Lax comma categories: descent and exponentiability

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Janelidze's categorical Galois theory [4] provides a unifying perspective on various Galois-type theorems, which include, for instance, Magid's Galois theory for commutative rings, and Grothendieck's Galois theory for étale coverings of schemes. In fact, such results were generalized and found applications in other settings [5] [6].

Aiming to develop a bidimensional counterpart of Janelidze's Galois theory, Clementino and Lucatelli Nunes initiated their research by establishing the core concepts of categorical Galois for lax comma 2-categories [1], which are the 2-dimensional analogue of comma categories. As these 2-categories play an important role, it then became apparent that their systematic study would provide some guiding principles in the development of such a higher dimensional notion of Galois theory.

We will give an overview our results from [2, 3] as well as ongoing work on various sorts of lax comma categories, of both a categorical and topological flavor. More specifically, we will discuss the effective descent morphisms and exponentiable objects in these classes of (2-)categories.

This talk is based on joint work with Maria Manuel Clementino, Dirk Hofmann and Fernando Lucatelli Nunes.

[1] M.M. Clementino and F. Lucatelli Nunes. Lax comma 2-categories and admissible 2-functors. *Theory Appl. Categ.*, 40(5):180–226, 2024.

[2] M.M. Clementino, F. Lucatelli Nunes and R. Prezado. Lax comma categories: cartesian closedness, extensivity, topologicity, and descent. *Theory Appl. Categ.*, 41(16):516–530, 2024.

[3] M.M. Clementino and R. Prezado. Effective descent morphisms of ordered families. *Quaest. Math.*, to appear, 2024.

[4] G. Janelidze. Pure Galois theory in categories. *J. Algebra*, 132(2):270–286, 1990

[5] F. Lucatelli Nunes. Pseudo-Kan extensions and descent theory. *Theory Appl. Categ.*, 33:15, 390–444, 2018

[6] B. Noohi, I. Tomašić. Galois theory of differential schemes. arXiv:2407.21147, 2024.