GEIGER COUNTING

Objectives

In this experiment we will characterize and use a Geiger-Mueller counter (GM tube). In the first part, we'll use a radioactive source to study the GM tube itself and the associated counting system. In the second part, we'll examine the radioactive decay and test to see if it obeys the statistics expected for a random source.

Background

1) Review radioactive decay in your modern physics textbook and the Background Notes at the end of this experiment.
2) Review Chapter 11 (Poisson statistics) in Taylor.
3) Read the document on Radiation Safety (available on PolyLearn and in the lab).
4) Read pages 485-488 in Melissinos on Radiation Safety and pages 295-298 and 320-333 on radiation experiments and GM tubes.

Note: Completing the radiation safety reading for this lab and signing the document stating that you have done so and understood it is a *mandatory requirement*. You cannot start this experiment until doing so.

Theory and Apparatus

A. Output of a Geiger-Mueller tube.

An end-window GM tube and the counting electronics are shown in Fig. 1. Ionization is produced when a high-energy particle or photon passes through the tube. The electrons are collected by the anode, which is biased with a positive high voltage, thus creating a voltage pulse that passes through the blocking capacitor and is amplified by the preamp. The output pulses are observed on the oscilloscope and counted by the counter/timer.

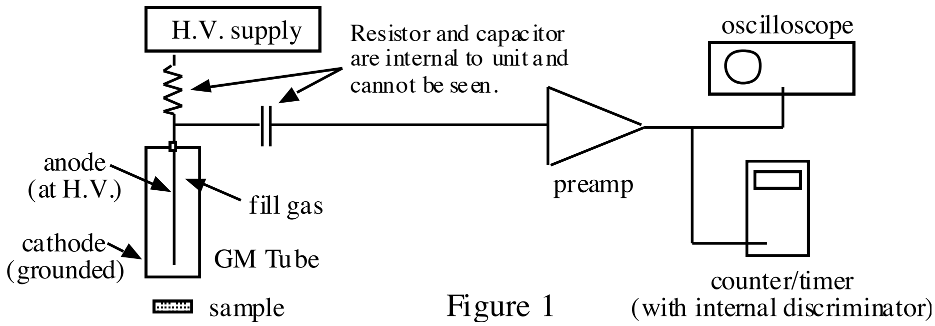


Figure 1

When a charged particle enters the Geiger tube it produces a track of initial ionization in the fill gas. The size or height ΔV of the voltage pulse that is subsequently produced depends mainly on the tube's high voltage setting V_{tube}, the nature of the fill gas, and the gain of the preamp. There are three more or less distinct regions of operation, depending on the tube voltage V_{tube}.

(1) When the tube high voltage V_{tube} is low, the pulse height ΔV is determined only by the collection of the initial ionization and is independent of V_{tube}.

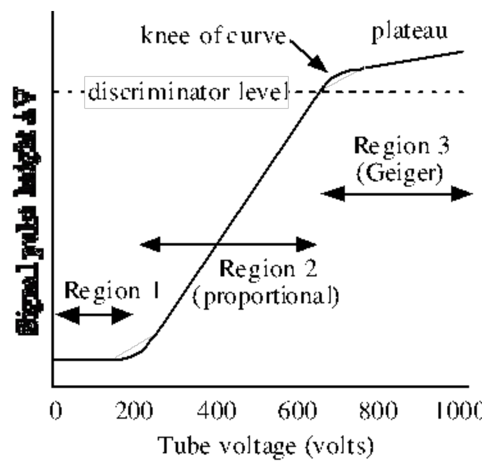


Figure 2

(2) As V_{tube} is increased to a few hundred volts, secondary ionization of the fill gas by the accelerated electrons causes a multiplication of the number of electron-ion pairs in the gas. The resulting voltage pulse may be many times bigger than the pulse that would have been produced by the original ionization. This is the "proportional" region, because the output is proportional to the ionization, and a GM tube operated in this mode is called a "proportional detector." Figure 2, which is a pulse-height vs tube-voltage graph for α-particles, shows how the pulse height is increasing as V_{tube} increases. Your curve will not be as clearly defined as this one.

(3) As V_{tube} is increased still more (to ≈ 700 V in our GM tubes), the "Geiger" region begins. In this region, an ion plasma spreads over the entire high-field region along the anode. In this situation, the pulse height no longer depends on the amount of initial ionization and is only weakly dependent on the tube voltage V_{tube}. Consequently, all pulses are of nearly equal height, and the height increases only slowly with V_{tube}. This is the **plateau region** of Fig. 2. The sudden change of slope as you enter the plateau region is called the **knee** of the curve.

We will operate the GM tubes as a Geiger counter rather than a proportional detector, so the first part of the experiment involves mapping the plateau region.

B. Discriminators and counters.

In experiments with radiation, we often want to "count" the decay particles and for this purpose we use a **counter**. The counter can be set to count output pulses for a fixed amount of time, ranging from fractions of a second to many seconds. However, it is important to count just "real" events and not pulses that might be due to electrical noise or some spurious event. In an ideal experiment, all "real" pulses exceed some threshold voltage V_0 while all "noise" pulses are less than V_0. Real experiments aren't this nicely divided, but one purpose of locating the **operating point** of a GM tube or other detector is to get as close to ideal as possible. Note that noise pulses aren't the same as **background** pulses, which are very real pulses coming from a source other than the sample.

To separate signal pulses from noise pulses, the pulses are passed through a **discriminator**. This is a comparator circuit that compares the pulse height to a preset voltage called the **discriminator level**. If the input pulse exceeds the discriminator level, the pulse is sent through to be counted. The high voltage at which this occurs is called the **threshold voltage**, and it will correspond to the beginning of the plateau region of the system. If the pulse is less than the discriminator level, the pulse is blocked and not counted. Thus the discriminator acts as a filter to block unwanted pulses. Other discriminators that we will use in later experiments have both a **lower level** and an **upper level**, and they pass only pulses that fall between the two levels.

Some experiments will use a stand-alone discriminator where you can control the levels. In this experiment, the discriminator is built into the "counter/timer" and has already been set. You will measure the discriminator level, but you can't change it.

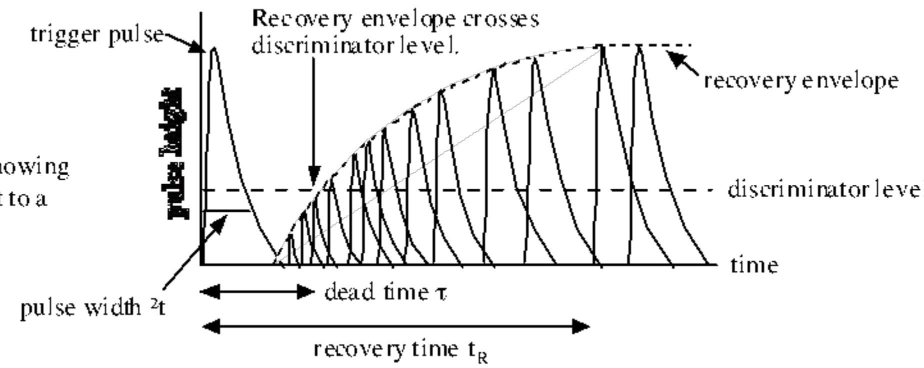
Note: Make sure you understand the difference between the *discriminator level* and the *threshold voltage*.

C. Dead-time and recovery time.

Following a Geiger pulse, a conducting plasma is formed around the anode and the electric field is reduced to a value too low to initiate a new pulse if a second particle enters the GM tube too soon. As the plasma moves out to the cathode, it is "quenched" and the field once more reaches a value at which pulses are big enough to be counted. Even then, there is a period of time in which pulses are big enough to be counted but have not yet recovered to their "normal" height.

Figure 3

Oscilloscope display showing the three times relevant to a Geiger counter.



This is shown in Fig. 3, which is the result of many superimposed oscilloscope traces. The trigger pulse is a full-size pulse that triggers an oscilloscope sweep. Subsequent pulses occur at random times after this pulse and are seen on the scope as a series of many overlapped pulses at random times. Very importantly, Fig. 3 also shows the discriminator level. Pulses are counted only if they exceed this level.

As you can see, there is a period of time τ after the trigger pulse during which any subsequent pulses are less than the discriminator level and hence will not be counted. This period of time is called the **dead time** of the GM tube. The dead time τ is measured as the time from the beginning of the trigger pulse until pulses again begin to exceed the discriminator level. You can measure τ directly from the oscilloscope.

Even after pulses again start to be counted, it takes a longer time t_R until the pulses recover to the full height of the trigger pulse. This is called the **recovery time**. For a tube operating in the Geiger mode, the recovery time is much less interesting and important than the dead time.

A third time of significance, also shown in Fig. 3, is the **pulse width** Δt. The pulse width is determined by both the GM tube characteristics and the electronics. We don't need a precise definition of Δt, but since the pulses have an approximately exponential decay we can use the e^{-1} point (≈37% of peak height) as a reasonable point to measure Δt.

Because of the dead time, a certain fraction of the particles producing ionization in the tube will be missed and not counted. However, we can correct for this. Suppose we "observe" and count R_{obs} pulses per second – that is R_{obs} is the **count rate**. Each observed pulse causes the GM tube to go dead for τ seconds, so the counter "misses" a fraction $R_{obs}\tau$ of each second. Thus the fraction of each second that the tube is actually "on" is $1 - R_{obs}\tau$. If the "true" count rate is R_{true} but the tube actually counts for only time $1 - R_{obs}\tau$ out of each 1 second, then

$$R_{obs} = R_{true}(1 - R_{obs}\tau)$$

This is easily solved to yield

$$R_{true} = \frac{R_{obs}}{1 - R_{obs}\tau}$$

If we count for time T, then the actual counts (rate × time) are

$$N_{true} = \frac{N_{obs}}{1 - R_{obs}\tau} = CN_{obs} \tag{1}$$

where $C = 1/(1 - R_{obs}\tau)$ is the **correction factor**. Notice that the correction factor depends on the **rate** R_{obs} (in counts per second), not simply on raw counts N_{obs}.

D. Counting statistics

If a counting experiment is repeated over and over, one can compute the average count $\overline{N}$ and the standard deviation σ.

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

$$\sigma = \sqrt{\overline{(x_i - \bar{x})^2}} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(x_i - \bar{x})^2}$$

This is an <u>experimental</u> value of σ. Many times, we only make one count and obtain value N. In that case, our "best estimate" is that $\overline{N} \approx N$ and $\sigma \approx \sqrt{N}$ This <u>theoretical</u> value of σ is based on the expectation that counts follow Poisson statistics. You should report the results of your single measurement as $N \pm \sqrt{N}$.

One way to *test* counting data for its randomness is to compute σ_{exp} from a series of measurements and to compare that value to $\sigma_{theory} = \sqrt{\overline{N}}$ The ratio $\sigma_{exp}/\sigma_{theory}$ is called the **reliability factor**. If it's fairly close to 1, you have confidence that your counts are random.

For low count rates, you can directly test data to see if they obey Poisson statistics. If the *average* number of counts per interval, μ, is small, then the probability of observing ν counts is given by the Poisson distribution

$$P_\mu(\nu) = e^{-\mu}\frac{\mu^\nu}{\nu!}. \tag{2}$$

If m observations are made, the *predicted* number of intervals that have ν counts is $m \cdot P_\mu(\nu)$.

This prediction can be compared to the *observed* number N_ν of intervals that have ν counts. To do so, note that your experimental determination N_ν is a count, so it has uncertainty $\pm\sqrt{N_\nu}$. If the data really is given by the Poisson distribution, then roughly two-thirds of your predicted values should fall within the error bars($\pm\sigma$) and essentially all predictions should fall within two error bars ($\pm2\sigma$).

For higher count rates, where the number of counts per interval is >>10, the Poisson distribution is approximated by a Gaussian distribution with $\sigma_{th} = \sqrt{\overline{N}}$. For a Gaussian distribution, we expect that 68% ≈ 2/3 of a repeated set of measurements should fall within the range from $\overline{N} - \sigma$ to $\overline{N} + \sigma$ and that 95% should fall within the range from $\overline{N} - 2\sigma$ to $\overline{N} + 2\sigma$. Testing these percentages is another way to test the randomness of your data.

CAUTION

A. Do not remove the GM tube from the housing. The end window is *very* thin and is easy to break.

B. Always make sure that the high voltage switch is off and the high voltage is turned down to zero when the power switch to the bin is turned on *and* when it is turned off.

C. **Do not exceed 1000 V** on the high voltage supplied to the GM tube.

Procedure

1. Check the wiring. Make sure the set-up looks like Figure 1. Enter pertinent data about instruments, radioactive sources, etc. in your lab notebook. Turn on the crate that holds the counter/timer. This provides power to the preamplifier. Set the oscilloscope initially on "automatic" trigger, DC input, 0.5 ms/div sweep, and 0.2 V/div gain.

2. The output from the Geiger tube ends up at a counter, probably an Ortec 994. Take a little time to familiarize yourself with this instrument. We will be using the input to counter B. The counter can be set to count pulses for a predetermined amount of time. This is achieved by setting an internal clock which, as you will note, has time units of either 0.01 seconds or 0.01 minutes. The number of these units is expressed as $MN\text{x}10^P$ where M and N go from 0 to 9 and P goes from 0 to 6 so the full range of the counter goes up to 99 million. You access these digits by pressing the DISPLAY button until the PRESET LED lights up. Select which of M, N or P you want using the SELECT button and then use ADVANCE to change the digits. If you set the digits to 000 the counter will count indefinitely until you stop it. If you want to set the counting time to, say, 12 seconds then the digits should read 122 with the time base at 0.01 seconds. Once the time is set then press the DISPLAY button again until the LED under B lights up.

Part 1: Plateau Determination

3. Place a ^{226}Ra radium source - yellow side UP - in the top slot beneath the counter. Set your counter's timer to 0-0, which causes it to count continuously without stopping. While observing the pulses on the oscilloscope, increase the Geiger tube high voltage (H.V.) in 100 V steps from zero to the point at which the counter starts to count. Then back off and home in as carefully as possible to the H.V. at which counting barely begins. Record the *pulse height* at which this occurs. This is the discriminator level, and the fact that pulses are starting to be counted means that it is now being exceeded. Also record the H.V. at which counting begins. This is the threshold voltage.

4. Set your counter to count for 10 s. Reduce the H.V. to 50 V below threshold, then round it the nearest 10 volts. Record the pulse height and the number of counts in 10 s. Raise the H.V. in 10 V steps, and at each step record the pulse height and the number of counts in 10 s. Continue until you are 100 V above the threshold voltage, then switch to 20 V steps and keep going to 900 V.

5. To determine the proper operating point of your GM tube, make graphs of counts-versus-H.V. and of pulse-height-versus-H.V. On each, locate the "knee" of the graph where the slope suddenly decreases. (It may not be the same on each graph. If not, pay more attention to the graph of counts.) Your **operating point** is about 100 V above the knee. (This point is chosen to make sure you're in the plateau region, but not at such a high voltage that other

effects begin to be a problem.) The pulse height *might* saturate. If that's the case, use your counts-versus high voltage graph to determine the operating point.

Part 2: Timing Measurements

6. Set your H.V. to the operating point determined in Part 5. Measure the pulse width Δt of an output pulse.

7. You should be observing a pattern of pulses similar to that in Fig. 3. Sketch a picture showing the initial pulse and the pulses in the "build-up" envelope. (You might use the persist feature on the oscilloscope.) Show time and voltage values on your axes. Add the discriminator level that you measured earlier as a horizontal line at the correct height. From your graph (and the scope), determine the dead time τ and the recovery time t_R.

Part 3: High Count Rate Statistics

8. Adjust your source to get a count rate of *approximately* 100 per second. You may be able to do this by turning the source over so that the black side is up. If this is too low, use the yellow side but pull the tray partially out until the rate drops to $\approx 100\ s^{-1}$. Set the timer to 10 seconds, then record the number of counts in 30 ten-second intervals.

Part 4: Background

9. Remove the sample and store in its container. Then record the background counts (mostly due to cosmic rays) for 100 seconds. Determine the average background count *rate* (i.e., counts per second).

Part 5: Low Count Rate Statistics

10. Based on your background rate from the previous step, determine a counting time interval that will give an average of 2 counts. Set your counter/timer as close as possible to this. Record the number of counts in 50 such intervals.

Analysis

1. Determine the range of voltages that make up the plateau. Determine the percent increase of counts per 100 V increase in the H.V. in the plateau region. (Note: This is not the same as the slope of the graph, although it's related.) A GM tube typically increases ≈15% per 100 V in the plateau region.

2. The detector records ≈10% of the total decays of the sample. Estimate the source activity of your sample in μCi (see the document on Radiation Safety).

3. Determine the value of τ from your drawing of the pulses. Calculate the count rate at which a 1% correction would be required. (Note: A 1% correction is not the same as $C = 0.01$)

4. Set up a speadsheet to analyze your high-count-rate data from Part 3. First, use Eq. 1 to correct the 30 high-count-rate values. Recall that the correction factor C uses the observed rate R_{obs}, not the observed counts N_{obs}. Then, using the corrected values, determine $\overline{N}$, σ_{exp}, σ_{theory}, and the reliability factor $\sigma_{exp}/\sigma_{theory}$ for your 30 values. Here $\sigma_{th} = \sqrt{\overline{N}}$ and σ_{exp} is the actual standard deviation of your 30 data points. For 30 repeated measurements, it can be shown that a reliability factor between 0.85 and 1.15 indicates a Gaussian distribution with 90% confidence.

5. Make a histogram of your 30 corrected counts by sorting them into "bins" 10 counts wide (e.g., 1051-1060, 1061-1070, 1071-1080, etc.). Then plot the number of counts in each bin as a function of the midpoint value of the bin (e.g., 1055 for 1051-1060, etc.). Does it look reasonably Gaussian?

6. Determine if approximately 2/3 of the counts occur between $\overline{N} \pm \sigma_{\text{exp}}$ and if approximately 95% occur between $\overline{N} \pm 2\sigma_{\text{exp}}$.

7. Compute the best value for the average counting rate and its uncertainty. (Question: Should you use the standard deviation or the standard deviation of the mean?)

8. For the low-count-rate data of Part 5, count the number of intervals in which you record 0 counts, 1 count, 2 counts, and so on. Display your results as a histogram.

9. *Test* your low-count-rate data to see if your counts obey Poisson statistics, using the method discussed earlier in the Theory section of this write-up.

Note: Saying that the histogram "appears to be" Poissonian is not a substitute for a statistical test.

Additional Questions

1. The GM tube consists of a cylindrical housing of diameter $2r_1 = 3$ cm that is grounded (0 V) and a coaxial center wire of diameter $2r_0 = 1$ mm at a high voltage V_0. The interior electric field points from the wire toward the housing. It's not hard to show that the electric field strength at radial distance r is

$$E = \frac{V_0}{r\ln(r_1/r_0)}$$

where the distances must all be in meters then E is in V/m.
Calculate E at $r = 1$ mm and at $r = 1$ cm for a typical high voltage $V_0 = 800$ V.

2. Electrons accelerate toward the center wire. For multiplication to occur, an electron must gain enough kinetic energy to ionize the next molecule it hits and kick out an additional electron. This requires a minimum of ≈ 20 eV of kinetic energy. Using the E you calculated above, calculate the distance needed by an electron at $r = 1$ cm to accelerate from rest up to a speed where it has 20 eV of kinetic energy. (The distance is small enough that you can assume a constant electric field.)

3. Compare the distance you calculated in Question 2 to the radius of the GM tube. Can you conclude that electron multiplication can occur in a GM with this much high voltage? Explain.

4. a) Look up the half life and decay mode of ^{226}Ra. (You'll find it in the CRC Handbook of Chemistry and Physics and in other handbooks.)
b) Based on the half-life and on your sample's activity (Step 2 of the analysis), determine the number of radium atoms in your sample.
c) Would you expect the decay particles of ^{226}Ra to pass through the window of the GM tube? If not, then how are ^{226}Ra decays being detected?

Nuclear physics background begins on next page…

Background Information: The Nucleus

Nucleons = Protons and Neutrons
Z = number of protons = atomic number
N = number of neutrons
$A = N+Z$ = nucleon number = atomic weight

Notation: $_Z\text{Element}^A$ or $^A\text{Element}$ (e.g.,: $_{55}\text{Cs}^{137}$ or ^{137}Cs)
Isotopes: Nuclei with the same Z (e.g., $_1\text{H}^1$, $_1\text{H}^2$, $_1\text{H}^3$)
Isobars: Nuclei with the same A (e.g., $_{55}\text{Cs}^{137}$, $_{56}\text{Ba}^{137}$, $_{57}\text{La}^{137}$)

Nuclear Energy Levels

a) Nuclei have quantized energy levels, just like atoms, only with energies of keV or MeV rather than just a few eV. Each energy level corresponds to a different nucleon configuration within the nucleus.
b) Energy level diagrams for all known nuclei are collected in the *Table of Isotopes*. Each state is designated by three quantum numbers: **energy**, **nuclear spin quantum number** I, and **parity** π.

The nuclear spin quantum number I is integer if A is even, half-integer if A is odd. The spin angular momentum of the nucleus has magnitude

$$|I| = \sqrt{I(I+1)}\hbar$$

The parity π is called "even" if $\pi = +1$ or "odd" if $\pi = -1$.

Alpha Decay: Emission of a helium nucleus

Heavy nuclei can decay by emission of an alpha particle, which is a helium nucleus $_2\text{He}^4$. The daughter nucleus has two less protons and two less neutrons than the decaying nucleus, so Z decreases by 2 and A decreases by 4.

Example: $_{95}\text{Am}^{241} \rightarrow \ _{93}\text{Np}^{237} + \alpha$

- Alphas have very low penetration - less than the thickness of a piece of paper.

Beta Decay: Transitions between isobars

In normal beta decay, called β-, a neutron is converted into a proton, an electron, and a neutrino: $n \rightarrow p + e^- + \nu$. Note that charge is conserved. The proton stays in the nucleus, so *Z* increases by 1 while *A* is unchanged (an isobar). The electron that is ejected is referred to as "beta radiation."

Example: $_{55}Cs^{137} \rightarrow {}_{56}Ba^{137} + e^-$ has $(N, Z) \rightarrow (N\text{-}1, Z\text{+}1) + e^- + \nu$.

A less common form of beta decay, called β^+ decay, converts a proton to a neutron, a positron, and a neutrino: $p \rightarrow n + e^+ + \nu$. In this case, *Z* decreases by 1.

Example: $_{59}Pr^{137} \rightarrow {}_{58}Ce^{137} + e^+$ has $(N, Z) \rightarrow (N\text{+}1, Z\text{-}1) + e^+ + \nu$.

Beta-plus decay ($Z \rightarrow Z$-1) can also occur by **electron capture**, in which an atomic electron, usually from the K shell, is "captured" by the nucleus as a proton turns into a neutron (e.g., $_{27}Co^{57}$ decays to $_{26}Fe^{57}$ by electron capture).

- Betas have moderate penetration - a few millimeters of lead.

Gamma Decay: Transitions between energy levels within a nucleus

Gamma rays: A nucleus in an excited state can make a transition to a lower energy state by emitting a high energy photon called a gamma ray. A gamma photon energy is typically in the range 10 keV < E_{gamma} < 10 MeV. The process is essentially the same as the emission of visible or ultraviolet photons by the valence electrons in an atom, but here the emission is by nucleons in a nucleus.

- Gammas have high penetration - several centimeters of lead.

Important note: Beta decay and alpha decay frequently populate excited states in the daughter nucleus. In this case, the alpha or beta decay is followed (very rapidly in most cases) by a gamma ray as the daughter nucleus de-excites. In many cases, alpha and beta emitters are not detected by observing the actual alpha or beta particle, but by observing the accompanying gamma ray photon.

Beta decay example: $^{137}Cs \rightarrow {}^{137}Ba + e^-$, followed by a 662 keV γ-ray from ^{137}Ba.

Alpha decay example: $^{241}Am \rightarrow {}^{237}Np + \alpha$, followed by a 60 keV γ-ray from ^{237}Np.