Countable and dependent choice
and countably distributive toposes

Alex Simpson

Faculty of Mathematics and Physics (FMF), University of Ljubljana
Institute for Mathematics, Physics and Mechanics (IMFM), Ljubljana

The axiom of countable choice and the stronger axiom of dependent choice are frequently used in mathematics, both classical and constructive. I shall consider various formulations of these axioms in elementary toposes and in Grothendieck toposes. One interesting case is that of atomic toposes, for which the two choice axioms can be related to simple conditions on the base category.

Countable choice can be viewed as a special case of a full countable distributivity property, enjoyed by *countaby distributive toposes*. I shall define this notion, provide several examples, and show that it enjoys a number of interesting characterisations.

An early incarnation of the material in the first half of the talk appeared in the master's dissertation of my former student Severin Mejak [1]. While much of this material may be folklore, I shall give a systematic presentation, which will serve the purpose of setting the scene for the second half of the talk.

References

[1] S. Mejak. *Topos models of set-theoretic principles.* Master's dissertation, Faculty of Mathematics and Physics, University of Ljubljana, 2019.