Photon Counting and the Statistics of Light

Objectives

In this lab we will use a photomultiplier tube (PMT) to directly count the number of photons arriving per unit time from two different light sources. The experiment will show the difference between the statistics of a constant intensity source and the Bose-Einstein statistics obtained from a pseudothermal source. This experiment underpins a number of light scattering techniques that are widely used in physics, biology and chemistry.

References

- "Photon Counting Statistics – An undergraduate experiment." Koczyk, P. *et al*. Am. J. Physics. **64**, 240-245, (1996).
- "Photon Counting Using PhotoMultiplier Tubes." Hamamatsu Technical Information, May 1998.
- "PhotoMultiplier Tubes R1527/R1527P" Hamamatsu Tech. Docs. Oct 1994.
- "Model SR400 Gated Photon Counter." Stanford Research Systems.

Background and theory

The photomultiplier tube

In explaining the photoelectric effect we imagine that a photon incident on a material can cause the ejection of an electron from the material's surface. This effect is utilized in the PMT where the ejected electron is accelerated through a potential difference and made to strike another surface so that several more electrons are produced. By repeating this acceleration/ejection process many times a large enough electrical pulse is generated to be seen on an oscilloscope and counted by a computer. The original photoelectron has been "multiplied." The amount of multiplication is referred to as the gain and gains of 10^6 to 10^7 are common. Note, however, that each photon incident on the PMT does not necessarily lead to an ejected photoelectron. Only a certain percentage of the incident photons do that and this percentage is referred to as the quantum efficiency (QE).

- The PMT data sheets are available in the lab. What is the gain of our phototube? What is the quantum efficiency of light at a wavelength of 633 nm?

Photon statistics

First consider a constant intensity laser beam. At high enough intensities it is not readily apparent that the beam is composed of discrete particles. However, at lower intensities the output pulses from a PMT form an irregular train which has a mean value proportional to the light intensity. But what are the details of the statistics of the pulses?

The probability of ejection of an electron from the PMT photocathode – the photoelectric effect – depends only on the intensity of the light. If the electron ejections take place randomly within a time interval δt that is very small compared to the time intervals between the ejections, then the probability of obtaining one count in a short time interval δt is

$$P(1, \delta t) = \lambda \delta t \tag{1}$$

where λ is the *photon count rate* (photons per second).

We are interested in calculating the probability of obtaining n counts in some large time interval T. To do this, let us divide the time T up into N intervals, each of duration δt, such that $N\delta t = T$. During each of these intervals we get either get 0 or 1 count. The probability of getting one count in any interval is $\lambda\delta t$ and the probability of not getting a count is ($1-\lambda\delta t$). Out of N trials you might say then that the probability of getting n counts is

$$(\lambda\delta t)^n(1-\lambda\delta t)^{N-n}$$

But this is only one way of getting n counts (n occurrences of counts and then N-n occurrences of no counts). We have to account for the fact that you can get the n counts in many different ways. How many? It would be the number of ways you can arrange N indistinguishable things taken n at a time. From statistics, this is given by

$$\binom{N}{n} = \frac{N!}{n!\,(N-n)!}$$

So finally, we have the probability of getting n counts as

$$P(n, N\delta t) = \binom{N}{n}(\lambda\delta t)^n(1-\lambda\delta t)^{N-n} \tag{2}$$

Now, if we let $N \to \infty$, $\delta t \to 0$, and recall that $N\delta t = T$ we can get

$$P(n, T) = \frac{(\lambda T)^n}{n!}\exp\left(-\lambda T\right) \tag{3}$$

Where we have used the identity

$$\lim_{N\to\infty}\left(1-\frac{x}{N}\right)^N = \exp\left(-x\right)$$

Because λ is the photon count rate, λT is the average number of photons counted in time T. We'll denote this average by n_{av}. Thus, the probability of detecting n photons in time T is

$$P(n, T) = \frac{{n_{av}}^n}{n!}\exp\left(-n_{av}\right) \tag{4}$$

You should recognize this as the Poisson distribution. In other words, a **constant-intensity light source**, such as a monochromatic laser beam, should cause electrons to be ejected from the cathode of the PMT (and thus produce photon counts) with a Poisson distribution.

- You should try to fill in the steps leading from equation (2) to equation (4)

Now, that was for the case of a constant intensity laser beam. What if the laser intensity varies with time? Of course the statistics will be changed depending on how the intensity varies. One particularly common and useful case occurs when the light at the detector is derived from a large number of independent, randomly phased sources. This occurs in a number of experimental situations – for example light scattering from microscopic particles like viruses suspended in a fluid.

In this lab we will scatter laser light from a moving ground glass screen. The inhomogeneities in the glass create random scattering centers (sources) while the movement of the glass assures that the phase relationship between the sources is continually changing. This type of light is often called "pseudothermal" since it has some of the characteristics of a traditional broad-spectrum light source.

In the lab you will be able to see the light scattered from the ground glass screen and note its granular appearance which is due to the constructive and destructive interference between the light from the scattering centers. The granular appearance is known as a laser speckle pattern. The fact that the light is sometimes bright and sometimes dim means that the photons don't continually arrive with a constant rate. Rather, they arrive with greater probability when the light is bright. This is known as **photon bunching**. The light intensity and photon picture for both cases of constant intensity and pseudothermal light is sketched in Figure 1.

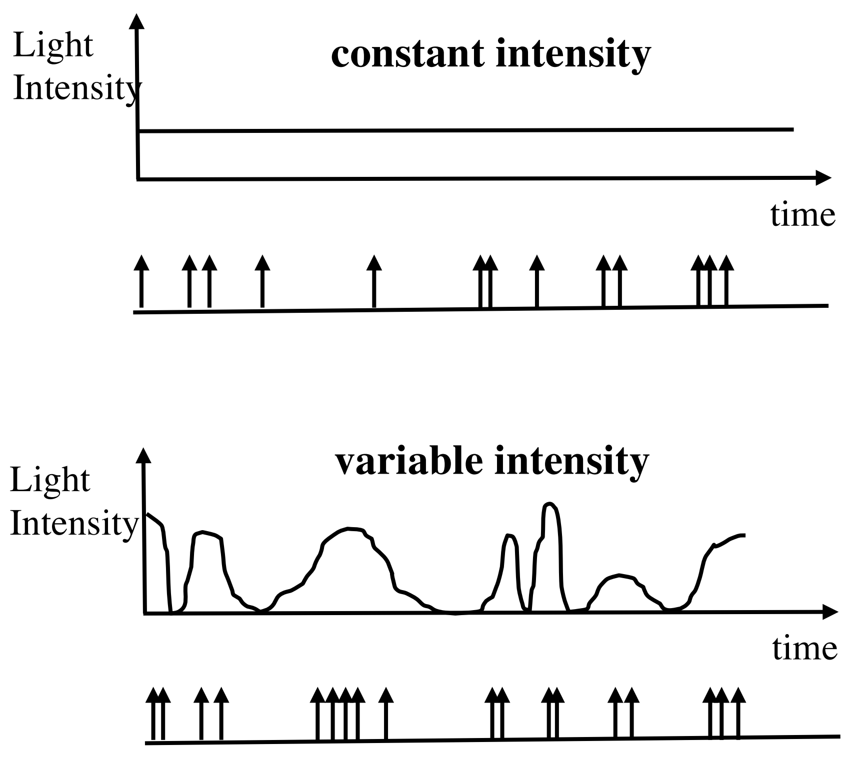

Figure 1. The upper sketch shows the case of light that is constant in intensity over time. The lower sketch shows a variable intensity source and the "bunching" of photons.

It is outside the scope of this course to derive the probability density function for the statistics of the speckle pattern, but one can show that the intensity of the light at any spatial location is distributed according to

$$P(I) = \frac{1}{I_{av}} \exp\left(-\frac{I}{I_{av}}\right) \tag{5}$$

Where I_{av} is the average intensity of the light. That is, the probability that at a randomly chosen point the intensity is between I and $I + dI$ is $P(I)dI$. Notice that the most probable intensity is zero. The intensity of the light is proportional to the photon count rate λ, so we can write

$$P(\lambda) = \frac{1}{\lambda_{av}} \exp\left(-\frac{\lambda}{\lambda_{av}}\right)$$

This is the probability that at a randomly chosen point the photon count rate is λ. Equation 3, which assumed that λ was constant, is still valid at points where the photon count rate is λ, but now we need to do a "weighted average" in which the probability of getting n counts is weighted by the probability that the photon count rate really is λ. This weighted average, which is an integral, gives us **the probability of detecting *n* photons in time *T***:

$$P(n,T) = \int_0^{\infty} \frac{(\lambda t)^n}{n!} e^{-\lambda t} \frac{e^{-\lambda/\lambda_{av}}}{\lambda_{av}} d\lambda = \frac{(\lambda_{av}T)^n}{(\lambda_{av}T+1)^{n+1}} = \frac{n_{av}^n}{(n_{av}+1)^{n+1}} \tag{6}$$

This is the Bose-Einstein distribution. Notice that the Bose-Einstein distribution always has zero counts as the most probable. You should be able to obtain equation (6), where you will need the standard integral

$$\int_0^{\infty} x^n e^{-ax} = \frac{1}{a^{n+1}} \Gamma(n+1) = \frac{n!}{a^{n+1}}$$

where Γ is the gamma function.

Both predictions, Equations (4) and (6), can be represented as histograms where the horizontal axis is "number of counts in time interval T" and the vertical axis is "number of occurrences".

- Use Matlab to make a plot of equations 4 and 6 from $n = 0$ to 10 and using $n_{av} = 4$.

Experimentally, you can *measure* the probability distribution by determining what fraction of your many measurements had 0 counts, what fraction had 1 count, what fraction had 2 counts, and so on. Your experimental probability distribution can also be portrayed as a histogram.

Procedure

There are several tasks to be accomplished. These include

a) Setting up the optical system.
b) Studying single-photon pulses on the scope.
c) Setting the discriminator on the photon counter.
d) Taking measurements of the statistics for constant intensity light.

e) Taking measurements of the statistics for pseudothermal light.

The apparatus we will use is shown in the Figure 2. The HeNe laser emits something like 10^{18} photons per second. We want to count a few thousand photons per second, so we need to reduce the intensity by something like 15 orders of magnitude. We do this in three steps.

First, we place a polarizer in front of the laser. Because the laser beam is polarized, a nearly crossed polarizer will nearly extinguish the laser beam. This polarizer is the "attenuator" shown in the figure.

Second, we expand the laser beam to a much larger diameter by sending it through a very-short-focal-length lens – a microscope objective. The beam will quickly come to a focus, then diverge after passing the focal point.

Third, the light passes through two pinholes (<100 μm diameter) at each end of a long metal tube. Any photons that survive are detected by the PMT.

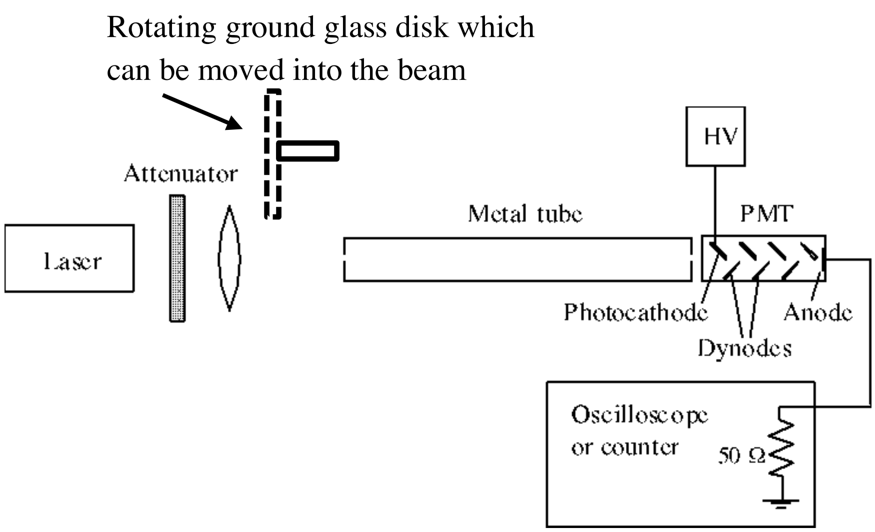

Figure 2. Schematic of experimental arrangement.

In order to have short, well-resolved pulses, the PMT output needs to be terminated with a 50 Ω resistor. This is accomplished with the 50 Ω input option on the oscilloscope OR the photon counter. The photon counter is a circuit that detects pulses above a certain threshold – the *discriminator level* – and counts the number of such pulses that arrive in a pre-set interval of time. You'll start by doing this manually, then collect a long series of samples under computer control.

Constant-intensity light

1. Set up the apparatus as shown. First install the polarizer – very close to the laser – and align things so the laser beam hits very near the center of the metal tube. Insert the microscope objective close to the polarizer, oriented so that its input – the side that would face a microscope slide – is *away* from the laser. Adjust its position so that the spread-out beam is centered on the pinhole. Adjust the polarizer so that the beam intensity is *very* weak, just barely visible on the face of the metal tube.

2. Attach the output from the PMT to the scope. Either set the scope to have a 50 Ω, DC-coupled input or, if the scope doesn't have a 50 Ω option, use a 50 Ω terminator on the cable. Set the timebase to 20 ns/div and set the scope to trigger on negative-going pulses on the input channel that you're using.

3. Turn off the room lights. Turn the HV to 1000 V and find the pulses on the scope. They are negative-going pulses. Adjust the high voltage so that the heights (in the negative direction) of the largest peaks are ≈ 200 mV. Use the trigger level to be sure you're looking at the largest pulses. Observe and report on what happens to the heights of the largest peaks as you change the high voltage to the PMT. **DO NOT EXCEED 1200 V**.

4. Adjust the scope trigger level so that you're looking at ≈ 50 mV pulses. These are more "typical" pulses than the largest ones. Sketch the pulse shape in your notebook (and later in your report), noting especially the FHWM and any "ringing" in the pulse shape.

5. Change the scope time base to 1 μs/div, so that it's 10 μs across the screen. Capture and hold one trace. With luck, you'll see several single-photon events. Adjust the laser intensity so that you can get ≈ 10 pulses in these 10 μs. If needed, adjust the high voltage so that a "typical" pulse is 30 to 50 mV.

Capture several traces and look at them carefully for similarities and differences. For example, are the pulses equally spaced? Does each trace (a 10 μs window) contain the same number of pulses? Can you see any obvious pattern to the pulses, or do they appear to be random? Report on this, and any conclusions that you can draw, in your written report.

6. On the photon counter use the A channel and set the mode to TSET. Switch to the A discriminator and set the discriminator level to –50 mV. Set the acquisition time to 1 sec (TSET = 1 s) and the pause between counts to 1 s (DWELL = 1 s). Make sure you're in the mode that counts over and over, rather than counting once and stopping. Adjust the light intensity of the laser until you get roughly 3000 counts/sec.

7. Now you are ready to set the discriminator on the photon counter. As you lower the discriminator (towards zero), you'll count more photons but you'll also let through more noise pulses due to the *dark rate* of the PMT. Let S/N = signal-to-noise ratio = real photon counts/noise counts. S/N decreases at very low discriminator settings because the noise level goes up. It also decreases at very high discriminator settings because you've blocked most real counts. Ideally, there's a discriminator setting that optimizes S/N. In practice, there's usually a fairly broad range over which S/N is reasonably constant.

First record the counts per second as you change the discriminator from –5 mV to –120 mV. Try going in 5 mV steps to –20 mV, then 10 mV steps to –60 mV, then 20 mV steps to –120 mV.

Then turn off the laser and all lights except for a weak flashlight to read the counter. Change the acquisition time to 10 s (for better statistics), then go through the same discriminator settings measuring the PMT dark rate. You can record N as the 10-second count divided by 10. If you are not getting any noise counts, increase the acquisition time accordingly.

Graph S/N versus the absolute value of the discriminator setting to locate the maximum and determine your optimum discriminator setting.

8. After setting your optimum discriminator level, you are ready to start looking at the statistics of the laser light. Turn the laser back on and verify that you are still getting *about* 3000 counts per second (you needn't be overly precise) at your final discriminator setting. If not, adjust the laser intensity. Then change the acquisition time (TSET) to 1 msec. Now you should get an average of ≈ 3 counts per interval. Finally, change the photon counter mode from Start to Stop.

9. Launch the Matlab program collect_photon_counts2014 on the desktop of computer. You should only modify four variables: *num_intervals, tset, discriminatorLevel,* and *delay.* **Do not change anything else!** Set the program to collect 100 1-msec samples with a *delay* setting of zero. You'll see counts appearing in the window and – if it worked correctly – the last count displayed on the computer should match the counts now showing on the photon counter.

If all is working, set it to collect 1000 1-msec samples. Save this data as a .csv file, then open it in Excel.

10. The raw date is simply a sequence of numbers with the number of photon counts per interval. Verify that you have 1000 numbers. Quickly compute the average and verify that it is near 3. If one or both of these isn't true, determine what's wrong.

11. Do two more runs, one with the laser intensity adjusted to give ≈ 1000 counts per second and one at ≈ 10,000 counts per second.

Pseudo-thermal light

1. Set up the diffusing ground glass screen as close as possible to the microscope objective. If you turn up the laser intensity, you should be able to see large speckles covering the front of the collection tube. The speckles will drift across the front of the tube when you make the screen move with the motorized stage. It is this intensity variation that has the statistics given by equation 5. The speckles must move sufficiently slowly that the intensity is essentially constant during the 1 msec counting interval, but changes between counting intervals.

2. With the motor running, adjust the laser intensity so that you are getting ≈ 3000 counts per second as you did with the constant-intensity source. There will be very large fluctuations from count to count, so you're looking for an *average* of ≈ 3000/s.

3. Set the program to collect 1000 1-msec samples, but this time set the delay to be ≈ 300 ms (take into consideration the Matlab script delay as mentioned in the code). The delay ensures that the speckle pattern is different for every sample. Start the data collection. Save the data.

5. Repeat at ≈ 1000 counts/sec and at ≈ 10,000 counts/sec.

Analysis and Report

You now have several data sets of the number of photon counts per millisecond interval. For each of them,

- Construct a *probability distribution* by counting the number of events that have no photons, one photon, two photons, etc. divided by the total number of events. Plot your results as a histogram. Because each histogram bar is a *count*, it has counting uncertainty ($\pm N^{1/2}$) associated with it. **Show your error bars**. Excel can put error bars on a histogram, but you'll have to figure out how to do it.
- Overlay a theoretical plot obtained from Equation 4 or 6. Be sure to first determine an accurate value of n_{av} from your data.

Looking at the paper by Koczyk *et. al.* should help you with your analysis.

You should have pretty good agreement between the experimental results and the theoretical plots if the experiment has been done correctly. But how can you quantify this? One statistical method to test if a hypothesis is supported by the data is to calculate the "chi-square," which is defined as

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}\frac{(y_i - y_{\exp i})^2}{\sigma_i^2} \tag{8}$$

where N is the total number of "bins" in your histogram, y_i is the number of counts in the i^{th} bin, and $y_{\exp\, i}$ is the number of counts expected from theory. σ_i is the standard deviation of the number of counts in the i^{th} bin and, because this is a counting experiment, is the square root of the expected number of counts ($\sigma_i = y_{\exp\, i}{}^{1/2}$). Don't forget to square σ_i! What is a good value for χ^2? *If* the model is a good fit to the data, then, on average, each value of $y_i - y_{\exp\, i}$ should be roughly σ_i. Thus each term in the sum should be roughly 1, so the sum should be $\approx N$. When divided by N, the whole of Equation (8) should be close to 1. If your value of χ^2 is roughly 1, then the hypothesis is supported by the data. If your value of χ^2 is significantly larger than 1, then the deviations of experiment from theory are consistently larger than the error bars and the model of photon counting is not supported by the data. If your value of χ^2 is significantly less than 1, then you've computed something wrong. Consult Melissinos (or another book on error analysis) if you want to learn more details about testing the value of χ^2.

In addition to reporting on the statistics,

- Report on the various observations you made using the scope.
- Use your observations and data to make a reasonable estimate of the number of electrons in the PMT output pulse. Compare this to the expected gain at the high voltage you were using. (You'll find this information in the Hamamatsu "PhotoMultiplier Tubes R1527/R1527P" booklet, which should be out.) Are they about the same, or different? Why?