Calculus Final Quiz (2015-2s-IE104)

Class: Seq No: Name:

Problem 1. Find **extrema** of $f(x,y) = 2 - xy$ with repect to the following conditions:

a) $x, y \geqslant 0$; (10%)

b) $x, y \geqslant 0, y \leqslant x, x + y \leqslant 2$. (10%)

c) $x^2 + y^2 \leqslant 2$. (10%)

Problem 2. Find the shortest distance from point $(1, 0, -2)$ to $x + 2y + z = 4$. (20%)

Problem 3. Describe the Fubini Theorem for double integration in Cartesian Coordinates under the assumption:

suppose that $f(x, y)$ is continuous on $D \subseteq \mathbb{R}^2$ and $D = \{(x, y) \in \mathbb{R}^2 |\ c \leqslant y \leqslant d,\ g_1(y) \leqslant x \leqslant g_2(y)\}$ where $g_1(y)$ and $g_2(y)$ are continuous. (5%)

Problem 4. Evaluate $\int\int_D x\,dA$ where $D = \{0 \leqslant y + 1 \leqslant x \leqslant 2\}$ (10%)

Problem 5.

a) The value of the folloing integral is

$$\int_0^\infty e^{-x^2/2} dx$$

i°) $\pi/2$, ii°) $\sqrt{\pi}/2$, iii°) $\sqrt{\pi/2}$, iv°) $\pi/\sqrt{2}$, v°) None of above. (5%)

b) Evaluate the double integral:

$$\iint_D e^{-x^2+xy-y^2}\, dA$$

where $D = \{(x, y) \in \mathbb{R}^2 | 0 \leqslant y < \infty\}$. (10%)

Problem 6. Evaluate the following integrals: (20%)

a)

$$\iiint\limits_{\{x^2+y^2 \leqslant 1, 0 \leqslant z \leqslant 1\}} x^2 y^2 z\, dV$$

b)

$$\iiint\limits_{\left\{x^2+y^2+z^2 \leqslant 4, \sqrt{x^2+y^2} \leqslant z\right\}} x^2 y^2 z\, dV$$

1. a) $2 - xy - z^2 + \lambda(1 - x^2 - y^2) + \mu(z^2 - xy)$

   b) $(-y - 2\lambda x - \mu y, -y - 2\lambda x - \mu y, -2z + 2\mu z)$

   c) $\begin{cases} (x, y, z) = (\pm 1, 0, 0), (0, \pm 1, 0) \\ (x, y, z) = \pm\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\right) \end{cases}$

   d) x, y and z are bounded.

   e) maximum: 2, minimum: 1

2. $\frac{5}{6}\sqrt{6}$

2. $\int\int_D f(x, y)\, dA = \int_c^d dy \int_{g_1(y)}^{g_2(y)} f(x, y) dx$

3.

$$\begin{aligned} \int\int_D x\, dA &= \int_0^2 dx \int_{-1}^{x-1} x\, dy \\ &= \int_0^2 x(x - 1 + 1) dx = 8/3 \end{aligned}$$

4.

$$
\begin{aligned}
I &= \int_0^{\infty} dy \int_{-\infty}^{\infty} e^{-(x-\frac{y}{2})^2} e^{-\frac{3}{4}y^2} dx \\
&= \sqrt{\pi} \int_0^{\infty} e^{-\frac{3}{4}y^2} dy \\
&= \sqrt{\pi} \frac{\sqrt{\pi}}{\sqrt{3}} = \frac{\pi}{\sqrt{3}}
\end{aligned}
$$

5.

$$
\begin{aligned}
\iiint\limits_{\{x^2+y^2\leqslant 1, 0\leqslant z\leqslant 1\}} x^2 y^2 z dV &= \int_0^{2\pi} d\theta \int_0^1 dr \int_0^1 (r\cos\theta)^2 (r\sin\theta)^2 z r dz \\
&= \frac{1}{2}\int_0^1 r^5 dr \int_0^{2\pi} (\cos\theta)^2 (\sin\theta)^2 d\theta \\
&= \frac{1}{12}\int_0^{2\pi} \left(\frac{1+\cos 2\theta}{2}\right)\left(\frac{1-\cos 2\theta}{2}\right) d\theta \\
&= \frac{1}{12}\int_0^{2\pi} \left(\frac{1-\cos^2 2\theta}{4}\right) d\theta \\
&= \frac{1}{48}\int_0^{2\pi} \left(1 - \frac{1+\mathrm{co}\cancel{\mathrm{s}}4\theta}{2}\right) d\theta = \frac{\pi}{48}
\end{aligned}
$$