Pretorsion Theories on Quasi-catégories

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In the 1960s, Spencer Dickson, in [3], axiomatizing properties of the category of abelian groups, presented a notion of *torsion theory* on abelian categories, which was soon generalized beyond the abelian setting (see, among many others, [1], [2]), and even, recently, in [4] and particularly [5], to general (and not necessarily pointed) $1-$categories. Classically, a *pretorsion theory* on a category $\mathbf{C}$ is a pair of full replete subcategories $(\mathbf{T}, \mathbf{F})$ such that every morphism between them factors through their intersection, $\mathbf{Z} := \mathbf{T} \cap \mathbf{F}$, and that there is a notion of short exact sequence consisting of a $\mathbf{Z}-$kernel and a $\mathbf{Z}-$cokernel that one may associate to every object in $\mathbf{C}$. Pretorsion theories satisfy multitudinous properties, including that $\mathbf{T}$, $\mathbf{F}$ and $\mathbf{Z}$ are closed under certain extensions, that $\mathbf{Z}-$kernels and -co-kernels are respectively monomorphisms and epimorphisms, and that $\mathbf{T}$ and $\mathbf{F}$ are respectively coreflective and reflective subcategories of $\mathbf{C}$.

In this presentation, we will introduce a paradigm for pretorsion theories for quasi-catégories (treated using the language of [6]) , and consider some of their properties in relation to those of $1-$categorical pretorsion theories, noting that many but not all of them hold. Critically, however, after passing to the homotopy category of any quasicategory equipped with one of our pretorsion theories, we obtain a one-categorical pretorsion theory. Through looking towards examples and almost-examples, we will also suggest some other generalizations of pretorsion theories in myriad directions.

References

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