Discovery of exact equations for integer sequences

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Introduction

Equation discovery for integer sequences with probabilistic grammars

- Equation discovery
- Probabilistic grammars
- Integer sequences

Experimental Evaluation (*linrec*)

Algorithm 1 For experimental evaluation of SINDy for *linrec* case

Input: Matrix $M = [X|\mathbf{y}]$ of observations $X = [\mathbf{a}_{n-1}|...|\mathbf{a}_{n-19}]$ and target $\mathbf{y} = \mathbf{a}_n$

Output: Equation $a_n = \sum_{i=1}^{19} c_i a_{n-i}$ or Failed

```
1: function SINDy's linrec(M)
2:     for p = 1, 2, ..., 19 do
3:         Add col_p(X) to M (i.e. X = [X|M_p])
4:         if SINDy_grid(M, d_max = 1) is not Failed then
5:             return SINDy_grid(M, d_max = 1)
6:         end if
7:     end for
8:     return Failed
9: end function
```

Experimental Evaluation (*linrec*)

Algorithm 2 For experimental evaluation of Diofantos for *linrec* case

Input: Matrix $M = [X|\mathbf{y}]$ of observations $X = [\mathbf{a}_{n-1}|...|\mathbf{a}_{n-19}]$ and target $\mathbf{y} = \mathbf{a}_n$

Output: Equation $a_n = \sum_{i=1}^{19} c_i a_{n-i}$ or Failed

```
1: function Diofantos's linrec(M)
2:     for p = 1, 2, ..., 19 do
3:         Add col_p(X) to M (i.e. X = [X|M_p])
4:         if Diofantos(M, d_max = 1) is not Failed then
5:             return Diofantos(M, d_max = 1)
6:         end if
7:     end for
8:     return Failed
9: end function
```

Experimental Evaluation (*linrec*)

Algorithm 3 For experimental evaluation of SINDy for *linrec* case

Input: Matrix $M = [X|\mathbf{y}]$ of observations $X = [\mathbf{a}_{n-1}|...|\mathbf{a}_{n-19}]$ and target $\mathbf{y} = \mathbf{a}_n$

Output: Equation $a_n = \sum_{i=1}^{19} c_i a_{n-i}$ or Failed

```
1: function SINDy's linrec(M)
2:     for p = 1, 2, ..., 19 do
3:         Add col_p(X) to M (i.e. X = [X|M_p])
4:         if SINDy_grid(M, d_max = 1) is not Failed then
5:             return SINDy_grid(M, d_max = 1)
6:         end if
7:     end for
8:     return Failed
9: end function
```

Experimental Evaluation (*core*)

Algorithm 4 For experimental evaluation of Diofantos for *core* case

Input: Matrix $M = [X|\mathbf{y}]$ of observations $X = [\mathbf{n}|\mathbf{a}_{n-1}|...|\mathbf{a}_{n-10}]$ and target $\mathbf{y} = \mathbf{a}_n$

Output: Equation $a_n = f(n, a_{n-1}, ...a_{n-10})$ or Failed

```
1: function Diofantos's core(M)
2:     M = [y]
3:     for p = 1, 2, ..., 10 do
4:         Add col_p(X) to M (i.e. X = [X|M_p])
5:         for d_max = 1, 2, 3 do
6:             if Diofantos(M, d_max) is not Failed then
7:                 return Diofantos(M, d_max)
8:             end if
9:         end for
10:    end for
11:    return Failed
12: end function
```

Experimental Evaluation (*core*) with SINDy

Algorithm 5 For experimental comparison of SINDy for *core* case

Input: Matrix $M = [X|\mathbf{a}_n]$ of observations $X = [\mathbf{n}|\mathbf{a}_{n-1}|...|\mathbf{a}_{n-10}]$

Output: Equation $a_n = f(n, a_{n-1}, ...a_{n-10})$ or Failed

```
1:  function SINDy's core(M)
2:      M = [n|y]
3:      for p = 1, 2, ..., 10 do
4:          Remove column n and add column a_{n-p} to M
5:          if SINDy_grid(M, d_max) is not Failed then
6:              return SINDy_grid(M, d_max = 1)
7:          end if
8:          Add column of observations corresponding to n to M
9:          if SINDy_grid(M, d_max) is not Failed then
10:             return SINDy_grid(M, d_max = 1)
11:         end if
12:     end for
13:     return Failed
```