ESE 4481 ASSIGNMENT #3: DYNAMICS

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Due Date: 10/03/23 Electronic submissions of all models, code, and writing should be included in Canvas before the deadline. Late work loses 10 points a day.

1. Academic Integrity

You may collaborate with your classmates, but write your own code, make your own model, and write your own report.

2. What to turn in

Document all work in a self-contained report and submit it to canvas. Include your code and models.

3. Assignment

3.1. **A Mixer.** Suppose that we wish to command the drone's motors to achieve a ground-truth value of wrench

$$\begin{bmatrix} Z_{T,gt} \\ L_{T,gt} \\ M_{T,gt} \\ N_{T,gt} \end{bmatrix}^{\mathrm{b}} = \begin{bmatrix} -|a|\left(\nu_{fl}+\nu_{fr}+\nu_{br}+\nu_{bl}\right) \\ |b|\left(\nu_{fl}-\nu_{fr}-\nu_{br}+\nu_{bl}\right) \\ |c|\left(\nu_{fl}+\nu_{fr}-\nu_{br}-\nu_{bl}\right) \\ |d|\left(-\nu_{fl}+\nu_{fr}-\nu_{br}+\nu_{bl}\right) \end{bmatrix}, \tag{3.1}$$

$$n_{fl} = \sqrt{|\nu_{fl}|}, \tag{3.2}$$

$$n_{fr} = \sqrt{|\nu_{fr}|}, \tag{3.3}$$

$$n_{br} = \sqrt{|\nu_{br}|}, \tag{3.4}$$

$$n_{bl} = \sqrt{|\nu_{bl}|}. \tag{3.5}$$

We have a linear system of equations in motor speed squared,

$$W_{gt} = \mathbb{M}\vec{\nu}, \tag{3.6}$$

where W_{gt} is ground-truth wrench command, $\mathbb{M}$ is the matrix parameterized by coefficients a, b, c, d that relate motor speed to force and moment,

$$W_{gt} = \left(Z_{T,gt}, L_{T,gt}, M_{T,gt}, N_{T,gt}\right)^T,$$

and where $\vec{\nu}$ is a vector of the signed motor speeds squared. Note that f signifies front, b signifies back, r signifies right, and l signifies left.

Implement a system in Symbolic Matlab that takes commanded aerodynamic wrench

$$W_c = \left(Z_{T,c}, L_{T,c}, M_{T,c}, N_{T,c}\right)^T$$

and determines the motor speeds required to achieve the request:

$$\vec{\nu} = \mathbb{M}^{-1}W_c. \tag{3.7}$$

Additionally, implement the equation for ground truth wrench (4) in Symbolic Matlab.

We will refer to this system as the *mixer*. The mixer inverts (4). Please note that to avoid taking square roots of negative numbers, the nominal commanded force in the z axis should be equal and opposite to gravity.

4. Create a block that inputs commanded wrench and outputs motor speeds

For this next section, please note that you can easily generate Matlab functions from symbolic matlab functions using the command. Example syntax is here:

```
syms x y z t
v = (x + y/2 + z/3)*exp(-t);
matlabFunction(v,"File","myfile","Vars",{t,[x y z]});
```

Using the equations of motion that you calculated in Symbolic Matlab for your previous homework and your mixer equations that you just created, generate a simulation using ode45 that simulates the equations of motion:

$$\dot{p}^E = V^E \tag{4.1}$$

$$\dot{\Theta} = H(\Theta)^{-1}\omega^b_{b/E} \tag{4.2}$$

$$\dot{V}^b = \frac{1}{m}\left(F_T^{b,gt} + F_A^b + F_G^b - \omega^b_{b/E} \times mV^b\right), \tag{4.3}$$

$$\dot{\omega}^b_{b/E} = J^{-1}\left(M_T^{b,gt} + M_A^b + M_\Omega^b - \omega^b_{b/E} \times J\omega_{b/E}\right). \tag{4.4}$$

These equations connect to the equation for ground truth motor commands:

$$W_{gt} = \mathbb{M}\vec{\nu},.$$

These equations connect to the equation for your mixer:

$$\vec{\nu} = \mathbb{M}^{-1}W_c.$$

The open loop aircraft dynamics take $Z_{T,c}$, L_c, M_c. and N_c commands and output $p^E, V^b, \Theta, \omega^b_{\mathrm{b/E}}$. Simulate the system from an equilibrium condition. Demonstrate that this system is in force and moment balance before the input is added to the system. Simulate the system when the $Z_{T,c}$ input steps by 0.01 N step. Then one-by-one, simulate the system when L_c, M_c. and N_c step by 0.004 N-m.

References

[1] Morelli, Eugene A., and Vladislav Klein. Aircraft system identification: theory and practice. Vol. 2. Williamsburg, VA: Sunflyte Enterprises, 2016.

[2] Stevens, Brian L., Frank L. Lewis, and Eric N. Johnson. Aircraft control and simulation: dynamics, controls design, and autonomous systems. John Wiley & Sons, 2015.

[3] Brekelmans, G. H. Extended quadrotor dynamics: from simulations to experiments. Technical report, MSc Thesis, Eindhoven University of Technology, Department of Mechanical Engineering, Dynamics and Control Group, Eindhoven, 2019.

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