Calculus, 2017-2-IE-3

Name: _____ Sequence Number: _____

(Due Time 80 minutes.)

1°). Evaluate the following double integrals. (total 50%, each 25%)

a°). $\iint_{\{(x,y)|0\le x,y \text{ and } x+y\le 1\}} (x+y)dA$:

b°). $\iint_{\{(x,y)|0\le x,y \text{ and } x^2+y^2\le 6\}} (x+y)dA$:

2°). Evaluate the following triple integrals. (total 50%, each 25%)

a°). $\iiint_{\{(x,y,z)\in\mathbb{R}^3 | x,y,z\leq 0 \text{ and } x+2y+z\geq -4\}} (x+y+z)dV$

b°). $\iiint_{\{(x,y,z)\in\mathbb{R}^3 | x,y,z\leq 0 \text{ and } x^2+(2y)^2+z^2\leq 6\}} \frac{dV}{\sqrt{x^2+4y^2+z^2}}$

1 Answer

1. a)

$$\iint_{\{(x,y)|0\leq x,y \text{ and } x+y\leq 1\}} (x+y)dA = \int_0^1 dx \int_0^{1-x} (x+y)dy = 1/3$$

1. b)

$$\iint_{\{(x,y)|0\leq x,y \text{ and } x^2+y^2\leq 6\}} (x+y)dA = \int_0^{\pi/2} d\theta \int_0^{\sqrt{6}} r^2(\cos\theta+\sin\theta)dr =$$

2. a)

$$\iiint_{\{(x,y,z)\in\mathbb{R}^3 | x,y,z\leq 0 \text{ and } x+2y+z\geq -4\}} (x+y+z)dV = \int_{-2}^0 dy \int_{-4-2y}^0 dx$$

2. b)

$$\iiint_{\{(x,y,z)\in\mathbb{R}^3 | x,y,z\leq 0 \text{ and } x^2+(2y)^2+z^2\leq 6\}} \frac{dV}{\sqrt{x^2+4y^2+z^2}} = \frac{1}{2}\cdot\int_{\pi/2}^{\pi} d\phi \int$$

In [3]:
```
from sympy import log,exp,cos,sin,diff,integrate,symbo
```

```
x,y,z=symbols("x y z")
r,t= symbols("r t")
```

In [1]:

```
from sympy import symbols,pprint,integrate,pi,sqrt,sin
x,y,z=symbols("x y z")

W  = '\033[0m'  # white (normal)
K  = '\033[30m' # black
R  = '\033[31m' # red
G  = '\033[32m' # green
O  = '\033[1;33m' # orange
B  = '\033[34m' # blue
P  = '\033[35m' # purple
T =  '\033[1;33;47m' #Title

def doubleInt3(f,X,xr,yr):
    Iy=integrate(f,[X[1],yr[0],yr[1]])
    I=integrate(Iy,[X[0],xr[0],xr[1]])
    #print("  %s \t  %s" %(xr[1],yr[1]))
    yrs=str(yr[1])
    xrs=' '+str(xr[1])
    print(xrs.ljust(9,' ')+yrs)
    print("∫   d",R+'{}'.format(X[0]),K+"∫ ",B+"{}".fo

    yrs0=str(yr[0])
    xrs0=str(xr[0])
    print(xrs0.ljust(8,' ')+yrs0)
    return I
```

In [12]:

```
# 1. a)
f=x+y
I=doubleInt3(f,[x,y],[0,1],[0,1-x])
```

```
 1       -x + 1
⌠    d x ⌠  x + y  d y  =  1/3
⌡        ⌡
0        0
```

In [4]:

```
# 1. b)
from sympy import pi,sqrt,sin,cos

g=r*r*(cos(t)+sin(t))
I=doubleInt3(g,[t,r],[0,pi/2],[0,sqrt(6)])
```

```
pi/2    sqrt(6)
```

```
 0       0
∫   d t ∫   r**2*(sin(t) + cos(t))  d r  =  4*sqrt(6)
```

2.

In [5]:
```
def tripleInt3(f,X,xr,yr,zr):
    Iz=integrate(f,[X[2],zr[0],zr[1]])
    Iy=integrate(Iz,[X[1],yr[0],yr[1]])
    Ix=integrate(Iy,[X[0],xr[0],xr[1]])
    zrs=' '+str(zr[1])
    yrs=str(yr[1])
    xrs=' '+str(xr[1])
    print(xrs.ljust(9,' ')+O+yrs.ljust(7,' ')+B+zrs)
    #print("  %s \t  %s  \t  %s" %(xr[1],yr[1],zr[1]))
    print("∫   d",R+'{}'.format(X[0]),
        K+"∫   d",R+'{}'.format(X[1]),
        K+"∫ ",B+"{}".format(f),K+"  d",R+"{}".format(
    zrs1=' '+str(zr[0])
    yrs1=str(yr[0])
    xrs1=' '+str(xr[0])
    print(xrs1.ljust(8,' ')+O+yrs1.ljust(7,' ')+B+zrs1
    #print(" %s \t %s  \t %s" %(xr[0],yr[0],zr[0]))
    return Ix
```

In [7]:
```
#2. a)
tripleInt3(x+y+z,[y,x,z],[-2,0],[-4-2*y,0],[-4-2*y-x,0
```

```
 0       0       0
∫   d y ∫   d x ∫  x + y + z   d z  =  -40/3
 -2     -2*y - 4 -x - 2*y - 4
```

Out[7]: -40/3

In [6]:
```
#2. b)
r,t,p=symbols("r t p")
h=r*sin(p)/2
tripleInt3(h,[r,t,p],[0,sqrt(6)],[pi,3*pi/2],[pi/2,pi]
```

```
 sqrt(6) 3*pi/2  pi
∫   d r ∫   d t ∫  r*sin(p)/2   d p  =  3*pi/4
 0      pi      pi/2
```

Out[6]: 3*pi/4

In [ ]: