Missingness

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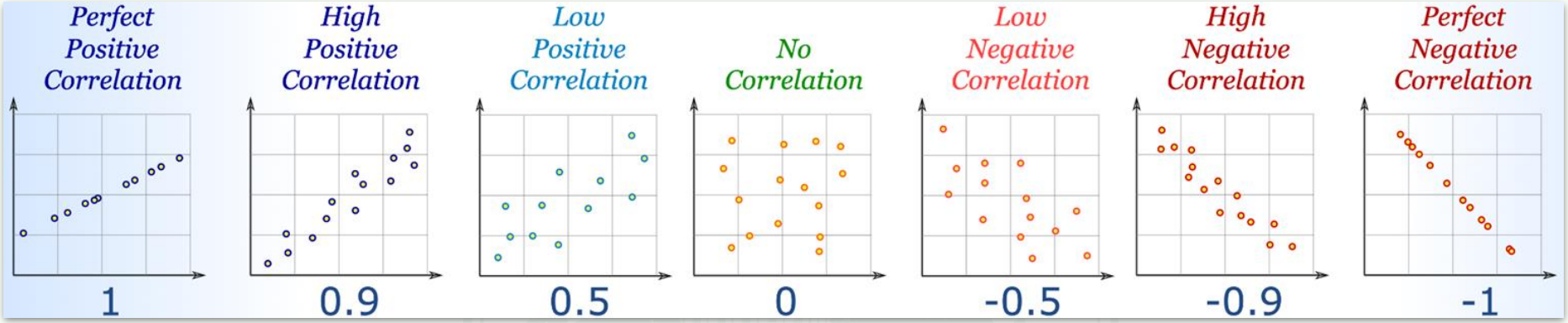
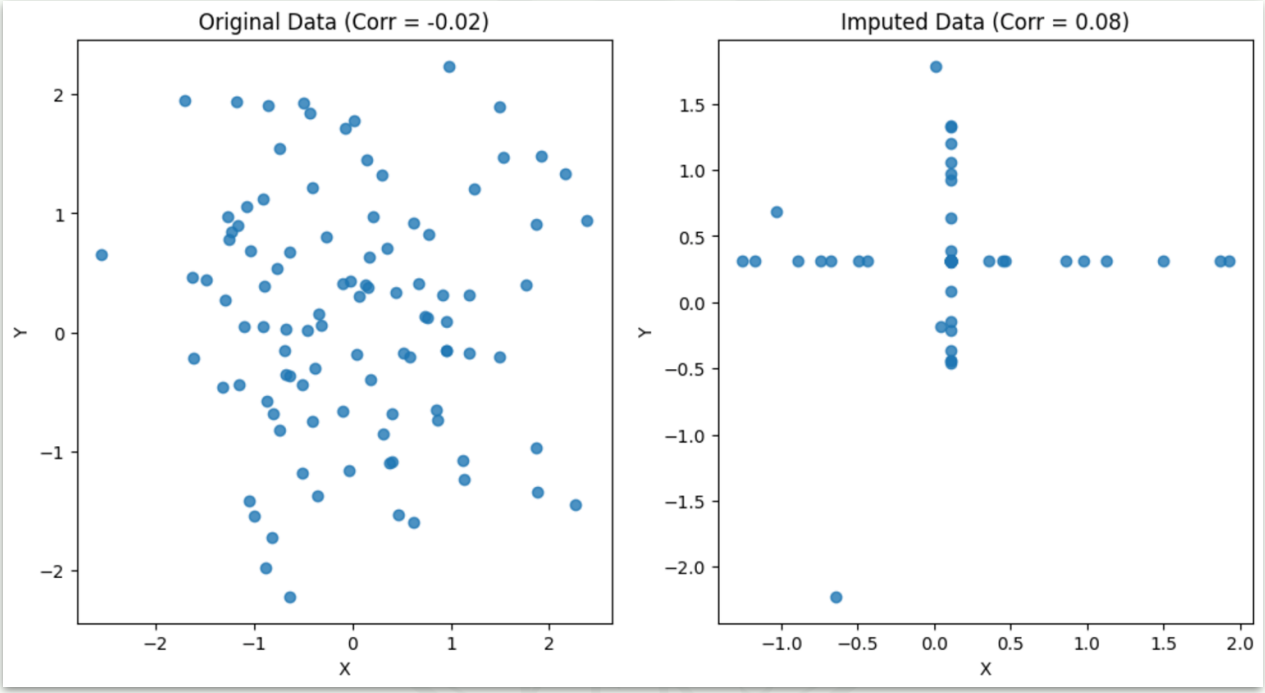
Loss of Correlations from Mean Imputation



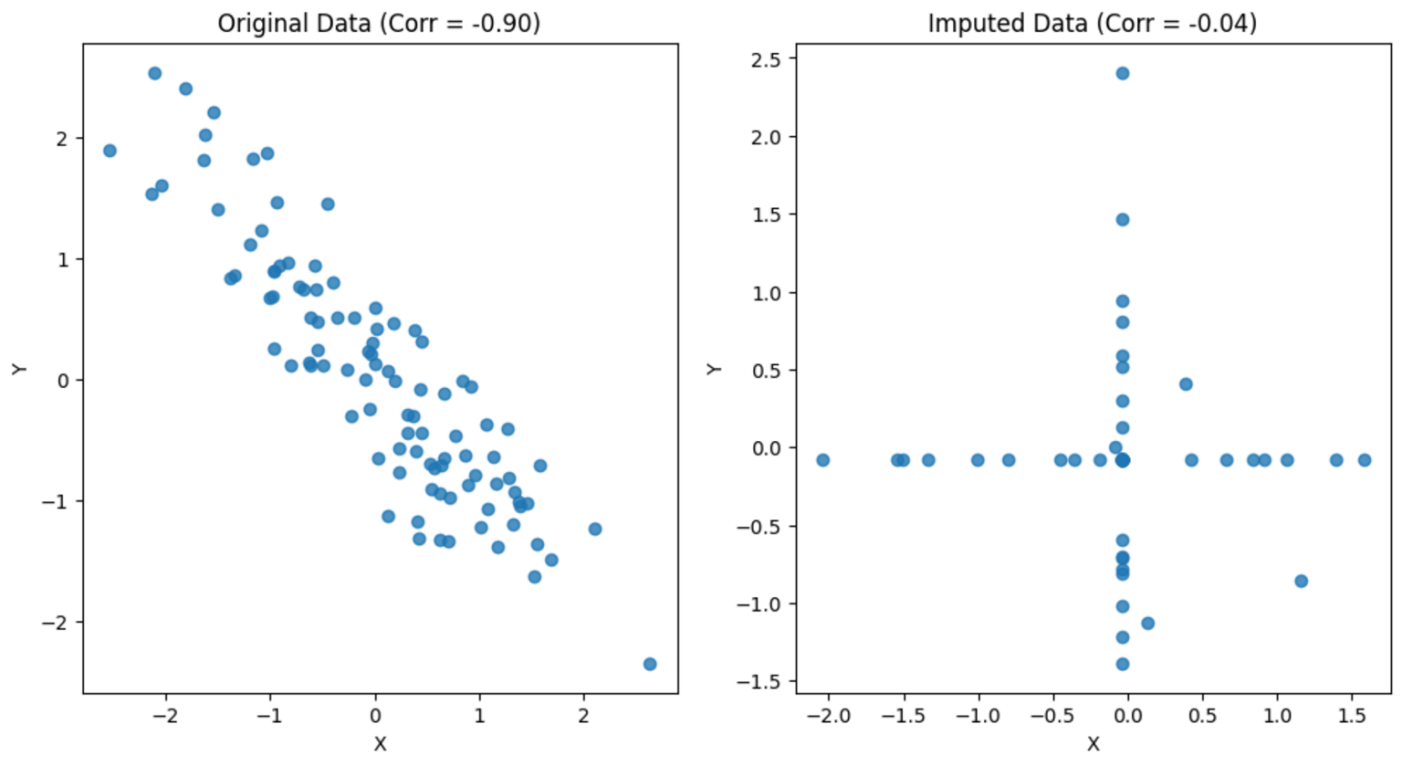
$$\rho_{X,Y} = \frac{\mathbb{E}[(X - \mu_X)(Y - \mu_Y)]}{\sigma_X \sigma_Y}$$

Loss of Correlations from Mean Imputation: Examples

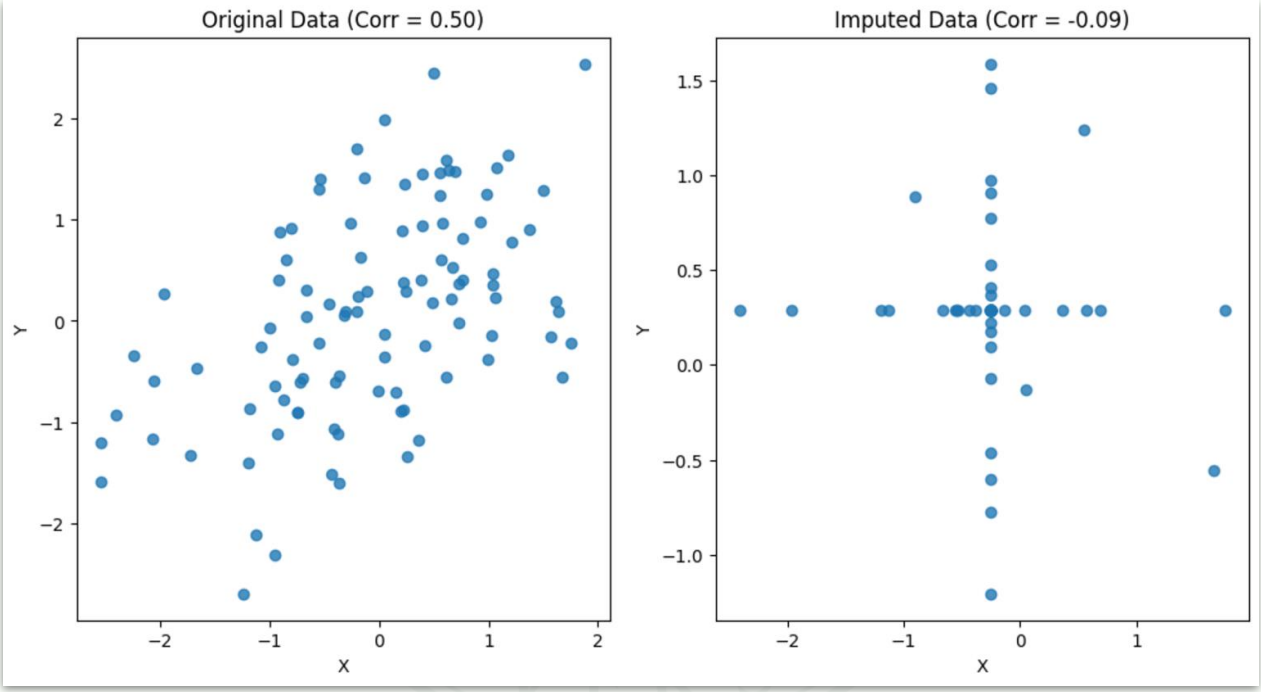
missing rate = 0.8



Loss of Correlations from Mean Imputation: Examples



Loss of Correlations from Mean Imputation: Examples



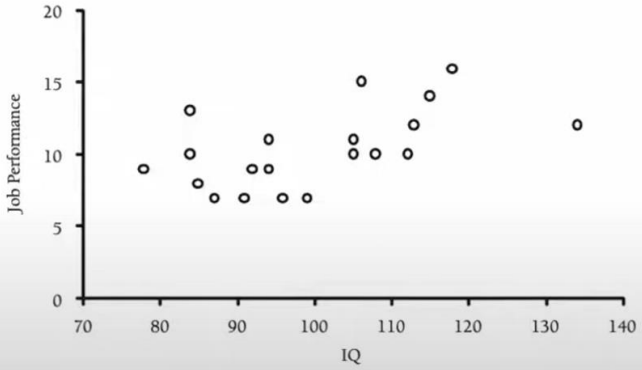
But First....Three Topics From Last Time

- impact of mean imputation on correlations
- stochastic regression
- diagnosing missingness
- visualization

Stochastic Regression: What is the Goal?

Here is the raw data:

Example dataset



2.1. Complete-data scatterplot of the IQ and job performance scores from Table 2.1.

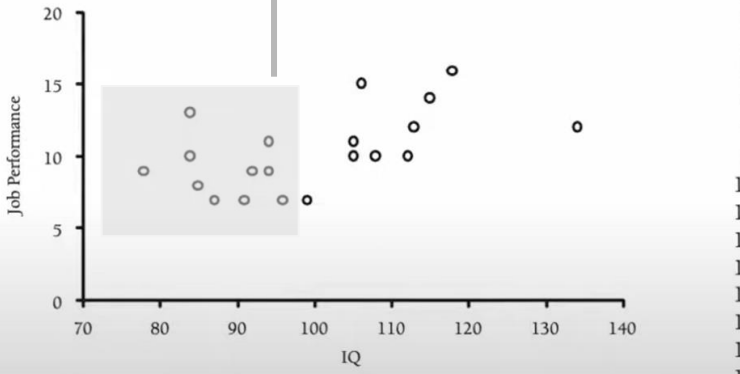
TABLE 2.1. Employee Selection Data Set

Complete data		Missing data
IQ	Job performance	Job Performance
78	9	—
84	13	—
84	10	—
85	8	—
87	7	—
91	7	—
92	9	—
94	9	—
94	11	—
96	7	—
99	7	7
105	10	10
105	11	11
106	15	15
108	10	10
112	10	10
113	12	12
115	14	14
118	16	16
134	12	12

Stochastic Regression: What is the Goal?

Here is the synthetic data:

Example dataset



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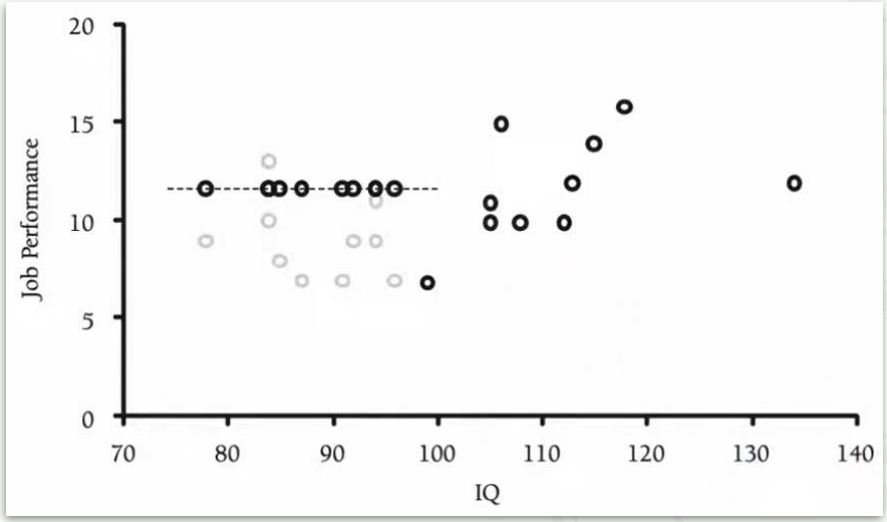
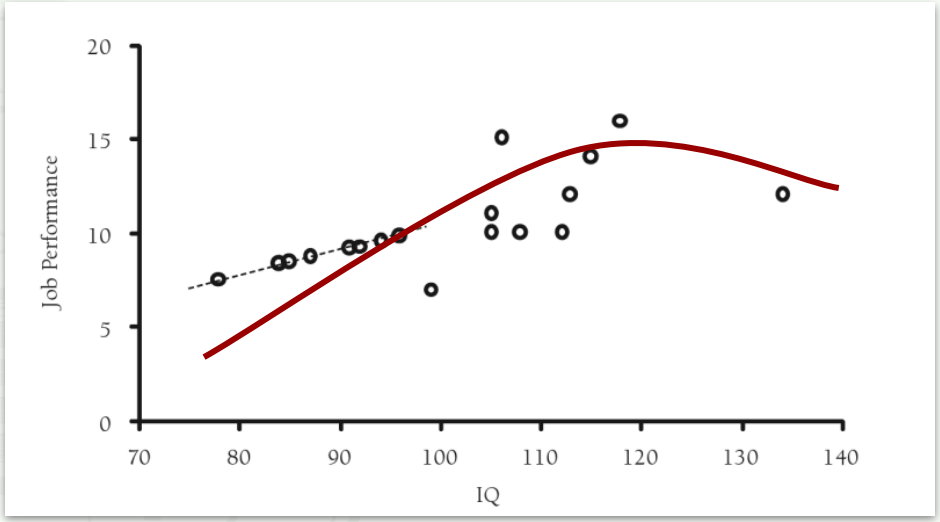
Fit Data and Extrapolate



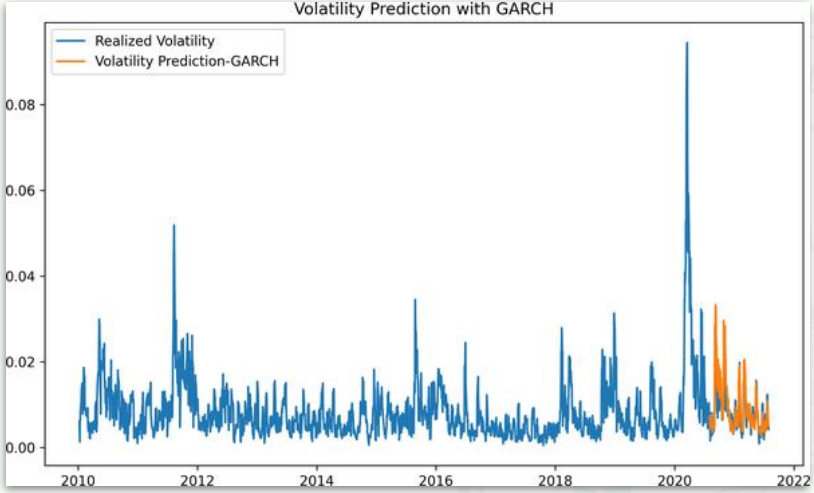


Point #1: mean imputation should “never” be done

Point #2: fitting is better, and perhaps good enough?

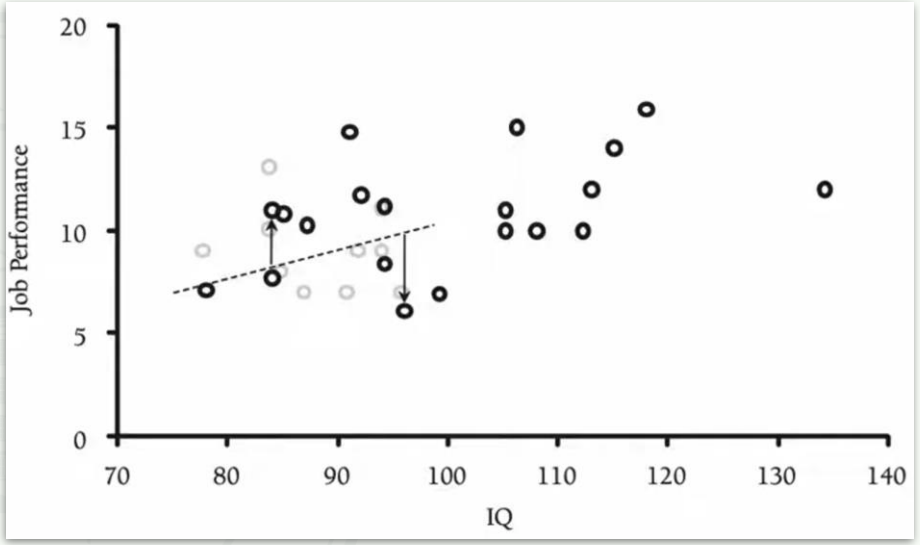
Point #3: don’t need to use a line (first-order polynomial)

Variance/Volatility



Very often we want to extrapolate the "volatility" in our data.

Sometimes, we only have "volatility".



Point #4: when we impute, we want to preserve the mean, trend and variance

multiple imputation can change the conclusion!

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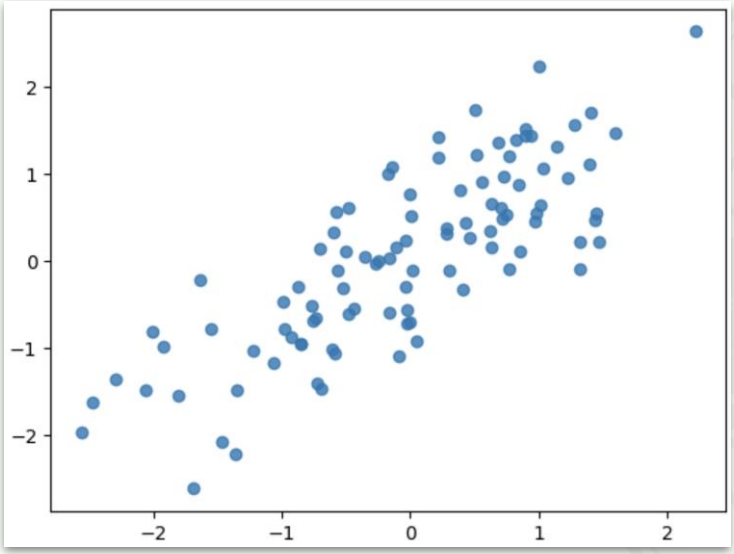
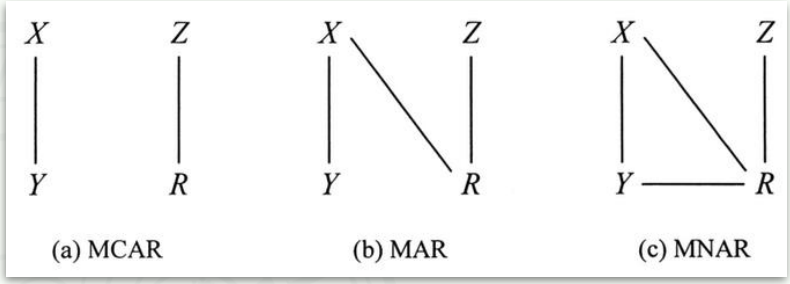
More Details on Missingness



original




```
19 # Introduce MCAR missingness
20 missing_rate = 0.2  # Proportion of missing values
21 df_mcar = df_original.copy()
22 for col in df_mcar.columns:
23     missing_indices = random.sample(range(n), int(missing_rate * n))
24     df_mcar.loc[missing_indices, col] = np.nan
25
26 # Introduce MAR missingness
27 df_mar = df_original.copy()
28 missing_indices_mar = df_original[df_original['X'] > df_original['X'].quantile(0.75)].index
29 df_mar.loc[missing_indices_mar, 'Y'] = np.nan
30
31 # Introduce MNAR missingness
32 df_mnar = df_original.copy()
33 missing_indices_mnar = df_original[df_original['Y'] > 0].index
34 df_mnar.loc[missing_indices_mnar, 'Y'] = np.nan
```

add missingness to the data.

Missingness in Terms of Conditional Probabilities

MCAR:

$$P(\text{missing}|\text{complete}) = P(\text{missing})$$

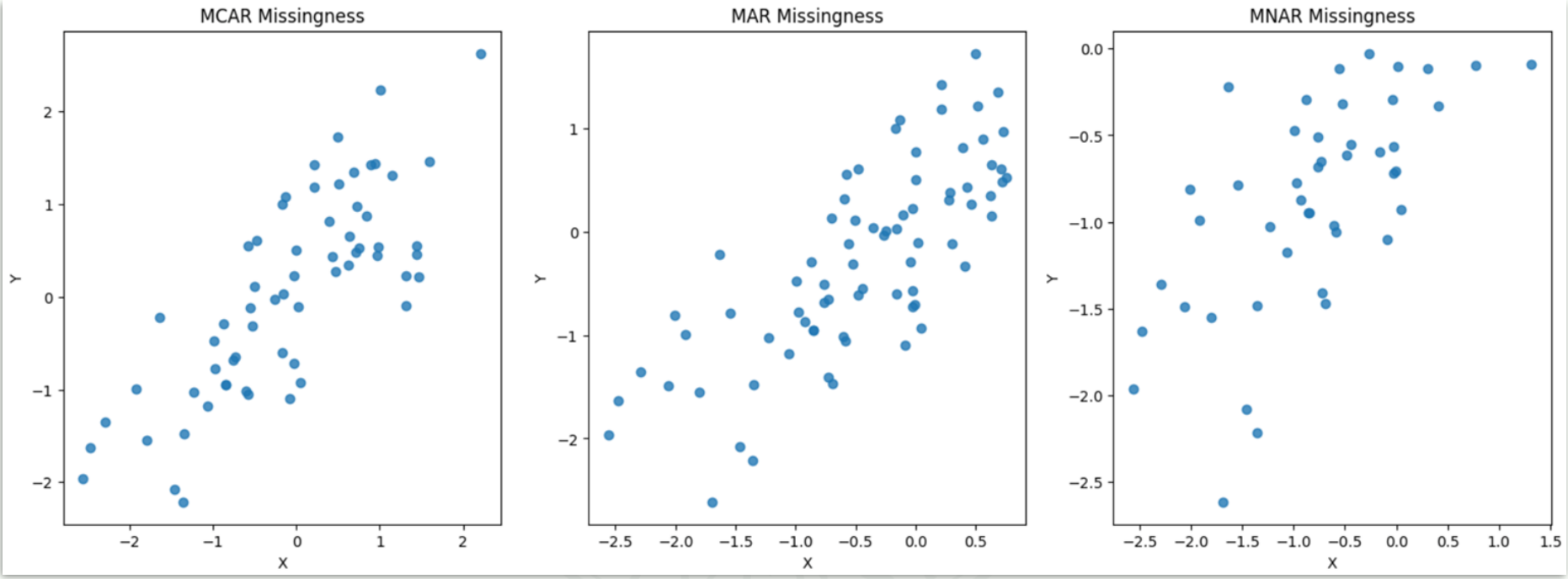
MAR:

$$P(\text{missing}|\text{complete}) = P(\text{missing}|\text{observed})$$

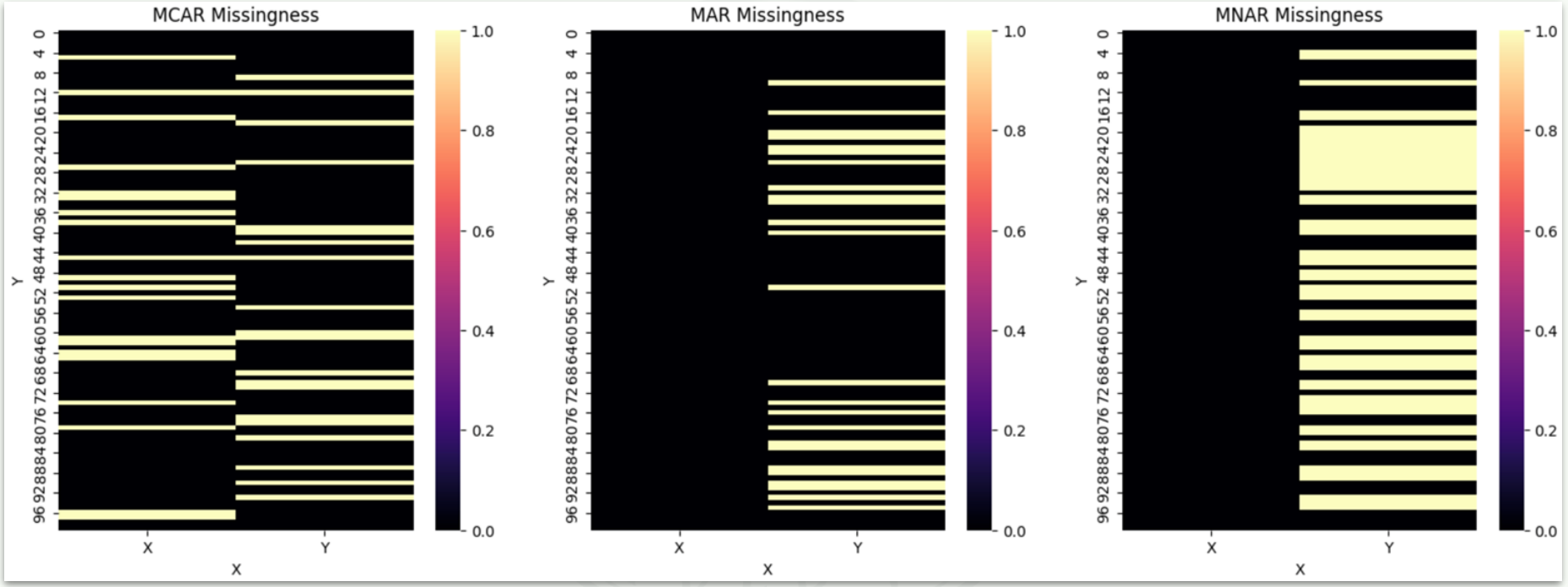
MNAR:

$$P(\text{missing}|\text{complete}) \neq P(\text{missing}|\text{observed})$$

More Details on Missingness



Visualizing Synthetic Missingness



Sorted by X Value