Some characterizations of lax idempotency for pseudomonads

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In this talk we will discuss various equivalent conditions for lax idempotency of a pseudomonad.

The first one involves a 2-functor associated to any pseudomonad that sends left adjoints to the components of the pseudomonad's unit to colax algebras. A pseudomonad is lax-idempotent if and only if this 2-functor restricts to an isomorphism between reflectors and normal colax algebras.

The second one involves a "lax idempotentiation" process that can be applied to any 2-monad on a suitable 2-category – it is a higher-dimensional analogue of [1] that can be used to turn (suitable) 2-monads into lax-idempotent ones. A 2-monad is lax-idempotent if and only if this construction can be applied to it and at the same time does nothing to it.

The third one involves colax adjunctions and the pseudomonad's Kleisli 2-category. A pseudomonad on a 2-category $\mathcal{K}$ is lax-idempotent if and only if any biadjunction between $\mathcal{K}$ and some other 2-category $\mathcal{L}$ whose left biadjoint factorizes through its Kleisli 2-category "descends" to a colax adjunction between the Kleisli 2-category and $\mathcal{L}$.

We also describe how these results specialize to characterizations of idempotent 1-monads. If time permits, we will also mention the relationship of lax-idempotency of a pseudomonad and the presence of certain (enriched weak) colimits in its Kleisli 2-category.

The results in the first and second paragraph may be found in the author's recent PhD thesis currently awaiting a review [3, Theorem 2.5.8, Proposition 6.1.14]. One direction of the result in the third paragraph may be found either in the thesis or in the author's arXiv preprint [2, Theorem 4.15].

References

[1] Fakir, Sabah. "Monade idempotente associée à une monade" C. R. Acad. Sci. Paris Ser. A-B 270.A99–A101 (1970)

[2] Štěpán, Miloslav. "Colax adjunctions and lax-idempotent pseudomonads" arXiv preprint arXiv:2405.00488 (2024)

[3] Štěpán, Miloslav. *Lax structures in 2-category theory.* Online. Doctoral theses, Dissertations. Brno: Masaryk University, Faculty of Science. Available from: https://is.muni.cz/th/qlooc/.