

(THE SEMANTIC LAX DESCENT)
FACTORIZATION OF A
FUNCTOR

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109TH PSSL

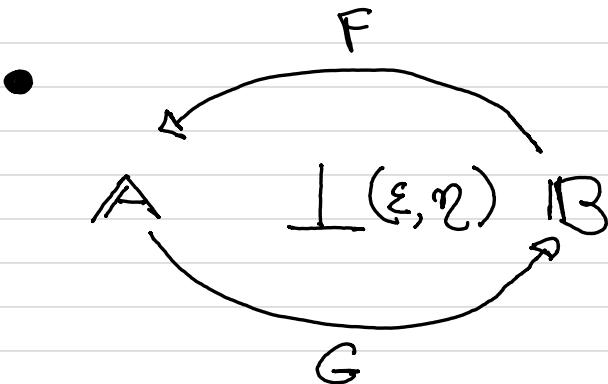
NOVEMBER 17TH, 2024

LEIDEN, THE NETHERLANDS

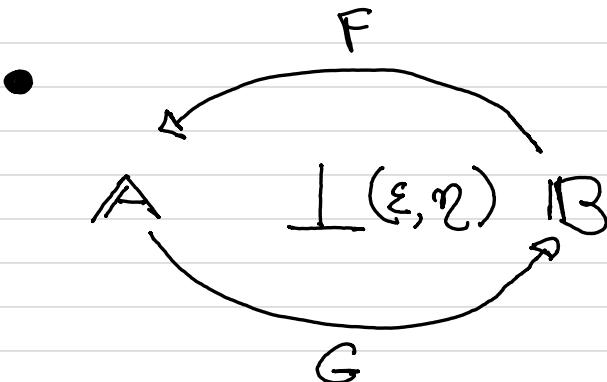
- AIM OF THE TALK
- LAX REGULAR IMAGE OF A FUNCTOR
(SEMANTIC LAX DESCENT FACTORIZATION)
- Point out THAT THE (Co)EILENBERG MOORE
FACTORIZATION OF A (LEFT) RIGHT ADJOINT
FUNCTOR.

* LUCATELLI NUNES,
Semantic Factorization and Descent
APPLIED CATEG. STRUCTURES, 2022

A \xrightarrow{G} B functor



adjoint situation

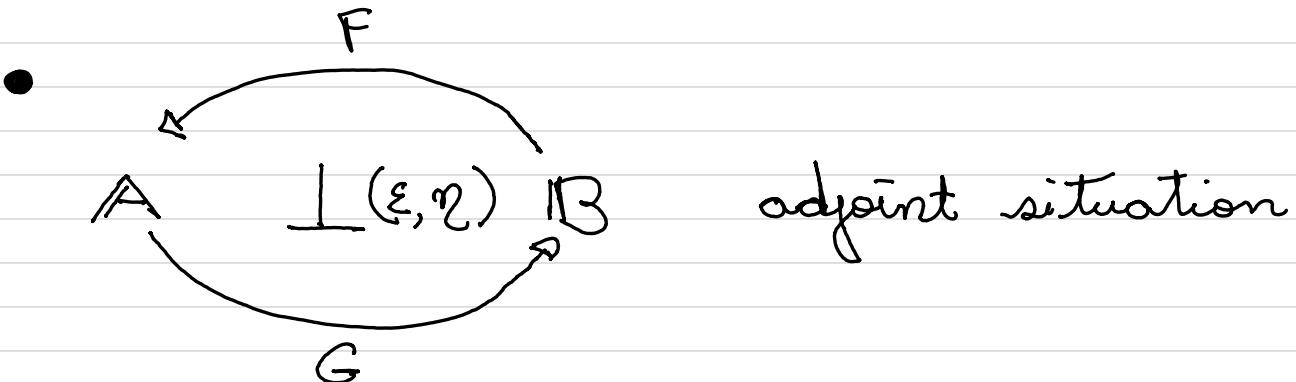


adjoint situation

→ monad $(GF, G\epsilon F, \eta)$

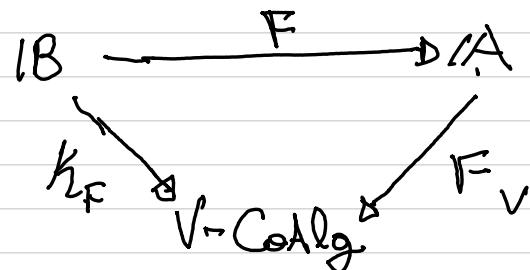
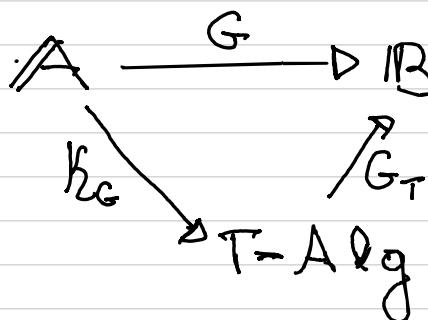
→ comonad $(FG, F\eta G, \epsilon)$

(P. S. Hirsch, 64)



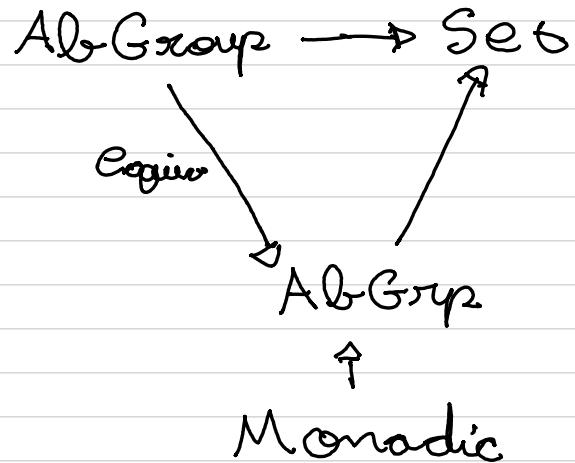
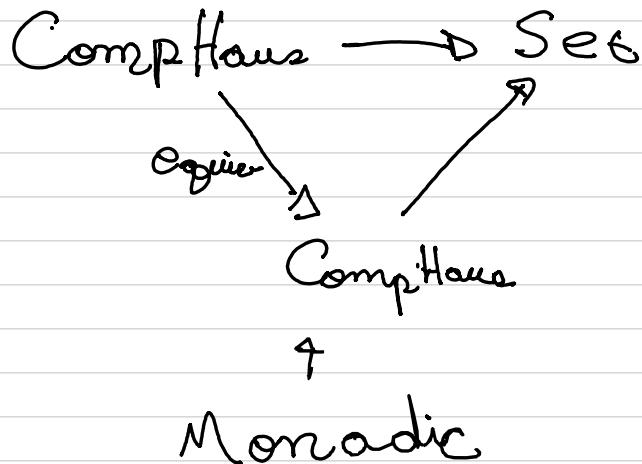
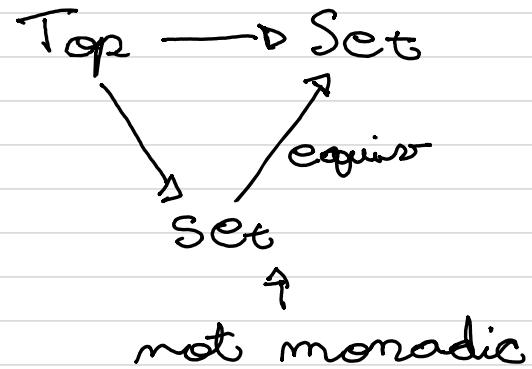
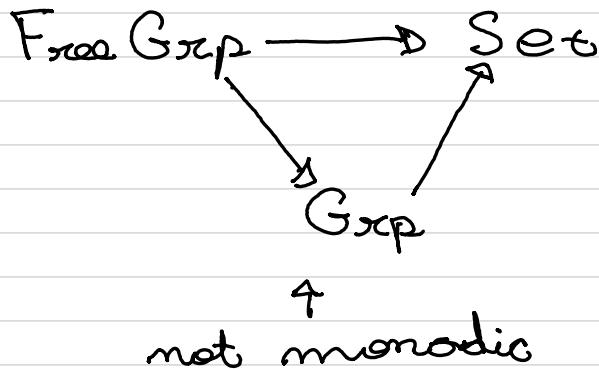
- monad $(GF, G \varepsilon F, \eta) = T$
- comonad $(FG, F\eta G, \varepsilon) = V$

• Ei LEN BERG - MOORE FACTORIZATIONS:

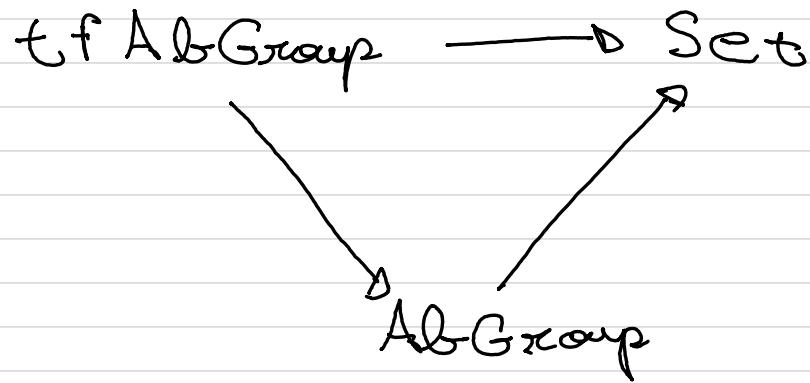


(Eilenberg and Moore , 1965)

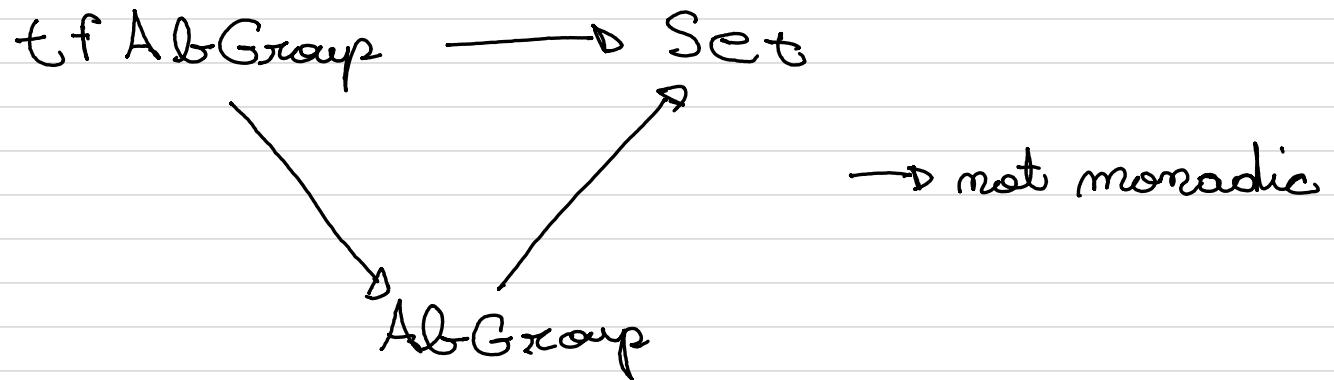
- Examples (Eilenberg - Moore Factorizations)



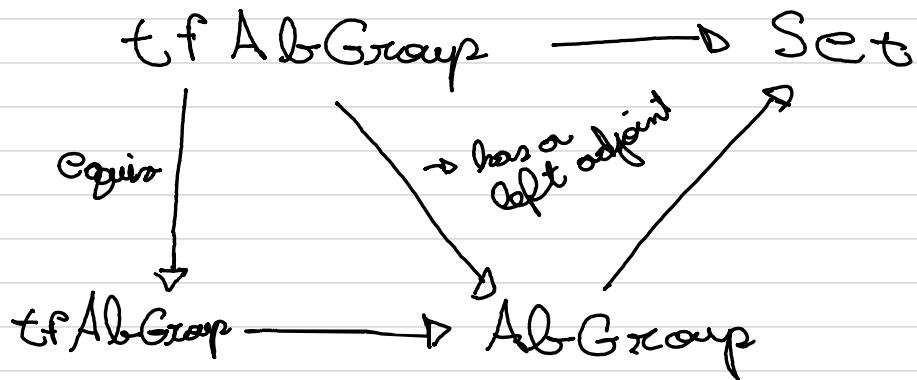
- Further Examples (Eilenberg - Moore Factorizations)



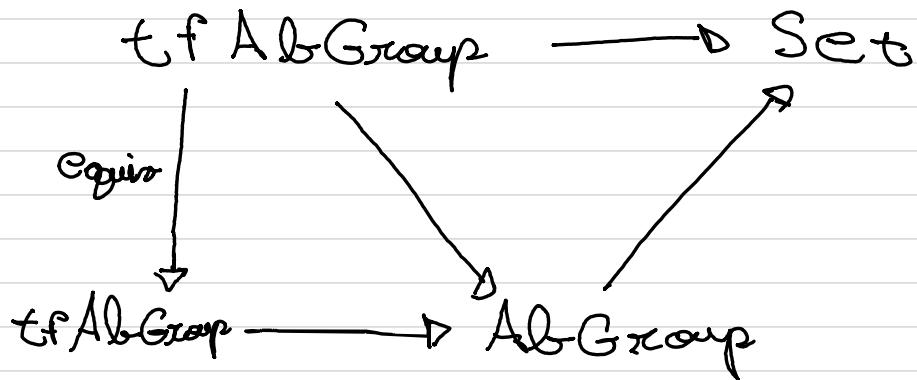
- Further Examples (Eilenberg - Moore Factorizations)



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- Further Examples (Eilenberg - Moore Factorizations)



COMPOSITION OF MONADIC FUNCTORS
MIGHT NOT BE MONADIC!

• LAST EXAMPLE :

$S \rightarrow$ small category
 $C \rightarrow$ cocomplete and complete

$$\begin{array}{ccc} \text{Fun}(S, C) & \longrightarrow & \text{Fun}(S_0, C) \\ \text{cois} \searrow & & \nearrow \\ & \text{Fun}(S, C) & \end{array}$$

MONADIC
AND
COMONADIC

A L G E B R A S
A N D
C O A L G E B R A S
COINCIDE.

- Getting back to the general setting:

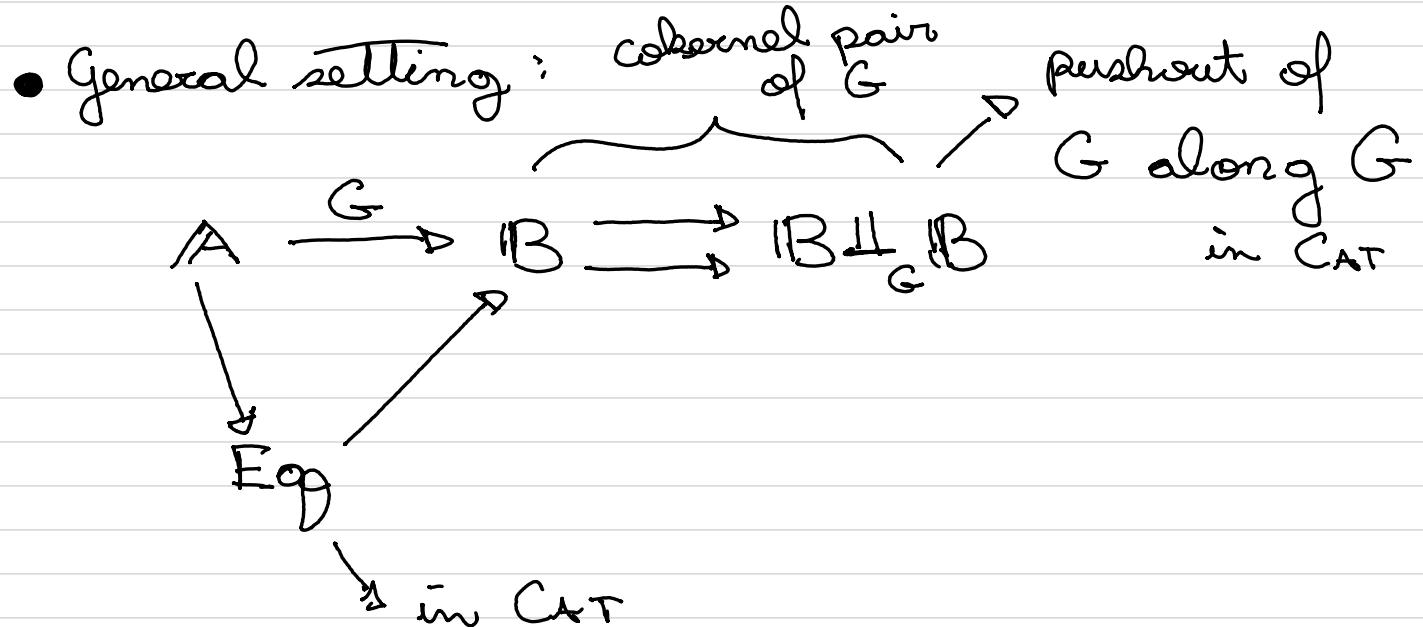
$$A \xrightarrow{G} B$$

- General setting:

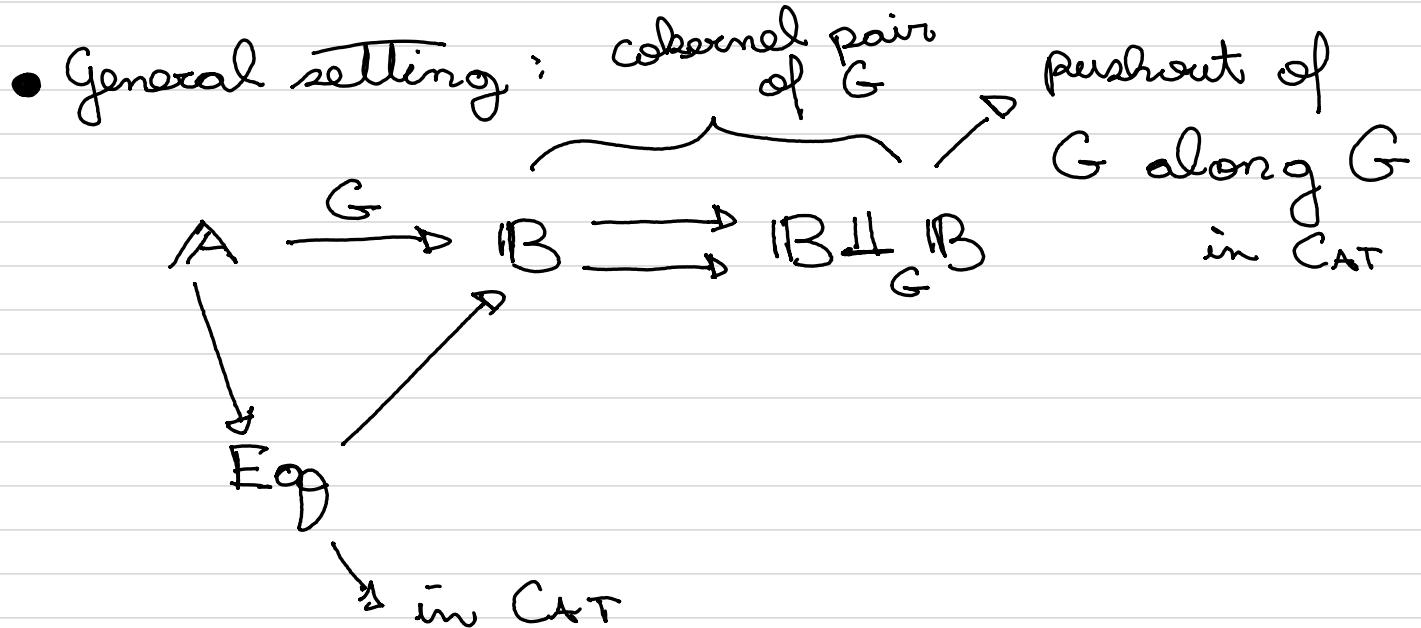
$$A \xrightarrow{G} B$$

DOES NOT HAVE EILENBERG
- MOORE Factorization

- General setting: cokernel pair of G  pushout of G along G in CAT



REGULAR IMAGE FACTORIZATION



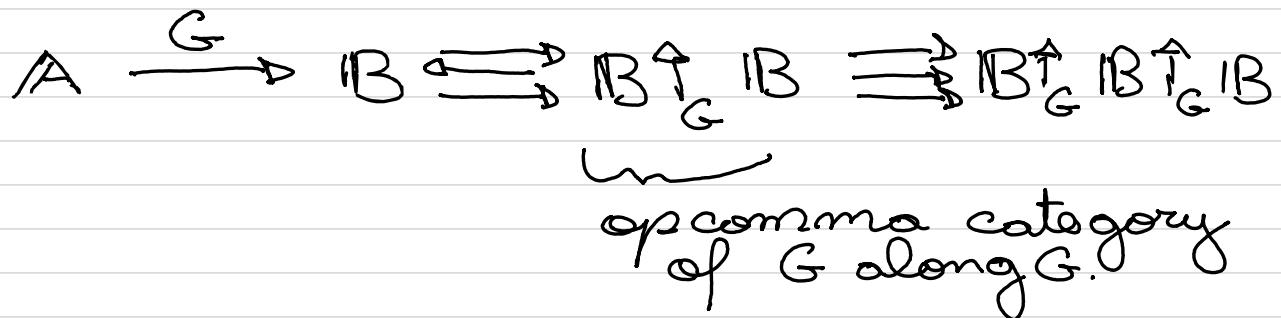
- CONSIDER THE 2-DIMENSIONAL STRUCTURE
- LET'S MAKE IT LAX

- General setting:

$$A \xrightarrow{G} B \rightleftarrows B^T_G B \xrightarrow{\exists} B^T_G B^T_G B$$

(LAX) TWO-DIMENSIONAL
COKERNEL DIAGRAM

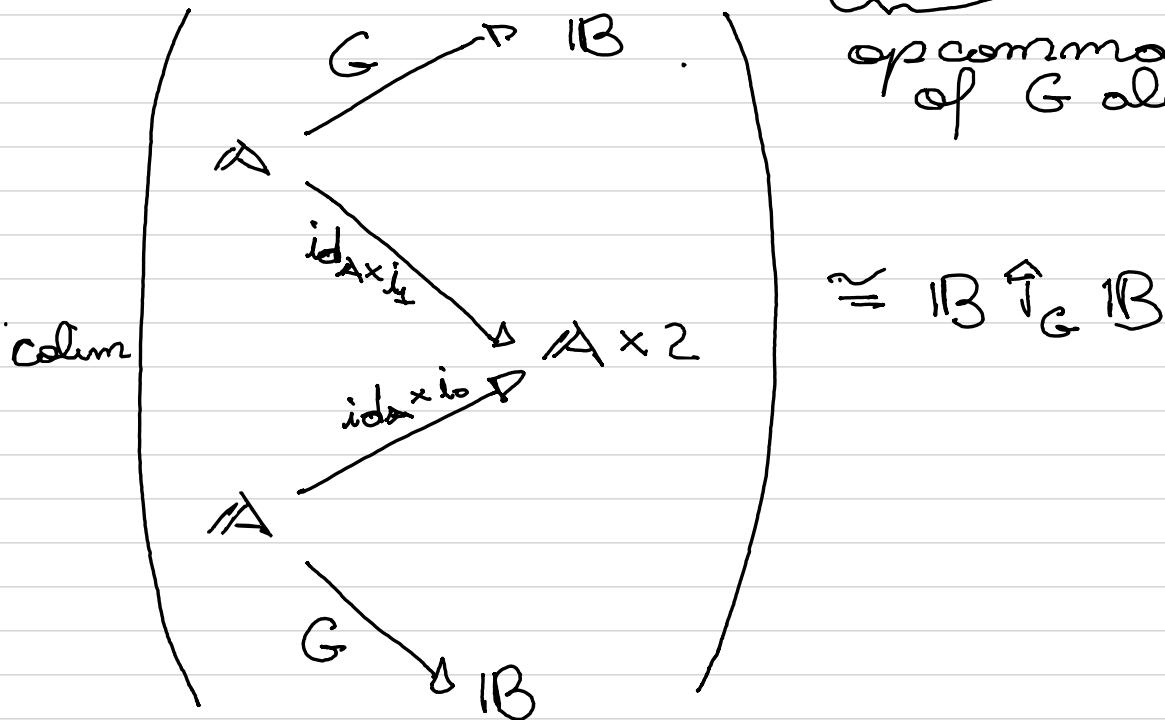
- General setting:



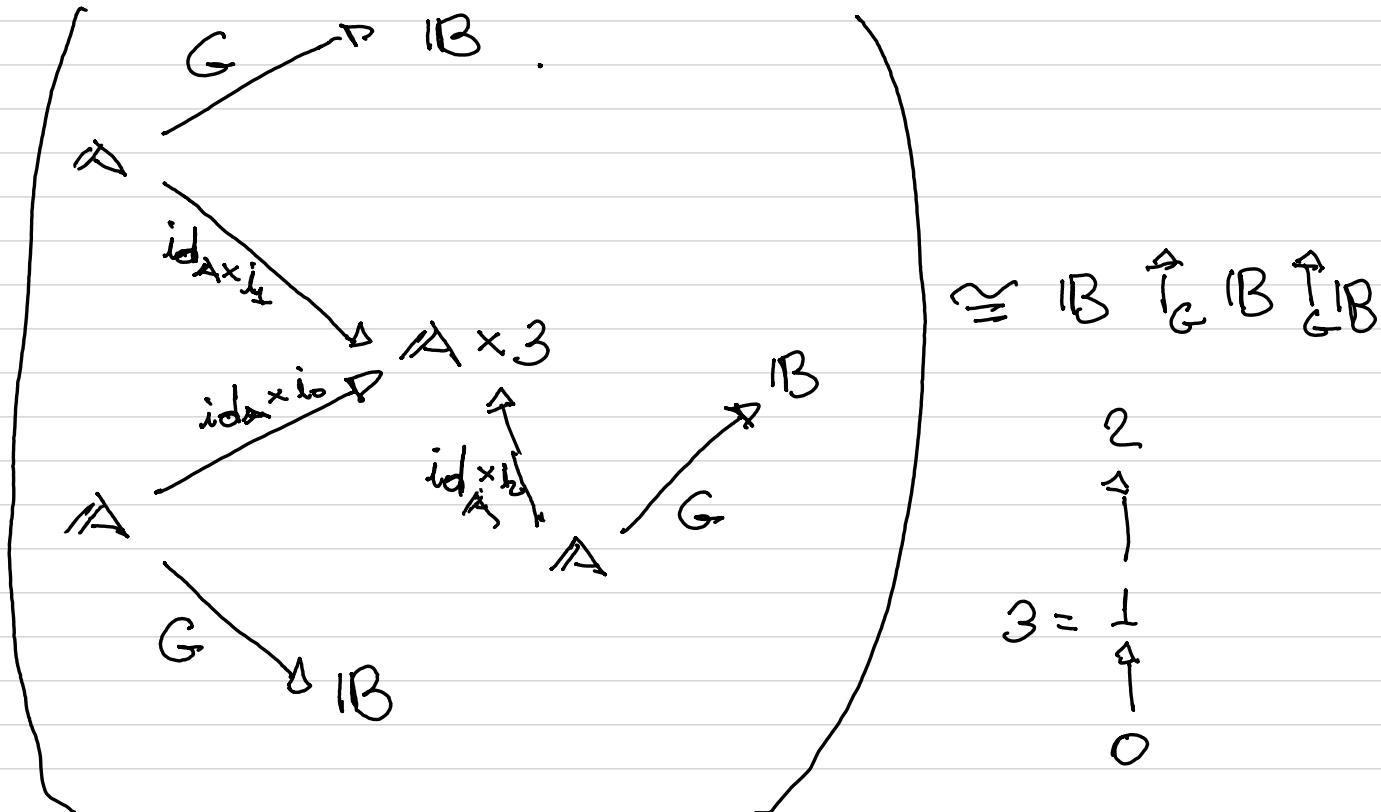
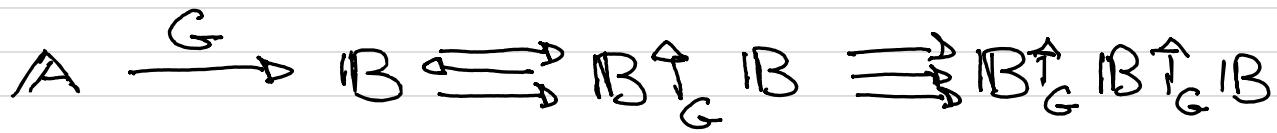
- General setting.:

$$A \xrightarrow{G} \mathbb{B} \Leftrightarrow \mathbb{B} \uparrow_G \mathbb{B} \rightrightarrows \mathbb{B} \uparrow_G \mathbb{B}$$

opcomma category
of G along G .



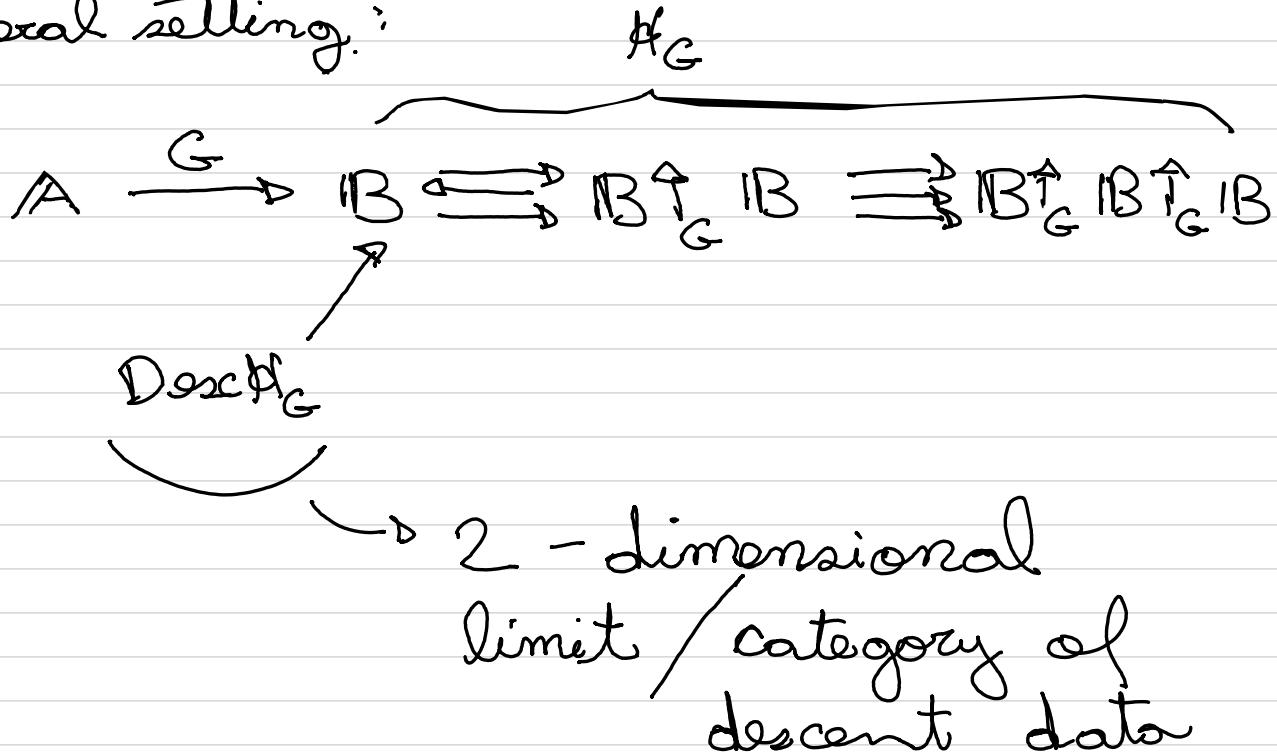
- General setting:



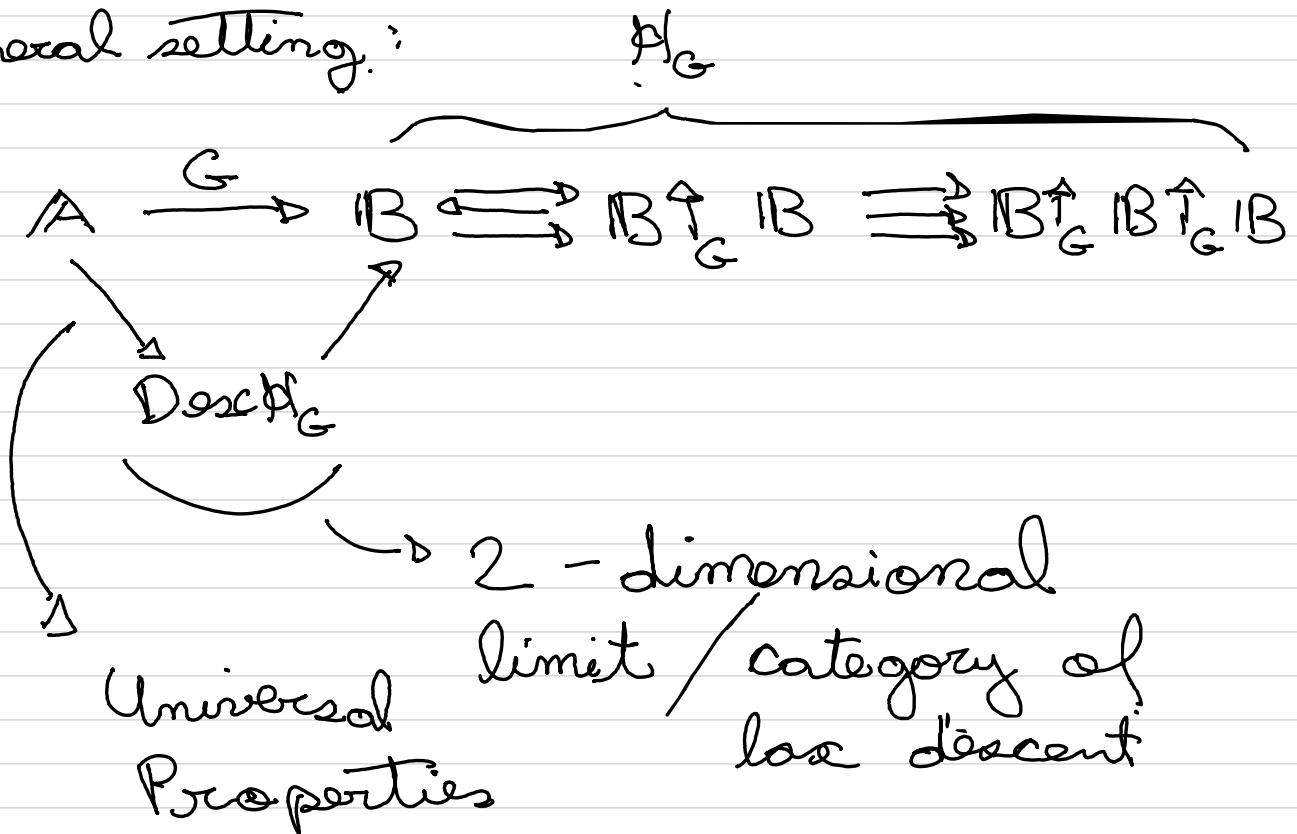
- General setting: 2-dimensional cokernel diagram of G

$$A \xrightarrow{G} B \rightleftarrows B^{\dagger}_G B \rightrightarrows B^{\dagger}_G B^{\dagger}_G B$$

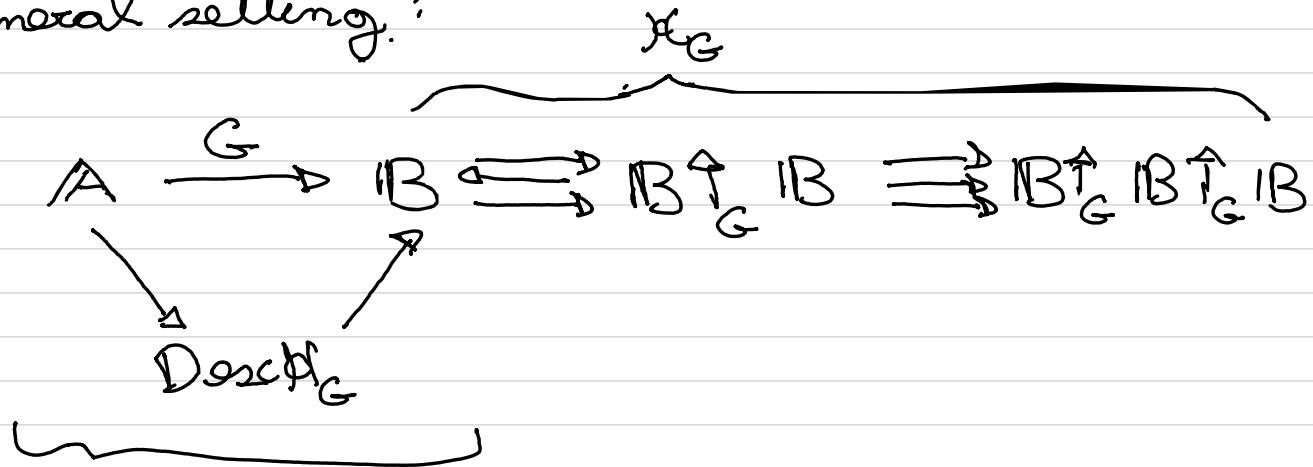
- General setting:



- General setting:

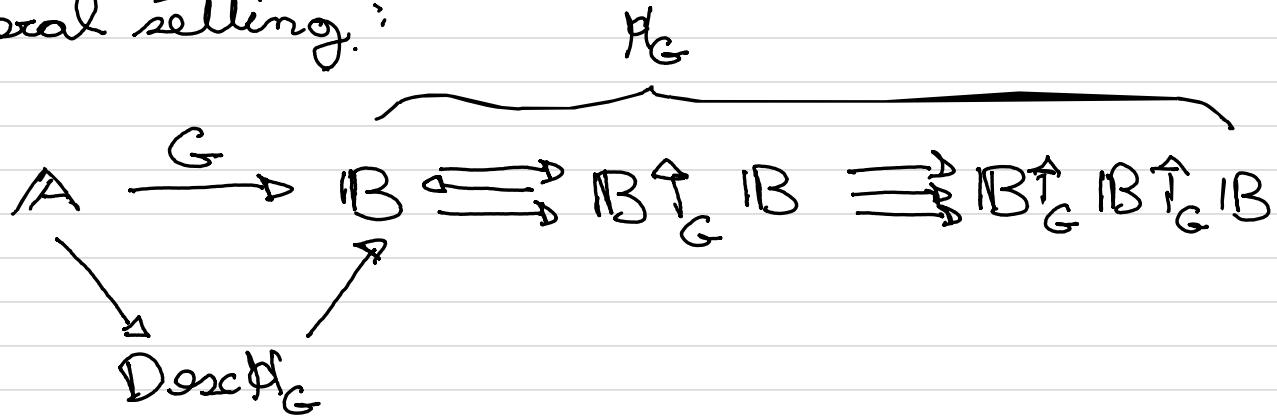


- General setting:



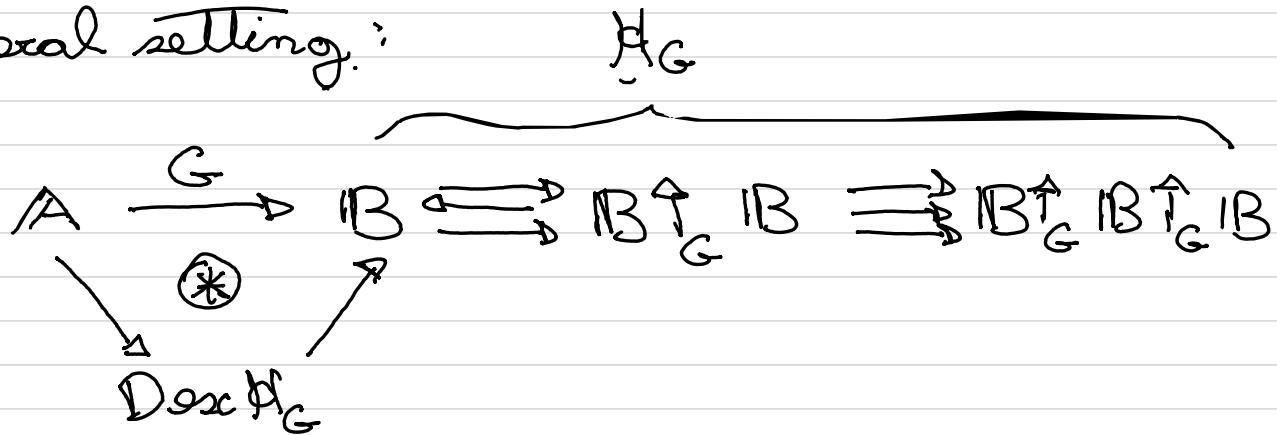
"Semantic lax descent factorization" of G

- General setting:



EVERY FUNCTOR HAS A SEMANTIC
LAX DESCENT FACTORIZATION!

- General setting:



MAIN THEOREM :

\$G\$ right adjoint \$\Rightarrow\$ (*) coincides w. EMF

\$G\$ left adjoint \$\Rightarrow\$ (*) coincides w. coEMF

Message :

THE SEMANTIC LAX DESCENT FACTORIZATION
OF ANY FUNCTOR

:

IS A NATURAL GENERALIZATION
OF THE

(co)EILEN BERG MOORE FACTORIZATIONS!

FURTHERMORE :

→ MAIN THEOREM HOLDS IN ANY 2-CATEGORY

(WITH SUITABLE 2-(CO)LIMITS.

→ CAN BE SEEN AS :

(A) FORMAL MONADICITY THEOREM;

(B) A COUNTERPART TO BÉNABOU-ROBAUD
THEOREM;

"DESCENT DATA IS A COMMON
GENERALIZATION OF BOTH ALGEBRAIC
AND CO ALGEBRAIC STRUCTURES"

Thank you
for your attention!