Final Quiz for Calculus II (MM)

Seq.No.:_____ Name:_____
Total points 100%

Problem 1. Describe Fubini Theorem.(10%)

Problem 2. Suppose that

$$f(x,y)=\exp(-\frac{x^2+y^2}{4}) \text{ where } -\infty<x,y<\infty$$

a) Evaluate $\int\int_{\mathbb{R}^2} f(x,y)\,dA$ where $\mathbb{R}^2=\{(x,y)\,|\,-\infty<x,y<\infty\}$. (10%)

b) Evaluate $\int\int_{\mathbb{R}^2} x\,f(x,y)\,dA$. (10%)

Ans: a) 4π b) 0

Problem 3. Evaluate the double integral:

$$\int\int_A \frac{\sin y}{x}\,dA$$

where $A=\{(x,y)\in\mathbb{R}^2|0\leqslant x\leqslant 1, x\leqslant y\leqslant 1\}$. (10%)
Ans: fails to exist

Problem 4. Evaluate the double integrals: (30%)

a) $\int\int_A 1\,dA$ where $A=\{(x,y)\,|\,0\leqslant x\leqslant 1, 1+x\leqslant y\leqslant 2\}$

b) $\int\int_A 1\,dA$ where $A=\{(x,y)\,|\,0\leqslant x, x^2+y^2\leqslant 4\}$

c) $\int\int_A x\,dA$ where $A=\{(x,y)\,|\,-1\leqslant x\leqslant 1, x^2+y^2\leqslant 4\}$

Ans: a) 1/2 b) 2π c) 0 since x is odd in A

Problem 5. Evaluate the triple integrals: (30%)

a) $\int\int\int_V xyz\,dV$ where $V=\{(x,y,z)\,|\,0\leqslant x\leqslant 1, 1+x\leqslant y\leqslant 2, 1\leqslant z\leqslant 2\}$

b) $\int\int\int_V \frac{x}{x^2+y^2}\,dV$ where $V=\{(x,y,z)\,|\,0\leqslant x, x^2+y^2+z^2\leqslant 4, x^2+y^2\leqslant 1\}$

c) $\int\int\int_V \frac{1}{x^2+y^2+z^2}\,dV$ where $V=\{(x,y,z)\,|\,-1\leqslant x\leqslant 1, x^2+y^2+z^2\leqslant 4, x^2+y^2\leqslant z^2\}$

Ans: a) 7/16 b) $4\int_0^{\pi/2}d\theta\int_0^1 dr\int_0^{\sqrt{4-r^2}}\frac{r\cos\theta}{r^2}\,r\,dz=8\sin^{-1}\frac{1}{2}+\sqrt{3}$ c)