The Compton Effect

James Amarel and Kevin Pederson

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1 Goal

To determine if Compton's theoretical description of gamma ray scattering is accurate and if so, calculate the electron rest mass. Additionally, we will compare the differential cross sections of Thomson and Klein-Nishina in order to determine which better describes gamma scattering off electrons.

2 Introduction/Background

Gamma rays traversing a material are absorbed by atomic electrons, which recoil and subsequently emit a new photon of a different wavelength [1]. This wavelength shift is not explained by classical electrodynamics, as the wave frequency is a characteristic property that cannot be altered. However, if one considers this event as the collision between a photon and electron, conservation of energy-momentum leads to a relation for the change in wavelength across the interaction in terms of the scattering angle, known as the Compton shift

$$\lambda' - \lambda = \frac{hc}{mc^2}\big(1 - \cos\theta\big) \tag{1}$$

where h is Planck's constant, c is the speed of light, θ is the scattering angle, and we shall use $mc^2 = 511$ keV to be the rest energy of an electron. Gamma rays are a prime choice for scattering experiments because their energy is much larger than the electron binding energy, so the gamma does not lose a significant portion of its energy in ejecting the bound electron and thus we may approximate the interaction as being with a free electron. Additionally, for extreme scattering angles the gamma ray energy may be reduced by a factor of 3, which allows clearer measurements than for rays of larger energy, such as X-rays, which would only see an energy reduction of approximately 1% [2].

Although the electron is believed to be a point particle, its electric field gives rise to an effective area where incident rays are scattered. By calculating the radiated power per solid angle due to the plane wave induced acceleration an electron, Thomson classically determined the electron's differential cross section

$$\frac{d\sigma}{d\Omega} = r_o^2\left(\frac{1 + \cos^2\theta}{2}\right) \tag{2}$$

where $r_o = 2.82 \times 10^{-15}$ m is called the classical electron radius, θ is the scattering angle, and by integrating all differential solid angles $d\Omega$ the total scattering cross section of an electron is found to be $\sigma = 6.66 \times 10^{-29}\ m^2$. Qualitatively, the cross section is an expression for the scattered flux per incident flux per unit area, thus the differential cross section is proportional to the intensity as measured by a detector occupying solid angle $d\Omega$.

Klein and Nishina then expanded upon Thomson's Equation 2 with a quantum mechanical derivation of the differential cross section, accounting for effects such as relativity and frequency dependent scattering

$$\frac{d\sigma}{d\Omega} = r_o^2\left(\frac{1+\cos^2\theta}{2}\right)\left(\frac{1}{(1+\gamma(1-\cos\theta))^2}\right)\left(1+\frac{\gamma^2(1-\cos\theta)^2}{(1+\cos^2\theta)(1+\gamma(1-\cos\theta))}\right) \quad (3)$$

where $\gamma = hf/mc^2$ is the ratio of photon energy to the electron rest energy. For low energy electromagnetic radiation or large electron rest energy, $\gamma \to 0$ and Equation 3 reduces to the classical result of Equation 2.

3 Procedures and Data

We used a 2 mCi ^{137}Cs source to produce γ rays that were collimated and directed by lead shielding at a 0.5 *in* diameter Aluminum rod as the scattering surface. As seen in Figure 1, scattered gammas are detected at an angle θ, after passing a 1 *in* diameter aperture, by an NaI scintillator crystal, which converts the energy of scintillation photons into a voltage pulse that is counted and binned according to height by the MCA.

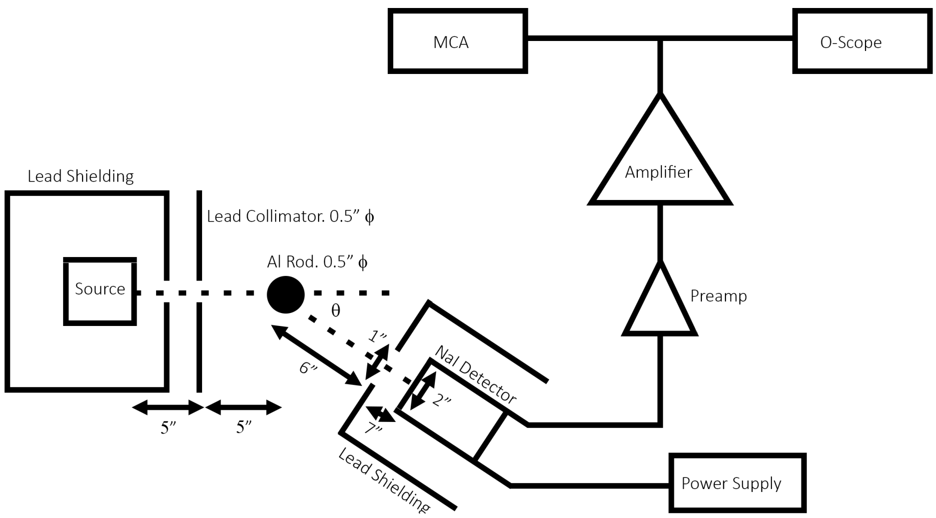

Figure 1: Block diagram of the experimental apparatus. The NaI detector can swivel to angle θ about the axis of the Al rod.

First, we calibrated the MCA, in the absence of scattering, by determining the photopeak channel for the accepted emission lines of ^{133}Ba, ^{57}Co, ^{24}Na, ^{137}Cs, and ^{54}Mn. The results of our calibration is shown in Figure 2, where we have plotted photopeaks on the horizontal axis, the accepted energy value on the vertical axis, and performed a quadratic fit to find a relationship between MCA channel number and ray energy.

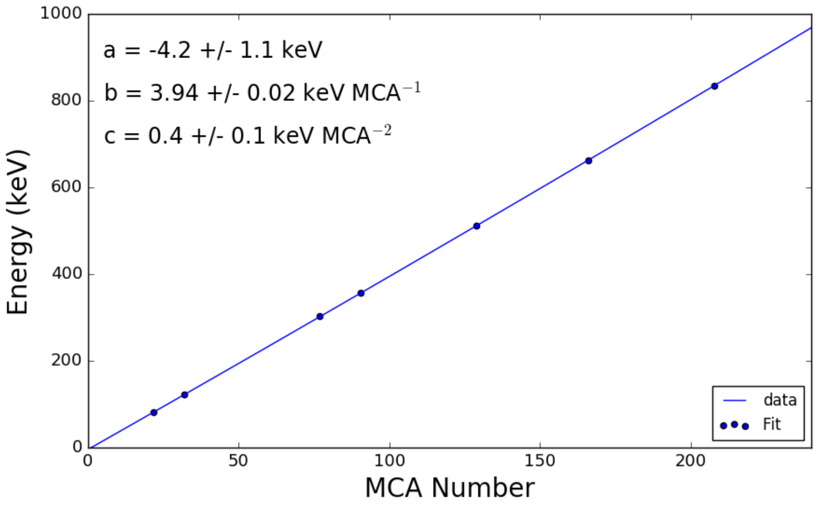

Figure 2: Quadratic fitting results of emission energy as a function of the registered MCA channel number.

Next, we stored and shielded all radiation sources from the detector and ran the MCA for ten minutes to find the background spectrum, which, as seen in Figure 3, contains a single peak at approximately 75 keV which is not in the region where we will be detecting scattered gamma rays. Then we introduced the radioactive ^{133}Cs source and recorded the photopeak channel for various scattering angles and the count rate for each photopeak. From the calibration relation, we then determined the measured energy of the scattered gamma rays at each angle, which is shown in Figure 4 in comparison with the expected energy as predicted by Compton scattering in Equation 1.

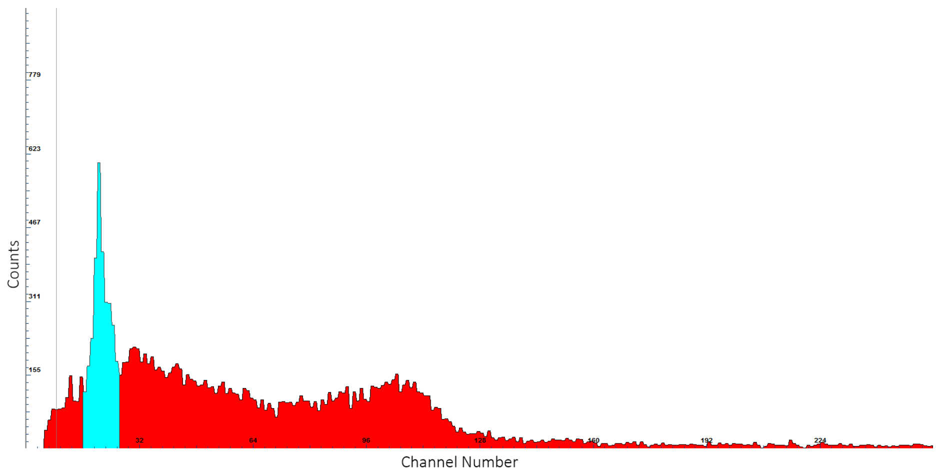

Figure 3: Ten minute background spectrum, with a single peak at channel 20, corresponding to approximately 75 keV.

The effective detector width is determined by the one-inch aperture placed prior to the detector face, this way gamma rays are more likely to travel through the center of the detector, which may improve measurement quality. Then the solid angle of the detector is $\Delta\Omega = \Delta A/d^2 = 0.087 \pm 0.001$ Steradians, where ΔA is the area of the

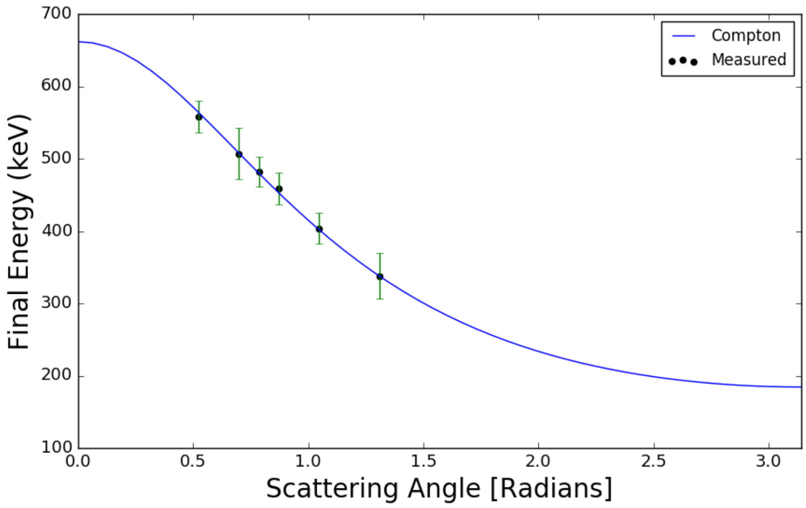

Figure 4: Measured energy of scattered rays compared with the value predicted by Equation 1. Where uncertainty in the number of counts and fit parameters is propagated to find the total uncertainty according to the product and sum formulas of standard error analysis [3].

aperture face at distance $d = 6\ in$ from the Al rod and the uncertainty is determined by

$$\delta\Omega = \frac{\delta A}{d^2} = \frac{2\pi r \delta r}{d^2} = \frac{2\pi r}{d}\delta\theta \tag{4}$$

where $r = 0.5\ in$ is the aperture radius and $\delta\theta = 0.25^o$ is the estimated uncertainty in scattering angle.

4 Analysis and Discussion

Our results of the scattered gamma energy vs. scattering angle, seen earlier in Figure 4, is in agreement with Compton's scattering predictions, as all points appear to land on the Compton line and our uncertainty is reasonably low. As such, we now use each measured energy, with its corresponding scattering angle, to determine the electron rest energy through a linear least squares fit with Equation 1. These results are shown in Figure 5, where the linear equation is the result of dividing both sides of Equation 1 by hc and using the relation for photon energy $E = hc/\lambda$, where λ is the photon wavelength. We found $mc^2 = 515 \pm 4$ keV, which is a satisfactory value as it is within one standard deviation of the accepted value of 511 keV.

Lastly, we use our measured count rate for each photopeak (at different scattering angles) in effort to determine whether the differential cross section of Thomson in Equation 2 or of Klein and Nishina in Equation 3 better describes the scattering of gamma rays off electrons. As the differential cross section is proportional to the intensity at the detector, we have arbitrarily and equally scaled the two cross sections so that they are on the same order as the count rate, which we have adjusted to correct for the detector efficiency. We then superimpose the count rate at each scattering angle over the scaled cross sections, as seen in Figure 6, and find that the Klein-Nishina cross

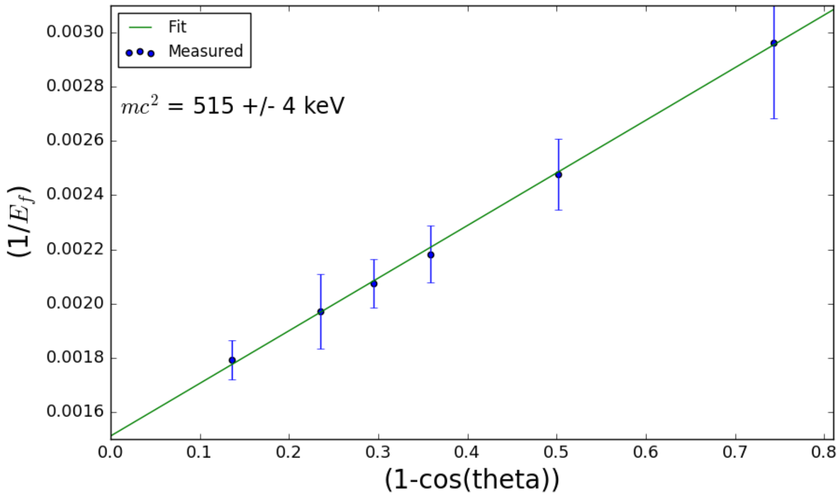

Figure 5: Least squares fit to find the electron mass from the measured final energy E_f after a scattering event at angle θ with a linear form of Equation 1.

section appears more accurate. But this is not a very strong claim because there are limited measurements and none of the data is within a few standard deviations of the Klein-Nishina curve. Also, there is a stray point at scattering angle 30^o which we believe was caused by non-scattered gamma rays traveling through the detector. Measurements here could be improved by longer data collection intervals and possibly by choosing more symmetric regions around the photopeak when determining the amount of counts in a peak.

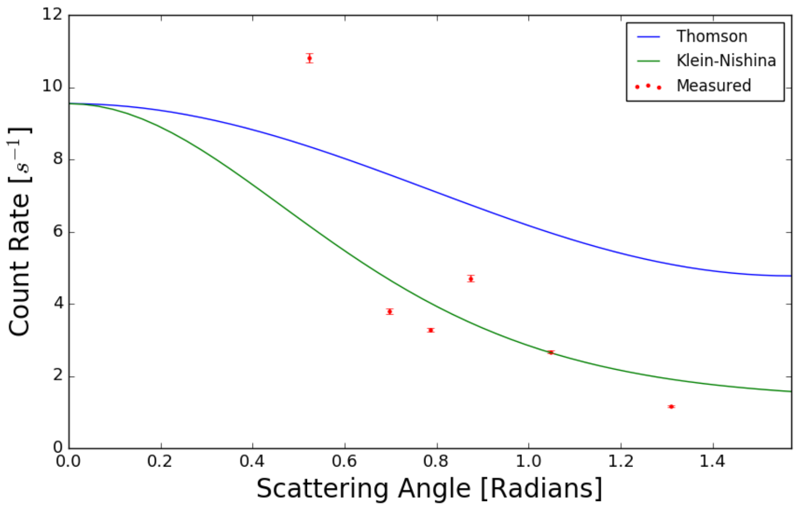

Figure 6: Thomson and Klein-Nishina differential cross sections in comparison with the net count rate of scattered gammas rays as a function of the scattering angle.

5 Conclusion

By measuring the energy of scattered gamma rays off an aluminum rod at various scattering angles, we have provided evidence for the validity of Compton's formula in gamma ray scattering and used this fact to determine the electron rest energy to be 514 ± 4 keV, which is within one standard deviation of the accepted value of 511 keV. Additionally, we measured the intensity of gamma rays incident upon the detector at each scattering angle and found possible evidence that the Klein-Nishina differential cross section is a better description of gamma-electron scattering.

References

[1] David Griffiths. *Introduction to Elementary Particles.* Wiley, 2004.

[2] Adrian C. Melissinos. *Experiments in Modern Physics.* Academic Press, 2003.

[3] John R. Taylor. *An Introduction to Error Analysis.* University Science Books, 1997.