1

Name: **Sequence Number:**

1˚). Determine whether the following sequence is convergent or divergent? Why? (total 50%)

$$\sum_{n=1}^{\infty} \left(\frac{\ln n}{n} \right)^n$$

2˚). Find the Taylor series of $f(x)$ below and its convergent radius (50%):

$$f(x) = \frac{1}{1 + 3x} \text{ at } c = 2$$

Hint:

$$\frac{1}{1 + 3x} = \frac{1}{7} \frac{1}{1 + 3(x - 2)/7}$$

Ans:

1. $\sum\limits_{n=1}^{\infty} \left(\frac{\ln n}{n} \right)^{n}$ convergent absolutely since $\frac{\ln n}{n} \to 0 < 1$ by root test.

Ans:

2.

$$f(x) = \frac{1}{1 + 3x} = \frac{1}{7} \frac{1}{1 + 3(x - 2)/7}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n} 3^{n}}{7^{n+1}} (x - 2)^{n}$$

where $|3(x - 2)/7| < 1$, $\left(|x - 2| < \frac{7}{3} \right)$, i.e. convergent radius: 7/3.

In [ ]: