Radioactive Decay

James Amarel and Gabriel Oman

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1 Goal

To determine the half life for the radioactive decay of isotopes Silver-110, Silver-108, and Indium-116.

2 Introduction/Background

An excited nucleus will undergo radioactive decay in order to reach a more desirable state by emitting energetic particles. Radioactive decay processes represent irreversible transformations upon a parent nucleus, which result in fundamentally new objects. But decay events are inherently random, so it is necessary to describe the likelihood of one occurring in terms of a probability per unit time. All excited quantum states have a constant probability per unit time to decay, which means, if A represents the probability proportionality factor, then AΔt is the probability that a state will decay in time interval Δt [2]. Therefore, if there are N excited nuclei, the change in N during interval Δt can be written as

$$\Delta N = AN\Delta t \tag{1}$$

where ΔN is the number of decay events. Dividing both sides of Equation 1 by Δt and taking the limit as Δt approaches zero gives the rate of change relationship

$$R = \frac{dN}{dt} = -AN \tag{2}$$

where R is the time rate change of the number of excited nuclei, and a minus sign is introduced to reflect the fact that particles are lost in the decay. Equation 2 is a separable differential equation, which can be solved by integration to yield

$$N(t) = N_o e^{-At} \tag{3}$$

where N_o is the result of an integration constant and represents the number of radioactive nuclei at time $t = 0$.

As A determines the rate at which a nucleus decays, we can define a characteristic property called the mean life (or lifetime) as $\tau = 1/A$, which is the elapsed time when N has been reduced to approximately 33% of N_o. Most informational sources instead choose to use an alternative timescale, called the half life. The half life, $T_{1/2}$, is directly related to the mean life by $T_{1/2} = \tau \log(2)$ and is the time necessary for the sample to decay to 50% of its original population [3].

In this lab we investigate the radioactive decay of isotopes Silver-108, Silver-110, and Indium-116. These isotopes are not naturally occurring and the accepted value for the half life of all three isotopes makes it impractical to place a shipping order, which means samples must be created on site. We utilize the process of neutron activation, where stable Silver and Indium samples are flooded with neutrons until many of the nuclei are excited into an unstable state. Neutron activation is a type of nuclear interaction in which a neutron source, we used a mixture of Plutonium and Beryllium, is used to irradiate a sample. When thin foil samples are placed in a neutron irradiation oven, we can approximate that radioactive nuclei are produced at an essentially constant rate. But this charging rate competes with the natural decay of the newly created unstable isotopes, so the net rate of change of N goes as

$$\frac{dN}{dt} = p - AN \tag{4}$$

where p is the constant charging rate. After integrating Equation 4 to find the solution, we see N achieves its maximum value, $N = p/A$, at large t, which occurs approximately when $t = 5T_{1/2}$, where $T_{1/2}$ is the half life of the unstable isotope that is being created.

3 Procedures and Data

Before measuring the radioactive decay of our excited Silver and Indium samples, we placed placed a radioactive Cs-137 source in a shielding enclosure, and within range of an NaI crystal scintillation detector, which was connected to a photomultipler tube (PMT), as seen in Figure 1.

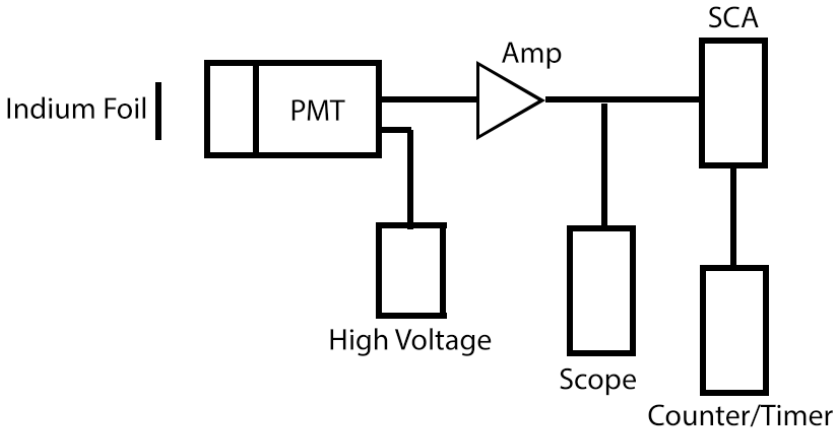

Figure 1: Block diagram of the experimental setup for measuring the decay of Indium-116.

The PMT functions to capture a portion of the crystal scintillations, then convert scintillation electrons into an amplified current pulse and direct that current into a singlechannel-analyzer (SCA). The SCA registers the voltage of incoming pulses, arranges them into to 1024 bins, where each bin corresponds to a voltage in the range 0 - 10 V and is proportional to the source radiation intensity. Then, we set the PMT voltage to 500 V and measured the pulse height of the Cs-137 radiation to be $\approx$ 3.5 V. The accepted value of the energy associate with Cs-137 decay is 662 KeV, so we used the lower level discriminator function within our SCA to filter out any noise beneath 200 KeV.

Then to begin the Indium procedure, we took a one-minute measurement of the background radiation with an inactive Indium sample, which we found to be 23 $\pm$ 5 counts in one minute. Then we investigated a radioactive Indium sample that had baked in a neutron irradiation oven long enough to ensure near full activation and had been allowed a five minute cooling

period to eliminate noise from any short-lived decays. Twelve measurements were made, as seen in Figure 2, by recording the counts reported by the SCA during one-minute intervals, with ten minute windows in between each measurement.

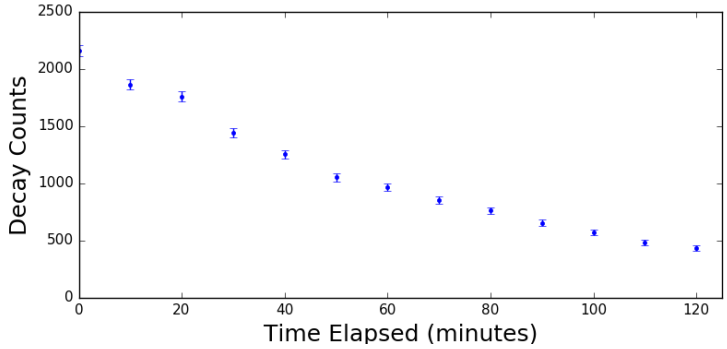

Figure 2: Scatter plot of $C_{tot} \pm \delta C_{tot}$ for our twelve measurements of Indium decay.

With the radioactive measurements completed, our last action in the Indium procedure was to remove the active Indium sample and take a secondary measurement of background radiation due to inactive Indium, which we found to be 20 ± 4 counts in one minute.

The accepted value of the half life for ^{108}Ag is approximately 2.5 minutes, and that of ^{110}Ag is nearly 30 seconds. Due to the quick half lives of these samples, we need to record the number of counts in time intervals Δt that are much less than the 30 seconds. So, it isn't feasible to use a simple counter as we did with Indium above, instead, we used a multichannel-analyzer in mutichannel-scaler (MCS) mode, as seen in Figure 3.

Then, we set the dwell time to 4 seconds, which is the amount of time the MCS will wait and count before switching channels and beginning a new count. The MCS had 256 channels, which means it records the number of decay counts in 256 consecutive 4 second windows, which allowed us enough precision to measure the exponential decay of our samples. The measurement process involved removing a Silver foil from the neutron activation oven, and as quickly as possible, placing it a shielding block equipped with a geiger counter. Decays are detected by the geiger tube, which we set to 700 V, in order to be certain the geiger tube was operating in the plateau region. Output from the geiger tube was passed through a series of amplifiers and

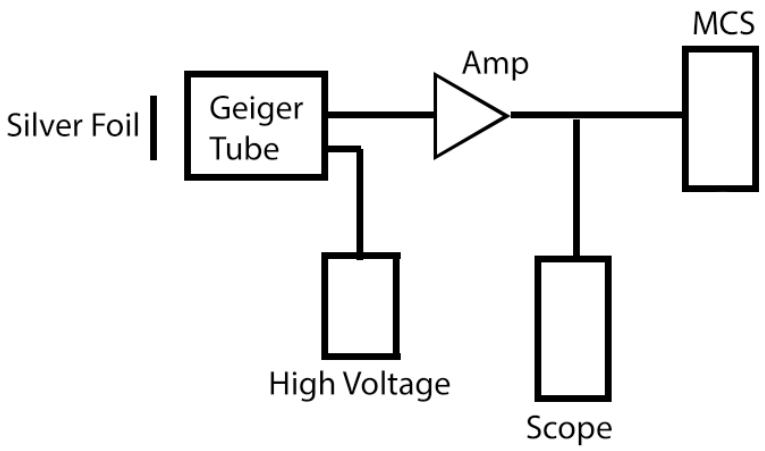

Figure 3: Block diagram of the experimental setup for measuring the decay of Silver-110 and Silver-108.

fed to the MCS, which counted the number of events per dwell time window. The MCS was equipped with its own computer software, and as soon as the hot Silver sample was placed in the measurement chamber, we immediately began recording data through the computer program and then we saved the data output after all 256 channels had been filled, which is seen in Figure 4.

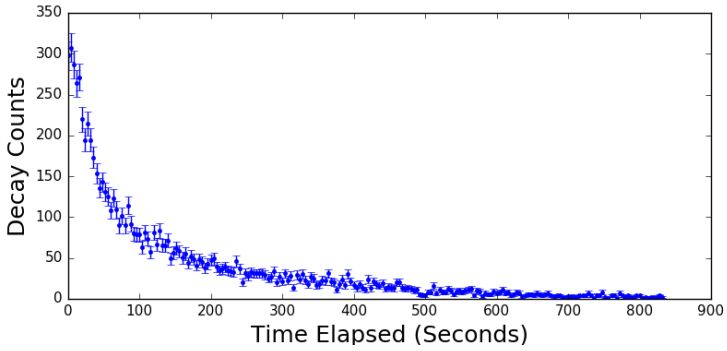

Figure 4: Scatter plot of $C_{tot} \pm \delta C_{tot}$ for the decay of isotopes ^{108}Ag and ^{110}Ag.

Finally, we removed the radiating Silver sample from the measurement

chamber and used the MCS to take a background measurement, which gave $\bar{C}_{avg} = 1 \pm 1$ for one minute intervals. This is a relatively low background count rate, a possible explanation for this may be that the MCS software is incorrectly filtering noise.

4 Analysis and Discussion

From the two background readings of the inactive Indium sample, we determined the background count rate to be $R_{bkd} = 22 \pm 5$ per minute. Then, we subtracted $C_{bkd} = 22$, from our measurements of C_{tot} to find C_{net}, for each one minute measurement. Next, we linearized Equation 3 into

$$\ln(C_{net}) = \ln(C_o) - At \tag{5}$$

where C_o is the initial number of counts at $t = 0$.

Figure 5 shows the results of a weighted least squares fit on our counts Indium decay, from Figure 2 to Equation 5. From the fitting algorithm, we found the half life of Indium-116 to be 51.6 ± 0.9 minutes. This is a satisfactory result, as it is within three standard deviations from the accepted value of 54.29 minutes [1].

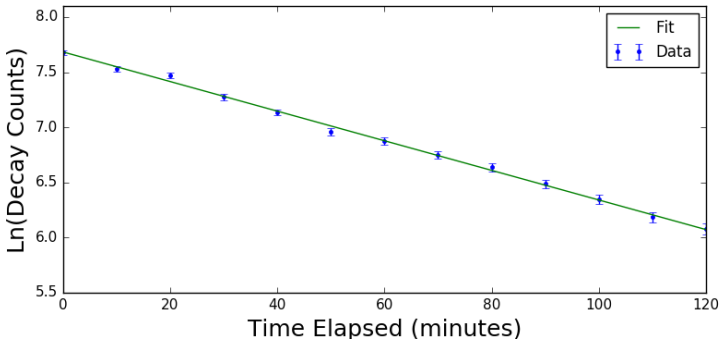

Figure 5: Plot of $\ln(C_{net}) vs. \ln(C_o) - At$, where C_{net} and t are from our measurements of Indium decay, with a least squares fit on parameters A and C_o.

If we were to write Equation 3 and account for the fact that our Silver sample contained two different radioactive isotopes, the resulting expression

would not be linearizable because there are two different half lives at play. Our intent was to determine the half lives of each isotope, and therefore if we wish to use a linear fitting algorithm, we can estimate the time at which the net decay was being dominated by the isotope with the longer half life. During the region in which one decay mechanism is dominating, a scatter plot of $\ln(C_{net})$ will be approximately linear in time. If there are two decay mechanisms, then there are two linear regions which are connected by a transition between them. So, we made a scatter plot of $\ln(C_{net}) vs. t$ seen in Figure 6, and estimated that the cutoff point at which the contribution from ^{110}Ag faded occurred after 128 seconds had elapsed.

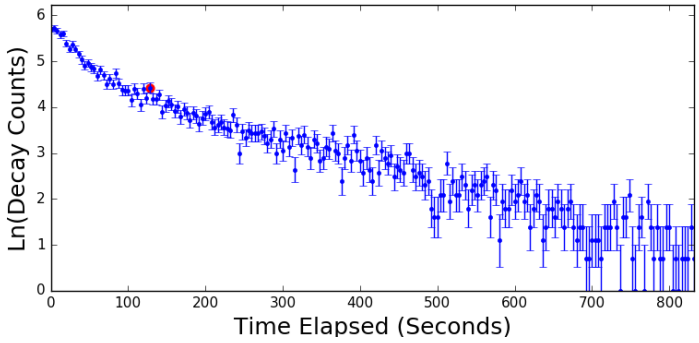

Figure 6: Plot of $\ln(C_{net})$ vs. t. Where the red dot at time 128 seconds signifies the location of our choice for a cutoff time.

Then, we performed a weighted fit with all data after 128 seconds, using the Equation 5 as we did with Indium, and found the half life of ^{108}Ag to be 153 ± 4 seconds, which is within three standard deviations from the accepted value of 142 seconds [1].

Then, we used our fitting result of the long lived ^{108}Ag data to calculate an expectation value for the contribution of ^{108}Ag to the number of counts in the short time data. This allowed us to subtract the contribution of ^{108}Ag from the net number of counts and determine the remaining contribution from the shorter lived isotope, ^{110}Ag. Then we performed the same weighted fit, as was used previously, on the data from ^{110}Ag and found its half life to be 23.6 ± 1.4 seconds. The fit was much more successful at predicting this half life, possibly due to the larger uncertainty at long time, landing within one standard deviation from the accepted value of 24.6 seconds.

Additionally, we used a non-linear fitting algorithm as a secondary method of determining the half life of each isotope. A non-linear algorithm fit the equation

$$C(t)_{net} = C_1 e^{-t/\tau_1} + C_2 e^{-t/\tau_2} \tag{6}$$

where C_i and τ_i are the initial count rate and half life for the i'th isotope and there are two terms to account for both ^{108}Ag and ^{110}Ag. We fit all of our Silver data to Equation 6, the result of this fit is seen in Figure 7,

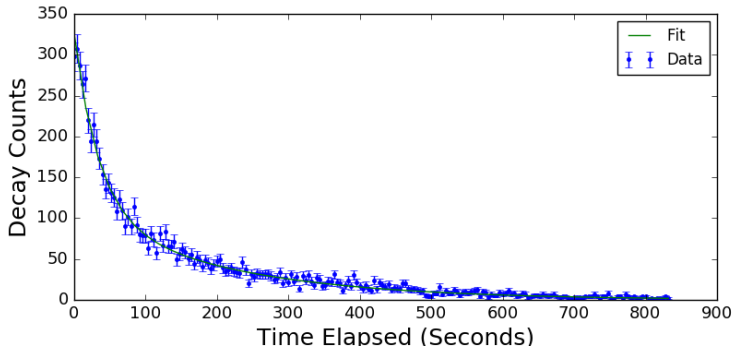

Figure 7: Nonlinear fit on the entirety of the Silver decay counts, where two of the fitting parameters determined the half life of the Silver isotopes.

where we found the half life of ^{108}Ag to be 143 ± 4 seconds and the half life of ^{110}Ag to be 24 ± 2 seconds. The answers are reasonably similar to the results we found from estimating a cutoff point. Both methods suggest the half life of ^{110}Ag is approximately 24 seconds, except the non-linear algorithm performed significantly better at predicting the half life of ^{108}Ag. This is likely due to our separation method to obtain two linearizable equations, and the fuzziness of the transition region. A better method may have been to use a smaller dwell time, increasing the number of measurements, and work without data location in the transition region.

5 Conclusion

We used the technique of neutron activation to create isotopes Silver-110, Silver-108, and Indium-116. And we implemented two different radiation detection techniques, namely a geiger tube and a scintillation crystal three

radioactive isotopes, which allowed us to count decay events. We then found the half life of each isotope by counting the number of decays within a set of spaced time intervals. The results of our measurements were as follows, ^{110}Ag: 24 ± 4 seconds, ^{108}Ag: 143 ± 4 seconds, and ^{116}In: 51.6 ± 0.9 minutes, where all three values were in decent agreement with the accepted value.

References

[1] Brookhaven National Lab. Nudat 2. URL `http://www.nndc.bnl.gov/nudat2/chartNuc.jsp`.

[2] Randy Harris. *Modern Physics*. Pearson, 1998.

[3] Adrian C. Melissinos. *Experiments in Modern Physics*. Academic Press, 2003.