

GENERALIZED FOX’S THEOREM AND PSEUDOCOMMUTATIVITY

Classically, Fox’s theorem tells us that for a symmetric monoidal category \mathcal{C} , the category $\mathbf{CMon}(\mathcal{C})$ of commutative monoids in \mathcal{C} and homomorphisms admits finite coproducts. We can observe a certain pattern: we are considering commutative monoids in \mathcal{C} , which is itself a commutative pseudomonoid in the 2-category \mathbf{Cat} . We propose a vast generalization of Fox’s result stated in the language of Lawvere 2-theories: symmetric monoidal categories are replaced by models of a 2-theory \mathcal{T} in \mathbf{Cat} , and instead of commutative monoids, we refer to internal algebras in a model. For all this to make sense, we define a notion of pseudocommutativity for Lawvere 2-theories using the Gray tensor product – which both matches and simplifies the standard definition in the setting of 2-monads. This is a work in progress, joint with John Bourke.