Opgaver til forelæsning uge 23

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Opg. 12.1

On a part-time job, you are asked to bring a cylindrical iron rod of length 85,8 cm and diameter 2,85 cm from a storage room to a machinist. Will you need a cart? (To answer, calculate the weight of the rod.)

Først findes volumenet af stangen vha. formlen for volumenet af en cylinder som

$$V = \pi \cdot h \cdot r^2 = \pi \cdot \left(\frac{2{,}85\,\mathrm{cm}}{2}\right)^2 \cdot 85{,}8\,\mathrm{cm} = 5{,}473 \cdot 10^{-4}\,\mathrm{m}^3.$$

Og dermed kan massen af stangen findes vha. densiteten for jern $\rho = 7{,}8 \cdot 10^3\,\frac{\mathrm{kg}}{\mathrm{m}^3}$ som

$$m = \rho \cdot V = 7{,}8 \cdot 10^3\,\frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 5{,}473 \cdot 10^{-4}\,\mathrm{m}^3 = 4{,}27\,\mathrm{kg}.$$

Altså behøver jeg ikke en vogn, fordi jeg er en stærk dreng, der godt kan løfte 4,27 kg.

Opg. 12.5

A uniform lead sphere and a uniform aluminum sphere have the same mass. What is the ratio of the radius of the aluminum sphere to the radius of the lead sphere?

Idet vi har at

$$m \sim r^3.$$

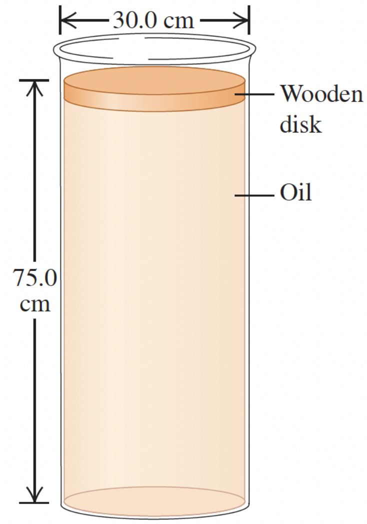
For de to sfærer har vi at

$$\begin{aligned}
\rho_l \cdot r_l^3 &= \rho_a \cdot r_a^3 \\
\frac{r_a}{r_l} &= \sqrt[3]{\frac{\rho_l}{\rho_a}} \\
&= \sqrt[3]{\frac{11{,}3 \cdot 10^3\,\frac{\mathrm{kg}}{\mathrm{m}^3}}{2{,}7 \cdot 10^3\,\frac{\mathrm{kg}}{\mathrm{m}^3}}} \\
&= 1{,}612.
\end{aligned}$$

Altså skal en kugle af aluminium have en radius, der er omkring 60% større end en kugle med bly for at de to kugler får samme masse.

Opg. 12.19

Figur 1:



A cylindrical disk of wood weighing 45,0 N and having a diameter of 30,0 cm floats on a cylinder of oil of density $0{,}850\,\mathrm{g/cm^3}$ (**Figur 1**). The cylinder of oil is 75,0 cm deep and has a diameter the same as that of the wood.

(a)

What is the gauge pressure at the top of the oil column?

Generelt er tryk givet som

$$p = \frac{F}{A}.$$

Sættes tallene fra opgaven ind fås

$$p_0 = \frac{45{,}0\,\mathrm{N}}{\pi \cdot \left(\frac{30{,}0\,\mathrm{cm}}{2}\right)^2} = 637\,\mathrm{Pa}.$$

(b)

Suppose now that someone puts a weight of 83,0 N on top of the wood, but no oil seeps around the edge of the wood. What is the change in pressure at

(i)

the bottom of the oil and

Ændringen i trykket findes som i opgaven ovenfor som

$$\Delta p = \frac{83{,}0\,\mathrm{N}}{\pi \cdot (15{,}0\,\mathrm{cm})^2} = 1174\,\mathrm{Pa}.$$

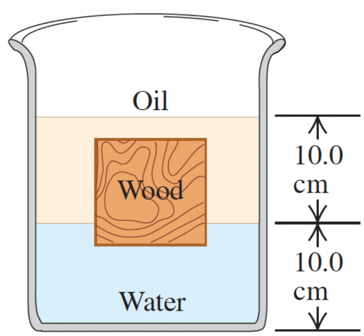
(ii)

halfway down in the oil?

Her er svaret som ovenfor, da det tilføjede tryk vil fordele sig ens igennem hele væsken.

Opg. 12.33

Figur 2:



A cubical block of wood, 10,0 cm on a side, floats at the interface between oil and water with its lower surface 1,50 cm below the interface (**Figur 2**). The density of the oil is $790\,\mathrm{kg/m^3}$.

(a)

What is the gauge pressure at the upper face of the block?

Trykket p kan findes ud fra formlen for tryk

$$p = \rho \cdot g \cdot h = 790\,\frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 9{,}80\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 1{,}50\,\mathrm{cm} = 116\,\mathrm{Pa}.$$

(b)

What is the gauge pressure at the lower face of the block?

Først findes trykket ved grænsefladen mellem olie og vand som

$$p_{ov} = 790\,\frac{\text{kg}}{\text{m}^3} \cdot 9{,}80\,\frac{\text{m}}{\text{s}^2} \cdot 10\,\text{cm} = 774{,}2\,\text{Pa}.$$

Og trykket fra vandet kan da findes som

$$p_v = 1000\,\frac{\text{kg}}{\text{m}^3} \cdot 9{,}80\,\frac{\text{m}}{\text{s}^2} \cdot 1{,}50\,\text{cm}147\,\text{Pa}.$$

Altså er det samlede *gauge-pressure* ved bunden af blokken givet som

$$p_b = p_{ov} + p_2 = 116\,\text{Pa} + 774{,}2\,\text{Pa} + 147\,\text{Pa} = 921\,\text{Pa}.$$

(c)

What are the mass and density of the block?

Først findes volumenet af blokken indeholdt i hhv. vandet og olien som

$$\begin{aligned} V_o &= 10\,\text{cm} \cdot 10\,\text{cm} \cdot 8{,}5\,\text{cm} = 8{,}5 \cdot 10^{-4}\,\text{m}^3 \\ V_v &= 10\,\text{cm} \cdot 10\,\text{cm} \cdot 1{,}5\,\text{cm} = 1{,}5 \cdot 10^{-4}\,\text{m}^3. \end{aligned}$$

Opdrift er generelt givet som

$$F_{op} = \rho \cdot V \cdot g.$$

Vi har altså en samlet opdrift på

$$F_{op} = g(\rho_o \cdot V_o + \rho_v \cdot V_v) = 9{,}80\,\frac{\text{m}}{\text{s}^2} \cdot \left(790\,\frac{\text{kg}}{\text{m}^3} \cdot 8{,}5 \cdot 10^{-4}\,\text{m}^3 + 1000\,\frac{\text{kg}}{\text{m}^3} \cdot 1{,}5 \cdot 10^{-4}\,\text{m}^3\right) = 8{,}05\,\text{N}.$$

Idet der er statisk ligevægt må det gælde at

$$F_{op} = F_g = 8{,}05\,\text{N}.$$

Altså er massen af blokken

$$m = \frac{F}{g} = \frac{8{,}05\,\text{N}}{9{,}80\,\frac{\text{m}}{\text{s}^2}} = 0{,}821\,\text{kg}.$$

Og blokkens densitet bliver da

$$\rho = \frac{m}{V} = \frac{0{,}821\,\text{kg}}{(0{,}10\,\text{m})^3} = 821\,\frac{\text{kg}}{\text{m}^3}.$$

Opg. 12.35

A rock is suspended by a light string. When the rock is in air, the tension in the string is 39,2 N. When the rock is totally immersed in water, the tension is 28,4 N. When the rock is totally immersed in an unknown liquid, the tension is 21,5 N. What is the density of the unknown liquid?

Først findes massen af stenen som

$$m = \frac{F_g}{g} = \frac{39{,}2\,\mathrm{N}}{9{,}80\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 4{,}00\,\mathrm{kg}.$$

Idet stenen nedsænkes i vandet falder den effektive tænkte med en værdi tilsvarende opdriften på blokken $F_{op} = T_0 - T_1 = 39{,}2\,\mathrm{N} - 28{,}4\,\mathrm{N} = 10{,}8\,\mathrm{N}$. Formlen for opdrift kan nu bruges til at finde volumenet af stenen som

$$F_{op} = \rho V g \implies V = \frac{F_{op}}{\rho \cdot g} = \frac{10{,}8\,\mathrm{N}}{1000\,\frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 9{,}80\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 0{,}0011\,\mathrm{m}^3.$$

Idet stenen nedsænkes i den ukendte væske får den en opdrift på $F_{op_2} = 39{,}2\,\mathrm{N} - 21{,}5\,\mathrm{N} = 17{,}7\,\mathrm{N}$. Vi kan da finde densiteten af den ukendte væske som

$$\rho = \frac{F_{op_2}}{Vg} = \frac{17{,}7\,\mathrm{N}}{0{,}0011\,\mathrm{m}^3 \cdot 9{,}8\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 1641{,}9\,\frac{\mathrm{kg}}{\mathrm{m}^3}.$$

Opg. 12.55

A swimming pool is 5,0 m long, 4,0 m wide, and 3,0 m deep. Compute the force exerted by the water against

(a)

the bottom

Trykket på bunden er givet ved den almindelige trykformel som

$$p_b = \rho \cdot g \cdot h = 1000\,\frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 9{,}80\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 3{,}00\,\mathrm{m} = 2{,}94 \cdot 10^4\,\mathrm{Pa}.$$

Og kraften på bunden bliver da

$$F_b = p_b \cdot A = 29\,400\,\mathrm{Pa} \cdot 5{,}0\,\mathrm{m} \cdot 4{,}0\,\mathrm{m} = 5{,}88 \cdot 10^5\,\mathrm{N}.$$

(b)

either end. (*Hint:* Calculate the force exerted on a thin, horizontal strip at a depth h, and integrate this over the end of the pool.) Do not include the force due to air pressure.

Vha. formlen for tryk kan trykket på en tynd stribe som funktion af højden opstilles som

$$p(h) = \rho \cdot g \cdot h.$$

Kraften bliver da

$$F(h) = b \cdot \rho \cdot g \cdot h.$$

Denne kraft kan integreres over hele højden for at få

$$\begin{aligned} F &= b \cdot \rho \cdot g \int_{0\,\mathrm{m}}^{3{,}00\,\mathrm{m}} h \,\mathrm{d}h \\ &= 4{,}00\,\mathrm{m} \cdot 1000\,\frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 9{,}80\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot \frac{1}{2} \cdot (3{,}00\,\mathrm{m})^2 \\ &= 1{,}76 \cdot 10^5\,\mathrm{N}. \end{aligned}$$

Opg. 12.65

A large, 40,0 kg cubical block of wood with uniform density is floating in a freshwater lake with 20,0% of its volume above the surface of the water. You want to load bricks onto the floating block and then push it horizontally through the water to an island where you are building an outdoor grill.

(a)

What is the volume of the block?

Vi opskriver flydeligevægten som

$$\begin{aligned} F_b &= F_g \\ 0{,}8 V \rho g &= mg \\ V &= \frac{m}{0{,}8\rho} \\ &= \frac{40{,}0\,\mathrm{kg}}{0{,}8 \cdot 1000\,\frac{\mathrm{kg}}{\mathrm{m}^3}} = 5{,}00 \cdot 10^{-2}\,\mathrm{m}^3. \end{aligned}$$

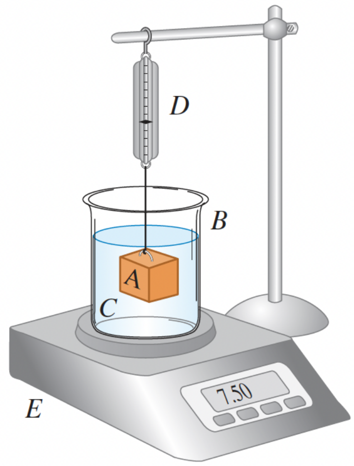
(b)

What is the maximum mass of bricks that you can place on the block without causing it to sink below the water surface?

Vi kan igen opskrive ligevægten fra før dog med en ekstra masse denne gang så vi får at

$$\begin{aligned} (m+M)g &= V\rho g \\ m + M &= V\rho g \\ M &= V\rho - m \\ &= 5{,}00 \cdot 10^{-2}\,\mathrm{m}^3 \cdot 1000\,\frac{\mathrm{kg}}{\mathrm{m}^3} - 40{,}0\,\mathrm{kg} \\ &= 10\,\mathrm{kg}. \end{aligned}$$

Figur 3:



Opg. 12.72

Block A in **Figur 3** hangs by a cord from spring balance D and is submerged in a liquid C contained in beaker B. The mass of the beaker is 1,00 kg; the mass of the liquid is 1,80 kg. Balance D reads 3,50 kg, and balance E reads 7,50 kg. The volume of the block A is $3{,}80 \cdot 10^{-3}\,\mathrm{m}^3$.

(a)

What is the density of the liquid?

(b)

What will each balance read if block A is pulled up out of the liquid.