Opgaver til forelæsning uge 21

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Opg. 11.1

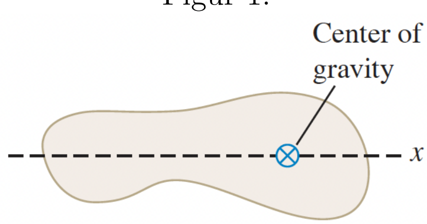
A 0,120 kg, 50,0 cm-long uniform bar has a small 0,055 kg mass glued to its left end and a small 0,110 kg mass glued to the other end. The two small masses can each be treated as point masses. You want to balance this system horizontally on a fulcrum placed just under its center of gravity. How far from the left end should the fulcrum be placed?

Vi bruger formlen for massemidtpunkt fra en række masser i 1 dimension som

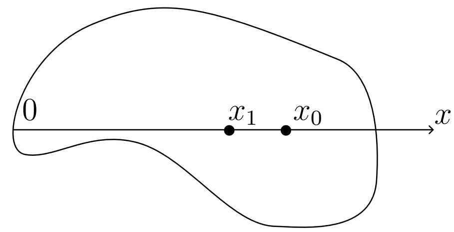
$$x_c = \frac{0{,}055\,\mathrm{kg} \cdot 0 + 0{,}120\,\mathrm{kg} \cdot \frac{50{,}0\,\mathrm{cm}}{2} + 0{,}110\,\mathrm{kg} \cdot 50{,}0\,\mathrm{cm}}{0{,}120\,\mathrm{kg} + 0{,}055\,\mathrm{kg} + 0{,}110\,\mathrm{kg}} = 29{,}8\,\mathrm{cm}.$$

Opg. 11.2

Figur 1:



The center of gravity of a 5,00 kg irregular object is shown in **Figur 1**. You need to move the center of gravity 2,20 cm to the left by gluing on a 1,50 kg mass, which will then be considered as part of the object. Where should the center of gravity of this additional mass be located?



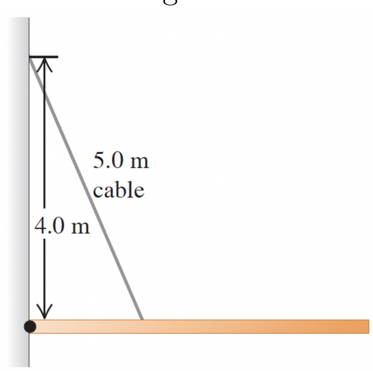
Figur 2: Fritlegemediagram for situationen

Vi har at

$$
\begin{aligned}
\Delta x = x_0 - 2{,}20\,\mathrm{cm} &= \frac{m x_1 + M x_0}{m+M} \\
x_0(m+M) - 2{,}20\,\mathrm{cm}(m+M) &= m x_1 + M x_0 \\
m x_1 &= x_0(m+M) - 2{,}20\,\mathrm{cm}(m+M) - M x_0 \\
m x_1 &= m(x_0 - 2{,}20\,\mathrm{cm}) - 2{,}20\,\mathrm{cm}\cdot M \\
x_1 &= x_0 - 2{,}20\,\mathrm{cm} - 2{,}20\,\mathrm{cm}\frac{M}{m} \\
x_1 &= \Delta x - 2{,}20\,\mathrm{cm}\frac{M}{m} \\
x_1 &= \Delta x - \frac{2{,}20\,\mathrm{cm}\cdot 5{,}00\,\mathrm{kg}}{1{,}50\,\mathrm{kg}} \\
x_1 &= \Delta x - 7{,}333\,\mathrm{cm} \\
x_1 &= x_0 - 9{,}533\,\mathrm{cm}.
\end{aligned}
$$

Opg. 11.19

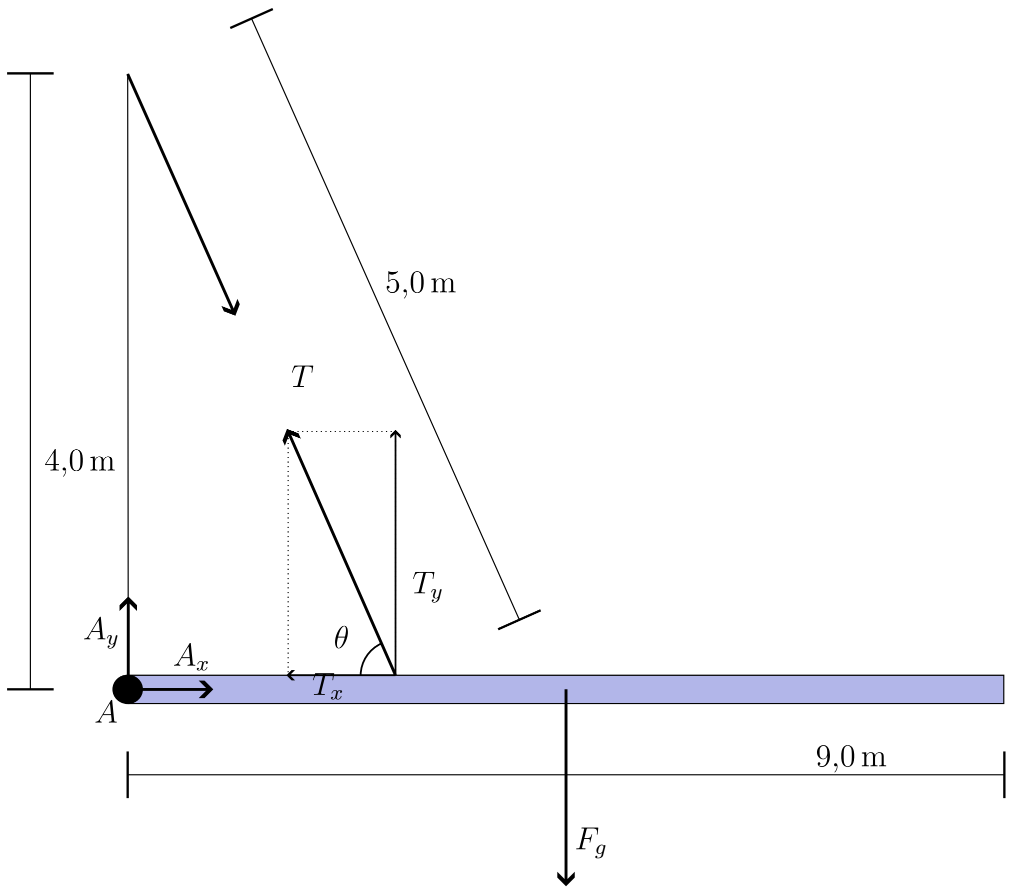
Figur 3:



A 9,00 m-long uniform beam is hinged to a vertical wall and held horizontally by a 5,00 m-long cable attached to the wall 4,00 m above the hinge (**Figur 3**). The metal of this cable has a test strength of 1,00 kN, which means that it will break if the tension in it exceeds that amount.

(a)

Draw a free-body diagram of the beam.



Figur 4: Fritlegemediagram for situationen.

Se **Figur 4**.

(b)

What is the heaviest beam that the cable can support in this configuration?

Kablet knækker ved $T = 1{,}00\,\text{kN}$. Vi skal altså finde størrelsen på F_g, der tilsvarer en spænding i kablet på $T = 1{,}00\,\text{kN}$. Idet der skal være statisk ligevægt har vi at

$$\sum \tau_A = 0.$$

Og dermed har vi at

$$
\begin{aligned}
0 &= T_y \cdot \underbrace{\sqrt{(5{,}0\,\mathrm{m})^2 - (4{,}0\,\mathrm{m})^2}}_{3{,}0\,\mathrm{m}} - F_g \cdot \frac{9\,\mathrm{m}}{2} \\
T_y &= F_g \cdot \frac{4{,}5\,\mathrm{m}}{3{,}0\,\mathrm{m}} \\
T &= F_g \cdot \frac{4{,}5\,\mathrm{m}}{3{,}0\,\mathrm{m} \cdot \sin(\underbrace{\theta}_{\tan^{-1}\left(\frac{4}{3}\right)})} \\
T &= 1{,}875 F_g \\
F_g &= \frac{1{,}00\,\mathrm{kN}}{1{,}875} = 0{,}533\,\mathrm{kN}.
\end{aligned}
$$

(c)

Find the horizontal and vertical components of the force the hinge exerts on the beam. Is the vertical component upward or downward?

Vi har, igen, statisk ligevægt og det gælder derfor at

$$\sum F_x = 0.$$

Vi har derfor

$$
\begin{aligned}
0 &= T_x - A_x \\
A_x &= T_x \\
&= T\cos(\theta) \\
&= 1{,}0\,\mathrm{kN} \cdot \frac{3}{5} = 600\,\mathrm{N}.
\end{aligned}
$$

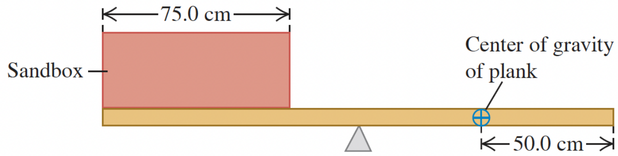
Og vi kan gøre det samme for y-retningen som

$$
\begin{aligned}
0 &= T_y + A_y - F_g \\
A_y &= 0{,}533\,\mathrm{kN} - \frac{4}{5}T \\
&= 0{,}533\,\mathrm{kN} - 0{,}800\,\mathrm{kN} = -0{,}267\,\mathrm{kN}.
\end{aligned}
$$

Opg. 11.47

A box of negligible mass rests at the left end of a 2,00 m, 25,0 kg plank (**Figur 5**). The width of the box is 75,0 cm, and sand is to be distributed uniformly throughout it. The center of gravity of the nonuniform plank is 50,0 cm from the right end. What mass of sand should be put into the box so that the plank balances horizontally on a fulcrum placed just below its midpoint?

Figur 5:

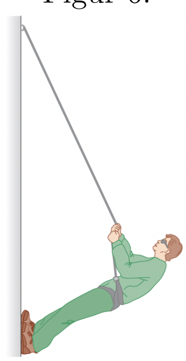


Vi sætter $x_{cm} = 0\,\text{cm}$ til at være netop ved det ønskede balancepunkt. Startmassemidtpunktet er da $x_p = \frac{200\,\text{cm}}{2} - 50{,}0\,\text{cm} = 50{,}0\,\text{cm}$. Massemidtpunktet for kassen er placeret ved $x_k = -\frac{200\,\text{cm}}{2} + \frac{75{,}0\,\text{cm}}{2} = -62{,}5\,\text{cm}$. Vi kan da opstille formlen for massemidtpunktet

$$
\begin{aligned}
x_{cm} = 0\,\text{cm} &= \frac{x_p \cdot m_p + x_k \cdot m_k}{m_p + m_k} \\
&= x_p \cdot m_p + x_k \cdot m_k \\
m_k &= \frac{-x_p \cdot m_p}{x_k} \\
&= \frac{-50\,\text{cm} \cdot 25{,}0\,\text{kg}}{-62{,}5\,\text{cm}} \\
&= 20{,}0\,\text{kg}.
\end{aligned}
$$

Opg. 11.49

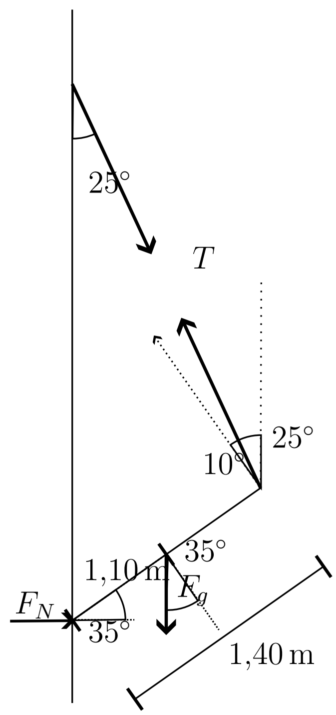
Figur 6:



Mountain Climbing. Mountaineers often use a rope to lower themselves down the face of a cliff (this is called *rappelling*). They do this with their body nearly horizontal and their feet pushing against the cliff (**Figur 6**). Suppose that an 82,0 kg climber, who is 1,90 m tall and has a center of gravity 1,1 m from his feet, rappels down a vertical cliff with his body raised 35,0° above the horizontal. He holds the rope 1,40 m from his feet, and it makes a 25,0° angle with the cliff face.

(a)

What tension does his rope need to support?



Figur 7: Fritlegemediagram for situationen.

Vi har statik og derfor må det gælde at

$$\sum \tau = 0.$$

Og dermed får vi at

$$\begin{aligned} 0 &= -F_g \cdot \cos(35°) \cdot l_{cm} + T \cdot \cos(10°) \cdot l_{tot} \\ T &= F_g \frac{\cos(35°) \cdot l_{cm}}{\cos(10°) \cdot l_{tot}} \\ T &= 0{,}786\,\mathrm{m} \cdot F_g = 525{,}191\,\mathrm{kN}. \end{aligned}$$

(b)

Find the horizontal and vertical components of the force that the cliff face exerts on the climber's feet.

Grundet samme overvejelser om statik som i opg. 11.19 har vi at

$$\sum F_x = 0.$$

Og vi får da, at

$$\begin{aligned}0 &= N - T \cdot \sin(25°)\\ N &= 221{,}955\,\text{kN}.\end{aligned}$$

Og det samme kan gøres i y-retningen som

$$\begin{aligned}0 &= T \cdot \cos(25°) - F_g + F_\mu\\ F_\mu &= F_g - T \cdot \cos(25°)\\ &= 82{,}0\,\text{kg} \cdot 9{,}8\,\frac{\text{m}}{\text{s}^2} - 525{,}191\,\text{kN} \cdot \cos(25°) = 327{,}788\,\text{kN}.\end{aligned}$$

(c)

What minimum coefficient of static friction is needed to prevent the climber's feet from slipping on the cliff face if he has one foot at a time against the cliff?

Gnidningskoefficienten findes ved brug af formlen for gnidningskraft som

$$\begin{aligned}F_\mu &= \mu \cdot N\\ \mu &= \frac{F_\mu}{N}\\ &= \frac{327{,}788\,\text{kN}}{221{,}955\,\text{kN}} = 1{,}4769\end{aligned}$$

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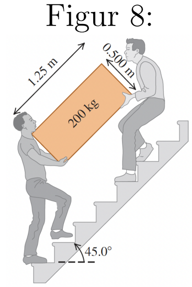
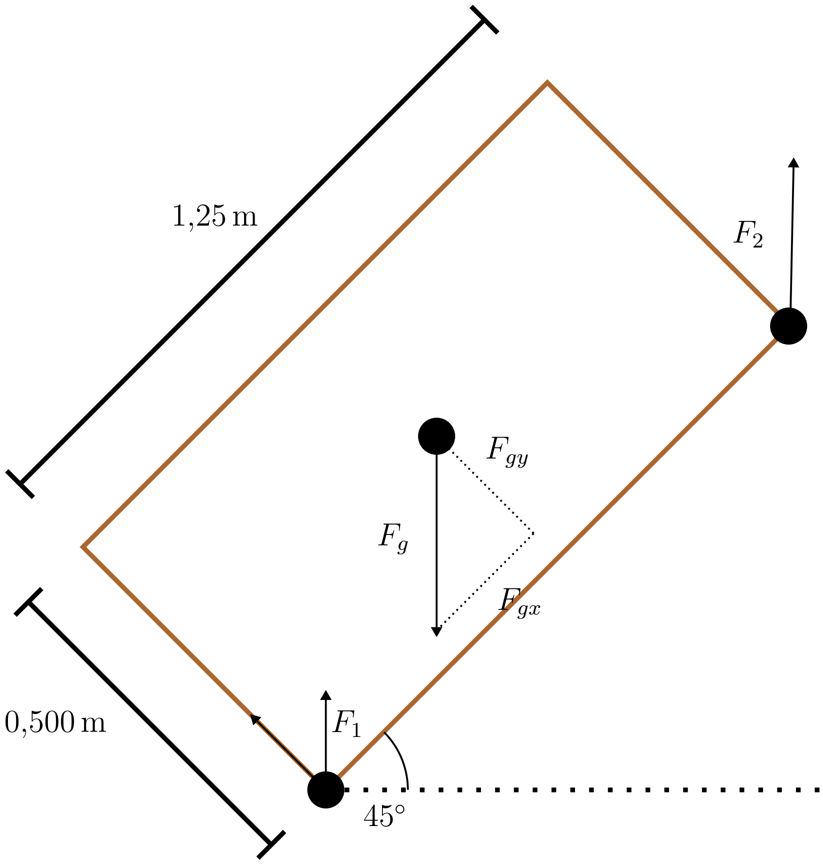
Opg. 11.71

Figur 8:



Two friends are carrying a 200 kg crate up a flight of stairs. The crate is 1,25 m long and 0,500 m high, and its center of gravity is at its center. The stairs make a 45,0° angle with respect to the floor. The crate also is carried at a 45,0° angle, so that its bottom side is parallel to the slope of the stairs (**Figur 8**). If the force each person applies is vertical, what is the magnitude of each of these forces? Is it better to be the person above or below on the stairs?



Figur 9: Fritlegemediagram

Vi sætter 0-punktet til at være netop det hjørne som den nederste person holder i. Vi får da at tyngdekraftet laver et moment τ_1 med udgangspunkt i tyngdekraften fra kassens massemidtpunkt og at den anden person laver et moment τ_2 med udgangspunkt i den kraft han løfter kassen med. Se evt. **Figur 9**.
Idet der er statik har vi, at

$$\sum F_y = F_1 + F_2 - F_g = 0$$

og

$$\sum \tau = F_2 \cdot 1{,}25\,\mathrm{m} \cdot \cos(45°) - \frac{1{,}25\,\mathrm{m}}{2} \cdot F_g \cdot \cos(45°) + \frac{0{,}500\,\mathrm{m}}{2} \cdot F_g \cdot \sin(45°) = 0.$$

Eftersom $\cos(45°) = \sin(45°)$ har vi at

$$\begin{aligned} F_2 \cdot 1{,}25\,\mathrm{m} \cdot \cos(45°) &= \frac{1{,}25\,\mathrm{m}}{2} \cdot F_g \cdot \cos(45°) - \frac{0{,}500\,\mathrm{m}}{2} \cdot F_g \cdot \sin(45°) \\ F_2 \cdot 1{,}25\,\mathrm{m} &= \frac{1{,}25\,\mathrm{m} - 0{,}500\,\mathrm{m}}{2} \cdot F_g \\ F_2 &= \frac{1{,}25\,\mathrm{m} - 0{,}500\,\mathrm{m}}{2 \cdot 1{,}25\,\mathrm{m}} \cdot F_g \end{aligned}$$

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