CALIFORNIA STATE UNIVERSITY, LONG BEACH

EE 381 – Probability and Statistics with Applications to Computing

Project on Conditional Probabilities

Communication through a Noisy Channel

0. Introduction and Background Material

0.1. Message transmission through a noisy communication channel

A digital message "M" is created and sent through a noisy communication channel (Figure 1). The signal "S" to be transmitted consists of a series of zeros and ones:

- symbol "0" appears in the signal with probability p_0
- symbol "1" appears in the signal with probability $p_1 = 1 - p_0$

The transmitted signal "S" is received at the other end of the channel as signal "R". Due to noise in the communication channel, a transmitted bit may change during transmission:

- a transmitted bit 0 may be received as 1 with probability ε_0 (probability of transmission error for symbol 0);
- a transmitted bit 1 may be received as 0 with probability ε_1 (probability of transmission error for symbol 1).

The errors for different symbol transmissions are independent.

In order to create one bit of the transmitted message “S” you have to:

- Generate a random number: $m =$ np.random.rand( )
- Generate the transmitted message “S” as: $S = \begin{cases} 0 & \text{if } m \le p_0 \\ 1 & \text{if } m > p_0 \end{cases}$
- Notice that this process can be implemented by using your own function:
  $S = \texttt{nSidedDie}([p_0, 1-p_0])$; $S = S - 1$

In order to create the received signal “R” you have to:

- Generate a random number: $t =$ np.random.rand( ). The random number t should be different than the previous random number m .
- Generate the received signal “R” as: $R = \begin{cases} 1 & \text{if } S=0 \text{ and } t \le \varepsilon_0 \\ 0 & \text{if } S=0 \text{ and } t > \varepsilon_0 \\ 1 & \text{if } S=1 \text{ and } t > \varepsilon_1 \\ 0 & \text{if } S=1 \text{ and } t \le \varepsilon_1 \end{cases}$
- This process can be implemented by using your own function:

  If $S = 1$: $R = \texttt{nSidedDie}([\varepsilon_1, 1-\varepsilon_1])$; $R = R - 1$

  If $S = 0$: $R = \texttt{nSidedDie}([1-\varepsilon_0, \varepsilon_0])$; $R = R - 1$

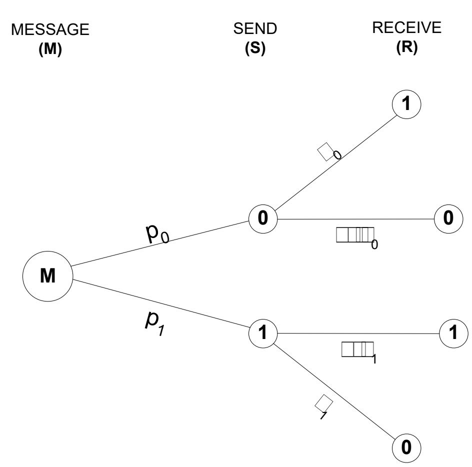

Figure 1. Probabilities for symbol transmission error

Reference: "*Introduction to probability*", D. Bertsekas and N. Tsitsiklis, 2nd Edition, Athena Scientific, 2008.

1. Probability of erroneous transmission

Consider the following experiment, where the required probabilities p_0 ; ε_0; and ε_1 *have been provided to you in a separate document*.

- You transmit a one-bit message S and look at the received signal R. If $R = S$, the experiment in considered a success., otherwise it is a failure.
- You repeat this experiment $N = 100,000$ times and count the number of failures.
- **Find the probability** that the transmitted bit will be received incorrectly, i.e. the probability of failure.
- **SUBMIT** your report in a Word or PDF file. *Use the table below for your answer.* Note: You will need to replicate the table in your Word file, in order to provide the answer in your report. Points will be taken off if you do not use the table.

Probability of transmission error	
Ans.	***p* =**

2. Conditional probability: $P(R=1|S=1)$

Use the same probabilities p_0 ; ε_0; and ε_1 as before and consider the following experiment:

- You create and transmit a one-bit message S as you did before. The goal is to calculate the conditional probability $P(R=1|S=1)$. This means that you will focus only in those transmissions where $S=1$.
- For all the events for which the transmitted signal is $S=1$, look at the received bit R. If $R=1$, the experiment is a success, i.e. success is defined as the conditional event: $(R=1|S=1)$
- You repeat this experiment N=100,000 times and count the number of successes.
- **Find the conditional probability** $P(R=1|S=1)$, i.e. the probability that if you transmit the symbol $S=1$, it will be received correctly.
- **SUBMIT** your report in a Word or PDF file. *Use the table below for your answer.* Note: You will need to replicate the table in your Word file, in order to provide the answer in your report. Points will be taken off if you do not use the table.

Conditional probability $P(R=1|S=1)$	
Ans.	***p* =**

3. Conditional probability: $P(S=1|R=1)$

Use the same probabilities p_0 ; ε_0; and ε_1 as before and consider the following experiment:

- You create and transmit a one-bit message S as you did before. The goal is to calculate the conditional probability $P(S=1|R=1)$. This means that you will only be interested in those messages where the received signal is $R=1$.
- For all the events for which the received signal is $R=1$, look at transmitted bit S. If $S=1$, the experiment is a success, i.e. success is defined as the conditional event: $(S=1|R=1)$
- You repeat this experiment N=100,000 times and count the number of successes.
- **Find the conditional probability** $P(S=1|R=1)$, i.e. the probability that if you receive the symbol $R=1$, you can correctly conclude that it actually came from a transmitted signal of $S=1$.
- **SUBMIT** your report in a Word or PDF file. *Use the table below for your answer*. Note: You will need to replicate the table in your Word file, in order to provide the answer in your report. Points will be taken off if you do not use the table.

Conditional probability $P(S=1\|R=1)$	
Ans.	***p*** **=**

4. Enhanced transmission method

Use the same probabilities p_0 ; ε_0; and ε_1 as before and consider the following experiment:

- You create and transmit a one-bit message S as before. In order to improve reliability, the same bit "S" is transmitted three times (S S S) as shown in Figure 2.
- The received bits "R" are not necessarily the same as the transmitted bits "S" due to transmission errors. The three received bits, shown as (R1 R2 R3) in Figure 2 will be equal to one of the following eight triplets:
  (R1 R2 R3) ={ (000), (001), (010), (100), (011), (101), (110), (111) }
  When you look at the received triplet (R1 R2 R3) you must decide what was the bit "S" originally transmitted by using voting and the majority rule. Here are some examples of the majority rule.
- For example, if the three received bits are (R1 R2 R3)=(001), then the majority rule will decide that the bit must be a "0". We denote this as the decoded bit D=0.
- As another example if the three received bits are (R1 R2 R3)=(101), then the majority rule will decode the bit as D=1.
- Another example: If you send S=0 three times, i.e. (S S S) = (0 0 0) and the received string is (R1 R2 R3) = (000), (001), (010), or (100) then the symbol will be decoded as D=0 and the experiment is a success, otherwise it is a failure.
- Another example: If you transmit S=1 three times, i.e. (S S S) = (1 1 1) and the received string is (011), (101), (110), or (111) the symbol will be decoded as D=1 and the experiment is a success, otherwise it is a failure.
- This procedure as described above is considered one experiment.
- Repeat the experiment N=100,000 times and count the number of successes.
- **Find the probability** that the transmitted bit "S" will be **received and decoded incorrectly**.
- **Comment** on whether the voting method used in this problem provides any improvement as compared to the method of Problem 1.
- **SUBMIT** your report in a Word or PDF file. *Use the table below for your answer.* Note: You will need to replicate the table in your Word file, in order to provide the answer in your report. Points will be taken off if you do not use the table.

Probability of error with enhanced transmission	
Ans.	***p* =**

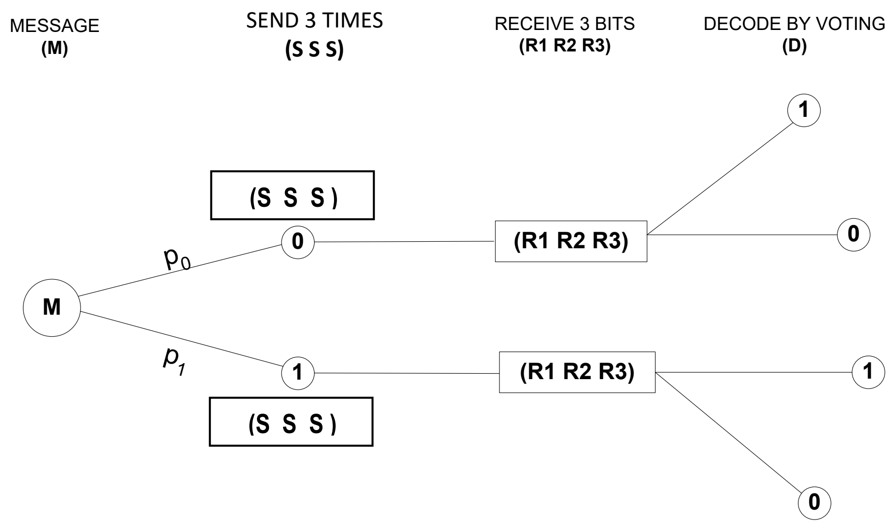

Figure 2. Enhanced message transmission method