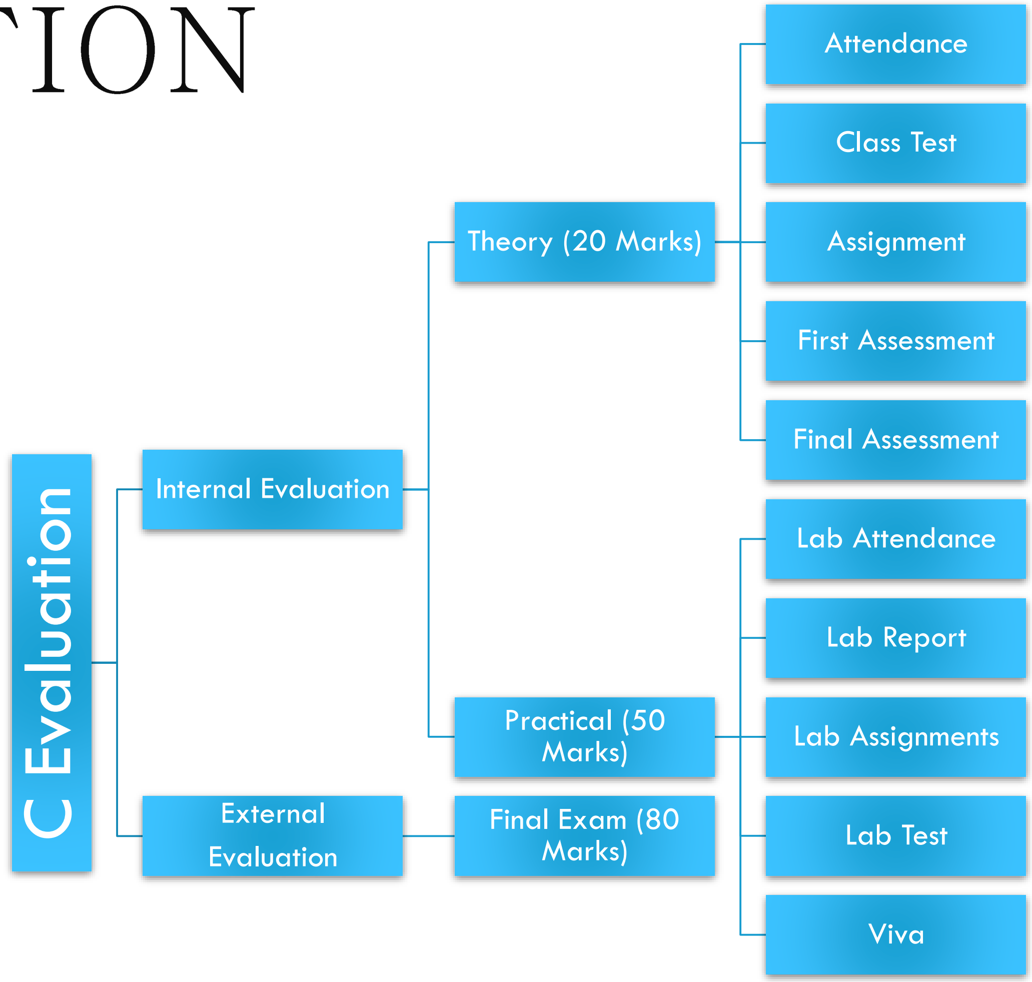
NUMERICAL METHODS

github.com/KCE/NM

Er. Shiva K. Shrestha
Head, Computer Department
Khwopa College of Engineering

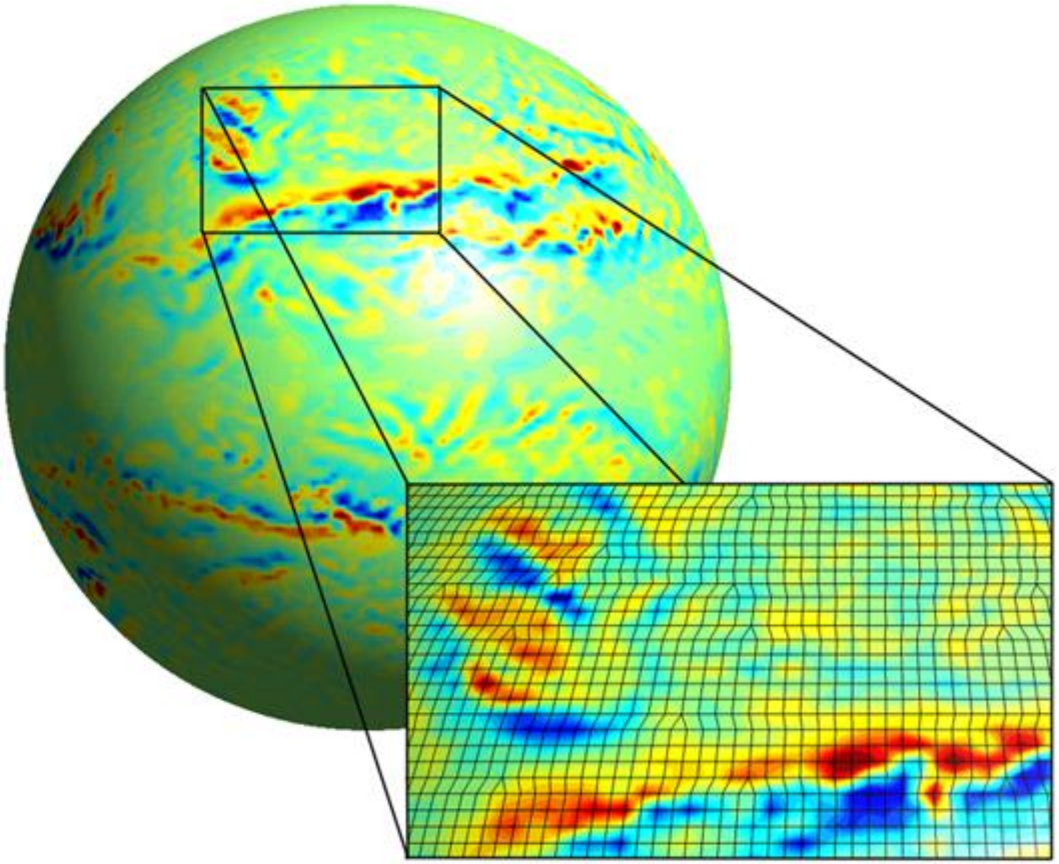
EVALUATION

Evaluation Criteria	Details
Internal Evaluation	✓ Theory (20) ✓ Practical (50) ✓ Lab Performance (30)
Final Evaluation	✓ Final Exam (80)

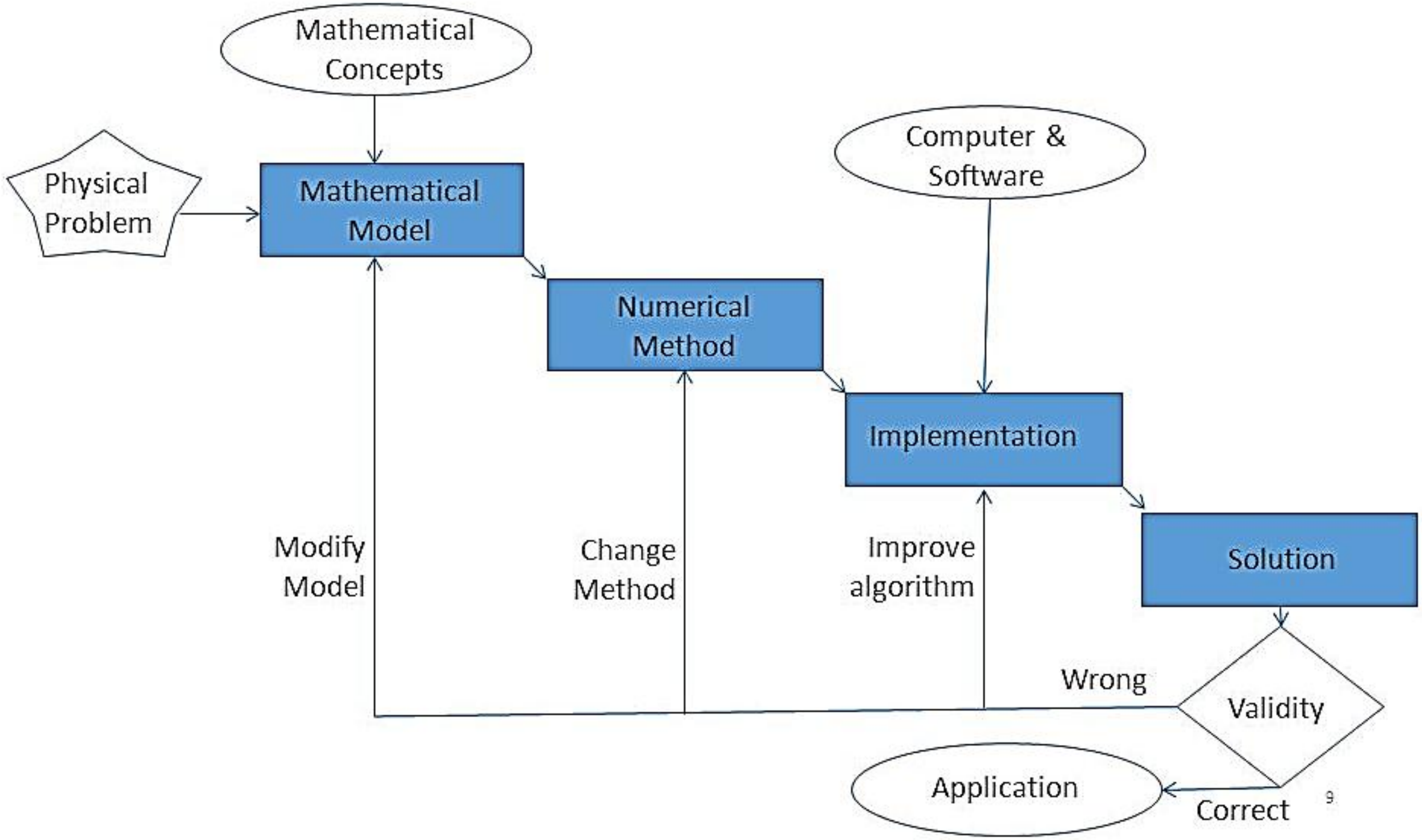


NUMERICAL COMPUTING

- Numerical computations play an indispensable role in solving real life mathematical, physical & engineering problems.
- It is an approach for solving complex mathematical problems using only simple arithmetic operations.



NUMERICAL COMPUTING PROCESS

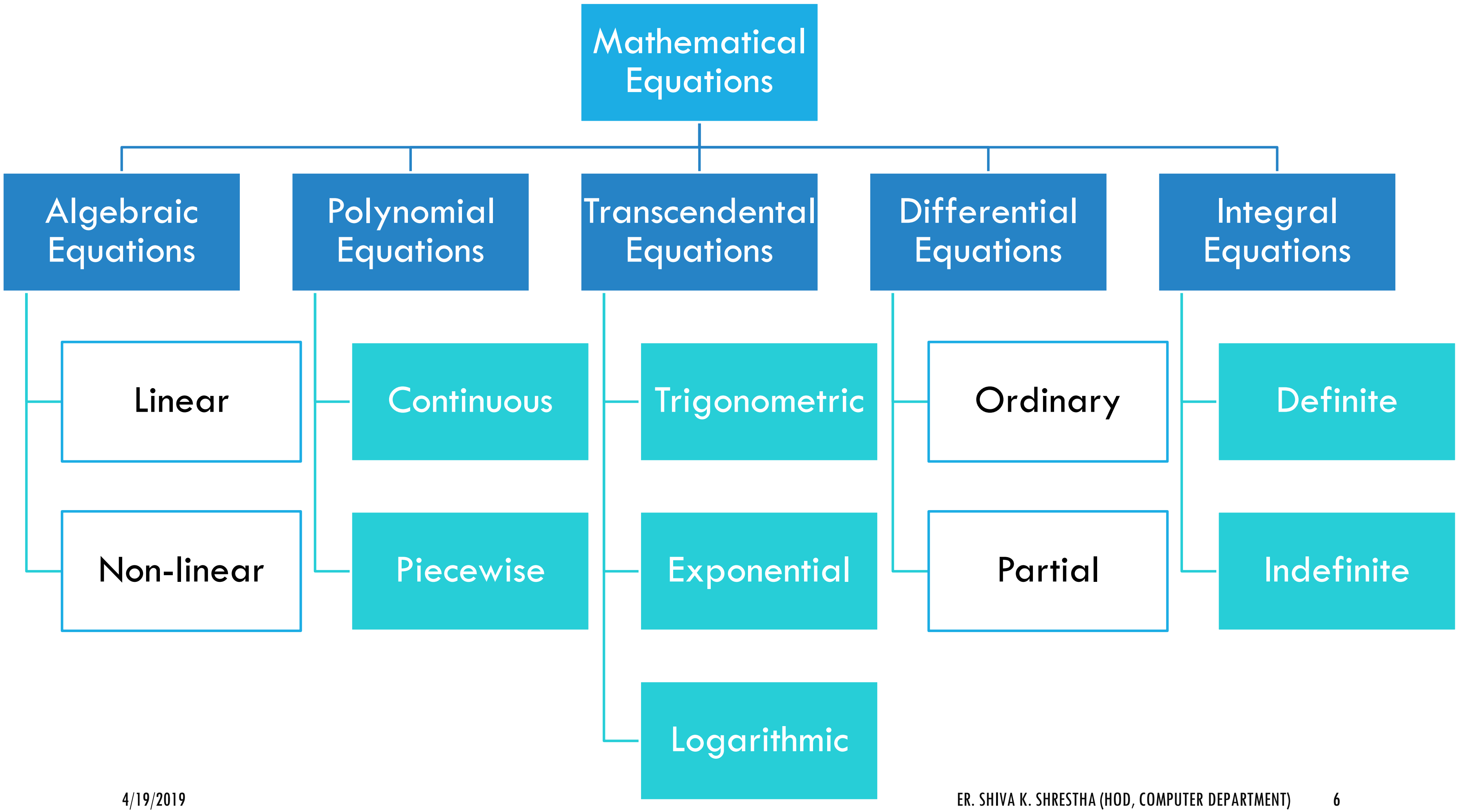


NM DEALS

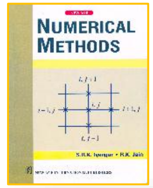
Traditional Numerical Computing Methods deals with:

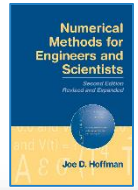
- ✓ Finding Roots of Equation
- ✓ Solving Systems of Linear Algebraic Equations
- ✓ Interpolation & Regression Analysis
- ✓ Numerical Integrations
- ✓ Numerical Differentiation
- ✓ Solution of Differential Equations
- ✓ Boundary Value Problems
- ✓ Solution of Matrix Problems

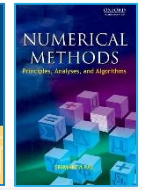
FORMS OF MATH EQUATIONS

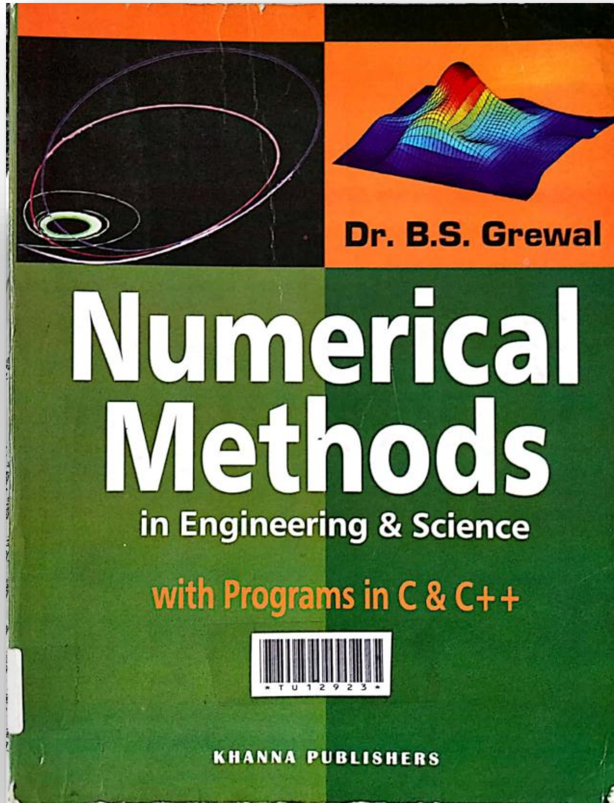


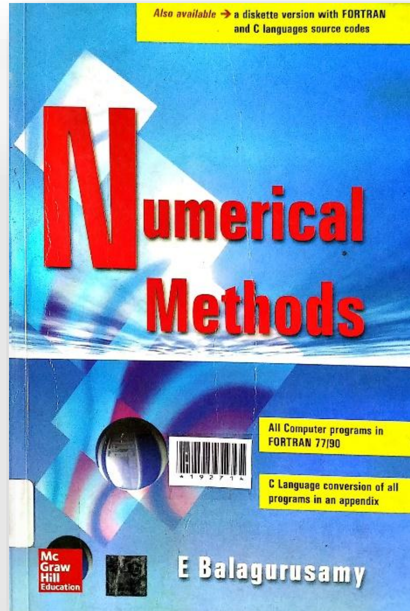
NM BOOKS

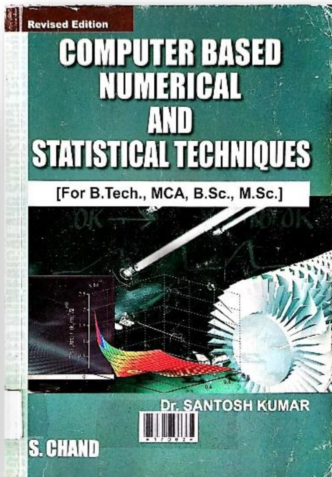

NUMERICAL METHODS
S.R.K. Iyengar • R.K. Jain

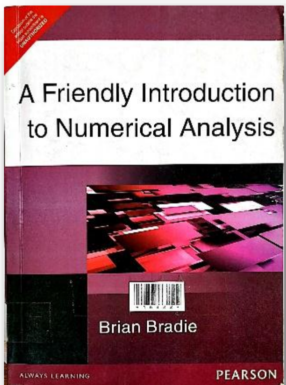

Numerical Methods for Engineers and Scientists
Second Edition
Revised and Expanded
Joe D. Hoffman

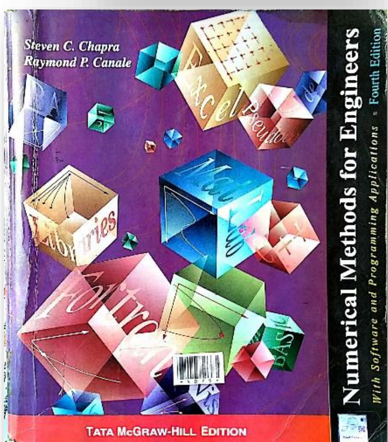

NUMERICAL METHODS
Principles, Analyses, and Algorithms

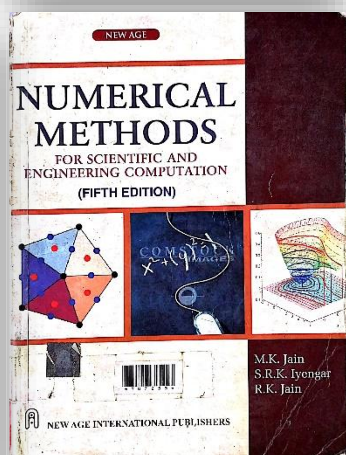

Dr. B.S. Grewal
Numerical Methods
in Engineering & Science
with Programs in C & C++
KHANNA PUBLISHERS

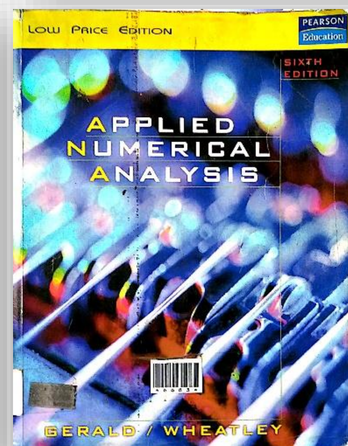

Also available → a diskette version with FORTRAN and C languages source codes
Numerical Methods
All Computer programs in FORTRAN 77/90
C Language conversion of all programs in an appendix
Mc Graw Hill Education
E Balagurusamy

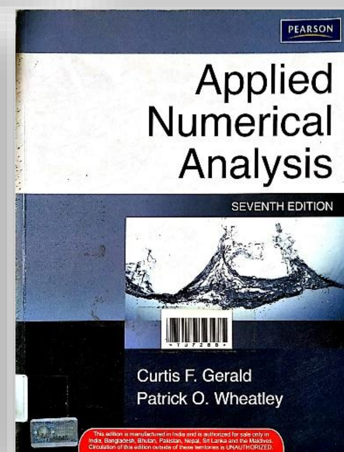

Revised Edition
COMPUTER BASED NUMERICAL AND STATISTICAL TECHNIQUES
[For B.Tech., MCA, B.Sc., M.Sc.]
Dr. SANTOSH KUMAR
S. CHAND

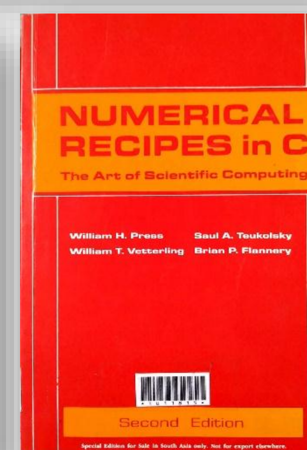

A Friendly Introduction to Numerical Analysis
Brian Bradie
ALWAYS LEARNING
PEARSON


Steven C. Chapra
Raymond P. Canale
Numerical Methods for Engineers
With Software and Programming Applications
Fourth Edition
TATA McGRAW-HILL EDITION


NEW AGE
NUMERICAL METHODS
FOR SCIENTIFIC AND ENGINEERING COMPUTATION
(FIFTH EDITION)
M.K. Jain
S.R.K. Iyengar
R.K. Jain
NEW AGE INTERNATIONAL PUBLISHERS


Low Price Edition
PEARSON Education
SIXTH EDITION
APPLIED NUMERICAL ANALYSIS
GERALD / WHEATLEY


PEARSON
Applied Numerical Analysis
SEVENTH EDITION
Curtis F. Gerald
Patrick O. Wheatley


NUMERICAL RECIPES in C
The Art of Scientific Computing
William H. Press
Saul A. Teukolsky
William T. Vetterling
Brian P. Flannery
Second Edition

COURSE CONTENT

1. Introduction, Approximation and Errors of Computation
2. Solutions of Non-linear Equations
3. Solution of System of Linear Algebraic Equations
4. Interpolation
5. Numerical Differentiation and Integration
6. Solution of Ordinary Differential Equations (ODE)
7. Numerical Solution of Partial Differential Equation (PDE)

Chapter 1

INTRODUCTION, APPROXIMATION AND ERRORS OF COMPUTATION (4)

1.1. Introduction, Importance of Numerical Methods

1.2. Approximation and Errors in Computation

1.3. Taylor's Series

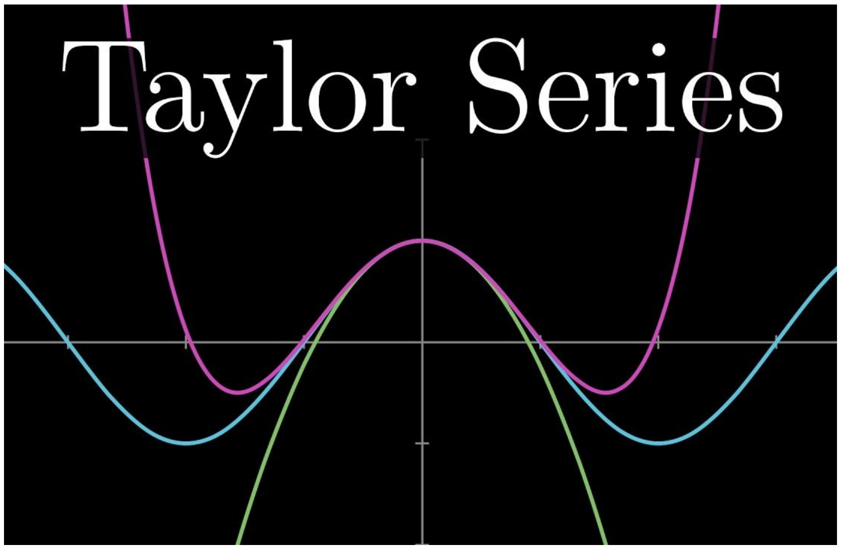
1.4. Newton's Finite Differences (Forward, Backward, Central Difference, Divided Difference)

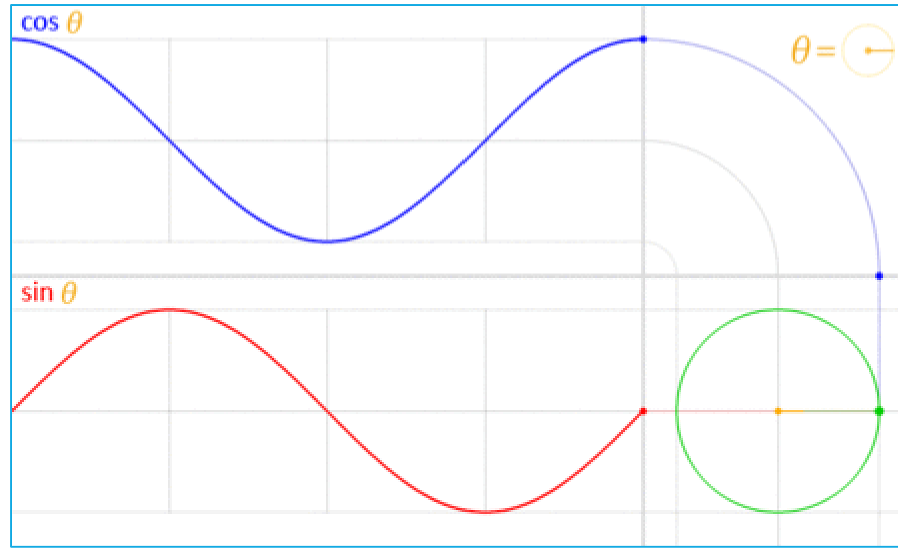
1.5. Difference Operators, Shift Operators, Differential Operators

1.6. Uses and Importance of Computer Programming in Numerical Methods.

Chapter 1

INTRODUCTION, APPROXIMATION AND ERRORS OF COMPUTATION (4)





Forward difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
x_0	y_0					
		Δy_0				
x_1 $(= x_0 + h)$	y_1		$\Delta^2 y_0$			
		Δy_1		$\Delta^3 y_0$		
x_2 $(= x_0 + 2h)$	y_2		$\Delta^2 y_1$		$\Delta^4 y_0$	
		Δy_2		$\Delta^3 y_1$		$\Delta^5 y_0$
x_3 $= (x_0 + 3h)$	y_3		$\Delta^2 y_2$		$\Delta^4 y_1$	
		Δy_3		$\Delta^3 y_2$		
x_4 $= (x_0 + 4h)$	y_4		$\Delta^2 y_3$			
		Δy_4				
x_5 $= (x_0 + 5h)$	y_5					

Backward difference table

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
x_0	y_0					
		∇y_1				
x_1 $(= x_0 + h)$	y_1		$\nabla^2 y_2$			
		∇y_2		$\nabla^3 y_3$		
x_2 $(= x_0 + 2h)$	y_2		$\nabla^2 y_3$		$\nabla^4 y_4$	
		∇y_3		$\nabla^3 y_4$		$\nabla^5 y_5$
x_3 $(= x_0 + 3h)$	y_3		$\nabla^2 y_4$		$\nabla^4 y_5$	
		∇y_4		$\nabla^3 y_5$		
x_4 $(= x_0 + 4h)$	y_4		$\nabla^2 y_5$			
		∇y_5				
x_5 $(= x_0 + 5h)$	y_5					

Chapter 2

SOLUTIONS OF NON-LINEAR EQUATIONS (5 HRS.)

2.1. Bisection Method

2.2. Newton Raphson Method (Two Equation Solution)

2.3. Regula-Falsi Method, Secant Method

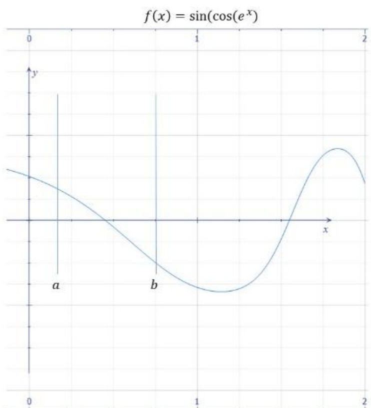
2.4. Fixed-point Iteration Method

2.5. Rate of Convergence and Comparisons of These Methods

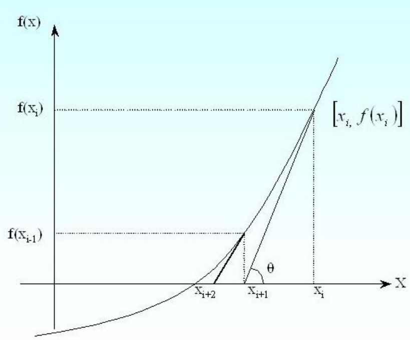
Chapter 2

SOLUTIONS OF NON-LINEAR EQUATIONS (5 HRS.)

Bisection Method



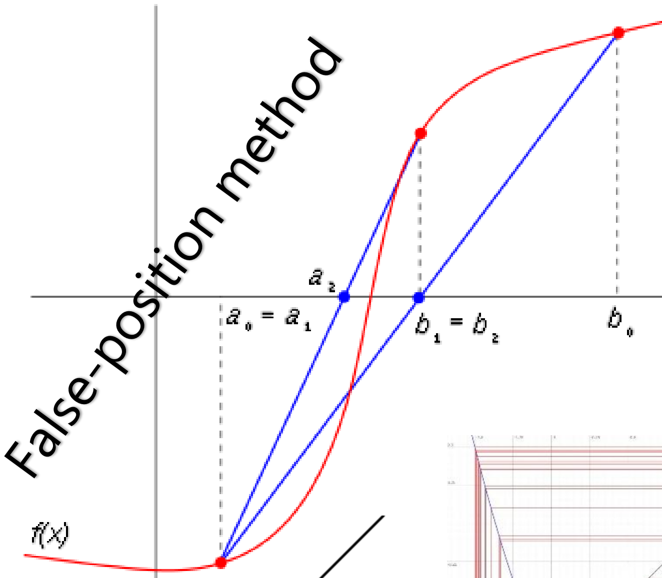
Newton-Raphson Method

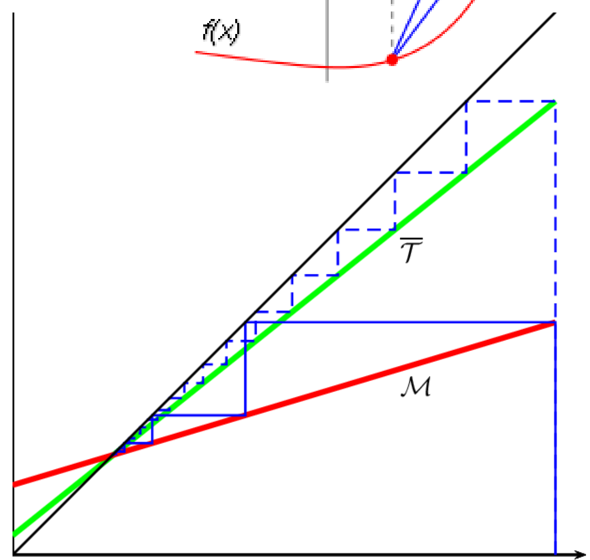


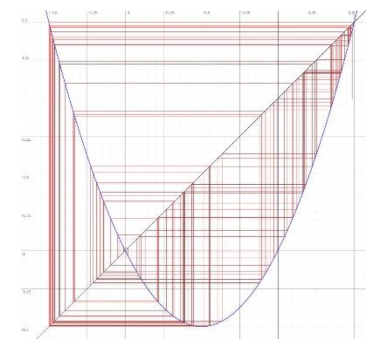
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Figure 1 Geometrical illustration of the Newton-Raphson method.

False-position method







Fixed Point Method

SOLUTIONS OF NON-LINEAR EQUATIONS (5 HRS.)

Applications:

Computer/Electrical Engineering

- Design of an Electric Circuit

Civil/Environmental Engineering

- Open-Chanel Flow, Greenhouse Gases & Rainwater

Mechanical/Aerospace Engineering

- Vibration Analysis, Pipe Friction

Chemical/Bio Engineering

- Ideal & Non-ideal Gas Laws

Chapter 3

SOLUTION OF SYSTEM OF LINEAR ALGEBRAIC EQUATIONS (8 HRS.)

3.1. Gauss Elimination Method with Pivoting Strategies

3.2. Gauss-Jordan Method

3.3. LU Factorization

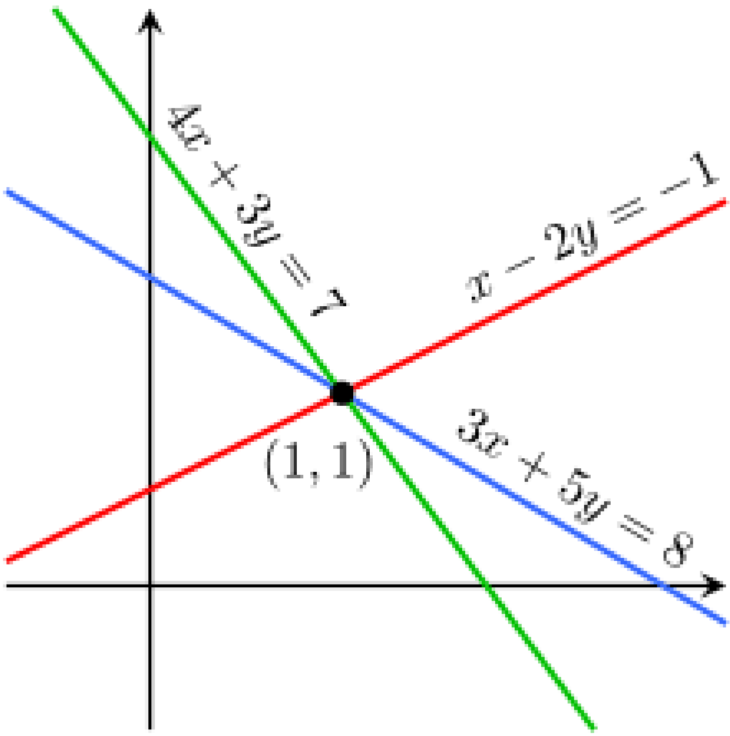
3.4. Iterative Methods

3.4.1. Jacobi Method,

3.4.2. Gauss-Seidel Method

3.5. Eigen Value and Eigen Vector

using Power Method



Chapter 3

SOLUTION OF SYSTEM OF LINEAR ALGEBRAIC EQUATIONS (8 HRS.)

Gauss-Jordan elimination

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & * \\ 0 & 1 & 0 & * \\ 0 & 0 & 1 & * \end{array}\right]$$

reduced row echelon form

Jacobi iteration

$$\begin{aligned} a_{11}\boxed{x_1} + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}\boxed{x_2} + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}\boxed{x_n} &= b_n \end{aligned}$$

$$x^0 = \begin{bmatrix} x_1^0 \\ x_2^0 \\ \vdots \\ x_n^0 \end{bmatrix}$$

$$x_1^1 = \frac{1}{a_{11}}(b_1 - a_{12}x_2^0 - \cdots - a_{1n}x_n^0)$$

$$x_2^1 = \frac{1}{a_{22}}(b_2 - a_{21}x_1^0 - a_{23}x_3^0 - \cdots - a_{2n}x_n^0)$$

$$x_n^1 = \frac{1}{a_{nn}}(b_n - a_{n1}x_1^0 - a_{n2}x_2^0 - \cdots - a_{nn-1}x_{n-1}^0)$$

$$x_i^{k+1} = \frac{1}{a_{ii}}\left[b_i - \sum_{j=1}^{i-1} a_{ij}x_j^k - \sum_{j=i+1}^{n} a_{ij}x_j^k\right]$$

Example: Power Method

Consider $A = \begin{bmatrix} 2 & 8 & 10 \\ 8 & 3 & 4 \\ 10 & 4 & 7 \end{bmatrix}$

Start with $z^{(1)} = x_0 = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}$

Assume all eigenvalues are equally important, since we don't know which one is dominant

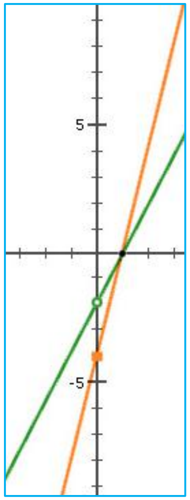
$$Az^{(1)} = \begin{bmatrix} 2 & 8 & 10 \\ 8 & 3 & 4 \\ 10 & 4 & 7 \end{bmatrix}\begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix} = \begin{Bmatrix} 20 \\ 15 \\ 21 \end{Bmatrix} = (21)\begin{Bmatrix} 0.9524 \\ 0.7143 \\ 1.0 \end{Bmatrix}$$

Eigenvalue estimate   Eigenvector

$$\begin{bmatrix} A00 & A01 & A02 \\ A10 & A11 & A12 \\ A20 & A21 & A22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ L10 & 1 & 0 \\ L20 & L21 & 1 \end{bmatrix}\begin{bmatrix} U00 & U01 & U02 \\ 0 & U11 & U12 \\ 0 & 0 & U22 \end{bmatrix}$$

Lower Triangular

Upper Triangular



Gauss-Seidel Method: Example 1

Rewriting each equation

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$$a_1 = \frac{106.8 - 5a_2 - a_3}{25}$$

$$a_2 = \frac{177.2 - 64a_1 - a_3}{8}$$

$$a_3 = \frac{279.2 - 144a_1 - 12a_2}{1}$$

SOLUTION OF SYSTEM OF LINEAR ALGEBRAIC EQUATIONS (8 HRS.)

Applications:

Computer/Electrical Engineering

- Current & Voltages in Resistor Circuits

Civil/Environmental Engineering

- Analysis of a Statically Determinate Truss

Mechanical/Aerospace Engineering

- Spring-Mass Systems

Chemical/Bio Engineering

- Steady-State Analysis of a System of Reactors

Chapter 4

INTERPOLATION (8 HRS.)

4.1. Newton's Interpolation (Forward, Backward)

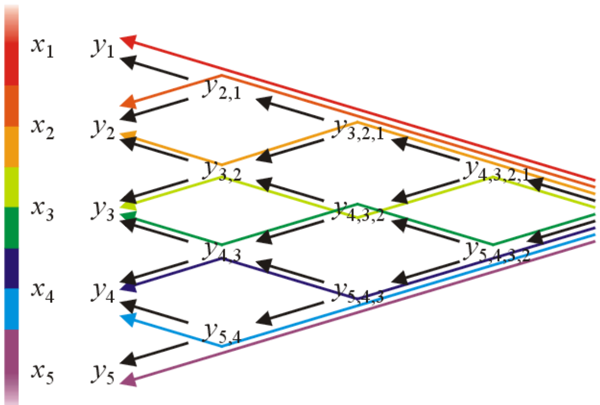
4.2. Central Difference Interpolation: Stirling's Formula, Bessel's Formula

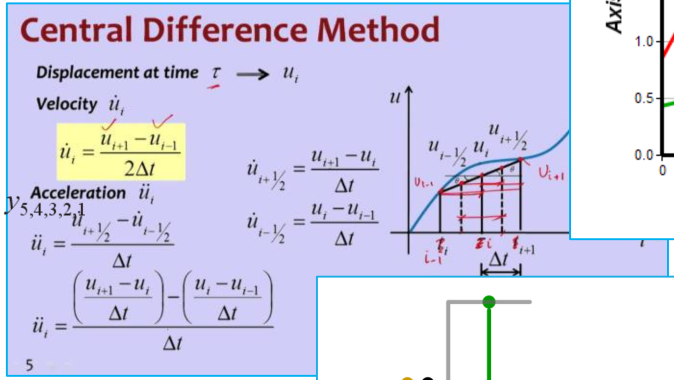
4.3. Lagrange Interpolation

4.4. Least Square Method of Fitting Linear and Nonlinear Curve for Discrete Data and Continuous Function

4.5. Spline Interpolation (Cubic Spline)

INTERPOLATION (8 HRS.)





Central Difference Method

Displacement at time $\tau \longrightarrow u_i$

Velocity $\dot{u}_i$

$$\dot{u}_i = \frac{u_{i+1} - u_{i-1}}{2\Delta t}$$

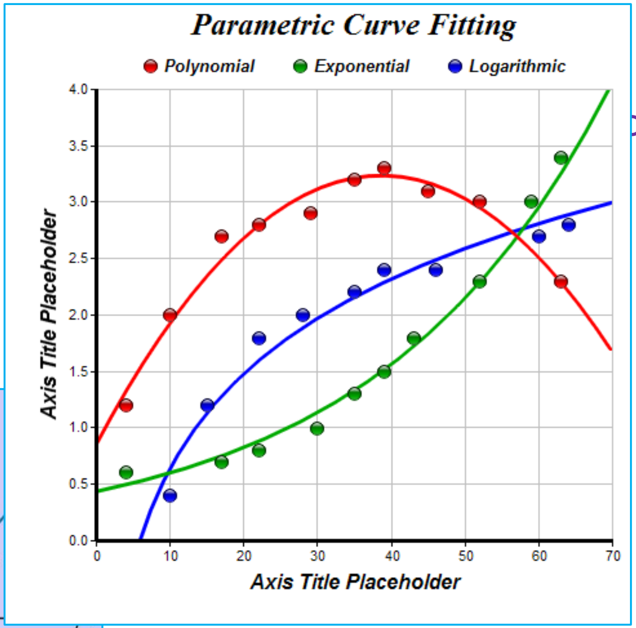
$$\dot{u}_{i+\frac{1}{2}} = \frac{u_{i+1} - u_i}{\Delta t} \qquad \dot{u}_{i-\frac{1}{2}} = \frac{u_i - u_{i-1}}{\Delta t}$$

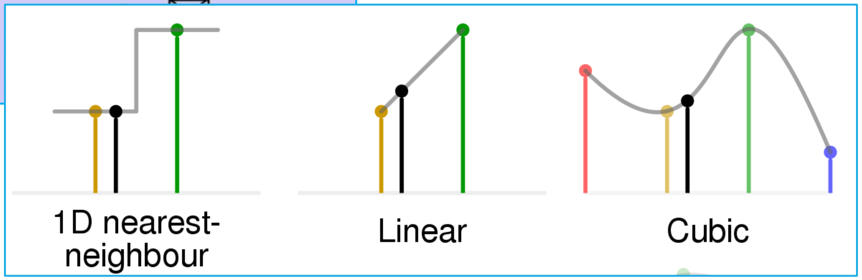
Acceleration $\ddot{u}_i$

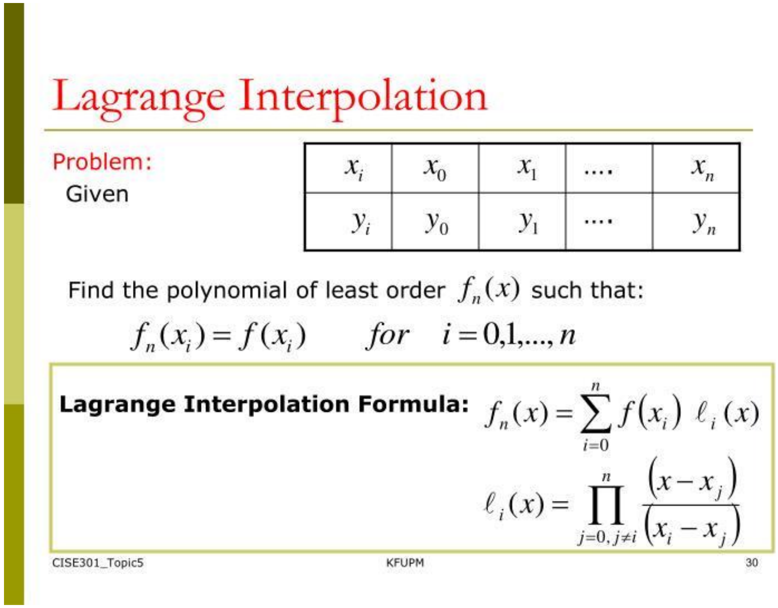
$$\ddot{u}_i = \frac{\dot{u}_{i+\frac{1}{2}} - \dot{u}_{i-\frac{1}{2}}}{\Delta t}$$

$$\ddot{u}_i = \frac{\left(\frac{u_{i+1} - u_i}{\Delta t}\right) - \left(\frac{u_i - u_{i-1}}{\Delta t}\right)}{\Delta t}$$

5







Lagrange Interpolation

Problem:
Given

x_i	x_0	x_1	....	x_n
y_i	y_0	y_1	....	y_n

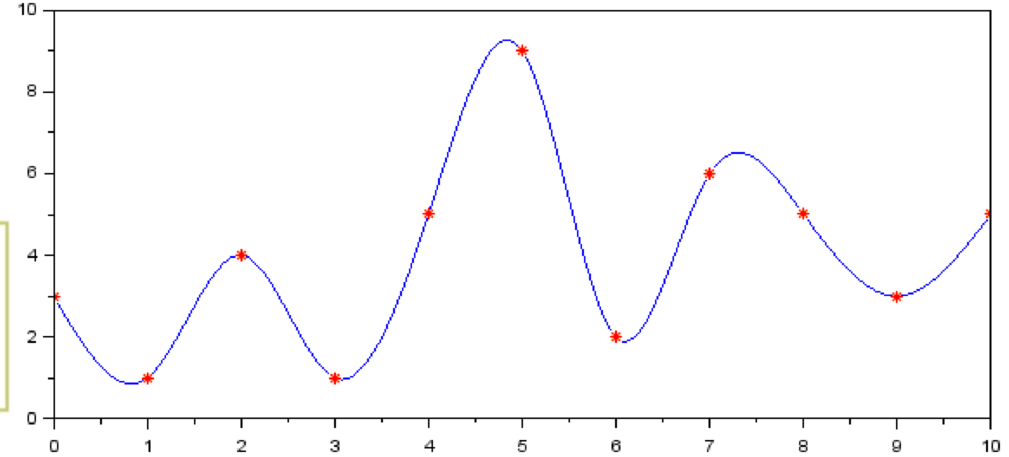
Find the polynomial of least order $f_n(x)$ such that:

$$f_n(x_i) = f(x_i) \qquad for \quad i = 0,1,\dots,n$$

Lagrange Interpolation Formula: $f_n(x) = \sum_{i=0}^{n} f(x_i)\ \ell_i(x)$

$$\ell_i(x) = \prod_{j=0,\, j\neq i}^{n} \frac{(x - x_j)}{(x_i - x_j)}$$

CISE301_Topic5 KFUPM 30



Chapter 4

INTERPOLATION (8 HRS.)

Applications:

Computer/Electrical Engineering

- Fourier Analysis

Civil/Environmental Engineering

- Use of Spline to Estimate Heat Transfer

Mechanical/Aerospace Engineering

- Analysis of Experimental Data

Chemical/Bio Engineering

- Linear Regression and Population Model

Chapter 5

NUMERICAL DIFFERENTIATION AND INTEGRATION (6 HRS.)

5.1. Numerical Differentiation Formulae

5.2. Maxima and Minima

5.3. Newton-Cote General Quadrature Formula

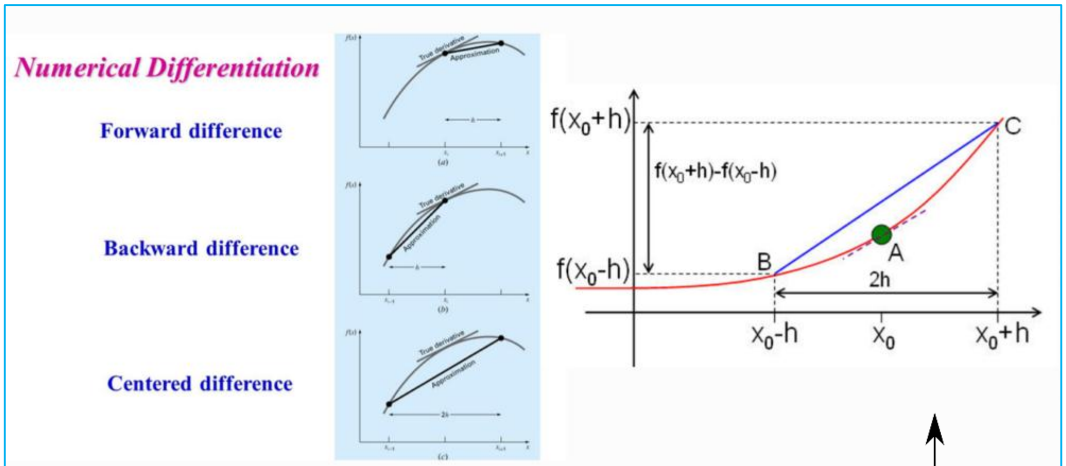
5.4. Trapezoidal, Simpson's 1/3, 3/8 Rule

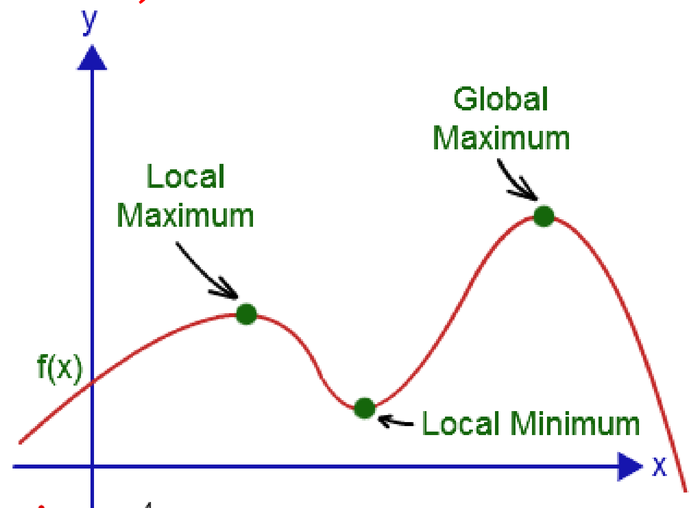
5.5. Romberg Integration

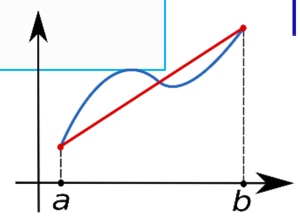
5.6. Gaussian Integration (Gaussian – Legendre Formula 2-point & 3-point)

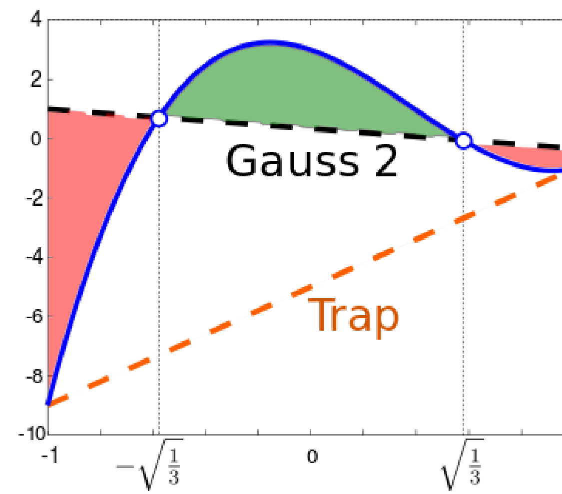
Chapter
5

NUMERICAL DIFFERENTIATION AND INTEGRATION (6 HRS.)

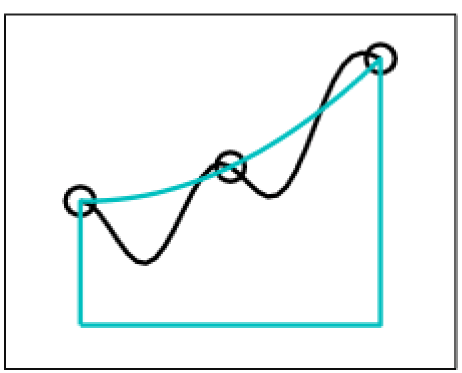




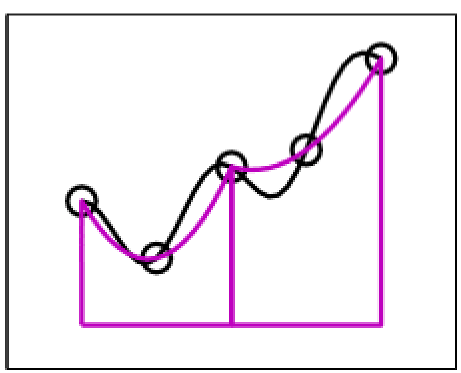


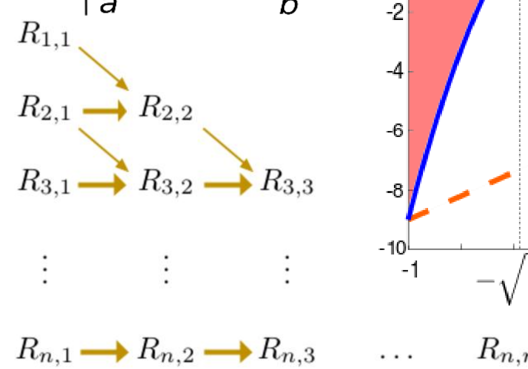


Simpson's rule



Composite Simpson's rule





NUMERICAL DIFFERENTIATION AND INTEGRATION (6 HRS.)

Applications:

Computer/Electrical Engineering

- Root-Mean-Square (RMS) Current by Numerical Integration

Civil/Environmental Engineering

- Effective Force on the Mast of Racing Sailboat

Mechanical/Aerospace Engineering

- Numerical Integration to Compute Work

Chemical/Bio Engineering

- Integration to Determine the Total Quantity of Heat

Chapter 6

SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS (6 HRS.)

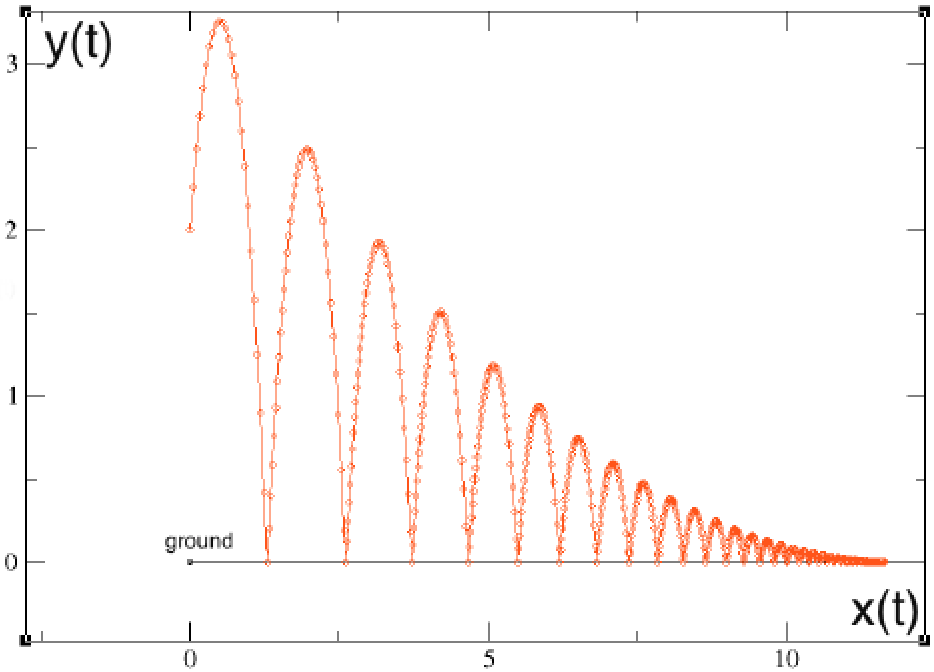
6.1. Euler's & Modified Euler's Method

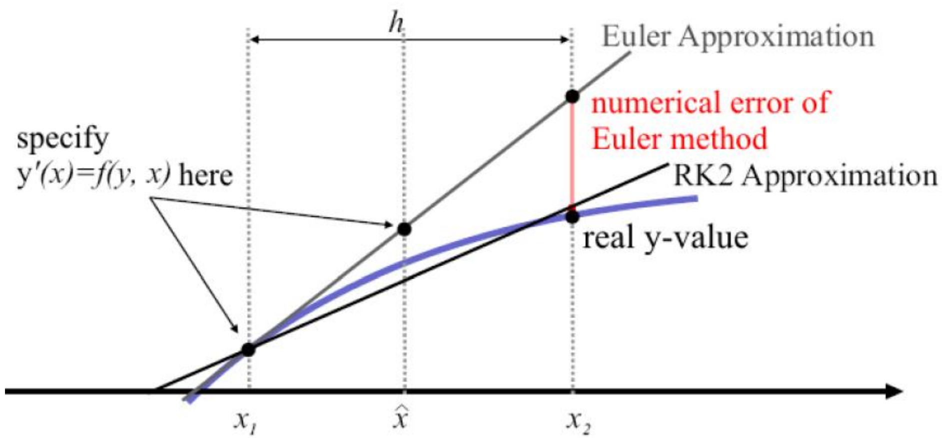
6.2. Runge Kutta Methods for 1st and 2nd Order Ordinary Differential Equations

6.3. Solution of Boundary Value problem by Finite Difference Method & Shooting Method

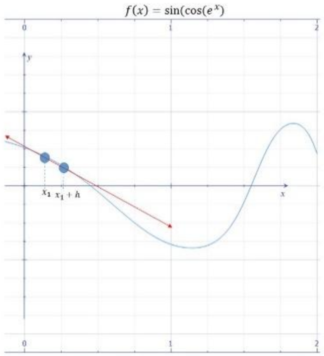
Chapter 6

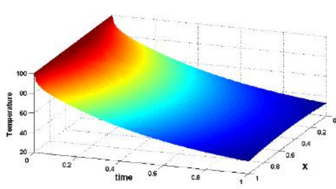
SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS (6 HRS.)

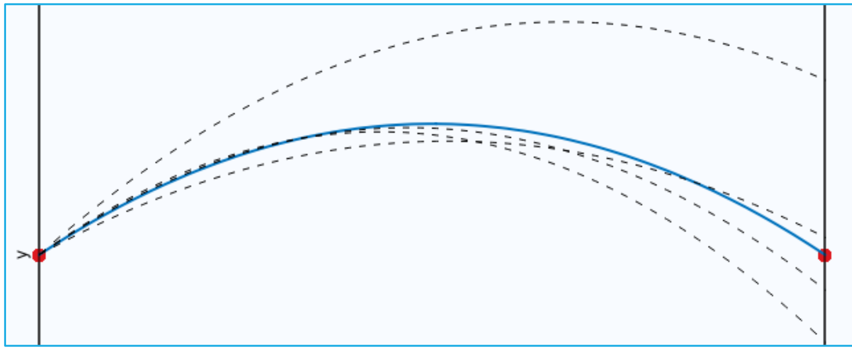




Finite Difference Method







Chapter 6

SOLUTION OF ORDINARY DIFFERENTIAL EQUATIONS (6 HRS.)

Applications:

Computer/Electrical Engineering

- Simulating Transient Current for an Electric Circuit

Civil/Environmental Engineering

- Predator-Prey Models & Chaos

Mechanical/Aerospace Engineering

- The Swinging Pendulum

Chemical/Bio Engineering

- Using ODEs to analyze the Transient Response of a Reactor

Chapter 7

NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATION (8 HRS.)

7.1. Classification of Partial Differential Equation (Elliptic, Parabolic, and Hyperbolic)

7.2. Solution of Laplace Equation (Standard 5-Point Formula with Iterative Method)

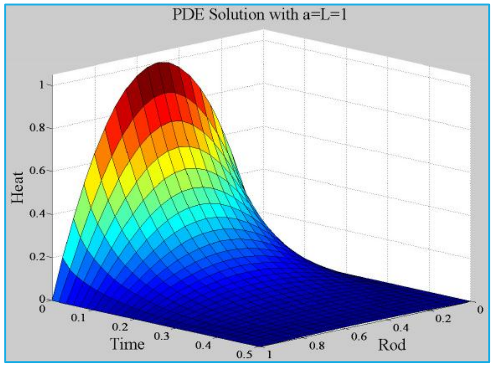
7.3. Solution of Poisson Equation (Finite Difference Approximation)

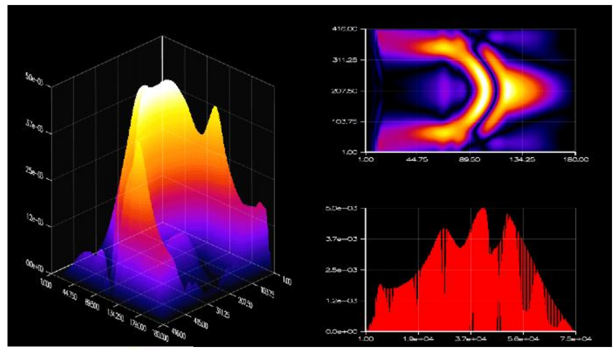
7.4. Solution of Elliptic Equation by Relaxation Method

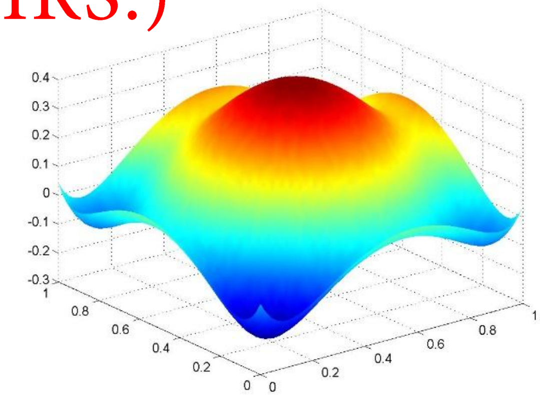
7.5. Solution of One-Dimensional Heat Equation by Schmidt Method

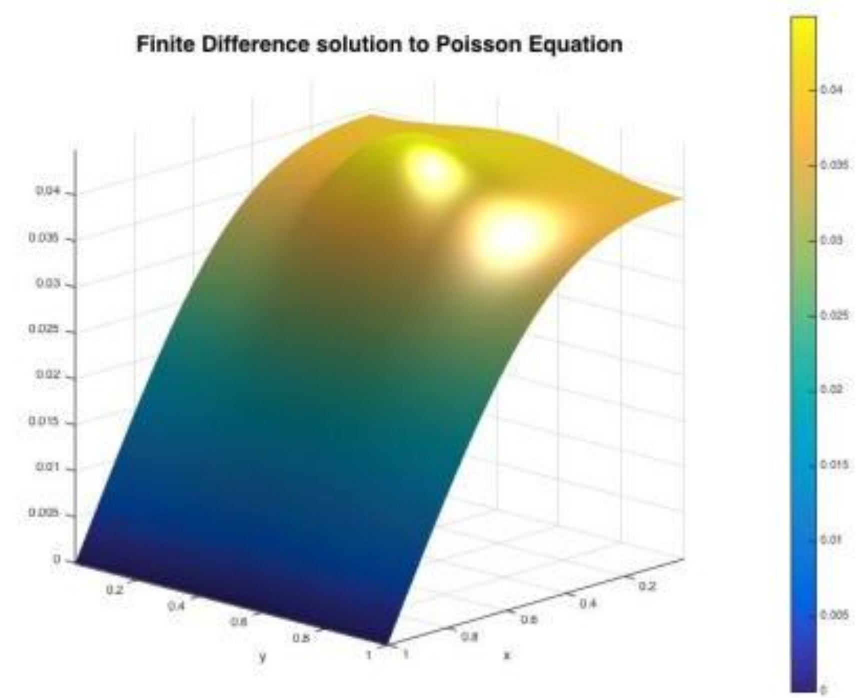
Chapter 7

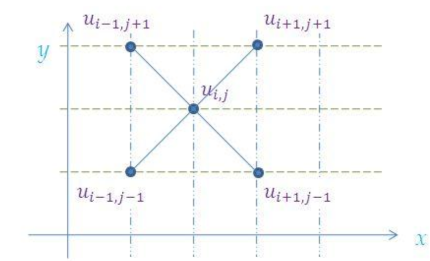
NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATION (8 HRS.)











Laplace Equation

Diagonal Five-Point Formula: $u_{xx} + u_{yy} = 0$

Laplace equation remains invariant under rotation of 45°.

Value of u at any grid point is the average of its values at four diagonal points.

$$u_{i,j} = \frac{1}{4}\left[u_{i-1,j+1} + u_{i-1,j-1} + u_{i+1,j-1} + u_{i+1,j+1}\right]$$

Error in Diagonal formula is FOUR TIMES than that in Standard formula.

Standard Five-point Formula should be preferred , if possible.

Chapter 7

NUMERICAL SOLUTION OF PARTIAL DIFFERENTIAL EQUATION (8 HRS.)

Applications:

Computer/Electrical Engineering

- 2D Electrostatic Field Problems

Civil/Environmental Engineering

- Deflections of a Plate

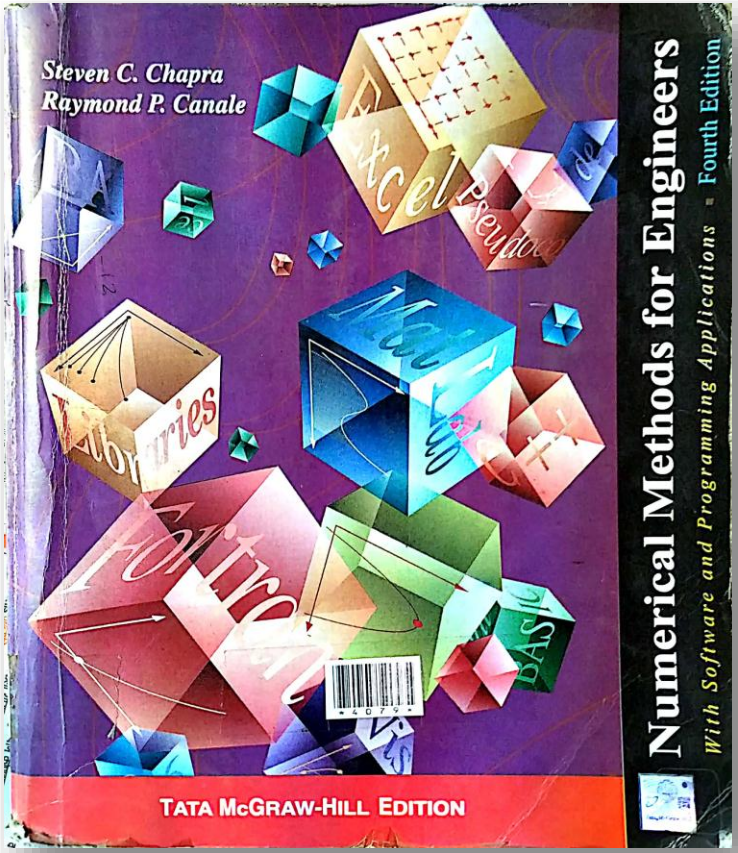
Mechanical/Aerospace Engineering

- Finite-Element Solution of a Series of Springs

Chemical/Bio Engineering

- 1D Mass Balance of a Reactor

WANT TO STUDY NM APPLICATIONS IN DETAIL?



PRACTICAL

Algorithm & Program Development in C Programming Language of Following:

1. Generate Difference Table.
2. At least two from Bisection Method, Newton-Raphson Method, Secant Method
3. At least one from Gauss Elimination Method or Gauss Jordan method. Finding Largest Eigen Value and Corresponding Vector by Power Method.
4. Lagrange Interpolation, Curve Fitting by Least Square Method.
5. Differentiation by Newton's Finite Difference Method. Integration using Simpson's 3/8 Rule
6. Solution of 1st Order Differential Equation using RK-4 Method
7. Partial Differential Equation (Laplace Equation)
8. Numerical Solutions using MATLAB.

REFERENCES

1. Dr. B.S.Grewal, “Numerical Methods in Engineering and Science”, Khanna Publication, 7th edition.
2. E Balagurusamy, “Numerical Methods”, Mc Graw Hill Education
3. Dr. Santosh Kumar, “Computer Based Numerical & Statistical Techniques”, S. Chand
4. Steven C. Chopra, Raymond P. Canale, “Numerical Methods for Engineeris”, TATA McGraw Hill
5. W.H. Press and et. Al., “Numerical Recipes in C”, Cambridge
6. Robert J schilling, Sandra l harries Applied Numerical Methods for Engineers using MATLAB and C”, 3rd edition Thomson Brooks/cole.
7. Richard L. Burden, J.Douglas Faires, “Numerical Analysis 7th edition” , Thomson / Brooks/cole
8. John. H. Mathews, Kurtis Fink , “Numerical Methods Using MATLAB 3rd edition” ,Prentice Hall publication
9. JAAN KIUSALAAS , “Numerical Methods in Engineering with MATLAB”, Cambridge Publication

Q/A?

Thank You!

Er. Shiva K. Shrestha

computer.khwopa@gmail.com

nm.khwopa@gmail.com

github.com/KCE/NM