THE ZEEMAN EFFECT

OBJECTIVES

To measure the Zeeman splitting of the degenerate energy levels in cadmium atoms and to make an experimental determination of the Bohr magneton. The very small wavelength shifts are measured using a high-resolution Fabry-Perot interferometer.

REFERENCES

Modern physics textbooks usually have a brief introduction to the Zeeman effect. Operation of the Fabry-Perot interferometer and details of the Zeeman experiment are covered in Melissinos (Sections 6.2 and 6.5) and in Preston and Dietz, *The Art of Experimental Physics*.

INTRODUCTION

The Zeeman Effect

Consider an atom with an electron in an excited state (we'll call it state 2). If the electron makes a transition to lower state (say state 1), it will emit a photon of energy equal to the energy difference between the states:

$$E_{\text{photon}} = hf_0 = hc/\lambda_0 = E_2 - E_1. \tag{1}$$

If a magnetic field is applied to the atom, the excited state will split into $2J_2+1$ levels and the lower state into $2J_1+1$ levels, where J_2 and J_1 are the quantum numbers of total angular momentum for the excited state and the lower state. As a result, the spectral line will exhibit "Zeeman splitting." The effect of magnetic fields on spectral lines was discovered by the Dutch physicist Pieter Zeeman, for which he shared the Nobel prize in 1902 with H.A. Lorentz.

We will observe this splitting in the 643.8 nm line in cadmium. Cadmium has two electrons outside the filled 4d shell. We're interested in electron configurations where the electrons are paired with opposing spins, so that $\mathbf{S} = \mathbf{s}_1 + \mathbf{s}_2 = 0$ and the total spin quantum number is $S = 0$. (Note that we use **bold** letters to represent angular momentum vectors, regular letters to represent quantum numbers.) These configurations have *multiplicity* $2S + 1 = 1$ and are called *singlet states*.

The $\lambda_0 = 643.8$ nm line in cadmium is a transition from level 5s6d ($^1D_2$) to level 5s6p ($^1P_1$). This is shown in Fig. 1. Recall the atomic physics notation

$$^{2S+1}L_J,$$

where the orbital angular momentum L is represented by a letter rather than a number. Thus $^1D_2$ has $S = 0$ (no net spin), $L = 2$, and $J = 2$. $^1P_1$ has $L = 1$ and $J = 1$. No net spin means that there's no distinction between orbital angular momentum $\mathbf{L}$ and total angular momentum $\mathbf{J}$.

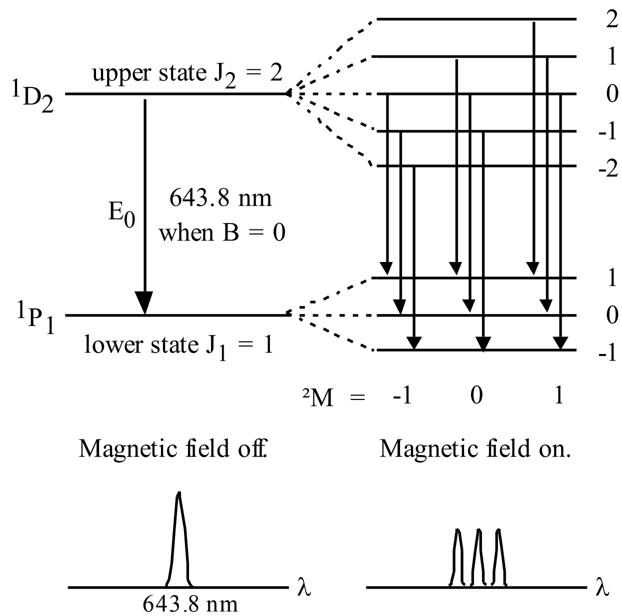

Figure 1. Zeeman effect in the 643.8 nm line of cadmium in a magnetic field. The single spectral line splits into three closely spaced lines.

Figure 1 also shows the splitting that occurs upon application of a magnetic field. In zero field, all states have the same energy - they're degenerate - but in a magnetic field, states with different M have different energies.

We've chosen singlet states for this experiment because the splitting when $S = 0$ (called the "normal" Zeeman effect) is simpler than if electron spins are involved. It gives rise to a symmetrical three-line Zeeman pattern in which the middle line wavelength is the same as in zero magnetic field.

Even though the upper term is split into five levels and the lower term is split into three levels, there are only 9 transitions with just 3 different transition energies, corresponding to $\Delta M = +1, 0, -1$. This simplification is due to:

- The quantum mechanical selection rule that transitions have $\Delta M = 0$ or ± 1, and
- The fact that the Zeeman shifts in both levels are the same.

Consequently, the original spectral line at wavelength λ_0 is split into three lines with slightly different wavelengths.

The Zeeman shift is due to the interaction of the atom's magnetic moment with the magnetic field. The magnetic energy U of a magnetic moment **m** in a field **B** is

$$U = -\mathbf{m}\cdot\mathbf{B}. \qquad (2)$$

The magnetic moment of an orbiting electron with angular momentum **J** is

$$\mathbf{m} = -\mu_B \mathbf{J}/\hbar \qquad (3)$$

where μ_B is the Bohr magneton. The Bohr magneton is a constant of fundamental importance because it determines the size of magnetic effects in atoms. The theoretical value is $\mu_B = eh/4\pi m = 9.27 \times 10^{-24}$ J /T, where m is the electron mass.

Combining Eqs. 2 and 3, the magnetic energy is

$$U = -\mu_B \mathbf{J}\cdot\mathbf{B}/\hbar = -\mu_B J_z B/\hbar = -M\mu_B B, \tag{4}$$

where from quantum mechanics $J_z = M\hbar$ and where we've assumed that the magnetic field **B** points in the z direction. M is called the *magnetic quantum number*.

From Fig. 1, the transition from upper state M_2 to lower state M_1 has energy

$$E_{\text{photon}} = E_2 - E_1 = (E_{2\ \text{initial}} + M_2\mu_B B) - (E_{1\ \text{initial}} + M_1\mu_B B) = E_0 - \Delta M\mu_B B \tag{5}$$

where $\Delta M = M_2 - M_1$. The energy shift, relative to the original zero-field line, is

$$\Delta E = h\Delta f = \Delta M\mu_B B. \tag{6}$$

According to quantum mechanics, ΔM *has* to be 0, or ±1. The three transitions with $\Delta M = 0$ are unshifted – their wavelength is the same as in zero field. All three transitions with $\Delta M = +1$ are shifted up in frequency ($\Delta f = \mu_B B/h$), while all three with $\Delta M = -1$ are shifted down by the same amount. There are only three distinct wavelengths in the Zeeman-split spectral line, even though there are nine transitions.

An important additional feature of the Zeeman spectral lines, justified in quantum mechanics textbooks, is that they are *polarized* relative to the magnetic field direction. The polarization directions are shown in Fig. 2. If the light is observed transverse to **B**, as we will be doing, then the frequencies and polarizations are:

ΔM	Frequency	Polarization relative to **B**
–1	$f_- = f_0 - \mu_B B/h$	perpendicular
0	f_0	parallel
1	$f_+ = f_0 + \mu_B B/h$	perpendicular.

If you observe the light through a **polarizer**, you can see *only* the single f_0 emission when the polarizer transmission axis is parallel to the magnetic field or *only* the two lines at f_+ and f_-, if the polarizer is perpendicular to the magnetic field.

One can easily calculate the expected shifts in frequency or wavelength that will be measured in the experiment. For a field $B \approx 0.3$ T, the frequency *shift* corresponding to $\Delta M = \pm 1$ is $\Delta f = \mu_B B/h \approx 4\times 10^9$ Hz. The 644 nm line has $f_0 \approx 4\times 10^{14}$ Hz, so the fractional shift $\Delta f/f_0$ is $\approx 10^{-5}$. We need to use high-resolution spectroscopy.

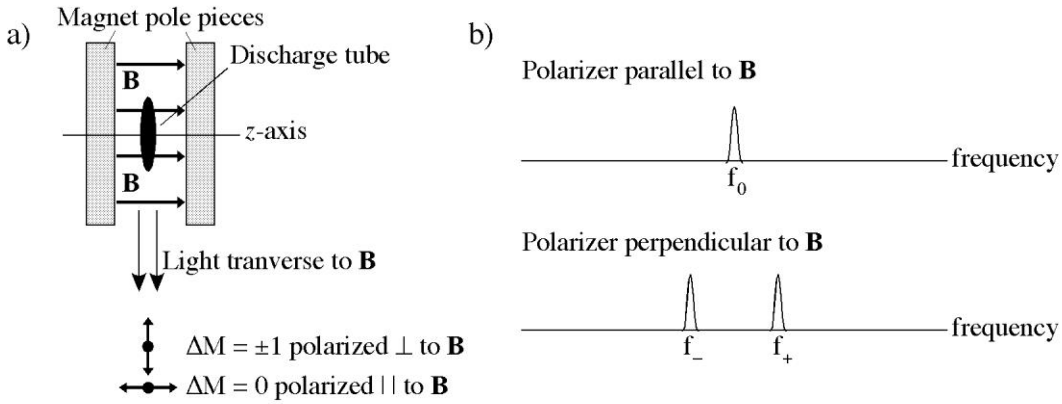

Figure 2. a) The emission transverse to the magnetic field is polarized parallel to or perpendicular to **B**. b) Observing the light through a polarizer reveals a single line at f_0 or two shifted lines at f_+ and $f_–$.

The Fabry-Perot Interferometer

A Fabry-Perot interferometer – also called an **etalon** – consists of two highly reflective ($R >$ 95%) surfaces placed exactly parallel to each other with separation t. In our etalon the medium between the mirrors is glass with a refractive index of n=1.46 at a wavelength of 644 nm. Because the surfaces are so reflective, very little light can pass through from left to right. The net transmission is $\approx(0.05)^2 = 0.0025$. **But**, transmission can be extremely high – essentially 100% – if the light waves set up standing waves between the reflecting surfaces.

Figure 3a shows the on-axis light that is perpendicular to the reflectors. You saw similar pictures of standing waves in Physics 132. The distance between two nodes is $\lambda/2$, so the condition for a standing-wave resonance is

$$t = m\cdot(\lambda/2n) \quad \text{or} \quad f = mc/2nt \quad \text{(on-axis transmission),} \tag{7}$$

where m is an integer and n is the refractive index. An optical analysis shows that this condition gives **constructive interference** (hence this is an *interferometer*) in the forward direction, and hence high transmission. The integer m for standing waves on strings is usually very small. But m for an optical interferometer m can be very large. The etalon in this experiment has $t \approx 1$ cm. Because $\lambda = 644$ nm, you can easily confirm that $m \approx 45{,}000$.

The etalon transmission is essentially 100% when Eq. 7 is satisfied. The transmission is ≈0% if the frequency is shifted slightly off resonance. Suppose that we monitor the on-axis transmission through a Fabry-Perot etalon while we slowly vary the frequency of the light. Figure 3b shows a periodic series of transmission maxima. These are the frequencies that satisfy Eq. 7 with integers m–1, m, m+1, m+2, ...

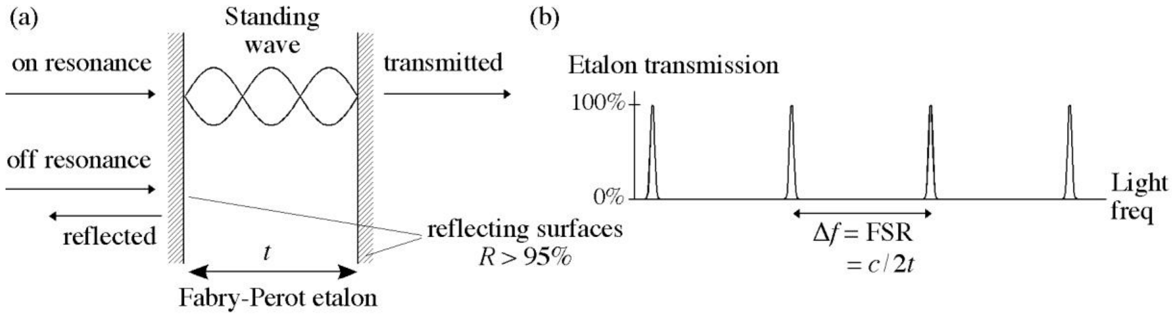

Figure 3. a) On-axis transmission of a Fabry-Perot etalon occurs when the light wavelength is able to set up a standing wave. b) Transmission as the frequency is changed shows periodic bursts of high transmission. The frequency spacing is called the free spectral range (FSR).

The frequency change needed to go through one cycle ($\Delta m = 1$), from peak to peak, is seen (from Eq. 7) to be

$$\Delta f = \mathrm{FSR} = c/2nt. \tag{8}$$

This is called the **free spectral range** of the etalon, and it depends only on the reflector spacing t. For $t = 1.0$ cm, the FSR is 1×10^{10} Hz. This is very close to the expected Zeeman shift, which we estimated to be 0.4×10^{10} Hz at $B = 0.3$ T.

In a practical set-up the output from the Fabry-Perot is obtained as follows.

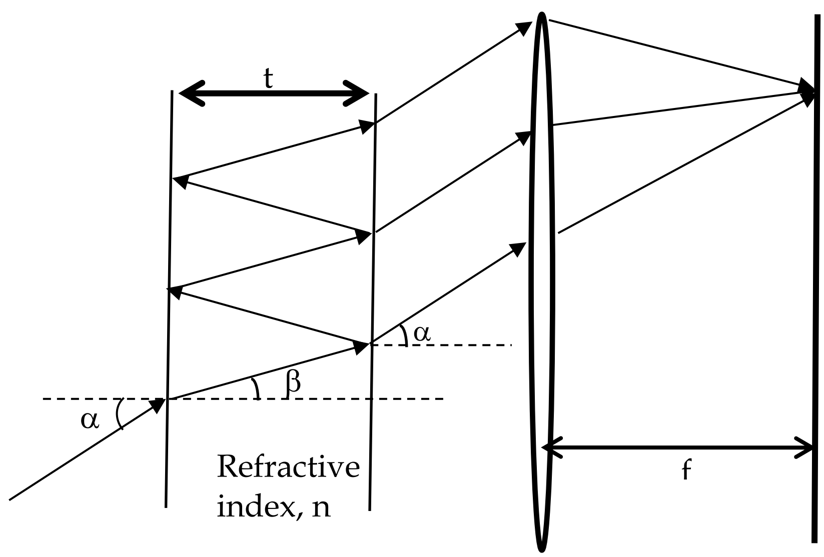

Figure 4. Diagram showing the entrance of a single ray into the Fabry-Perot interferometer. The mirrors are separated by a distance t and the medium between the mirrors has refractive index n. The medium outside is air with refractive index 1. Parallel rays exiting the Fabry Perot are collected by the lens of focal length f and brought together at the lens focal plane.

Figure 4 shows the geometry where a single ray of light is entering the interferometer at angle α. From Snell's law we have

$$\sin(\alpha) = n \sin(\beta) \tag{9}$$

And the optical path difference between adjacent beams exiting the interferometer is

$$\delta = 2nt \cos(\beta) \tag{10}$$

where for constructive interference (a bright fringe) this must be an integral number of wavelengths of the light.

If we now imagine that the wavelength of the input light is initially λ_1, creating a bright fringe at some angle β_1 and that this wavelength changes to λ_2 yielding a bright fringe at β_2 then

$$\lambda_1 = \frac{2n}{m} t \cos(\beta_1) \qquad \text{and} \qquad \lambda_2 = \frac{2n}{m} t \cos(\beta_2) \tag{11}$$

where *m* is some integer. Then we get

$$\frac{\lambda_2 - \lambda_1}{\lambda_1} = \frac{\Delta\lambda}{\lambda} = \frac{\cos(\beta_2)}{\cos(\beta_1)} - 1 \tag{12}$$

And from this we can calculate the change in energy as the wavelength changes.

$$\Delta E = -\frac{hc}{\lambda}\frac{\Delta\lambda}{\lambda} \tag{13}$$

Where h is Planck's constant and c is the speed of light.
Before you go on with the experiment check the following: show that if the external angles you measure are $\alpha_1 = 0.740^o$ and $\alpha_2 = 0.788^o$ then the change in energy is 10.1 μeV.

In practice, because the source is extended there will be many rays entering the interferometer and you will see a set of circular fringes at the focal plane.

PROCEDURES

The arrangement for observing the interference pattern is shown in Figure 5.

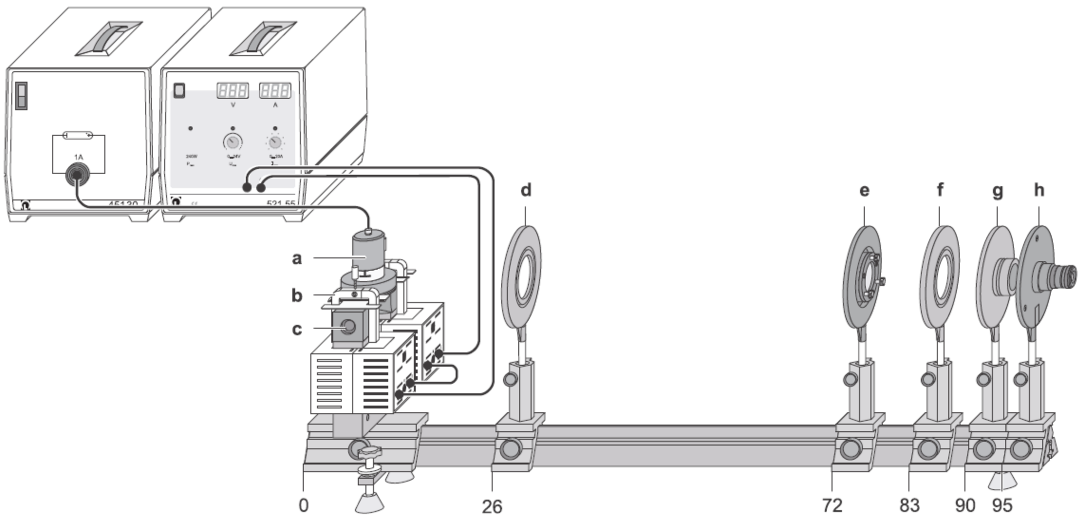

Figure 5. Experimental arrangement.
a Cadmium lamp with holding plate. b Clamps. c Pole pieces. d Positive lens, f=150 mm (Condenser Lens).
e Fabry-Perot etalon. f Positive lens, f=150 mm (Imaging lens). g Color filter (transmits 644 nm light).
h Ocular to view the fringe pattern. This is later replaced with the video camera.

You will find it useful to consult the Leybold Physics Leaflet "Normal Zeeman effect", available at
http://www.ld-didactic.de/literatur/hb/e/p6/p6274_e.pdf

First you need to determine the magnetic field between the pole pieces as a function of the current (the cadmium lamp is removed). This is done by measuring the field using the Gauss meter provided. Your data should look similar to that on the graph shown as *diagramm 1*in the leaflet, reproduced here in Figure 6. After the magnetic field is determined the cadmium lamp is put back between the pole pieces.

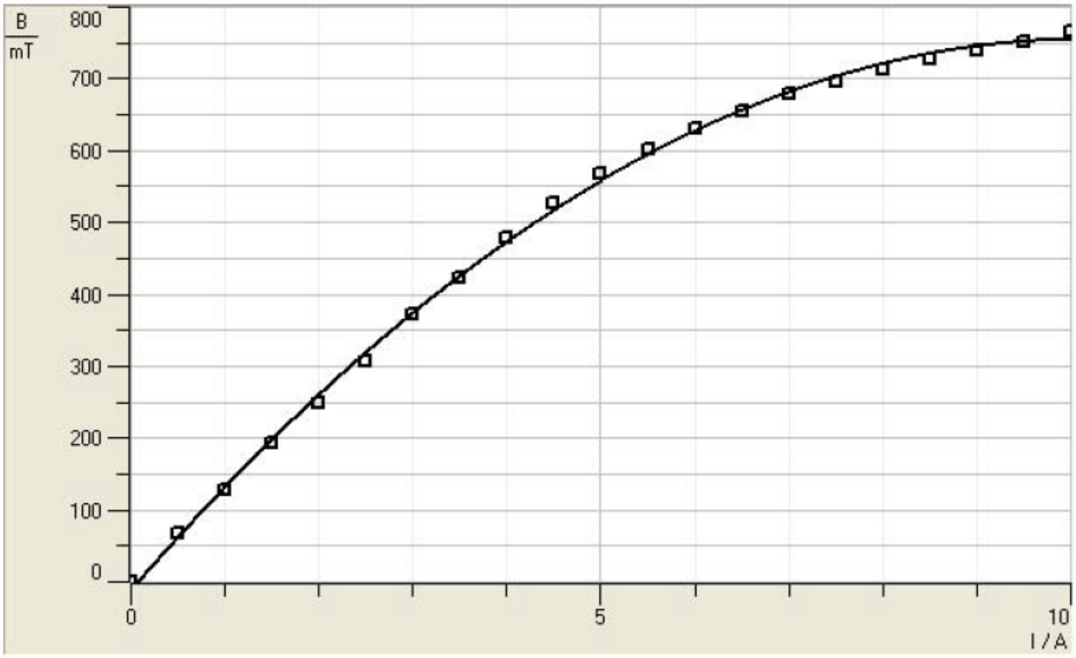

Figure 6. Typical results for the magnetic field strength as a function of the current. Note that your measurements will NOT be exactly the same as these.

Set up the optical system as indicated in Figure 5. Using the eyepiece provided make sure you can see the distinct Fabry-Perot circular pattern. Adjust the optics and the Fabry-Perot until you have a clear, centered pattern. Observe what happens as you turn up the current to the magnet – each bright circular ring will divide and turn into three rings. Place a polaroid in the optical path. Note that the unshifted ring (the π ring) is polarized at right-angles to the shifted rings (the σ_+ and σ_- rings). Leave the polaroid in place.

Now remove the eyepiece and replace it with the camera. Note that this camera has just one line of detector elements. Ensure the camera lens is removed as you are going to project the Fabry-Perot pattern straight on to the sensor. The output from the camera can be monitored via USB on the PC. To see the camera output make sure the camera power is on, the USB cable is attached, and that you have double clicked the ***VideoCom Intensities*** icon on the desktop.

Set the camera so that the sensor is about 1 focal length (150 mm) from the last lens. Adjust the camera position until the output seen on the computer screen is similar to Figure 7. (Your data might not have quite as much contrast between maxima and minima as that shown.)

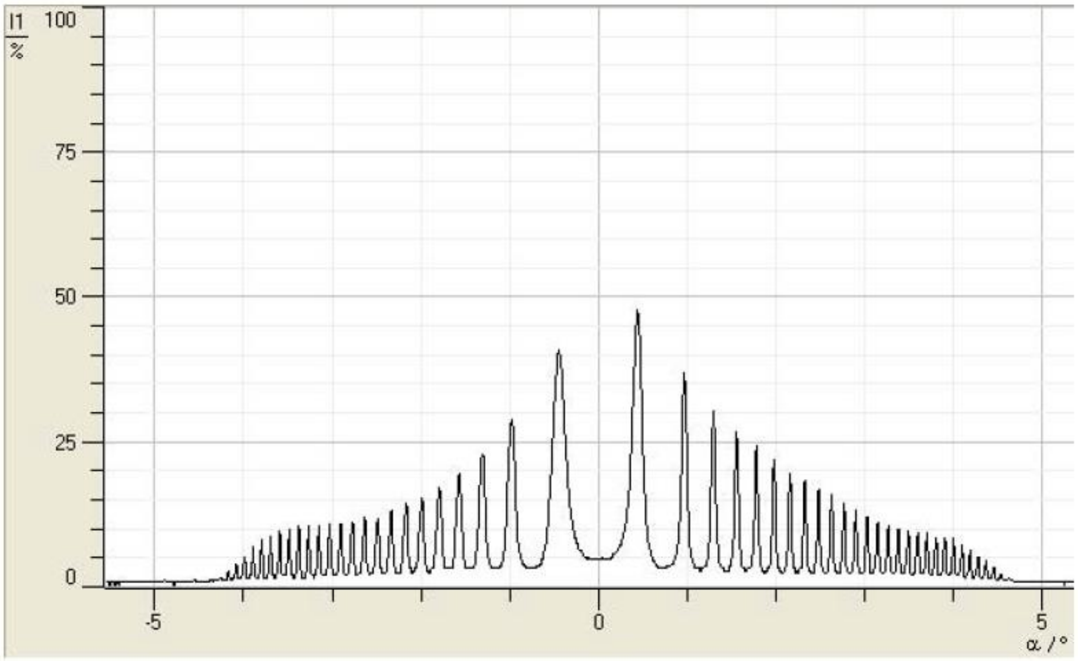

Figure 7. Typical output from the video camera linear array

In order to measure the wavelength shift as a function of magnetic field one needs to know the angular position of the constructive interference (bright rings) of the light that passes through the Fabry-Perot.

Because the lens focal length is known, position on the sensor is proportional to the angle of the output light from the Fabry-Perot. However, we do need to tell the camera/software what the focal length is and to set the zero of the angular output. This is done as follows

- Click the calibration icon on the VideoCom toolbar and enter the lens focal length. Hit return and note that the bottom horizontal scale has changed to degrees.
- Now go to the two central peaks in turn and record their angular positions. You do this by placing the cursor on the peak and looking at the columns on the left hand side of the intensity display where you will see the angular position highlighted.
- Take the average of these two numbers and, returning to the calibration icon, enter this average as its *negative* and hit return. This will change the scale so that zero degrees is now half-way between the two peaks.

You are now ready to start taking data.

Before you apply any magnetic field, record the angular position of one of the un-shifted peaks. Set the polaroid so that the π light is extinguished. Turn up the current to the magnet – from your earlier calibration you can translate this current into a magnetic field. You will notice that about 1500 G the central peak will divide into two so that you can see two peaks and using your cursor you can measure the angular positions of the shifted peaks (both σ_+

and σ_-). Note that you can zoom in on the peaks and that there is an averaging option on the toolbar which will help smooth out the peak positions.

Record the angular positions of the peaks as a function of the applied field, making an estimate of the uncertainty in the angle. From the angles you can deduce the energy shifts, as described above. You will be able to get two energy shifts at each magnetic field setting – one for the upshifted energy and one for the downshifted. As you can see from equation (6) a plot of energy shift vs. magnetic field should lie on a straight line and have a slope equal to the Bohr magneton. You can use the MATLAB program appended here to plot and fit the data.

Appendix A: Program to plot and fit the Zeeman experiment data. See Taylor's book on error analysis for a derivation of fitting equations.

```
%  Zeeman Experiment
%   weighted fitting of a line to the energy vs magnetic field
%   the raw data is the angle obtained from the video camera.
%   You will supply an estimate of the angular uncertainty for each angle
%   measurement. Some typical data has been supplied below in the variable
%   x (array of magnetic field values, in Gauss)
%   alpha1 (initial angle of an unshifted bright fringe)
%   alpha (array of angles you measure as a function of the magnetic
field)
%   delta_alpha1 (uncertainly in the initial angle)
%   delta_alpha (uncertainty in the shifted angles)

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clear all; clf;
h=6.63e-34; c=3e8; lam=644e-9;
% enter the data.
x = [1503 1803 2108 2271 2556 2824 2990 3251 3490 3605 3806]; %the indep.
variable (magnetic field)
%which we convert from Gauss to Tesla
x=x/10000;

alpha1=0.74; %angle of unshifted spectral line (degrees)
%convert to radians
alpha1=alpha1*pi/180;
%the next line is the angle of one of the shifted lines corresponding to
%the various magnetic fields
alpha = [0.772 0.777 0.788 0.793 0.799 0.804 0.809 0.815 0.82 0.82 0.825];
%the angle
%convert to radians
alpha=alpha*pi/180;

beta1=asin(sin(alpha1)/1.46); %this is the internal angle in the Fabry
Perot
beta=asin(sin(alpha)/1.46);

% now calculate the energies
del_lam_over_lam=(cos(beta)/cos(beta1))-1;
E = -(h*c/lam)*del_lam_over_lam;
%and convert to micro_eV
E=1e6*E/1.6e-19;

%Get the uncertainties in the energies. This follows from the
%uncertainties in the angles, alpha1 and alpha
delta_alpha1=0.005;
delta_alpha = [0.005 0.001 0.001 0.001 0.001 0.001 0.001 0.001 0.001 0.001
0.001]; %angle uncertainty
%convert to radians;
delta_alpha1=delta_alpha1*pi/180; delta_alpha=delta_alpha*pi/180;
%and find the uncertainty in beta
delta_beta1=delta_alpha1*(cos(alpha1)/(1.46*cos(beta1)));
delta_beta=delta_alpha.*(cos(alpha)./(1.46*cos(beta)));
```

```
deltaE=-(h*c/lam)*sqrt(((sin(beta)/cos(beta1)).*delta_beta).^2 +...
    (cos(beta)*sin(beta1)./(cos(beta1)).^2.*delta_beta1).^2);
deltaE=1e6*deltaE/1.6e-19; %the uncertainty in the energy in micro_eV

%plot the data

errorbar(x,E,deltaE,'k.');
xlabel('Magnetic Field (T)');
ylabel('Energy Shift (\mu eV)');
hold on

%Now compute the coefficients for the weighted fit. See equations 8.37,38
%and 39 in Taylor
w=1./(deltaE.^2); %the weights
del=sum(w*sum(w.*(x.^2)))-(sum(w.*x))^2;
A=(sum(w.*(x.^2))*sum(w.*E)-sum(w.*x)*sum(w.*x.*E))/del;
B=(sum(w)*sum(w.*x.*E)-sum(w.*x)*sum(w.*E))/del;

Fit=A+B*x;
plot(x,Fit);

%What are the uncertainties in A and B?
%These are given by formulas quoted in problem 8.19
sig_A=sqrt(sum(w.*(x.^2))/del);
sig_B=sqrt(sum(w)/del);

disp(['the intercept is ' num2str(A) ' with uncertainty ' num2str(sig_A)])
disp(['the slope is ' num2str(B) ' with uncertainty ' num2str(sig_B)])
```