Calculus, 2017-2-IE-2

Name: **Sequence Number:**

Due Time 80 minutes

1°). Suppose that $\mathbf{f(x,y)} = \frac{\mathbf{x}}{\sqrt{\mathbf{x^2+y^2}}}$ (total 30%)
a°). Find the directional derivative of $\mathbf{f(x,y)}$ at $\mathbf{(x,y) = (1,2)}$ and in the direction $(3,4)$; (20%)
b°). Find the direction with which $\mathbf{f(x,y)}$ owns the maximal directional derivative at $\mathbf{(x,y) = (1,2)}$. (10%)

2°). Find critical points of following functions if any, classify the types of (local and absolute) extrema or saddle point: (total 70%)
a°). $\mathbf{f(x,y) = x^2y((4 - x - y)}$ for $\mathbf{(x,y)} \in \mathbb{R}^2$. (40%)
b°). Find out extrema of $\mathbf{f}$ within $\mathbf{D} = \{\mathbf{(x,y)} \in \mathbb{R}^2 \big| \mathbf{0 \le x, y \le 1}\}$ (30%).

Answer

1.

a). $\nabla f = (y^2, -xy)/(x^2 + y^2)^{3/2}$ (5%)

- $\nabla f(1,2) = (4,-2)/5^{3/2}$ (5%)
- $\mathbf{u} = (3,4) \rightarrow \mathbf{e_u} = (3/5, 4/5)$, (5%)
- $\mathbf{\nabla_e f(1,2) = (4,-2)/5^{3/2} \bullet (3/5, 4/5) = \frac{4}{5^{5/2}}}$ (5%)

b). Since the direction of maximal directional derivative is parallell to $\nabla f(2,1)$, it is in the direction of $\mathbf{(4,-2)}$. (10%)

2.

a). $\nabla f = [8xy - 3x^2y - 2xy^2, 4x^2 - x^3 - 2x^2y]$ (5%)

- $\nabla f = [0,0]$
  - $x = 0, (y \in \mathbb{R})$, (5%)
  - $y = 0, 4 - x - 2y = 0 \rightarrow (x,y) = (4,0)$ (5%)
  - $8 - 3x - 2y = 0, 4 - 2x - 2y = 0, \rightarrow (x,y) = (2,1)$. (5%)
- (5%)

$$|H|(x,y) = \begin{vmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial y \partial x} \\ \frac{\partial^2 f}{\partial x \partial y} & \frac{\partial^2 f}{\partial y^2} \end{vmatrix} = \begin{vmatrix} 8y - 6xy - 2y^2 & 8x - 3x^2 - 4xy \\ 8x - 3x^2 - 4xy & -2x^2 \end{vmatrix}$$

  - $x = 0 \rightarrow z = 0$ none of extremum, (5%)
  - $(x,y) = (4,0) \rightarrow |H| = -16$: $(4,0,0)$ is saddle point, (5%)
  - $(x,y) = (2,1) \rightarrow |H| = 32$ with $f_{xx}(2,1) = -8<0$: $f(2,1) = 4$ is local maxima, (5%)

b).

- $l_1 : y = 0, 0 \leq x \leq 1 \rightarrow f = 0$ (5%)
- $l_2 : x = 1, 0 \leq y \leq 1 \rightarrow f = y(3-y) \rightarrow \max = 2$ and $\min = 0$ (5%)
- $l_3 : y = 1, 0 \leq x \leq 1 \rightarrow f = 3x^2 - x^3 \rightarrow \max = 3$ and $\min = 0$ (5%)
- $l_4 : x = 0 : f = 0$ (5%)
- $f(2,1) = 4$ in a) but $(2,1)$ is not in the considered domain.

Conclusion: Max:3, Min:0 (10%)