to lower states with a characteristic time called the **half-life**. Atoms decay by the emission of photons, as do some nuclei undergoing gamma decay. Alpha and beta decay processes are similar, but the energy is emitted as a particle instead of a photon.

A property of any excited quantum state is that it has a *constant* probability per unit time of decay. This **decay constant** is A, and it has standard units of s^{-1}. Since A is the probability *per second*, then $A\Delta t$ is the actual probability of decay during time Δt. If we have N excited atoms or nuclie, the number that decay during Δt is

$$\Delta N = AN\Delta t. \qquad\qquad (Eq.\ 1)$$

The decay constant A depends on the details of the excited state and the decay mode. While in principle it is possible to calculate A-values in quantum mechanics, such calculations are too difficult to be practical except for the simplest systems. In practice, A-values are determined experimentally by measuring half lives.

Strictly speaking, Eq. 1 is true only when Δt is small enough that $A\Delta t \ll 1$. It wouldn't make sense to have a probability greater than 1, and we would need set up an integral to find the probability for longer Δt. But Eq. 1 is certainly true if we let Δt get smaller and smaller, until $\Delta t \rightarrow dt$. Then $\Delta N \rightarrow dN$, and we need to include a minus sign since the population in the excited state is decreasing. So excited state decay is governed by the differential equation

$$\frac{dN}{dt} = -AN. \tag{Eq. 2}$$

This equation is easily integrated to give

$$N(t) = N_0 e^{-At}, \tag{Eq. 3}$$

where N_0 is the initial number of atoms or nuclei in the excited state. Excited state decay is another important example of an exponential decay.

First, define the **lifetime** of a state to be $\tau = 1/A$. Then

$$N(t) = N_0 e^{-t/\tau} \tag{Eq. 4}$$

so τ is the time at which the population has decayed to $e^{-1} \approx$ 37% of its initial value. This definition of lifetime, the same that as used to define *time constants* in electronics, is used in atomic physics to characterize excited states. But for historical reason, nuclear physics usually characterizes excited states by their half life. We define the **half-life** $T_{1/2}$ as the time at which half the sample has decayed. That is, $N(t = T_{1/2}) = N_0$. Using this definition with Eq. 3 gives

$$N(t = T_{1/2}) = \tfrac{1}{2}N_0 = N_0 e^{-AT_{1/2}}$$

$$\Rightarrow T_{1/2} = \frac{\ln 2}{A} = \tau \ln 2 = 0.693\tau$$

(Eq. 5)

So A, τ, and $T_{1/2}$ all convey the same information and can be converted from one to the other.
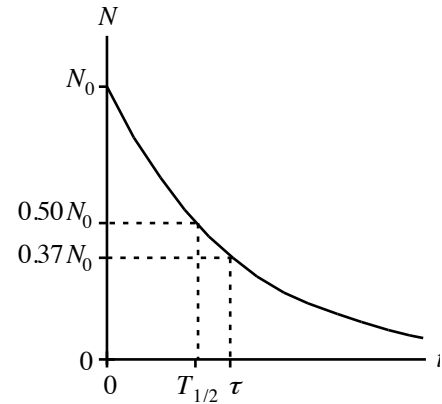
The **rate of decay**, or number of decays per second, is the absolute value of Eq. 2:

$$\#\,\text{decays per second} = \left|\frac{dN}{dt}\right| = AN$$

But we now know $N(t)$ from Eq. 3, so

$$\#\,\text{decays per second} = AN(t) = AN_0 e^{-At}$$

Suppose that a fraction f of all decays get counted. This will depend on the geometry of the source and detector as well as the detector's efficiency. The experimental **count rate** R at time t is

$$R(t) = f\,(\#\,\text{decays per second at time } t) = fAN_0 e^{-At}$$

While we don't know f, A, or N_0 individually, their product is just R_0, the initial count rate. So the experimental signal is

$$R(t) = R_0 e^{-At} \qquad\qquad\qquad \text{(Eq. 6)}$$

The experimental count rate decays exponentially with the same decay rate, and hence the same half life, as does the excited state we wish to measure.

Taking the natural logarithm of Eq. 6 gives

$$\ln R = \ln R_0 - At \qquad\qquad\qquad \text{(Eq. 7)}$$

A graph of $\ln R$-versus-t should be a straight line, and its *slope* gives the value of the decay constant A.

IF, that is, there's only a single decaying isotope. Suppose there are *two* radioactive isotopes in a sample, with decay constants A_1 and A_2. If R_{10} is the initial count rate due to the first isotope and R_{20} due to the second, then the *total* count rate seen by a detector is

$$R_{\text{total}} = R_1 + R_2 = R_{10}e^{-A_1 t} + R_{20}e^{-A_2 t} \qquad\qquad\qquad \text{(Eq. 8)}$$

The sum of two exponentials is *not* an exponential, and $\ln(R_{\text{total}})$ is *not* a straight line. (Because $\log(a+b) \neq \log a + \log b$.) This will be the situation you'll face with silver, and much of the analysis will be focused on how to deal with this.

B.   Neutron Activation

When certain nuclides are exposed to a thermal neutron flux, they capture neutrons and promptly release gamma-rays. The product nucleus, which has an atomic mass number raised by one unit, is usually radioactive, decaying mainly by β⁻ emission. These radioactive products begin to decay immediately, in accordance with Eqs. 3 and 4. The decay particles can be detected and yield a count rate R, given by Eq. 6 or 8. This method of creating radioactive isotopes is called **neutron activation**.

We will use thin foils of indium and silver and expose them to a strong neutron emitter (≈ 3 curies of plutonium mixed with beryllium). Indium has a single stable isotope, ^{115}In, which is activated to radioactive ^{116}In. Its half life is ≈ 1 hour. Silver, in contrast, has two stable isotopes, ^{107}Ag (51%) and ^{109}Ag (49%). These activate to ^{108}Ag ($T_{1/2} \approx 2.5$ min) and ^{110}Ag ($T_{1/2} \approx 30$ s).

How long do the foils need to be irradiated to activate them? The rate at which radioactive nuclei are produced in the foil is essentially constant at p activations per sec. But the newly created nuclei immediately begin to decay. The number of radioactive nuclei at the end of an

irradiation time, t_r, will be determined by the "competition" between production and decay. Thus, $dN/dt = p - AN$, which when integrated over the irradiation time t_r gives

$$N(t_r) = \frac{p}{A}\left(1 - e^{-At_r}\right) \qquad \text{(Eq. 9)}$$

When t_r becomes large, N approaches the maximum value of $N_{max} = p/A$. Thus, $AN_{max} = p$; that is, the decay rate exactly balances the production rate. Practically speaking, this condition will be reached when the activation continues for a time $t_r \approx 5T_{1/2}$. At this time about 97% of the maximum value is reached.

From this analysis, you can see that the silver foils can be activated in ≈15 minutes, but the indium foils take ≈5 hours. They'll be activated overnight.
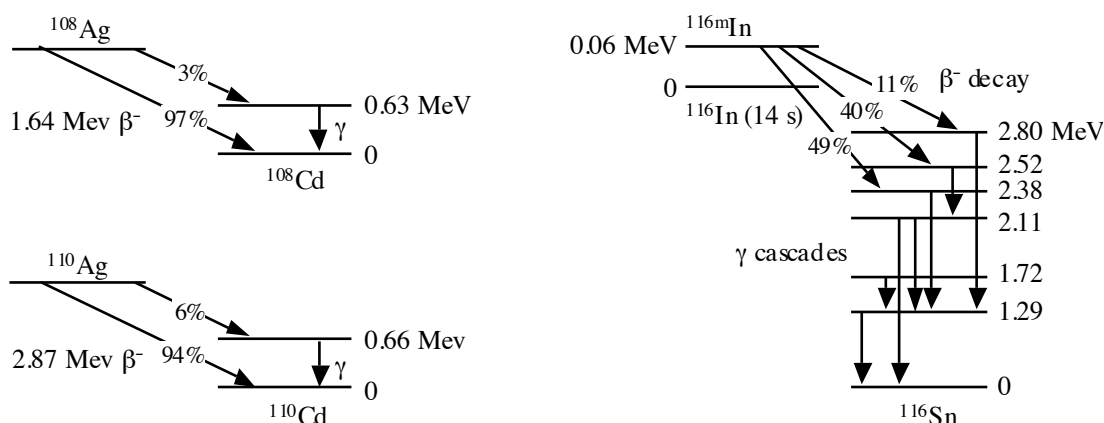

Figure 1. Decay schemes of silver and indium.

Figure 1 shows the decay schemes. The two silver isotopes undergo β⁻ decay to cadmium. The decays are overwhelmingly (97% and 94%) to the ground state of the cadmium nucleus, so there is essentially no gamma radiation. Because of this, we will detect the silver beta radiation with a Geiger tube.

Indium is more complicated. The ground state of ^{116}In has a half life of only 14 s. The decay we're going to observe is from an excited nuclear state in ^{116}In, designated ^{116m}In. It decays to excited nuclear states of ^{116}Sn. These excited nuclear states promptly emit lots of different energy gamma rays as they decay to the ground state of ^{116}Sn. Because gammas can be detected very efficiently with a scintillation detector, that's what we'll use to observe the ^{116}In decay.
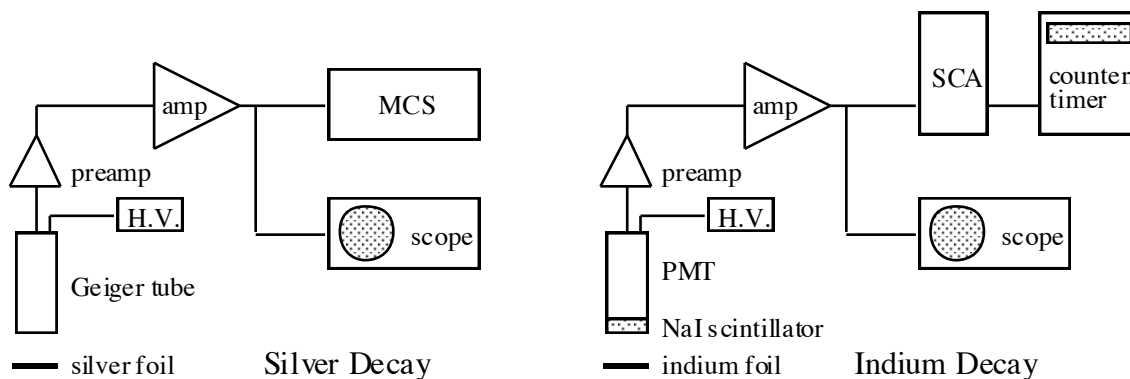
C.   Apparatus

Figure 2. Experimental arrangement.

Figure 2 shows the experimental setup for detecting these decays. For silver, the Geiger tube will produce the same kind of pulses you observed in Experiment 5. The pules will be sent to the MCA, but in this experiment we will use the analyzer in **multichannel scaler** mode, or **MCS**. In this mode, 256 channels are used as sequential counters. The MCS starts in channel 1, and it counts the number of pulses N_1 that arrive during a time interval Δt called the **dwell time**. It then switches to channel 2 and dwells for Δt, counting N_2 pulses. It then spends Δt counting N_3 pulses in channel 3, and so on. It takes $256 \Delta t$ for one complete *sweep* through all the channels.

If the sample is decaying, channel 1 gets more counts than channel 2, which gets more than channel 3, and so on. The display on the MCA is a *histogram* of counts (vertical) versus channels (horizontal). But since the *channel number* is directly proportional to the *elapsed time* since the experiment started, the display is really a **counts-versus-time** graph. The figure shows the basic idea, although you'll have many more channels.
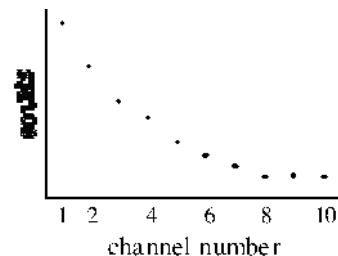
If the count rate at time t is $R(t)$, then the number of counts in that channel is $C(t) = R(t)\Delta t$. Since Δt is the same for all channels, we can use Eq. 6 to write

$$C(t) = C_0 e^{-At} \qquad\qquad \text{(Eq. 10)}$$

So the theory described above in Eqs. 6-8 applies directly to the measured counts.

An important feature of exponential decay is that Eq. 10 is valid for *any* choice of $t = 0$. In other words, we don't have to start our clock at the instant the foils finish activation. We can call the first channel $t = 0$, no matter when it starts, measure the counts C_0 in that channel, and Eq. 6 will

then describe the rest of the decay. So our first data point will be *defined* as $t = 0$, and the data in channel n is then at time

$t = (n - 1)\Delta t$.

For indium, the amplified pulses from the scintillation detector are sent to a device called a **single channel analyzer**, or SCA. Recall in Experiment 5 that the Geiger counter pulses were counted in a counter/timer with a built-in **discriminator**. This is a comparator circuit that compares the pulse height to a preset voltage called the **discriminator level**. If the input pulse exceeds the discriminator level, the pulse is sent through to be counted. The SCA does the same thing, but it has more flexibility and options than the built-in discriminator of the counter/timer. In this experiment you're going to use only a lower-level discriminator that the pulse must exceed to get counted. (In Physics 342 you'll do experiments in which the SCA passes only pulses that fall in a *window* between a lower-level discriminator and an upper-level discriminator.)

PROCEDURE

A.   Indium sample

1. Start with the indium foils, since these take much longer to acquire data. These will already have been activated for > 5 hours. Make sure your oscilloscope is triggering on and displaying the appropriate channel.

2. Set the PMT voltage at 500 V. Adjust the amplifier gain so that pulses from a ^{137}Cs calibration source are seen on the scope as $\approx$ 3.5 V pulses. Make sure the amplifier input is on "neg".

3. Notice the three turn knobs on the SCA. Make sure the upper-level discriminator is set to its maximum value (full clockwise), then lock the knob. The SCA input should be on "norm". Adjust the lower-level discriminator and observe the counter while using the ^{137}Cs source. You'll see that there's a fairly narrow range over which the count rate increases from very low to very high. Since you know that the ^{137}Cs pulses are $\approx$ 3.5 V, this setting corresponds to a discriminator level of $\approx$ 3.5 V. (The transition is not a perfect step function since the there's some variation in the ^{137}Cs pulses.) The ^{137}Cs energy is 662 keV, and (as you learned in Scintillation experiment) the scintillation detector output is linear in energy. Use this information to set the lower-level discriminator to pass pulses from gamma rays with > 200 keV.

4. Set the counter to 1 minute. Verify that everything is working while still using the ^{137}Cs source, then move the ^{137}Cs behind shielding so that it doesn't create background. Take a 1 minute background reading from an indium foil that has <u>not</u> been activated. (Why from an inactive foil, rather than with an empty sample chamber? Something to think about for your report.)

5. Remove an indium foil from the neutron source and place in the sample holder. Wait 5 minutes for the sample to "cool," meaning that the 14 s half life ground state and possible other very short-lived isotopes can decay away. Then take a 1-minute count every 10 minutes for at least 2 hours. For each data point, record a) the exact time at which you start, and b) the counts. The intervals need not be exactly 10 minutes, as long as you know exactly when each count really did start. But exact 10-minute intervals will simplify your analysis.

6. Take another background reading at the end.

B.   Silver sample

1. Set the MCA to multichannel scaler (MCS) mode. Set the dwell time to 1 s and the number of channels to 256. Make sure your oscilloscope is triggering on and displaying the appropriate channel.

2. Set the Geiger tube voltage at 700 V, which is high enough to be in the plateau region.

3. Start acquiring ^{137}Cs data with the MCA. You should see it move from channel to channel every second. Your data should be a horizontal line, except for statistical fluctuations, since ^{137}Cs isn't decaying on this short time scale. Stop the MCS, erase the data, and **set the dwell time to 4 s**. Acquire a few more channels of data, then stop the MCS, erase the memory, and remove the ^{137}Cs. You want to be *absolutely sure* that everything is working properly because the shorter half-life in silver is *so* short that you can't tolerate even the slightest delay in starting the data acquisition.

4. Remove a silver foil from the source, as quickly as possible place it bare metal side up in the sample holder, and immediately start data acquisition. (Check an inactive foil first so you'll know which side is which.) Save the data as a '.mca' file. This can then be opened using Excel,

the header and follower information stripped off, and the file resaved as a text file to be subsequently read into MATLAB for processing.

5. Remove the silver foil and measure the background rate. Why is it so low?

ANALYSIS:

Use the software of your choice (Excel, Matlab, Python etc).

A.   **Indium**

1. Do this analysis first, because it's simpler. Average your two background readings to find C_{bgd} and δC_{bgd}. Subtract the background from each reading to find C_{net} and δC_{net} at each time. Note that $\delta C_{net} \neq \sqrt{C_{net}}$! Find the time t for each reading in minutes, letting the first reading be $t = 0$. Make a graph of C_{net}-versus-t.

2. For each point, find $y = \ln(C_{net})$ and the corresponding δy. You may get to a point where C_{net} is negative, so $\ln(C_{net})$ isn't defined. At this point the "signal has disappeared into the background." Stop your analysis at this point. Don't just throw out negative points while keeping positive ones further along since that will skew your least-squares fit.

3. Use the "Weighted Least Squares" program that you previously used to perform a *weighted* least-squares fit. Print a graph showing both your data and the fitted line.

4. Extract values for the slope and its uncertainty from your least-squares fit. Use them to compute the half life $T_{1/2}$ and its uncertainty $\delta T_{1/2}$. Look up the accepted value of $T_{1/2}$ for ^{116m}In in the *Table of the Isotopes*. Compare your experimental value to the accepted value.

B.   **Silver**

1. The raw data may need a deadtime correction. Recall from Experiment 5 that Geiger tube data need a correction

$$C_{true} = \frac{C_{obs}}{1 - R_{obs}\tau}$$

where C_{obs} is the observed number of counts, C_{true} is the true number that should have been counted, τ is the deadtime, and R_{obs} is the observed count *rate*. Since you used 4-second channels, $R_{obs} = C_{obs}/4$. Use a value of $\tau = 400$ μs to correct the data for the tube deadtime.

2. As you did with indium, find $\ln(C_{net})$ and the corresponding δy for each point. Plot a graph of $\ln(C_{net})$-versus-t. Unlike the case with indium, this <u>won't</u> be a linear graph because you're observing two isotopes with different half lives. However, eventually the shorter-lived isotope will have decayed away and only the longer-lived will remain. Notice that your graph does become linear at longer times.

3. You can't do a linear fit to this data, and although there are more powerful routines available that can do this sort of fitting (you'll use them in Physics 341), we will try and extract the half-lives using the following procedure. First, inspect the log plot and see where (in time) the graph becomes linear. Then use the $\ln(C_{net})$ data after this time and make a weighted least-squares fit to it. The slope of the graph will of course be related to the half-life of the longer-lived isotope.

4. Once you have the equation for this graph you can then go back and use it to calculate the counts that, at earlier times, were due to the longer lived isotope. You then subtract these from the actual counts and what you are left with must have been due to the shorter lived isotope. NOTE: You must subtract actual counts, not logarithms of the counts. You can now take the log of this new data and fit it to a straight line.

5. Look up the accepted values of the two half lives in the *Table of the Isotopes* and compare your measured values to the accepted values.

6. Next, you will try a different approach. Use your complete, raw data set (no log plot) and the fitting routine you developed in *Introduction to Matlab*. Fit a double exponential to the data. How do your answers compare to #4? Are they similar? Why or why not?

Although this analysis is more involved than any you've done up until now, it's quite realistic as far as what "real world" data analysis is like.