ESE 4481 ASSIGNMENT #4: SIMPLIFIED LINEAR DYNAMICS

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Due Date: 10/19/23 Electronic submissions of all models, code, and writing should be included in Canvas before the deadline. Late work loses 20 points a day.

1. Academic Integrity

Please don't plagarize. Work together, but write your own reports.

2. What to turn in

Please submit all answers in a self-contained short report (<5 pages) that answers all questions. If you have pages of figures, these may be concisely summarized in an appendix (<5 pages). Additionally, submit all code as a supplement in canvas.

3. Back

This assignment reuses the notation from assignment 3, but make some minor modifications for clarity

The equations of motion can be written as follows [1]:

$$\dot{P}^E = R_B^E V^B, \tag{3.1}$$

$$\dot{\Theta} = H(\Theta)^{-1}\omega_{b/E}, \tag{3.2}$$

$$\dot{V}^b = \frac{1}{m}\left(F_t^b(\vec{n}) + F_a^b(\vec{n}) + F_g^b(\Theta) - \omega_{b/E} \times mV^b\right), \tag{3.3}$$

$$\dot{\omega}_{b/E} = J^{-1}\left(M_t^b(\vec{n}) + M_\Omega^b(\vec{n}) - \omega_{b/E} \times J\omega_{b/E}\right). \tag{3.4}$$

These equations depend on F_t^b and M_t^b which depend on the speed of the drones 4 motors. With the mixer in the loop, the drones dynamics can be simplified. We're going to design controllers that request force and moment, and assume that the mixer will perfectly determine the motor speeds to generate force and moments. As a result, the control input to our drone dynamics will be $\vec{u}$

$$\vec{u} = \begin{bmatrix} 0 \\ 0 \\ Z_c \\ M_c \\ L_c \\ N_c \end{bmatrix}$$

where Z_c, M_c, L_c, N_c are commanded thrust, rolling moment, pitching moment, and yawing moment.

This leads to a dynamics:

$$\dot{P}^E = R_B^E V^B, \tag{3.5}$$

$$\dot{\Theta} = H(\Theta)^{-1}\omega_{b/E}, \tag{3.6}$$

$$\begin{bmatrix} \dot{V}^b \\ \dot{\omega}_{b/E} \end{bmatrix} = \begin{bmatrix} \frac{1}{m}\left(F_a^b(\vec{n}) + F_g^b(\Theta) - \omega_{b/E} \times mV^b\right) \\ J^{-1}\left(M_\Omega^b(\vec{n}) - \omega_{b/E} \times J\omega_{b/E}\right). \end{bmatrix} + \begin{bmatrix} \frac{1}{m}I & 0 \\ 0 & J^{-1} \end{bmatrix} \vec{u} \tag{3.7}$$

4. Assignment

Assume that the UAV has its initial condition moving forward in the V_N direction at 1 m/s. Please assume that $v = 0$. Let the linearized control be the difference between the nonlinear control $\vec{u}$ and the trim control $\vec{u}_0$,

$$\delta\vec{u} = \vec{u} - \vec{u}_0.$$

Let the linearized state be the difference between the nonlinear state $\vec{x}$ and the trim state $\vec{x}_0$,

$$\delta\vec{x} = \vec{x} - \vec{x}_0.$$

Let the linearized state $\delta\vec{x}$ and control $\delta\vec{u}$ take the following form:

$$\delta\vec{x} = \begin{bmatrix} P_N - P_{N,0} \\ P_E - P_{E,0} \\ P_D - P_{D,0} \\ w - w_0 \\ v - v_0 \\ \phi - \phi_0 \\ p - p_0 \\ u - u_0 \\ \theta - \theta_0 \\ q - q_0 \\ \psi - \psi_0 \\ r - r_0 \end{bmatrix} = \begin{bmatrix} \delta P_N \\ \delta P_E \\ \delta P_D \\ \delta w \\ \delta v \\ \delta\phi \\ \delta p \\ \delta u \\ \delta\theta \\ \delta q \\ \delta\psi \\ \delta r \end{bmatrix}, \qquad \delta\vec{u} = \begin{bmatrix} Z_c - Z_0 \\ L_c - L_0 \\ M_c - M_0 \\ N_c - N_0 \end{bmatrix} = \begin{bmatrix} \delta Z \\ \delta L \\ \delta M \\ \delta N \end{bmatrix}. \tag{4.1}$$

5. Assignment

(1) We'll decouple this 12th order system linear system that you derived in Assignment 3 into 5 linear systems:
   (a) Thrust is mainly used to control position above the ground. Justify, argue for, or derive a simplified linear system that is useful for altitude control as follows:

$$\begin{bmatrix} \delta\dot{P}_D \\ \delta\dot{w} \end{bmatrix} = \delta A_{\delta Z} \begin{bmatrix} \delta P_D \\ \delta w \end{bmatrix} + B_{\delta Z}\delta Z, \tag{5.1}$$

$$\delta y = \begin{bmatrix} \delta P_D \\ \delta w \end{bmatrix}. \tag{5.2}$$

(b) Rolling moment will mainly be used to control v. Justify, argue for, or derive a simplified linear system that takes the form

$$\begin{bmatrix}\delta\dot{v}\\ \delta\dot{\phi}\\ \delta\dot{p}\end{bmatrix} = A_{\delta L}\begin{bmatrix}\delta v\\ \delta\phi\\ \delta p\end{bmatrix} + B_{\delta L}\delta L, \tag{5.3}$$

$$\delta y = \begin{bmatrix}\delta v\\ \delta\phi\\ \delta p\end{bmatrix}. \tag{5.4}$$

(c) Pitching moment will mainly be used to control u. Justify, argue for, or derive a simplified linear system that takes the form

$$\begin{bmatrix}\delta\dot{u}\\ \delta\dot{\theta}\\ \delta\dot{q}\end{bmatrix} = A_{\delta M}\begin{bmatrix}\delta u\\ \delta\theta\\ \delta q\end{bmatrix} + B_{\delta M}\delta M, \tag{5.5}$$

$$\delta y = \begin{bmatrix}\delta u\\ \delta\theta\\ \delta q\end{bmatrix}. \tag{5.6}$$

(d) Yawing moment will mainly be used to control ψ. Justify, argue for, or derive a simplified linear system that takes the form

$$\begin{bmatrix}\delta\dot{\psi}\\ \delta\dot{r}\end{bmatrix} = A_{\delta N}\begin{bmatrix}\delta\psi\\ \delta r\end{bmatrix} + B_{\delta N}\delta N, \tag{5.7}$$

$$\delta y = \begin{bmatrix}\delta\psi\\ \delta r\end{bmatrix}. \tag{5.8}$$

(e) Lastly, show that the linearized kinematics can be written as

$$\begin{bmatrix}\delta\dot{P}_N\\ \delta\dot{P}_E\end{bmatrix} = A_{\text{guid}}\begin{bmatrix}\delta P_N\\ \delta P_E\end{bmatrix} + B_{\text{guid}}\begin{bmatrix}\delta u\\ \delta\psi\end{bmatrix}, \tag{5.9}$$

$$\delta y = \begin{bmatrix}\delta P_N\\ \delta P_E\end{bmatrix} \tag{5.10}$$

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(2) For each of the 5 systems, determine the A and B matrices
(3) For each of the 5 systems, determine the transfer matrix from the input to all outputs
(4) For each of the 5 systems, determine the state transition matrix
(5) For each of the 5 systems, calculate the mode functions
(6) For each of the 5 systems, determine whether the system is controllable and find the controllable subspace of the state space
(7) For each of the 5 systems, determine whether they are bounded-input, bounded output stable
(8) For each of the 5 systems, determine a feedback matrix K for state-feedback ($u = -Kx$) that move all poles into the half-plane $s < -3$.

References

[1] Morelli, Eugene A., and Vladislav Klein. Aircraft system identification: theory and practice. Vol. 2. Williamsburg, VA: Sunflyte Enterprises, 2016.

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