The IPython Notebook: open, reproducible scientific computing

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June 30, 2013

Abstract

While computing has become a foundation of all it is challenging for researchers .

1 Introduction

2 The lifecycle of research

3 The IPython Notebook

3.1 Web application

3.2 Notebook document format

3.3 Installation

4 Collaboration

5 Broader ecosystem

6 Future directions

Dexy Snippets

The notebook `notebooks/Chapter1_Introduction.ipynb`ipynb— uses format `3.0`.

Markdown Content

Here is an example of a markdown cell:

3Ẇhat is the mean of λ_1 **given** we know τ is less than 45. That is, suppose we have new information as we know for certain that the change in behaviour occurred before day 45. What is the expected value of λ_1 now? (You do not need to redo the PyMC part, just consider all instances where 'tau$_samples < 45$'.)

Here is the same cell in a Verbatim block:

```
3\. What is the mean of $\lambda_1$ **given** we know $\tau$ is less than 45.  That is,
```

Here is the same cell presented as highlighted markup:

```
3\. What is the mean of $\lambda_1$ **given** we know $\tau$ is less than 45.  That is,
```

Python Content

```
"""
The book uses a custom matplotlibrc file, which provides the unique styles for matplotli
If executing this book, and you wish to use the book's styling, provided are two options
    1. Overwrite your own matplotlibrc file with the rc-file provided in the book's styl
       See http://matplotlib.org/users/customizing.html
    2. Also in the styles is  bmh_matplotlibrc.json file. This can be used to update the
       in only this notebook. Try running the following code:

        import json
        s = json.load( open("../styles/bmh_matplotlibrc.json") )
        matplotlib.rcParams.update(s)

"""

#the code below can be passed over, as it is currently not important.
%pylab inline
figsize( 11, 9)

import scipy.stats as stats

dist = stats.beta
n_trials = [0,1,2,3,4,5,8,15, 50, 500]
data = stats.bernoulli.rvs(0.5, size = n_trials[-1] )
x = np.linspace(0,1,100)

for k, N in enumerate(n_trials):
    sx = subplot( len(n_trials)/2, 2, k+1)
    plt.xlabel("$p$, probability of heads") if k in [0,len(n_trials)-1] else None
    plt.setp(sx.get_yticklabels(), visible=False)
```

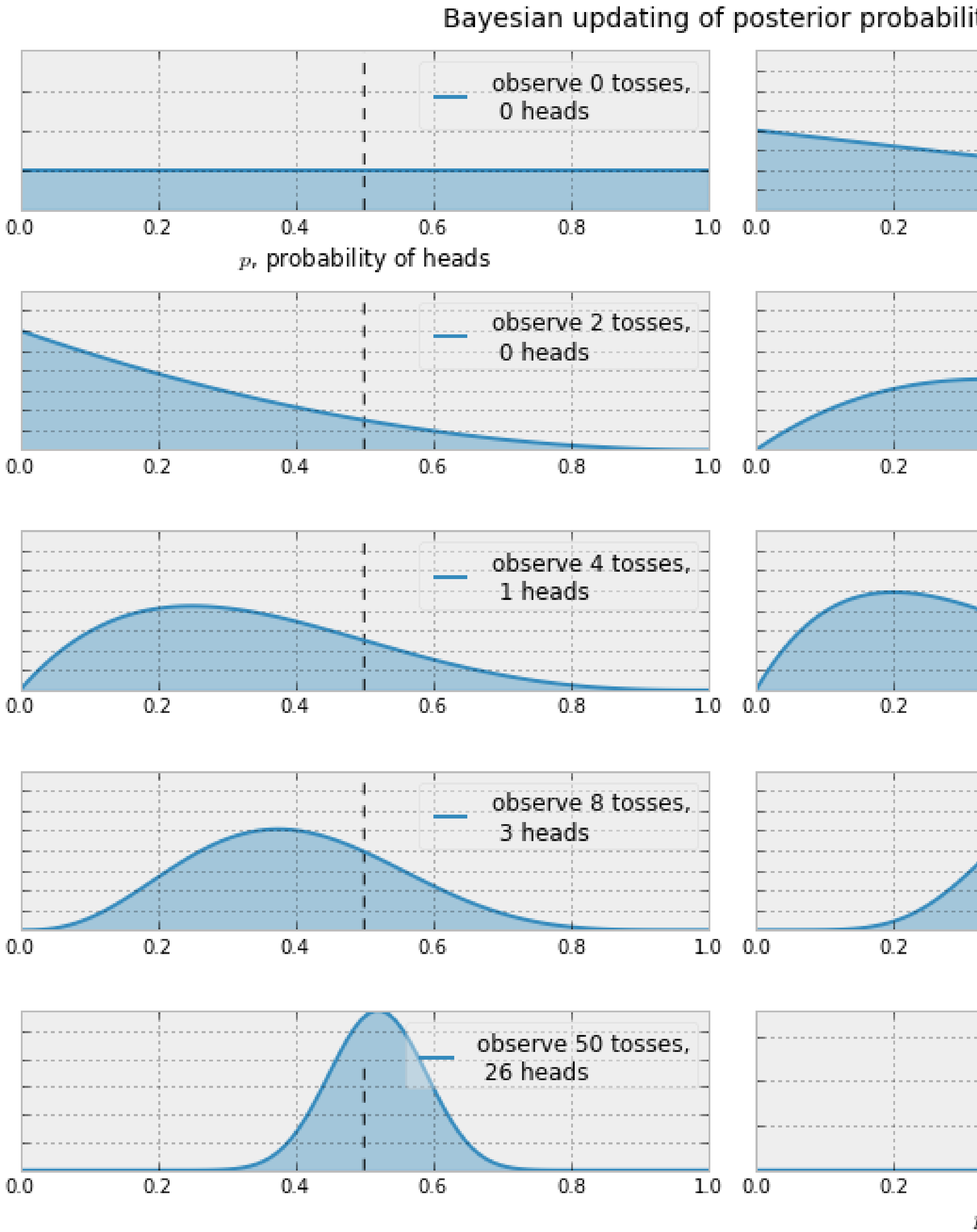
```
    heads = data[:N].sum()
    y = dist.pdf(x, 1 + heads, 1 + N - heads )
    plt.plot( x, y, label= "observe %d tosses,\n %d heads"%(N,heads) )
    plt.fill_between( x, 0, y, color="#348ABD", alpha = 0.4 )
    plt.vlines( 0.5, 0, 4, color = "k", linestyles = "--", lw=1 )

    leg = plt.legend()
    leg.get_frame().set_alpha(0.4)
    plt.autoscale(tight = True)


plt.suptitle( "Bayesian updating of posterior probabilities",
              y = 1.02,
              fontsize = 14);

plt.tight_layout()
```

```
Welcome to pylab, a matplotlib-based Python environment [backend: module://IPython.zmq.p
For more information, type 'help(pylab)'.
```


Bayesian updating of posterior probabilit
observe 0 tosses,
0 heads
p, probability of heads
observe 2 tosses,
0 heads
observe 4 tosses,
1 heads
observe 8 tosses,
3 heads
observe 50 tosses,
26 heads
0.0
0.2
0.4
0.6
0.8
1.0

Iterating over Cells and Documents

This notebook has 47 cells and 59 documents based on these cells.

Cells

Here is a list of cells:

- `notebooks/Chapter1_Introduction--0`
- `notebooks/Chapter1_Introduction--1`
- `notebooks/Chapter1_Introduction--2`
- `notebooks/Chapter1_Introduction--3`
- `notebooks/Chapter1_Introduction--4`
- `notebooks/Chapter1_Introduction--5`
- `notebooks/Chapter1_Introduction--6`
- `notebooks/Chapter1_Introduction--7`
- `notebooks/Chapter1_Introduction--8`
- `notebooks/Chapter1_Introduction--9`
- `notebooks/Chapter1_Introduction--10`
- `notebooks/Chapter1_Introduction--11`
- `notebooks/Chapter1_Introduction--12`
- `notebooks/Chapter1_Introduction--13`
- `notebooks/Chapter1_Introduction--14`
- `notebooks/Chapter1_Introduction--15`
- `notebooks/Chapter1_Introduction--16`
- `notebooks/Chapter1_Introduction--17`
- `notebooks/Chapter1_Introduction--18`
- `notebooks/Chapter1_Introduction--19`
- `notebooks/Chapter1_Introduction--20`
- `notebooks/Chapter1_Introduction--21`
- `notebooks/Chapter1_Introduction--22`

- notebooks/Chapter1_Introduction--23
- notebooks/Chapter1_Introduction--24
- notebooks/Chapter1_Introduction--25
- notebooks/Chapter1_Introduction--26
- notebooks/Chapter1_Introduction--27
- notebooks/Chapter1_Introduction--28
- notebooks/Chapter1_Introduction--29
- notebooks/Chapter1_Introduction--30
- notebooks/Chapter1_Introduction--31
- notebooks/Chapter1_Introduction--32
- notebooks/Chapter1_Introduction--33
- notebooks/Chapter1_Introduction--34
- notebooks/Chapter1_Introduction--35
- notebooks/Chapter1_Introduction--36
- notebooks/Chapter1_Introduction--37
- notebooks/Chapter1_Introduction--38
- notebooks/Chapter1_Introduction--39
- notebooks/Chapter1_Introduction--40
- notebooks/Chapter1_Introduction--41
- notebooks/Chapter1_Introduction--42
- notebooks/Chapter1_Introduction--43
- notebooks/Chapter1_Introduction--44
- notebooks/Chapter1_Introduction--45
- notebooks/Chapter1_Introduction--46

Documents

Here is a list of documents:

- notebooks/Chapter1_Introduction--0.md
- notebooks/Chapter1_Introduction--1.md
- notebooks/Chapter1_Introduction--2.md
- notebooks/Chapter1_Introduction--3.md
- notebooks/Chapter1_Introduction--4.md
- notebooks/Chapter1_Introduction--5.md
- notebooks/Chapter1_Introduction--6.md
- notebooks/Chapter1_Introduction--7-input.py
- notebooks/Chapter1_Introduction--7-output-0.txt
- notebooks/Chapter1_Introduction--7-output-1.png
- notebooks/Chapter1_Introduction--8.md
- notebooks/Chapter1_Introduction--9.md
- notebooks/Chapter1_Introduction--10.md
- notebooks/Chapter1_Introduction--11.md
- notebooks/Chapter1_Introduction--12-input.py
- notebooks/Chapter1_Introduction--12-output-0.png
- notebooks/Chapter1_Introduction--13.md
- notebooks/Chapter1_Introduction--14-input.py
- notebooks/Chapter1_Introduction--14-output-0.png
- notebooks/Chapter1_Introduction--15.md
- notebooks/Chapter1_Introduction--16.md
- notebooks/Chapter1_Introduction--17-input.py
- notebooks/Chapter1_Introduction--17-output-0.png
- notebooks/Chapter1_Introduction--18.md
- notebooks/Chapter1_Introduction--19-input.py

- notebooks/Chapter1_Introduction--19-output-0.png
- notebooks/Chapter1_Introduction--20.md
- notebooks/Chapter1_Introduction--21.md
- notebooks/Chapter1_Introduction--22-input.py
- notebooks/Chapter1_Introduction--22-output-0.png
- notebooks/Chapter1_Introduction--23.md
- notebooks/Chapter1_Introduction--24-input.py
- notebooks/Chapter1_Introduction--25.md
- notebooks/Chapter1_Introduction--26-input.py
- notebooks/Chapter1_Introduction--26-output-0.txt
- notebooks/Chapter1_Introduction--27-input.py
- notebooks/Chapter1_Introduction--28.md
- notebooks/Chapter1_Introduction--29-input.py
- notebooks/Chapter1_Introduction--30.md
- notebooks/Chapter1_Introduction--31-input.py
- notebooks/Chapter1_Introduction--31-output-0.txt
- notebooks/Chapter1_Introduction--31-output-1.txt
- notebooks/Chapter1_Introduction--32-input.py
- notebooks/Chapter1_Introduction--33-input.py
- notebooks/Chapter1_Introduction--33-output-0.png
- notebooks/Chapter1_Introduction--34.md
- notebooks/Chapter1_Introduction--35.md
- notebooks/Chapter1_Introduction--36-input.py
- notebooks/Chapter1_Introduction--36-output-0.png
- notebooks/Chapter1_Introduction--37.md
- notebooks/Chapter1_Introduction--38.md
- notebooks/Chapter1_Introduction--39-input.py

- notebooks/Chapter1_Introduction--40.md
- notebooks/Chapter1_Introduction--41-input.py
- notebooks/Chapter1_Introduction--42.md
- notebooks/Chapter1_Introduction--43-input.py
- notebooks/Chapter1_Introduction--44.md
- notebooks/Chapter1_Introduction--45-input.py
- notebooks/Chapter1_Introduction--46-input.py

Here are the contents of documents:

Here are the contents of `notebooks/Chapter1_Introduction--0.md`:

```
Probabilistic Programming
=====
and Bayesian Methods for Hackers
========

#####Version 0.1
Welcome to *Bayesian Methods for Hackers*. The full Github repository, and additional ch
```

Here are the contents of `notebooks/Chapter1_Introduction--1.md`:

```
Chapter 1
======
***
```

Here are the contents of `notebooks/Chapter1_Introduction--2.md`:

```
The Philosophy of Bayesian Inference
------

> You are a skilled programmer, but bugs still slip into your code. After a particularly

If you think this way, then congratulations, you already are a Bayesian practitioner! Ba
```

Here are the contents of `notebooks/Chapter1_Introduction--3.md`:

```
###The Bayesian state of mind


Bayesian inference differs from more traditional statistical inference by preserving *un

The Bayesian world-view interprets probability as measure of *believability in an event*
```

For this to be clearer, we consider an alternative interpretation of probability: *Frequ

Bayesians, on the other hand, have a more intuitive approach. Bayesians interpret a prob

Notice in the paragraph above, I assigned the belief (probability) measure to an *indivi

- I flip a coin, and we both guess the result. We would both agree, assuming the coin is

- Your code either has a bug in it or not, but we do not know for certain which is true

- A medical patient is exhibiting symptoms x, y and z. There are a number of dise

This philosophy of treating beliefs as probability is natural to humans. We employ it co

To align ourselves with traditional probability notation, we denote our belief about eve

John Maynard Keynes, a great economist and thinker, said "When the facts change, I chang

1\. $P(A): \;\;$ the coin has a 50 percent chance of being heads. $P(A | X):\;\;$ You lo

2\. $P(A): \;\;$ This big, complex code likely has a bug in it. $P(A | X): \;\;$ The

3\. $P(A):\;\;$ The patient could have any number of diseases. $P(A | X):\;\;$ Performi

It's clear that in each example we did not completely discard the prior belief after see

By introducing prior uncertainty about events, we are already admitting that any guess w

Here are the contents of notebooks/Chapter1_Introduction--4.md:

###Bayesian Inference in Practice

If frequentist and Bayesian inference were programming functions, with inputs being sta

For example, in our debugging problem above, calling the frequentist function with the a

> *YES*, with probability 0.8; *NO*, with probability 0.2

This is very different from the answer the frequentist function returned. Notice that th

```
####Incorporating evidence

As we acquire more and more instances of evidence, our prior belief is *washed out* by t


Denote $N$ as the number of instances of evidence we possess. As we gather an *infinite*

One may think that for large $N$, one can be indifferent between the two techniques sinc

> Sample sizes are never large. If $N$ is too small to get a sufficiently-precise estima

### Are frequentist methods incorrect then?

**No.**

Frequentist methods are still useful or state-of-the-art in many areas. Tools like Least


#### A note on *Big Data*
Paradoxically, big data's predictive analytic problems are actually solved by relatively

The much more difficult analytic problems involve *medium data* and, especially troubles
```

Here are the contents of `notebooks/Chapter1_Introduction--5.md`:

```
### Our Bayesian framework

We are interested in beliefs, which can be interpreted as probabilities by thinking Baye

Secondly, we observe our evidence. To continue our buggy-code example: if our code passe

\begin{align}
 P( A | X ) = & \frac{ P(X | A) P(A) } {P(X) } \\\\[5pt]
& \propto P(X | A) P(A)\;\; (\propto \text{is proportional to } )
\end{align}

The above formula is not unique to Bayesian inference: it is a mathematical fact with us
```

Here are the contents of `notebooks/Chapter1_Introduction--6.md`:

```
##### Example: Mandatory coin-flip example

Every statistics text must contain a coin-flipping example, I'll use it here to get it o
```

```
We begin to flip a coin, and record the observations: either $H$ or $T$. This is our obs

Below we plot a sequence of updating posterior probabilities as we observe increasing am
```

Here are the contents of `notebooks/Chapter1_Introduction--7-input.py`:

```python
"""
The book uses a custom matplotlibrc file, which provides the unique styles for matplotli
If executing this book, and you wish to use the book's styling, provided are two options
    1. Overwrite your own matplotlibrc file with the rc-file provided in the book's styl
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    2. Also in the styles is  bmh_matplotlibrc.json file. This can be used to update the
       in only this notebook. Try running the following code:

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"""

#the code below can be passed over, as it is currently not important.
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dist = stats.beta
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data = stats.bernoulli.rvs(0.5, size = n_trials[-1] )
x = np.linspace(0,1,100)

for k, N in enumerate(n_trials):
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    plt.xlabel("$p$, probability of heads") if k in [0,len(n_trials)-1] else None
    plt.setp(sx.get_yticklabels(), visible=False)
    heads = data[:N].sum()
    y = dist.pdf(x, 1 + heads, 1 + N - heads )
    plt.plot( x, y, label= "observe %d tosses,\n %d heads"%(N,heads) )
    plt.fill_between( x, 0, y, color="#348ABD", alpha = 0.4 )
    plt.vlines( 0.5, 0, 4, color = "k", linestyles = "--", lw=1 )

    leg = plt.legend()
    leg.get_frame().set_alpha(0.4)
    plt.autoscale(tight = True)
```
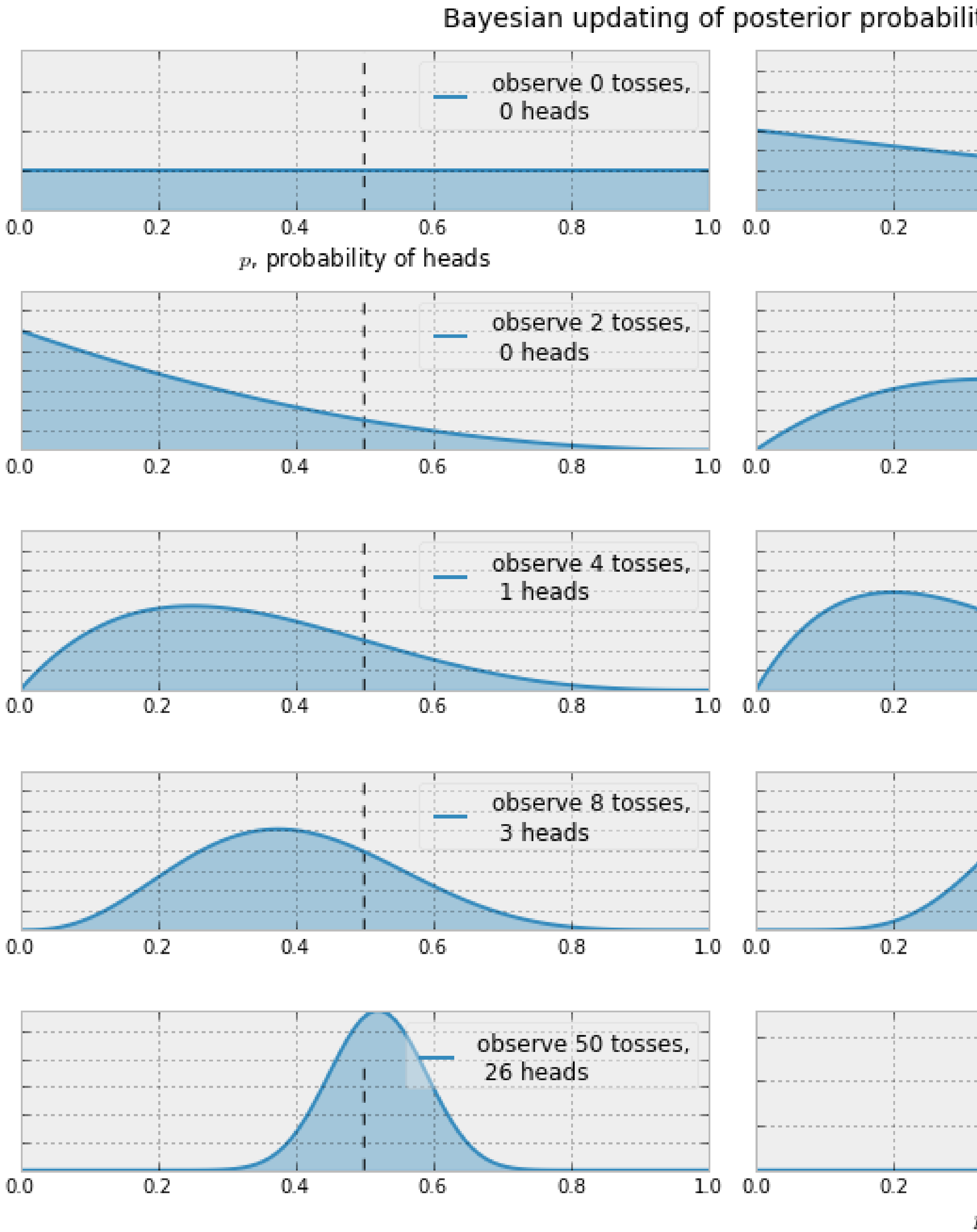

```
plt.suptitle( "Bayesian updating of posterior probabilities",
              y = 1.02,
              fontsize = 14);

plt.tight_layout()
```

Here are the contents of `notebooks/Chapter1_Introduction--7-output-0.txt`:

```
Welcome to pylab, a matplotlib-based Python environment [backend: module://IPython.zmq.p
For more information, type 'help(pylab)'.
```

Here are the contents of `notebooks/Chapter1_Introduction--7-output-1.png`:



Here are the contents of `notebooks/Chapter1_Introduction--8.md`:

```
The posterior probabilities are represented by the curves, and our confidence is proport

Notice that the plots are not always *peaked* at 0.5. There is no reason it should be: r

The next example is a simple demonstration of the mathematics of Bayesian inference.
```

Here are the contents of notebooks/Chapter1_Introduction--9.md:

```
#####Example: Bug, or just sweet, unintended feature?


Let $A$ denote the event that our code has **no bugs** in it. Let $X$ denote the event t

We are interested in $P(A|X)$, i.e. the probability of no bugs, given our debugging test

What is $P(X | A)$, i.e., the probability that the code passes $X$ tests *given* there a

$P(X)$ is a little bit trickier: The event $X$ can be divided into two possibilities, ev
```

Here are the contents of notebooks/Chapter1_Introduction--10.md:

```
\begin{align}
P(X ) & = P(X \text{ and } A) + P(X \text{ and } \sim A) \\\\[5pt]
 & = P(X|A)P(A) + P(X | \sim A)P(\sim A)\\\\[5pt]
& = P(X|A)p + P(X | \sim A)(1-p)
\end{align}
```

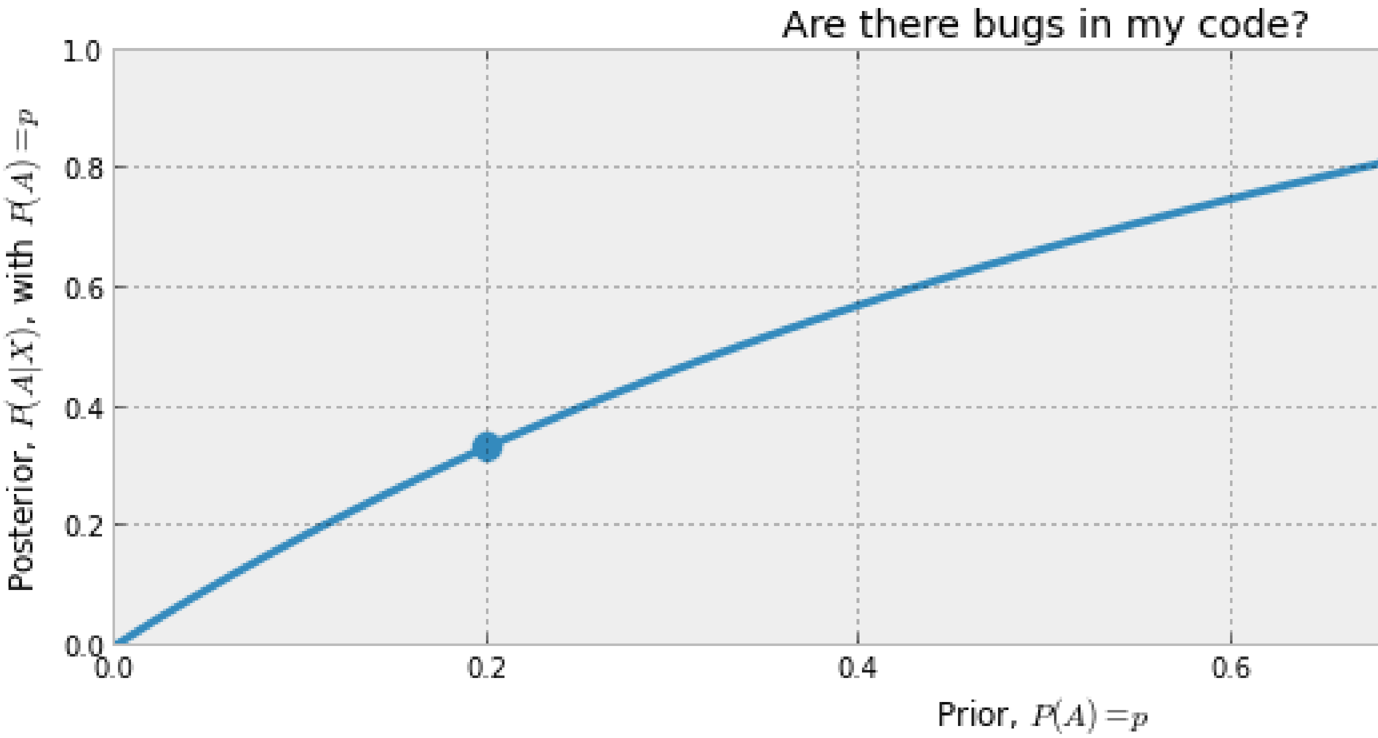
Here are the contents of notebooks/Chapter1_Introduction--11.md:

```
We have already computed $P(X|A)$ above. On the other hand, $P(X | \sim A)$ is subjectiv

\begin{align}
P(A | X) & = \frac{1\cdot p}{ 1\cdot p +0.5 (1-p) } \\\\
& = \frac{ 2 p}{1+p}
\end{align}
This is the posterior probability. What does it look like as a function of our prior, $p
```

Here are the contents of notebooks/Chapter1_Introduction--12-input.py:

```
figsize(12.5,4)
p = np.linspace( 0,1, 50)
plt.plot( p, 2*p/(1+p), color = "#348ABD", lw = 3 )
#plt.fill_between( p, 2*p/(1+p), alpha = .5, facecolor = ["#A60628"])
plt.scatter( 0.2, 2*(0.2)/1.2, s = 140, c ="#348ABD"  )
plt.xlim( 0, 1)
plt.ylim( 0, 1)
plt.xlabel( "Prior, $P(A) = p$")
plt.ylabel("Posterior, $P(A|X)$, with $P(A) = p$")
plt.title( "Are there bugs in my code?");
```

Here are the contents of notebooks/Chapter1_Introduction--12-output-0.png:



Here are the contents of notebooks/Chapter1_Introduction--13.md:

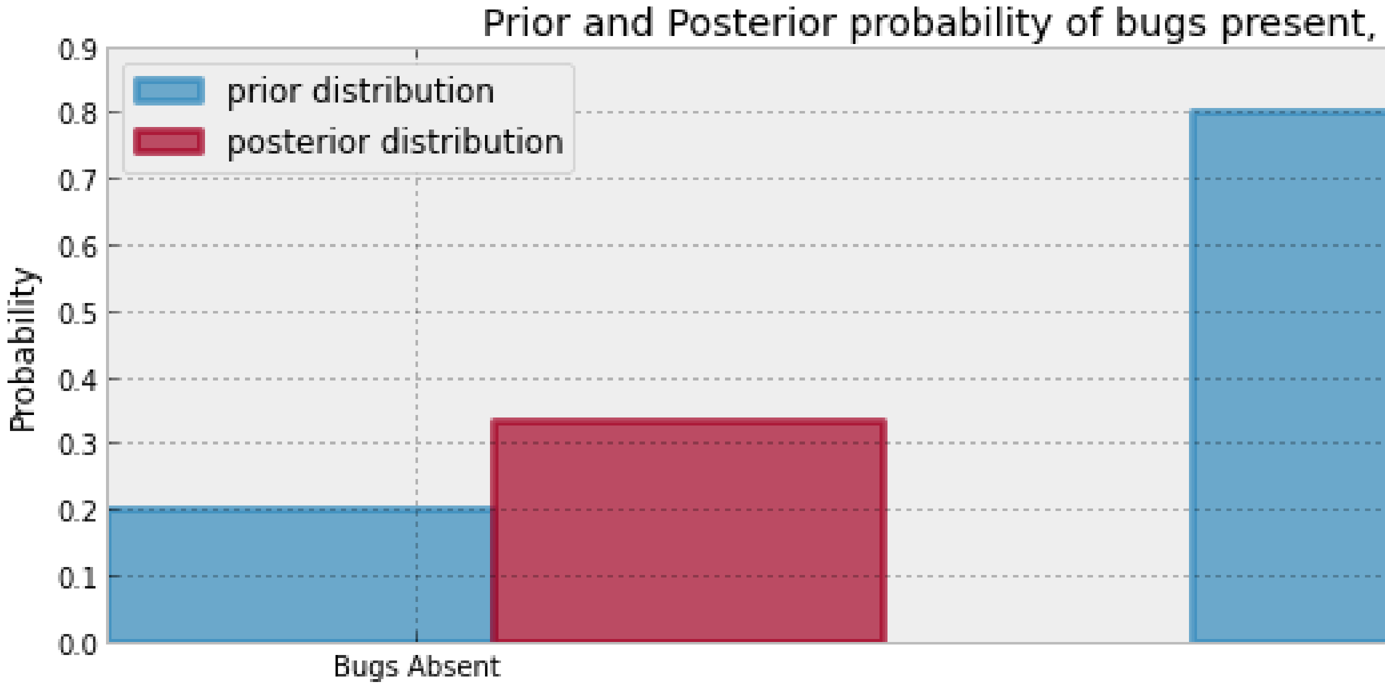
```
We can see the biggest gains if we observe the $X$ tests passed when the prior probabili

Recall that the prior is a probability: $p$ is the prior probability that there *are no

Similarly, our posterior is also a probability, with $P(A | X)$ the probability there is
```

Here are the contents of notebooks/Chapter1_Introduction--14-input.py:

```
figsize( 12.5, 4 )
colours = ["#348ABD", "#A60628"]

prior = [0.20, 0.80]
posterior =  [1./3, 2./3]
plt.bar( [0,.7], prior ,alpha = 0.70, width = 0.25, \
            color = colours[0], label = "prior distribution",
             lw = "3", edgecolor = colours[0])


plt.bar( [0+0.25,.7+0.25], posterior ,alpha = 0.7, \
          width = 0.25, color =  colours[1],
          label = "posterior distribution",
          lw = "3", edgecolor = colours[1])
```

```
plt.xticks( [0.20,.95], ["Bugs Absent", "Bugs Present"] )
plt.title("Prior and Posterior probability of bugs present, prior = 0.2")
plt.ylabel("Probability")
plt.legend(loc="upper left");
```

Here are the contents of `notebooks/Chapter1_Introduction--14-output-0.png`:



Here are the contents of `notebooks/Chapter1_Introduction--15.md`:

```
Notice that after we observed $X$ occur, the probability of bugs being absent increased.

This was a very simple example of Bayesian inference and Bayes rule. Unfortunately, the
```

Here are the contents of `notebooks/Chapter1_Introduction--16.md`:

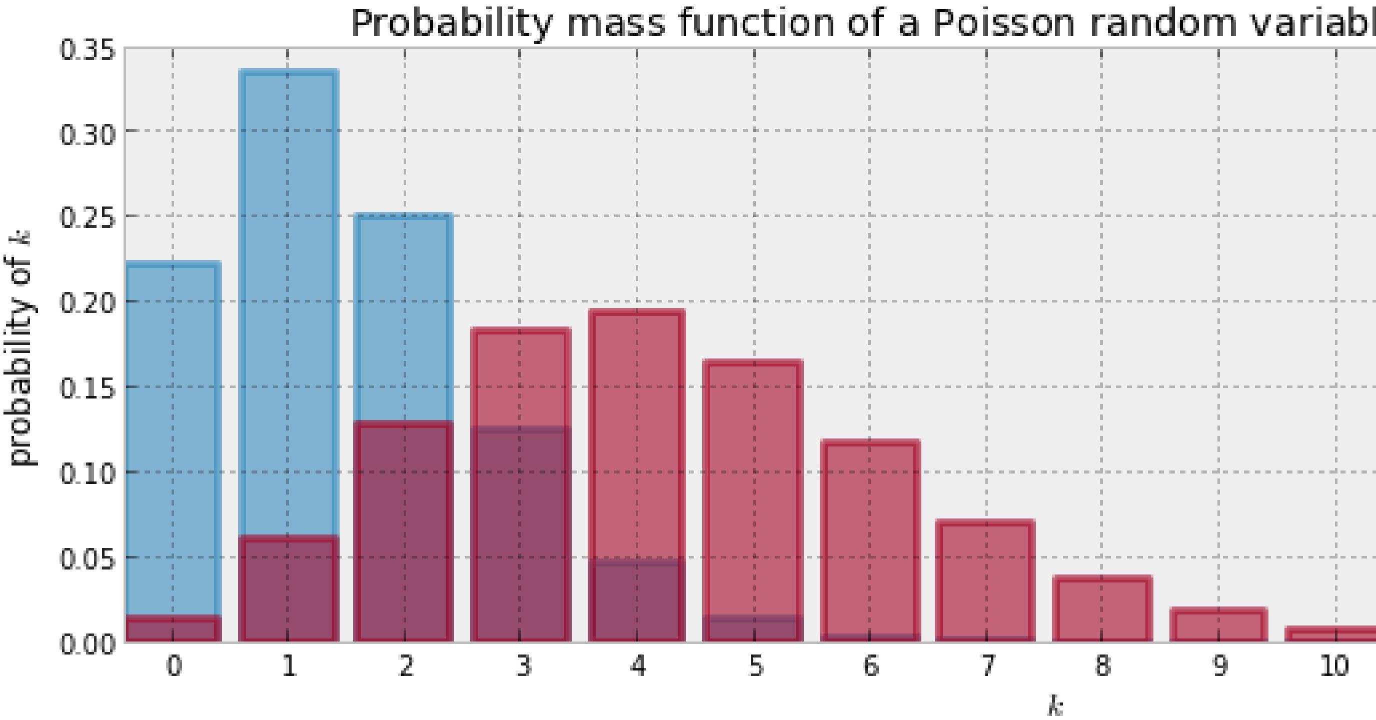
```
-------

##Probability Distributions


**Let's quickly recall what a probability distribution is:** Let $Z$ be some random vari

We can divide random variables into three classifications:

-   **$Z$ is discrete**: Discrete random variables may only assume values on a specified

-   **$Z$ is continuous**: Continuous random variable can take on arbitrarily exact valu
```

```
- **$Z$ is mixed**: Mixed random variables assign probabilities to both discrete and con

###Discrete Case
If $Z$ is discrete, then its distribution is called a *probability mass function*, which

$$P(Z = k) =\frac{ \lambda^k e^{-\lambda} }{k!}, \; \; k=0,1,2, \dots $$

What is $\lambda$? It is called the parameter, and it describes the shape of the distrib

Unlike $\lambda$, which can be any positive number, the value $k$ in the above formula m

If a random variable $Z$ has a Poisson mass distribution, we denote this by writing

$$Z \sim \text{Poi}(\lambda) $$

One very useful property of the Poisson random variable, given we know $\lambda$, is tha

$$E\large[ \;Z\; | \; \lambda \;\large] = \lambda $$

We will use this property often, so it's something useful to remember. Below we plot the
```

Here are the contents of `notebooks/Chapter1_Introduction--17-input.py`:

```
figsize( 12.5, 4)

import scipy.stats as stats
a = np.arange( 16 )
poi = stats.poisson
lambda_ = [1.5, 4.25 ]

plt.bar( a, poi.pmf( a, lambda_[0]), color=colours[0],
        label = "$\lambda = %.1f$"%lambda_[0], alpha = 0.60,
        edgecolor = colours[0], lw = "3")

plt.bar( a, poi.pmf( a, lambda_[1]), color=colours[1],
         label = "$\lambda = %.1f$"%lambda_[1], alpha = 0.60,
          edgecolor = colours[1], lw = "3")

plt.xticks( a + 0.4, a )
plt.legend()
plt.ylabel("probability of $k$")
plt.xlabel("$k$")
plt.title("Probability mass function of a Poisson random variable; differing \
$\lambda$ values");
```

Here are the contents of notebooks/Chapter1_Introduction--17-output-0.png:



Here are the contents of notebooks/Chapter1_Introduction--18.md:

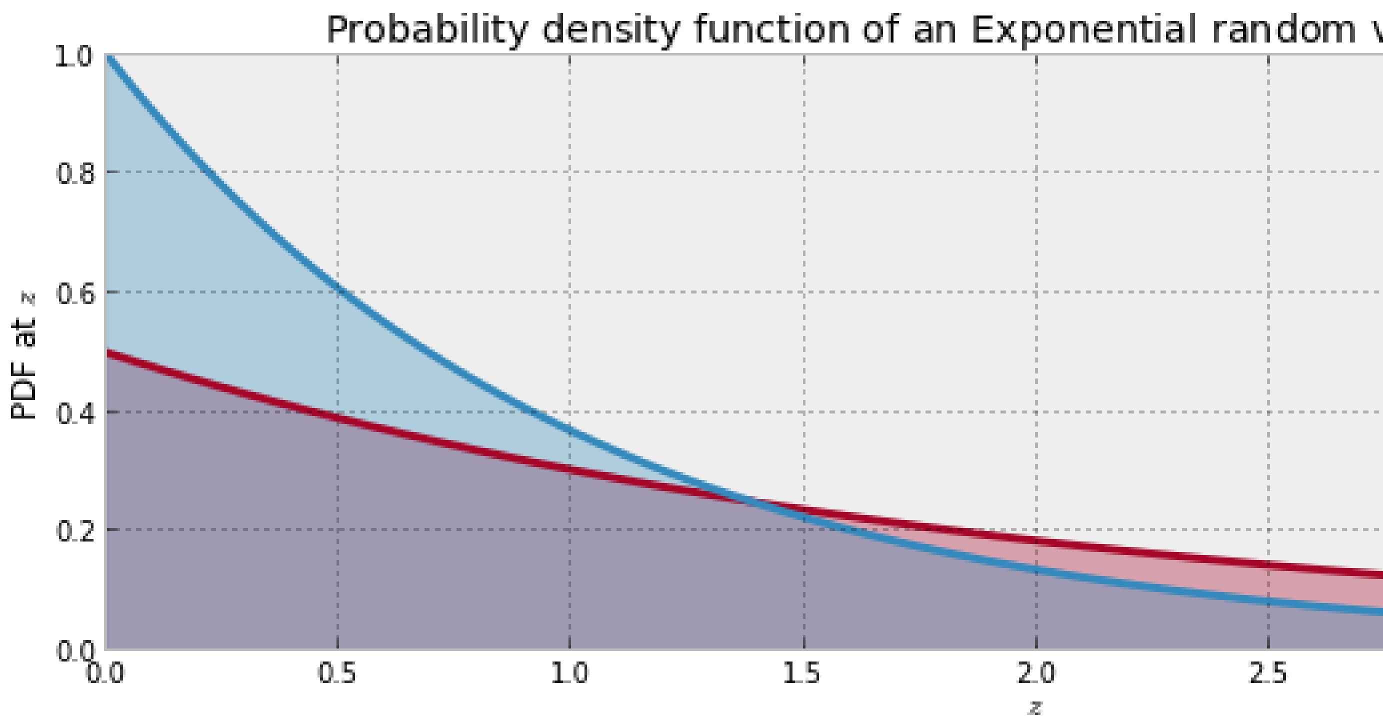
```
###Continuous Case
Instead of a probability mass function, a continuous random variable has a *probability

$$f_Z(z | \lambda) = \lambda e^{-\lambda z }, \;\; z\ge 0$$

Like the Poisson random variable, an exponential random variable can only take on non-ne

When a random variable $Z$ has an exponential distribution with parameter $\lambda$, we

$$Z \sim \text{Exp}(\lambda)$$

Given a specific $\lambda$, the expected value of an exponential random variable is equa

$$E[\; Z \;|\; \lambda \;] = \frac{1}{\lambda}$$
```

Here are the contents of notebooks/Chapter1_Introduction--19-input.py:

```
a = np.linspace(0,4, 100)
expo = stats.expon
lambda_ = [0.5, 1]

for l,c in zip(lambda_,colours):
```

```
    plt.plot( a, expo.pdf( a, scale=1./l), lw=3,
                color=c, label = "$\lambda = %.1f$"%l)
    plt.fill_between( a, expo.pdf( a, scale=1./l), color=c, alpha = .33)

plt.legend()
plt.ylabel("PDF at $z$")
plt.xlabel("$z$")
plt.title("Probability density function of an Exponential random variable;\
 differing $\lambda$");
```

Here are the contents of `notebooks/Chapter1_Introduction--19-output-0.png`:



Here are the contents of `notebooks/Chapter1_Introduction--20.md`:

```
###But what is $\lambda \;$?


**This question is what motivates statistics**. In the real world, $\lambda$ is hidden f

Bayesian inference is concerned with *beliefs* about what $\lambda$ is. Rather than try

This might seem odd at first: after all, $\lambda$ is fixed, it is not (necessarily) ran
```

Here are the contents of `notebooks/Chapter1_Introduction--21.md`:

```
##### Example: Inferring behaviour from text-message data
```

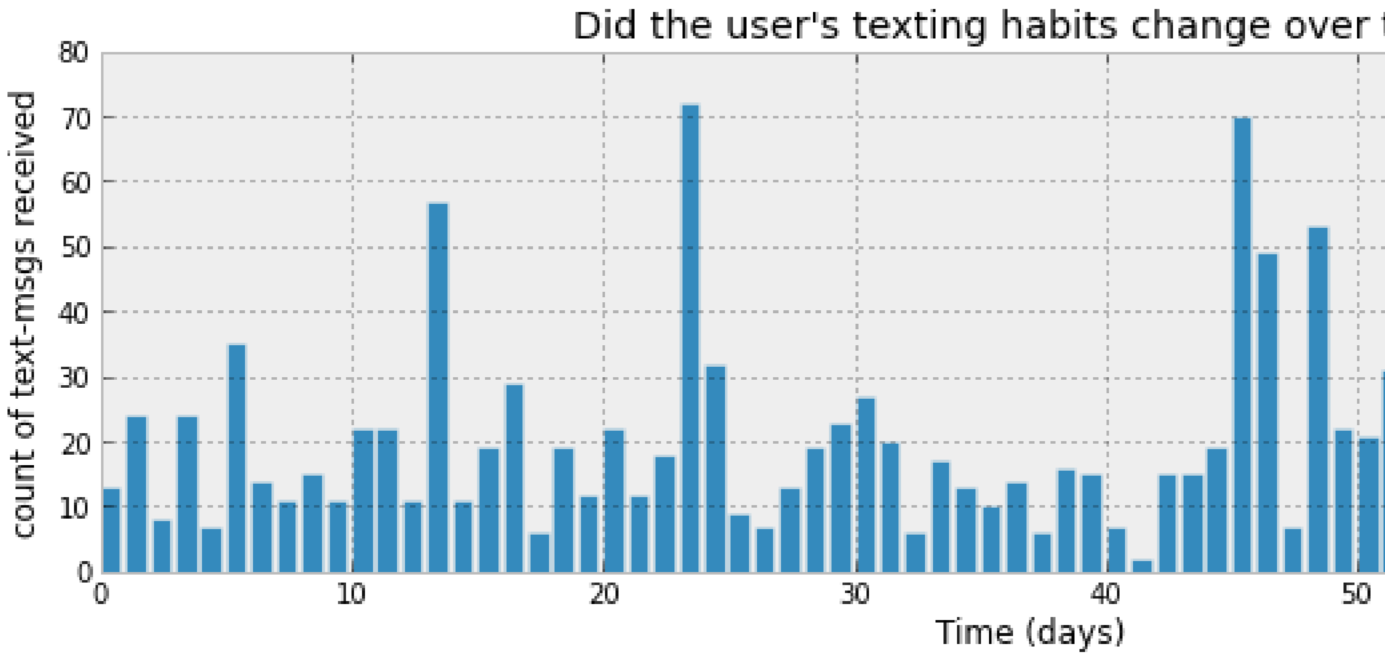
Let's try to model a more interesting example, concerning text-message rates:

> You are given a series of text-message counts from a user of your system. The data, p

Here are the contents of `notebooks/Chapter1_Introduction--22-input.py`:

```
figsize( 12.5, 3.5 )
count_data = np.loadtxt("data/txtdata.csv")
n_count_data = len(count_data)
plt.bar( np.arange( n_count_data ), count_data, color ="#348ABD"  )
plt.xlabel( "Time (days)")
plt.ylabel("count of text-msgs received")
plt.title("Did the user's texting habits change over time?")
plt.xlim( 0, n_count_data );
```

Here are the contents of `notebooks/Chapter1_Introduction--22-output-0.png`:



Here are the contents of `notebooks/Chapter1_Introduction--23.md`:

Before we begin, with respect to the plot above, would you say there was a change in beh
during the time period?

How can we start to model this? Well, as I conveniently already introduced, a Poisson ra

$$ C_i \sim \text{Poisson}(\lambda) $$

We are not sure about what the λ parameter is though. Looking at the chart above

How can we mathematically represent this? We can think, that at some later date (call it

```
$$
\lambda =
\begin{cases}
\lambda_1  & \text{if } t \lt \tau \cr
\lambda_2 & \text{if } t \ge \tau
\end{cases}
$$


 If, in reality, no sudden change occurred and indeed $\lambda_1 = \lambda_2$, the $\lam

We are interested in inferring the unknown $\lambda$s. To use Bayesian inference, we nee

\begin{align}
&\lambda_1 \sim \text{Exp}( \alpha ) \\\
&\lambda_2 \sim \text{Exp}( \alpha )
\end{align}

$\alpha$ is called a *hyper-parameter*, or a *parent-variable*, literally a parameter th

$$\frac{1}{N}\sum_{i=0}^N \;C_i \approx E[\; \lambda \; |\; \alpha ] = \frac{1}{\alpha}$

Alternatively, and something I encourage the reader to try, is to have two priors: one f

What about $\tau$? Well, due to the randomness, it is too difficult to pick out when $\t

\begin{align}
& \tau \sim \text{DiscreteUniform(1,70) }\\\\
& \Rightarrow P( \tau = k ) = \frac{1}{70}
\end{align}

So after all this, what does our overall prior for the unknown variables look like? Fran


Introducing our first hammer: PyMC
-----

PyMC is a Python library for programming Bayesian analysis [3]. It is a fast, well-maint

We will model the above problem using the PyMC library. This type of programming is call

B. Cronin [5] has a very motivating description of probabilistic programming:

>   Another way of thinking about this: unlike a traditional program, which only runs in
```

```
Due to its poorly understood title, I'll refrain from using the name *probabilistic prog
```

```
The PyMC code is easy to follow along: the only novel thing should be the syntax, and I
```

Here are the contents of notebooks/Chapter1_Introduction--24-input.py:

```
import pymc as mc



alpha = 1.0/count_data.mean() #recall count_data is
                              #the variable that holds our txt counts

lambda_1 = mc.Exponential( "lambda_1",  alpha )
lambda_2 = mc.Exponential( "lambda_2", alpha )

tau = mc.DiscreteUniform( "tau", lower = 0, upper = n_count_data )
```

Here are the contents of notebooks/Chapter1_Introduction--25.md:

```
In the above code, we create the PyMC variables corresponding to $\lambda_1, \; \lambda_
```

Here are the contents of notebooks/Chapter1_Introduction--26-input.py:

```
print "Random output:", tau.random(),tau.random(), tau.random()
```

Here are the contents of notebooks/Chapter1_Introduction--26-output-0.txt:

```
Random output: 58 31 17
```

Here are the contents of notebooks/Chapter1_Introduction--27-input.py:

```
@mc.deterministic
def lambda_( tau = tau, lambda_1 = lambda_1, lambda_2 = lambda_2 ):
    out = np.zeros( n_count_data )
    out[:tau] = lambda_1 #lambda before tau is lambda1
    out[tau:] = lambda_2 #lambda after tau is lambda2
    return out
```

Here are the contents of notebooks/Chapter1_Introduction--28.md:

```
This code is creating a new function `lambda_`, but really we think of it as a random va
```

Here are the contents of notebooks/Chapter1_Introduction--29-input.py:

```
observation = mc.Poisson( "obs", lambda_, value = count_data, observed = True)

model = mc.Model( [observation, lambda_1, lambda_2, tau] )
```

Here are the contents of notebooks/Chapter1_Introduction--30.md:

```
The variable `observation` combines our data, `count_data`, with our proposed data-gener

The below code will be explained in the Chapter 3, but this is where our results come fr
```

Here are the contents of notebooks/Chapter1_Introduction--31-input.py:

```
### Mysterious code to be explained in Chapter 3.
mcmc = mc.MCMC(model)
mcmc.sample( 40000, 10000, 1 )
```

Here are the contents of notebooks/Chapter1_Introduction--31-output-0.txt:

```
[****************100%******************]  40000 of 40000 complete
```

Here are the contents of notebooks/Chapter1_Introduction--31-output-1.txt:

Here are the contents of notebooks/Chapter1_Introduction--32-input.py:

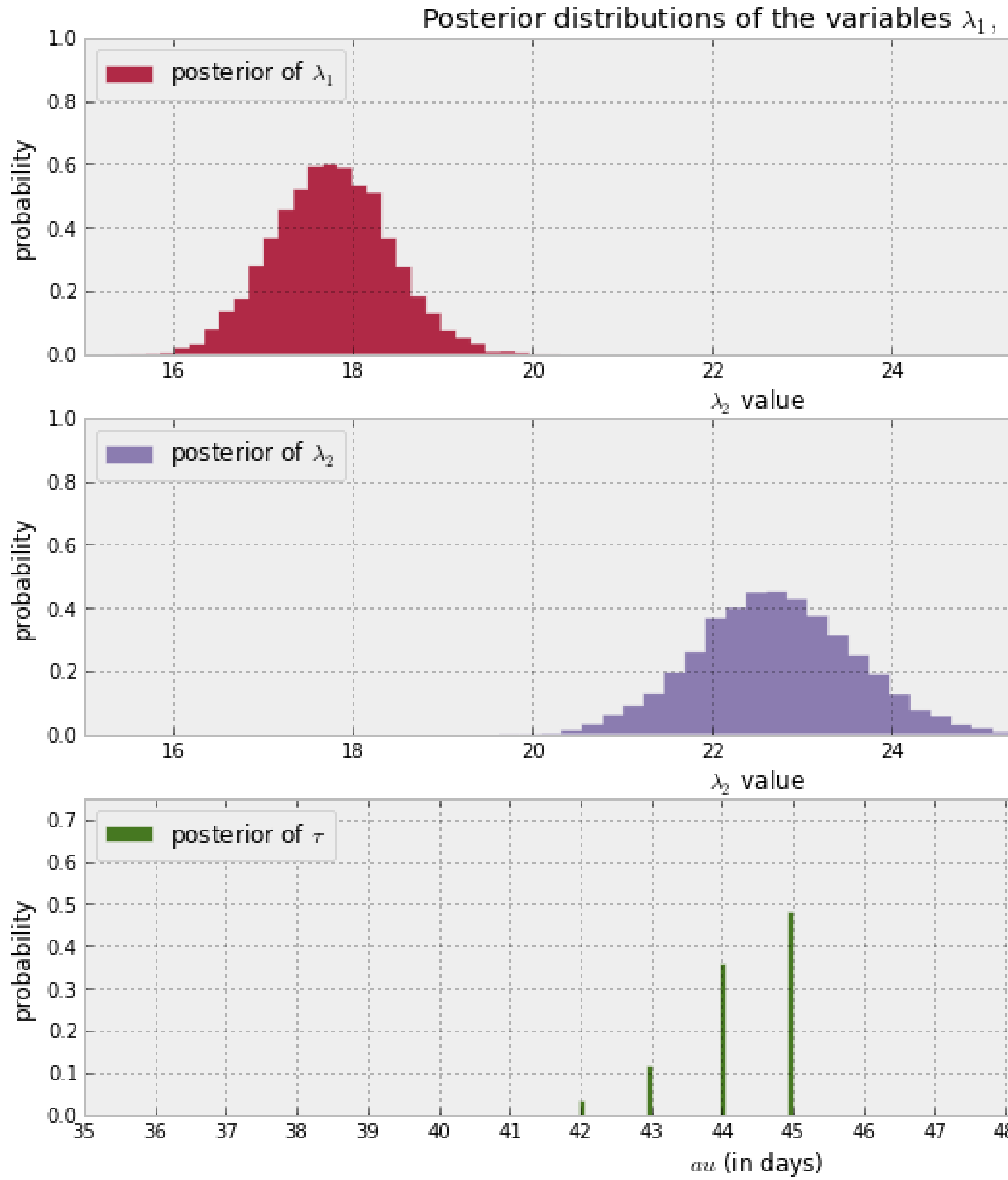
```
lambda_1_samples = mcmc.trace( 'lambda_1' )[:]
lambda_2_samples = mcmc.trace( 'lambda_2' )[:]
tau_samples = mcmc.trace( 'tau' )[:]
```

Here are the contents of notebooks/Chapter1_Introduction--33-input.py:

```
figsize(12.5, 10)
#histogram of the samples:

ax = plt.subplot(311)
ax.set_autoscaley_on(False)

plt.hist( lambda_1_samples, histtype='stepfilled', bins = 30, alpha = 0.85,
        label = "posterior of $\lambda_1$", color = "#A60628",normed = True )
plt.legend(loc = "upper left")
plt.title(r"Posterior distributions of the variables $\lambda_1,\;\lambda_2,\;\tau$")
plt.xlim([15,30])
plt.xlabel("$\lambda_2$ value")
plt.ylabel("probability")
```

```
ax = plt.subplot(312)
ax.set_autoscaley_on(False)

plt.hist( lambda_2_samples,histtype='stepfilled', bins = 30, alpha = 0.85,
          label = "posterior of $\lambda_2$",color="#7A68A6", normed = True )
plt.legend(loc = "upper left")
plt.xlim([15,30])
plt.xlabel("$\lambda_2$ value")
plt.ylabel("probability")

plt.subplot(313)


w = 1.0/ tau_samples.shape[0] * np.ones_like( tau_samples )
plt.hist( tau_samples, bins = n_count_data, alpha = 1,
         label = r"posterior of $\tau$",
         color="#467821", weights=w, rwidth =2. )
plt.xticks( np.arange( n_count_data ) )

plt.legend(loc = "upper left");
plt.ylim([0,.75])
plt.xlim([35, len(count_data)-20])
plt.xlabel("$\tau$ (in days)")
plt.ylabel("probability");
```

Here are the contents of `notebooks/Chapter1_Introduction--33-output-0.png`:



Here are the contents of `notebooks/Chapter1_Introduction--34.md`:

Interpretation

Recall that the Bayesian methodology returns a *distribution*, hence we now have distrib

Also notice that the posterior distributions for the λ's do not look like any ex

Our analysis also returned a distribution for what τ might be. Its posterior distri

Here are the contents of `notebooks/Chapter1_Introduction--35.md`:

###Why would I want samples from the posterior, anyways?

We will deal with this question for the remainder of the book, and it is an understateme

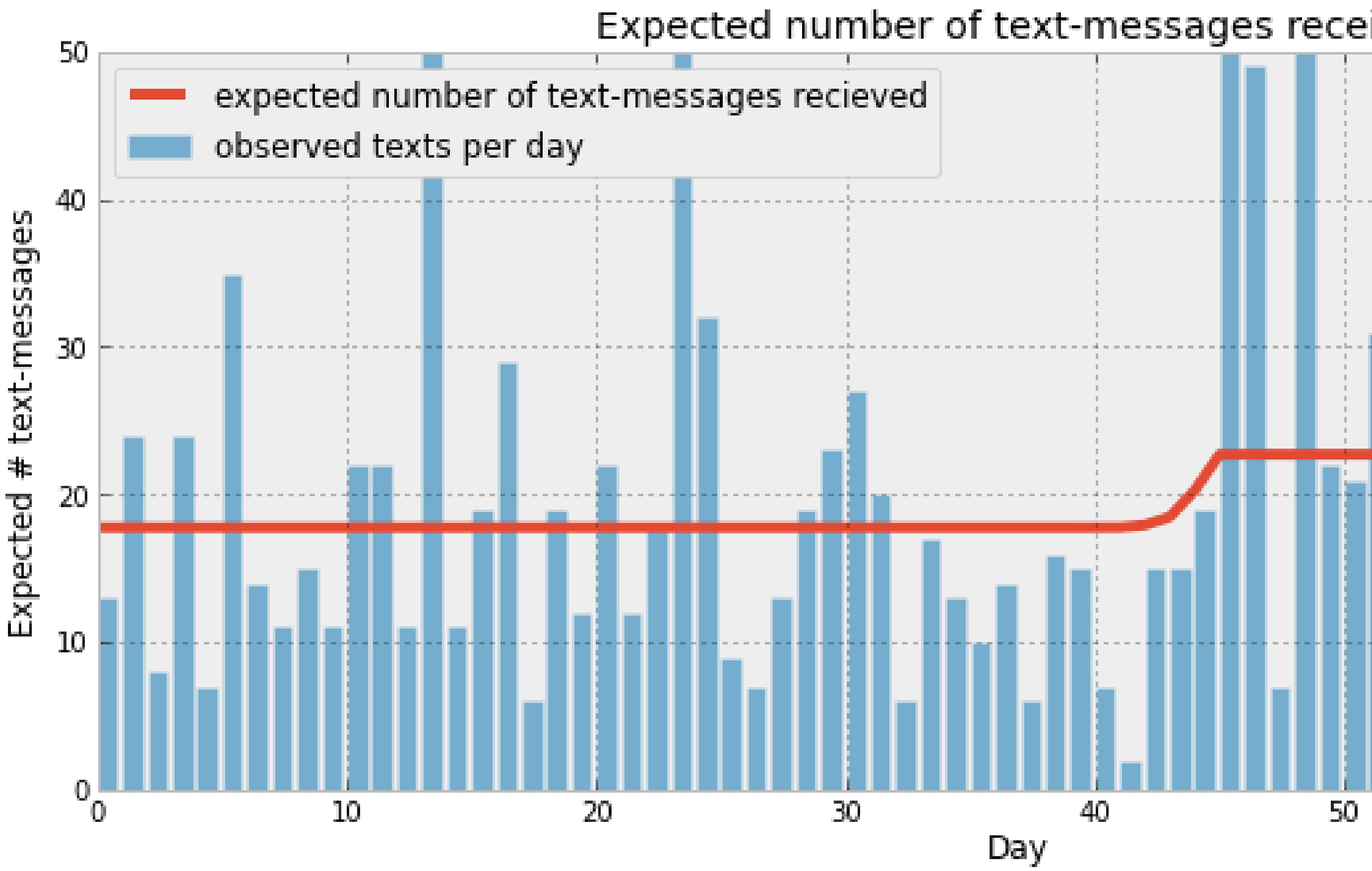
In the code below, we are calculating the following: Let i index samples from the post

Here are the contents of `notebooks/Chapter1_Introduction--36-input.py`:

```
figsize( 12.5, 5)
# tau_samples, lambda_1_samples, lambda_2_samples contain
# N samples from the corresponding posterior distribution
N = tau_samples.shape[0]
expected_texts_per_day = np.zeros(n_count_data)
for  day in range(0, n_count_data):
    # ix is a bool index of all tau samples corresponding to
    # the switchpoint occurring prior to value of 'day'
    ix = day < tau_samples
    # Each posterior sample corresponds to a value for tau.
    # for each day, that value of tau indicates whether we're "before"
    # (in the lambda1 "regime") or
    #  "after" (in the lambda2 "regime") the switchpoint.
    # by taking the posterior sample of lambda1/2 accordingly, we can average
    # over all samples to get an expected value for lambda on that day.
    # As explained, the "message count" random variable is Poisson distributed,
    # and therefore lambda (the poisson parameter) is the expected value of "message cou
    expected_texts_per_day[day] = (lambda_1_samples[ix].sum()
                                    + lambda_2_samples[~ix].sum() ) /N


plt.plot( range( n_count_data), expected_texts_per_day, lw =4, color = "#E24A33",
         label = "expected number of text-messages recieved")
plt.xlim( 0, n_count_data )
plt.xlabel( "Day" )
plt.ylabel( "Expected # text-messages" )
```

```python
plt.title( "Expected number of text-messages received")
plt.ylim( 0, 50 )
plt.bar( np.arange( len(count_data) ), count_data, color ="#348ABD", alpha = 0.65,
            label="observed texts per day")

plt.legend(loc="upper left");
```

Here are the contents of `notebooks/Chapter1_Introduction--36-output-0.png`:



Here are the contents of `notebooks/Chapter1_Introduction--37.md`:

```
Our analysis shows strong support for believing the user's behavior did change ($\lambda
```

Here are the contents of `notebooks/Chapter1_Introduction--38.md`:

```
##### Exercises

1\.  Using `lambda_1_samples` and `lambda_2_samples`, what is the mean of the posterior
```

Here are the contents of `notebooks/Chapter1_Introduction--39-input.py`:

```python
#type your code here.
```

Here are the contents of `notebooks/Chapter1_Introduction--40.md`:

```
2\.  What is the expected percentage increase in text-message rates? `hint:` compute the
```

Here are the contents of notebooks/Chapter1_Introduction--41-input.py:

```
#type your code here.
```

Here are the contents of notebooks/Chapter1_Introduction--42.md:

```
3\. What is the mean of $\lambda_1$ **given** we know $\tau$ is less than 45.  That is,
```

Here are the contents of notebooks/Chapter1_Introduction--43-input.py:

```
#type your code here.
```

Here are the contents of notebooks/Chapter1_Introduction--44.md:


```
### References


-  [1] Gelman, Andrew. N.p.. Web. 22 Jan 2013. <http://andrewgelman.com/2005/07/n_is_nev
-  [2] Norvig, Peter. 2009. [*The Unreasonable Effectiveness of Data*](http://www.csee.w
- [3] Patil, A., D. Huard and C.J. Fonnesbeck. 2010.
PyMC: Bayesian Stochastic Modelling in Python. Journal of Statistical
Software, 35(4), pp. 1-81.
- [4] Jimmy Lin and Alek Kolcz. Large-Scale Machine Learning at Twitter. Proceedings of
- [5] Cronin, Beau. "Why Probabilistic Programming Matters." 24 Mar 2013. Google, Online
```


Here are the contents of notebooks/Chapter1_Introduction--45-input.py:

```
from IPython.core.display import HTML
def css_styling():
    styles = open("../styles/custom.css", "r").read()
    return HTML(styles)
css_styling()
```

Here are the contents of notebooks/Chapter1_Introduction--46-input.py: