Millikan's Oil Drop

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1 Goal

The goal of this experiment is to determine the charge of an electron. We will recreate the Oil Drop measurements of Millikan in 1909 and extend these values to find the mass of an electron and Planck's constant from previously measured quantities. A stopwatch, camera, and ruled grid are used to determine the terminal velocity of droplets in the presence of an Electric field. Then, the terminal velocity is used with Newtonian mechanics to find the net droplet charge and we will show that net charges are always an integer multiple of a fundamental charge unit.

2 Introduction/Background

Modern physics and technologies desire to understand the quantized properties of physical phenomena. Many important experiments of the 20th century provide a description for the discrete characteristics of nature. For example, bound systems such as the Hydrogen atom display discrete energy levels and the photon behaves as the discrete packet of electromagnetic information. In 1909, Millikan used charged oil droplets to show that the ionic charge on a droplet is always an integer multiple of the smallest charge observed [1]. Millikan knew that oil droplets ejected from a nozzle would usually gain or lose a few electrons. In combination with the fact that the droplets soon reach terminal velocity due to air resistance, it becomes possible to observe droplet motion for prolonged periods of time. Droplets of this size and velocity will obey the stokes drag law, which predicts a drag force that is proportion to the drop speed. The droplets reach terminal velocity within milliseconds, and a measurement of the terminal velocity can be used with Stoke's law to determine the droplet radius

$$r = \sqrt{\frac{9\eta v_f}{2g\rho}} \tag{1}$$

where ρ is the droplet density, v_f is the terminal velocity, and η is the viscosity of air. Droplets are assumed to be spherical, so that the mass of the droplet can be estimated from the droplet radius and density. Therefore, it is now possible to use Newtonian dynamics and solve for the drag coefficient from

$$F_{net} = kv_f - mg = 0 \tag{2}$$

where k is the drag coefficient.

The terminal velocity of a droplet is slow enough that it is possible to suspend many droplets between a pair of electrodes and Millikan used this to take multiple measurements of the time for individual droplets to travel a certain distance in the presence of an electric field. By reversing the field between the plates, it is possible to observe the same droplet and take enough measurements to achieve great precision. In the presence of an upwards electric field, a droplet at terminal velocity will behave according to

$$F_{net} = qE - kv_u - mg = 0 \tag{3}$$

where q is the droplet charge, E is the electric field, and v_u is the terminal velocity when the electric force points upwards. After isolating q from Equation 3, it is necessary to correct for the breakdown of the fluid continuum approximation. The droplet sizes are comparable enough to the mean free path length for air molecules, so Millikan showed that Stoke's law could be modified with the correction factor

$$\gamma = (\frac{1}{1 + b/r})^{3/2} \tag{4}$$

where b is dependent on the pressure and temperature of the fluid. With the introduction of γ, the charge of an electron traveling upwards or downwards can be determined from two measurements of its terminal velocity, as shown by

$$q_u = \gamma \frac{mg}{E} (\frac{v_u}{v_f} + 1) \tag{5}$$

and

$$q_d = \gamma \frac{mg}{E}(\frac{v_d}{v_f} - 1) \tag{6}$$

where q_u and q_d are the charges calculated from the terminal velocity of upwards traveling and downwards traveling droplets, respectively.

After collecting data for the charge of many unique droplets, Millikan found that each total amount of charge could always be expressed as an integer multiple of some value that was common between all of the droplets [1]. We intend to reproduce Millikan's measurement by similar methods. Additionally, we will use our measured value of the electron's charge to calculate the electron mass and Planck's constant from the previously measured value of the electron's charge to mass ratio and the Rydberg energy.

3 Procedures and Data

In order to measure the fundamental charge of an electron, we observed and recorded the speeds of charged oil droplets in the presence of air drag and an Electric field. As seen in Figure 3, a voltage generator maintains the potential difference between the two electrodes, which are spaced to enclose a central chamber. The measured potential difference of the plates was 301.8 V (with negligible uncertainty) and we measured their spacing to be 7.63 mm. A camera looks in from the side to view droplets that are falling at terminal velocity. The lighting and camera focus were adjusted by using a focusing wire inserted through a gap in the top electrode. Droplets are side illuminated, which gives them the appearance of glowing stars floating across divisions of the grid paper.

The hole in the top electrode is large enough to deposit oil droplets from above. As the droplets were sprayed from a nozzle, an ionizing radiation source was aimed towards the center of the cavity. This way charge may accumulate on the droplets through friction in the nozzle and ionizing radiation. The ionizing source was closed before taking any measurements.

We used a reversing switch to change the polarity of the Electric field and bring the droplet of interest back into the camera's viewing region whenever necessary. First, we used a stopwatch and ipad camera to record the time required for droplets to fall 0.5 mm, which was known to be five grid divisions, in the absence of an electric field.

As seen in Table 3, we took ten measurements per drop for each travel

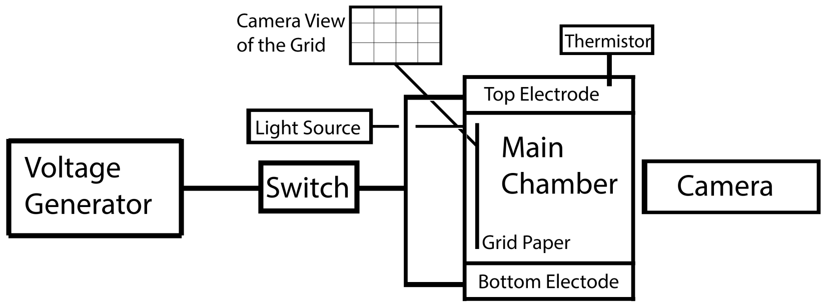

Figure 1: Block diagram of the apparatus and setup. The camera observes into the main chamber through a small hole where droplets are illuminated by the light source and seen in front of a backdrop grid.

time of interest. Then, in order to determine the electric force on the droplet we recorded the travel time of the same distance, but now with an applied Electric field. Column three of Table 3 shows the time required for a droplet to rise 0.5 mm when the Electric force opposed gravity and column four shows the recorded time for a droplet to fall 0.5 mm when the Electric force aligned with gravity. In total, we recorded measurements of four unique droplets. Some measurements, such as the last measurement in column three, were deleted if they show a large jump that represents an abrupt change in charge.

In order to determine the chamber temperature, which is necessary to calculate the air viscosity, we used a calibrated thermistor to detect any temperature fluctuations. The thermistor reported that droplet one measurements were recorded in air with temperature 24^oC, droplet two was at 25^oC, and droplet three/four were at 26^oC.

4 Analysis and Discussion

To begin the analysis we take the mean of each column in Table 3, for each droplet, and use the standard of that mean as the uncertainty in measurement times. To find the fall speed we can compute $v = h/t$, where h is always 0.5 mm. An estimate the uncertainty in the fall speed is found from the expression

$$\delta v = \frac{h}{t^2}\delta t \tag{7}$$

Table 1: Recorded travel times for our first oil drop to move the distance 0.5 mm when being acted on by an Electric field of three different configurations.

Oil Drop 1	Fall Time (s) $qE = 0$	Rise Time (s) qE points up	Fall Time (s) qE points down
1	13.60	3.79	2.89
2	14.57	2.82	2.65
3	14.60	4.37	2.73
4	15.51	4.36	2.80
5	13.69	4.67	2.34
6	14.80	4.51	2.45
7	13.78	4.64	2.45
8	14.60	4.12	2.21
9	13.90	4.43	2.60
10	14.09	~~7.90~~	2.37
Average	14.3 ± 0.2	4.2 ± 0.2	2.55 ± 0.07

where δt is calculated from the standard deviation of the mean for each column.

The electric field was specified by the measured quantities for the Voltage and plate separation from the standard equation $E = V/d$, where the values for V and d are listed in the previous section. Now we use Equation 1 to find the radius of each drop, where ρ =885 kg/m^3 and η is determined from the tabulated values for air viscosity at different temperatures

$$\eta = (1.800 + \frac{T(^oC) - 15^oC}{209})x10^{-5}\frac{Ns}{m^2} \tag{8}$$

Finally we can calculate the droplet mass from the equation $m = \frac{4}{3}\pi r^3\rho$ and the uncertainty in mass from the equation

$$\delta m = \pi r^2\sqrt{\frac{18\eta\rho}{g}}\frac{\delta v_f}{\sqrt{v_f}} \tag{9}$$

Where we have assumed no uncertainty in correction factor, γ.

Then we examined the rise time and fall time values for each drop and grouped like values of the same column if there was at least three. In this way we intend to account for any change of charge during the experiment. Using the same methods as outlined for the terminal droplet speed in the absence of an electric field, we calculated the terminal speed when the droplets were under the influence of an electric field. With these values we can calculate the charge on each drop from Equation 5 and Equation 6.

Then once the total charge on each droplet is known, we divide that total charge by integer values and search for the fundamental charge unit. Each droplet charge must be an integer multiple of the electron charge, and so the fundamental charge unit can be discovered by finding the largest factor in common with all of the total charges.

Table 2: The resulting total charge and units of fundamental charge calculated from falling measurements.

Drop Number	q [C]$x10^{19}$	δq [C]$x10^{19}$	n	e [C]$x10^{19}$	δe [C]$x10^{19}$
1	7.7	0.3	5	1.54	0.06
2	10.7	0.2	7	1.52	0.03
3.1	6.2	0.3	4	1.56	0.08
3.2	3.94	0.11	3	1.31	0.04
4	7.02	0.13	4	1.75	0.03

It appears one drop was exposed to the ionization source during our observations. This occurrence is shown in Table 4 by Drop Number 3.1 and 3.2, from the results in column four we expect this drop originally had four units of charge but lost an electron during our measurements.

As estimate of the uncertainty in the individual drop charge comes from the equations

$$\frac{\delta q_u}{q_u} = \sqrt{(\frac{\delta m}{m})^2 + (\frac{\delta v_u}{v_u + v_f})^2 + (\frac{v_u}{v_f}\frac{\delta v_f}{v_f + v_u})^2} \tag{10}$$

and

$$\frac{\delta q_d}{q_d} = \sqrt{(\frac{\delta m}{m})^2 + (\frac{\delta v_u}{v_u - v_f})^2 + (\frac{v_u}{v_f}\frac{\delta v_f}{v_f - v_u})^2} \tag{11}$$

where q_u is the charge calculated for an upwards moving droplet and q_d is the charge for a downward moving drop. Our final results for the charge of the electron $(1.54 \pm .04)x10^{-19}$ C. This value is the average of all values we calculated for the electron charge from falling and rising droplets. The uncertainty in electron charge is estimated from the standard deviation of the mean of all calculated electron charges. This appears to be a satisfactory result as it is within two standard deviations of the accepted value of $1.602x10^{-19}$ C.

In an earlier lab we measured the charge to mass ratio of an electron to be $e/m = (1.6 \pm 0.3)x10^{11}$ C/kg. Which we now use to calculate the electron mass as $(9.3 \pm 1.6)x10^{-31}$ kg. Where we estimate the uncertainty in electron mass from the quadratic sum of the uncertainty in both charge and the charge to mass ratio. This result is fairly acceptable as it lies within one standard deviation of the accepted value of $9.1x10^{-31}$ kg, but the fractional uncertainty is suspiciously high.

Additionally we can use the electron mass and electron charge to calculate Planck's constant from Rydberg energy that we also measured in an earlier lab. Planck's constant is found in the equation for the Hydrogen energy levels

$$E_r = \frac{me^4}{2(4\pi\epsilon_o)^2\hbar^2} \tag{12}$$

where E_r is the Rydberg energy. From Equation 12 and our experimentally determined value of $E_r = (13.77 \pm .01)$ eV, we calculate Plank's constant to be $h = (6.2 \pm 1.1)x10^{-34}$ Js. The uncertainty in h was also estimated by the quadratic sum of the uncertainty in electron charge, electron mass, and Rydberg energy. This result is in decent agreement with the accepted value, as our measured value is within one standard deviation of the accepted value of $h = 6.6x10^{-34}$ Js.

5 Conclusion

We have demonstrated that there is likely a fundamental unit of ionic charge that can be transferred between atoms. By tracking the motion of oil droplets at terminal velocity, we found discrete units of total accumulated charge on individual droplets, which is strong evidence that charge is a quantized property. The ability to take many measurements of a single droplet allowed for enhanced precision and a satisfactory result for the electron charge, $q = (1.54 \pm .04)x10^{-19}$ C. We also combined our result for the charge of the electron with previously measured values to determine the electron mass and Planck's constant. This experiment was of great significance in 1909, because Millikan was the first to demonstrate such a reliable method of determining the electron charge. Due to such high precision, this experiment is still continued today to search for any fluctuations of the electron charge, but so far all electrons are measured to have equivalent charges [1].

References

[1] Adrian C. Melissinos. *Experiments in modern physics.* Academic Press, 2003.