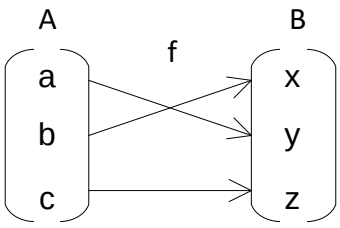
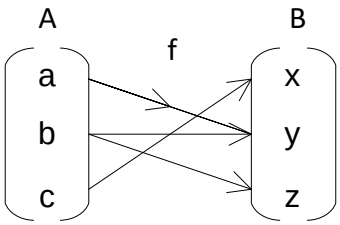
Islington College, Kathmandu
Module Code MA4001: Logic and problem solving
Tutorial: Week 11

1. Which of the following set mapping diagrams define functions $f: A \rightarrow B$?

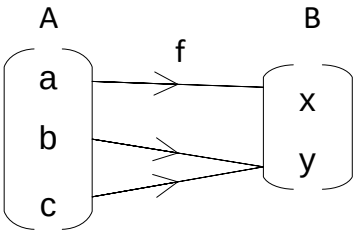
(i)



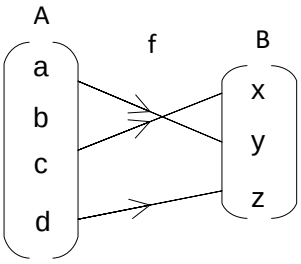
(ii)



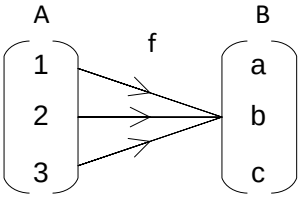
(iii)



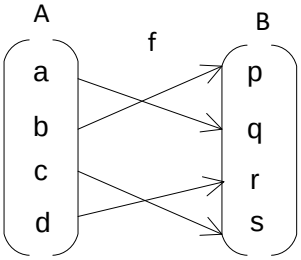
(iv)



(v)



(vi)



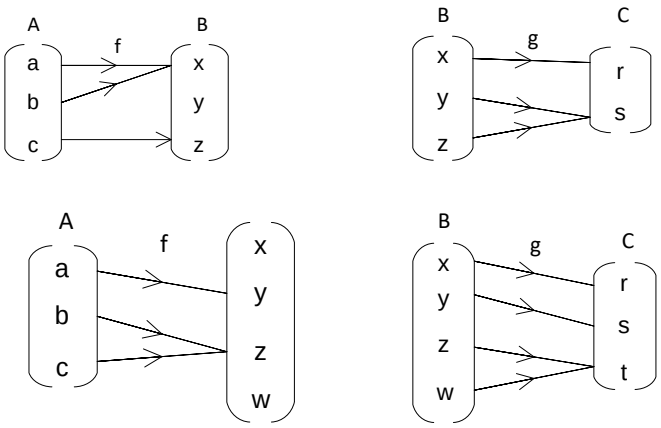
State the range of each function.

2. Let X = { 1, 2, 3, 4} and Y = { 5, 6, 7, 8}
Which of the following subsets of X × Y define the function f: X → Y. Determine drawing arrow diagram in each case.
   (i) { (1, 6), (2, 8), (3, 8), (4, 5) }
   (ii) { (2, 7), (4, 6), (1, 5), (3, 8) }
   (iii) { (1, 7), (2, 8), (3, 9), (4, 6) }
   (iv) { (2, 5), (3, 7), (2, 8), (1, 6), (4, 7) }
   (v) { (3, 7), (1, 6), (4, 8) }
   (vi) { (4, 5), (1, 7), (2, 8), (3, 6) }

   State the range of each function.

3. If f: A → B and g: B→C are defined by the following set mapping diagrams, determine the composite function g o f: A→C in each case, giving your answers

   i) as a set mapping diagram
   ii) as a subset of A×C



4. Functions $f: R \to R$ and $g: R \to R$ are defined by the formulae $f(x) = 1+2x$, $g(x) = x^2-1$. Find,

   a. $(g \circ f)(x)$ b. $(f \circ g)(x)$ c. $(f \circ f)(x)$ and d. $(g \circ g)(x)$.

5. Find the inverse of the following functions.
   (i) $f(x) = 2x - 1$
   (ii) $f(x) = 4x^2 - 1$
   (iii) $f(x) = \frac{x}{x-2}$

6. Let $X = \{1, 2, 3, 4, 5, 6\}$ and let $f: X \to X$ and $g: X \to X$ be defined by $f(x) = (2x+1) \pmod 6$ and $g(x) = (3x-1) \pmod 6$

   Find f o g(x): X→X and g o f(x): X→X giving your answers as subsets of X × X

7. Let $f, g: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 4x - 1$ and $g(x) = x^2 + 1$. Find :

i)	$f(2)$	vi)	$(g \circ g)(2)$
ii)	$g(2)$	vii)	$(f \circ g \circ f)(3)$
iii)	$(g \circ f)(2)$	viii)	$(g \circ f \circ g)(3)$
iv)	$(f \circ g)(2)$	ix)	$(g \circ f)(x)$
v)	$(f \circ f)(2)$	x)	$(f \circ g)(x)$

8. Let f, g and h be functions $\mathbb{R} \to \mathbb{R}$ defined respectively

$$f(x) = 2x + 1, \qquad g(x) = \frac{1}{x^2+1} \text{ and } h(x) = \sqrt{x^2+1}$$

   Find expressions for each of the following:

i)	$(g \circ f)(1)$	vi)	$(g \circ f)(x)$
ii)	$(f \circ g)(1)$	vii)	$(g \circ h)(x)$
iii)	$(g \circ h)(2)$	viii)	$(f \circ f)(x)$
iv)	$(h \circ f)(3)$	ix)	$((f \circ g) \circ h)(x)$
v)	$(f \circ g)(x)$	x)	$(f \circ (g \circ h))(x)$

9. Let $f(x) = \frac{1}{\sqrt{x^2+1}}$ and $g(x) = \frac{2}{x-3}$. Determine $(f \circ g)(x)$.