

Joyal's representation theorem for Heyting categories

An unpublished result by André Joyal, believed to have been formulated around 1970, extends the Stone representation theorem for Boolean algebras to Heyting categories. Specifically :

Every small Heyting category \mathcal{H} can be embedded into one of the form $\mathbf{Set}^{\mathcal{C}}$ via a Heyting, conservative functor.

Through the construction of syntactic categories built from first-order theories, Joyal's representation theorem offers completeness results for intuitionistic first-order logic. While a model-theoretic proof was originally presented by Makkai and Reyes in 1977, we introduce in this work a new, purely categorical approach.