Random Variable Symmetries

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Just trying to flesh out notations and how to implement these ideas computationally.

Take a set of random variables $\tilde{\mathcal{R}}$ for an inflation DAG and a set of random variables for the original DAG $\mathcal{R}$ and define the following:

The *copy set* $\tilde{\mathcal{R}}_i$ are all of the random variables in $\tilde{\mathcal{R}}$ with the particular copy index i.

$$\tilde{\mathcal{R}}_i = \left\{R_s \in \tilde{\mathcal{R}} \mid s = i\right\} \quad \tilde{\mathcal{R}}_i \subseteq \tilde{\mathcal{R}} \quad \tilde{\mathcal{R}} = \bigcup_{i \in \mathcal{I}(\tilde{\mathcal{R}})} \tilde{\mathcal{R}}_i$$

The *face set* $\tilde{\mathcal{R}}_X$ are all of the random variables in $\tilde{\mathcal{R}}$ with particular face X.

$$\tilde{\mathcal{R}}_X = \left\{R_s \in \tilde{\mathcal{R}} \mid R = X\right\} \quad \tilde{\mathcal{R}}_X \subseteq \tilde{\mathcal{R}} \quad \tilde{\mathcal{R}} = \bigcup_{X \in \mathcal{R}} \tilde{\mathcal{R}}_X$$

Permutation of Parties

For a particular copy set $\tilde{\mathcal{R}}_i$, say $\tilde{\mathcal{R}} = \{A_1, A_2, B_1, B_2, C_1, C_2, C_3, C_4\}$ where each has a number of outcomes $n\left(X_s\right)$. As an example we have,

$$\tilde{\mathcal{R}}_1 = \{A_1, B_1, C_1\}$$

The *permutation group* of S_n where $n = \left|\tilde{\mathcal{R}}_i\right|$ has $n-1$ generators $\sigma_1, \ldots, \sigma_{n-1}$ that has the properties,

$$\sigma_i^2 = 1 \quad \sigma_i\sigma_j = \sigma_j\sigma_i \text{ if } j \neq i \pm 1 \quad \left(\sigma_i\sigma_{i+1}\right)^3 = 1$$