Opgaver til forelæsning 10

Noah Rahbek Bigum Hansen

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Opg. 7.4

The *food calorie*, equal to 4186 J, is a measure of how much energy is released when the body metabolizes food. A certain fruit-and-cereal bar contains 140 food calories.

(a)

If a 65 kg hiker eats one bar, how high a mountain must he climb to "work off" the calories, assuming that all the food energy goes into increasing gravitational potential energy?

Den samlede energi i myslibaren findes som

$$E_{mysli} = 140\,\mathrm{kCal} \cdot 4186\,\frac{\mathrm{J}}{\mathrm{kCal}} = 586\,040\,\mathrm{J}.$$

Den potentielle energi i tyngdefeltet er givet ved

$$U_t = m \cdot g \cdot h \implies h = \frac{U_t}{m \cdot g} = \frac{586\,040\,\mathrm{J}}{65\,\mathrm{kg} \cdot 9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2}} = 0{,}92\,\mathrm{km}.$$

Opg. 7.8

$$e_{musli} = 140\,\mathrm{kcal} \cdot 4186\,\frac{\mathrm{J}}{\mathrm{kcal}} = 586\,040\,\mathrm{J}.$$

(b)

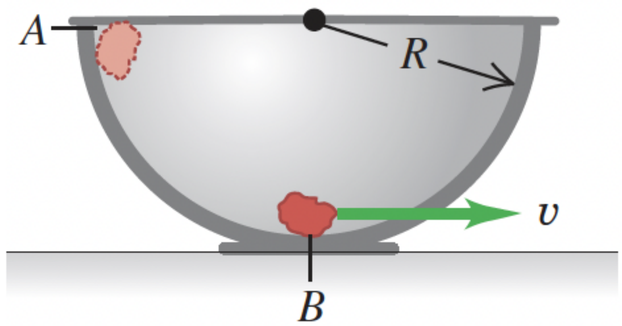
If, as is typical, only 20% of the food calories go into mechanical energy, what would be the answer to part (a)? (*Note*: In this and all other problems, we are assuming that 100% of the food calories that are eaten are absorbed and used by the body. This is not true. A person's "metabolic efficiency" is the percentage of calories eaten that are actually used; the body eliminates the rest. Metabolic efficiency varies considerably from person to person.)

Hvis kun 20% af energien optages må vi have at klatreren kan bevæge sig 20% af distancen fra før. Altså fås at

$$h_{20\%} = h \cdot 20\% = 0{,}92\,\mathrm{km} \cdot 20\% = 0{,}18\,\mathrm{km}.$$

Opg. 7.9

Figur 1:



A small rock with mass 0,20 kg is released from rest at point A, which is at the top edge of a large, hemispherical bowl with radius $R = 0{,}50\,\mathrm{m}$ (**Figur 1**). Assume that the size of the rock is small compared to R, so that the rock can be treated as a particle, and assume that the rock slides rather than rolls. The work done by friction on the rock when it moves from point A to point B at the bottom of the bowl has magnitude 0,22 J.

(a)

Between points A and B, how much work is done on the rock by

(i)

the normal force and

The normal force is always perpendicular to the direction in which the stone is travelling, thus the normal force can at no point do any work.

(ii)

gravity?

The work done by gravity is opposite the change in the potential energy of the rock. Thus we have

$$W_g = -\Delta U_t = -m \cdot g \cdot \Delta h = -0{,}20\,\mathrm{kg} \cdot 9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 0{,}50\,\mathrm{m} = 0{,}981\,\mathrm{J}.$$

(b)

What is the speed of the rock as it reaches point B?

Den kinetiske energi er summen af alt arbejdet udført på stenen. Friktionens arbejde må være modsatrettet stenens bevægelsesretning og tyngdekraftens arbejde må være i samme retning og det samlede arbejde må derfor være

$$k = W_{tot} = W_t - W_\mu.$$

Dermed kan hastigheden beregnes som

$$v = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2 \cdot (0{,}981\,\mathrm{J} - 0{,}22\,\mathrm{J})}{0{,}20\,\mathrm{kg}}} = 2{,}76\,\frac{\mathrm{m}}{\mathrm{s}}.$$

(c)

Of the three forces acting on the rock as it slides down the bowl, which (if any) are constant and which are not? Explain.

Tyngdekraften er konstant, men normalkraften er ikke, da den afhænger af skålens vinkel og da gnidningskraften afhænger af normalkraften er den ej heller konstant.

(d)

Just as the rock reaches point B, what is the normal force on it due to the bottom of the bowl?

Idet stenen netop når punkt B vil den opleve en normalkraft der modsvarer summen af de andre kræfter stenen oplever. Disse andre kræfter er hhv. tyngdekraften og centripetalkraften. Altså har vi

$$F_N = -(F_t + F_c) = -m(g + \frac{v^2}{r}) = -0{,}20\,\mathrm{kg}\left(-9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2} + \frac{(-2{,}76\,\frac{\mathrm{m}}{\mathrm{s}})^2}{0{,}50\,\mathrm{m}}\right) = 5{,}0\,\mathrm{N}.$$

Opg. 7.19

A spring of negligible mass has force constant $k = 800\,\frac{\mathrm{N}}{\mathrm{m}}$.

(a)

How far must the spring be compressed for 1,20 J of potential energy to be stored in it?

Formlen for en fjeders potentielle energi er

$$U_s = \frac{1}{2}kx^2.$$

Heri kan strækningen, x, isoleres som så

$$x = \sqrt{\frac{2U_s}{k}} = \sqrt{\frac{2 \cdot 1{,}20\,\mathrm{J}}{800\,\frac{\mathrm{N}}{\mathrm{m}}}} = 5{,}48\,\mathrm{cm}.$$

(b)

You place the spring vertically with one end on the floor. You then lay a 1,60 kg book on top of the spring and release the book from rest. Find the maximum distance the spring will be compressed.

Fjederen vil blive trykket sammen netop indtil arbejdet udøvet på fjederen netop har samme størrelse som tyngdekraftens arbejde. Altså har vi at

$$|W_t| = |U_s| \implies m \cdot g \cdot x = \frac{1}{2}k \cdot x^2 \implies x = \frac{2m \cdot g}{k} = \frac{2 \cdot 1{,}60\,\mathrm{kg} \cdot 9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2}}{800\,\frac{\mathrm{N}}{\mathrm{m}}} = 3{,}92\,\mathrm{cm}.$$

Opg. 7.29

A 62,0 kg skier is moving at 6,50 m/s on a frictionless, horizontal, snow-covered plateau when she encounters a rough patch 4,20 m long. The coefficient of kinetic friction between this patch and her skis is 0,300. After crossing the rough patch and returning to friction-free snow, she skis down an icy, frictionless hill 2,50 m high.

(a)

How fast is the skier moving when she gets to the bottom of the hill?

Først regnes arbejdet som friktionskraften har udført på skiløberen. Her har vi at

$$W_\mu = F_\mu \cdot s = F_N \cdot \mu \cdot s = m \cdot g \cdot \mu \cdot s = 62{,}0\,\mathrm{kg} \cdot 9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 0{,}300 \cdot 4{,}20\,\mathrm{m} = 766{,}357\,\mathrm{J}.$$

Dernæst kan arbejdet som tyngdekraften har udført på hende på det sidste stykke regnes. Denne er givet ved

$$W_t = m \cdot g \cdot h = 62{,}0\,\mathrm{kg} \cdot 9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 2{,}50\,\mathrm{m} = 1520{,}55\,\mathrm{J}.$$

Det samlede tilførte arbejde er altså

$$W_{tot} = W_t - W_\mu = 1520{,}55\,\mathrm{J} - 766{,}357\,\mathrm{J} = 754{,}193\,\mathrm{J}.$$

Hendes kinetiske energi før var hun ramte stykket med friktion var

$$k_0 = \frac{1}{2}m \cdot v_0^2 = \frac{1}{2} \cdot 62{,}0\,\mathrm{kg} \cdot \left(6{,}50\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = 1309{,}75\,\mathrm{J}.$$

Og hendes samlede kinetiske energi i bunden af bakken er derfor

$$k_{tot} = k_0 + W_{tot} = 2063{,}943\,\mathrm{J},$$

Hvilket svarer til en hastighed på

$$v = \sqrt{\frac{2k_{tot}}{m}} = \sqrt{\frac{2 \cdot 2063{,}943\,\mathrm{J}}{62{,}0\,\mathrm{kg}}} = 8{,}16\,\frac{\mathrm{m}}{\mathrm{s}}.$$

(b)

How much internal energy was generated in crossing the rough patch?

Dette er fundet som $W_\mu = 766\,\mathrm{J}$ i opgaven ovenfor

Opg. 7.32

The potential energy of a pair of hydrogen atoms separated by a large distance x is given by $U(x) = -\frac{C_6}{x^6}$, where C_6 is a positive constant. What is the force that one atom exerts on the other? Is this force attractive or repulsive?

Idet vi ved at $U = \int F\, \mathrm{d}s$ må det gælde at

$$F = -\frac{\mathrm{d}U}{\mathrm{d}x}.$$

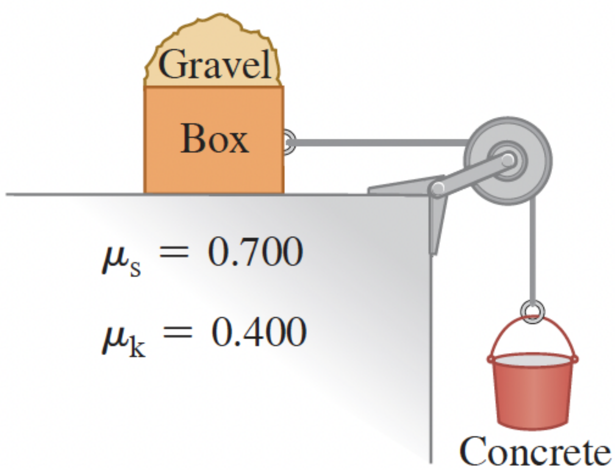
Altså har vi

$$F = -6C_6 \cdot x^{-7}.$$

Af det oprindelige udtryk for den potentielle energi kan ses at hvis afstanden mellem de to atomer, x, øges så stiger den potentielle energi og derfor må de to hydrogenatomer tiltrække hinanden.

Opg. 7.37

Figur 2:



At a construction site, a 65,0 kg bucket of concrete hangs from a light (but strong) cable that passes over a light, friction-free pulley and is connected to an 80,0 kg box on a horizontal roof (**Figur 2**). The cable pulls horizontally on the box, and a 50,0 kg bag of gravel rests on top of the box. The coefficients of friction between the box and roof are shown.

(a)

Find the friction force on the bag of gravel and on the box.

Den maksimale friktionskraft, F_{μ_max} er givet ved

$$F_{\mu_max} = F_N \cdot \mu = m \cdot g \cdot \mu = (80{,}0\,\mathrm{kg} + 50{,}0\,\mathrm{kg}) \cdot 9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 0{,}700 = 892{,}71\,\mathrm{N}.$$

Friktionskraften er dog kun så stor som den kraft der trækker i kassen, T

$$T = w_c = m_c \cdot g = 65{,}0\,\mathrm{kg} \cdot 9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2} = 638\,\mathrm{N}.$$

(b)

Suddenly a worker picks up the bag of gravel. Use energy conservation to find the speed of the bucket after it has descended 2.00 m from rest. (Use Newton's laws to check your answer.)

Vi har at

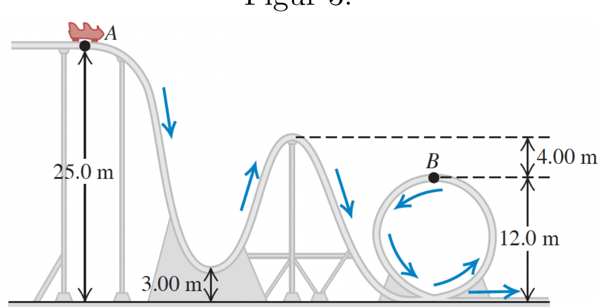
$$k_1 + U_1 + W_\mu = k_2 + U_2 \implies 0 + m_c \cdot g \cdot s + (-m_b \cdot g \cdot \mu_k \cdot s) = \frac{1}{2}(m_b + m_c)v^2.$$

Altså har vi at

$$v = \sqrt{\left(\frac{2(m_c - m_b\mu_k)sg}{m_b + m_c}\right)} = \sqrt{\left(\frac{2 \cdot (65{,}0\,\mathrm{kg} - 80{,}0\,\mathrm{kg} \cdot 0{,}400)2{,}00\,\mathrm{m} \cdot 9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2}}{65{,}0\,\mathrm{kg} + 80{,}0\,\mathrm{kg}}\right)} = 2{,}99\,\frac{\mathrm{m}}{\mathrm{s}}.$$

Opg. 7.41

Figur 3:



A 350 kg roller coaster car starts from rest at point A and slides down a frictionless loop-the-loop (**Figur 3**) . The car's wheels are designed to stay on the track.

(a)

How fast is this roller coaster car moving at point B?

Til punktet B har rutsjebanen bevæget sig sammenlagt $25{,}0\,\mathrm{m} - 12{,}0\,\mathrm{m} = 13{,}0\,\mathrm{m}$ ned i tyngdefeltet. Altså er den kinetiske energi her

$$k = \Delta U_t = m \cdot g \cdot h = 350\,\mathrm{kg} \cdot 9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 13{,}0\,\mathrm{m} = 44{,}636\,\mathrm{kJ}.$$

Dette tilsvarer en hastighed på

$$v = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2 \cdot 44{,}636\,\mathrm{kJ}}{350\,\mathrm{kg}}} = 15{,}97\,\frac{\mathrm{m}}{\mathrm{s}}.$$

(b)

How hard does it press against the track at point B?

Normalkraften til punktet B er givet som differensen mellem centripetalkraften her og tyngdekraften. Altså har vi

$$F_N = F_c - F_t \implies F_N = m\frac{v^2}{r} - mg = m\left(\frac{v^2}{r} - g\right).$$

$$F_N = 350\,\mathrm{kg}\left(\frac{\left(15{,}97\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2}{6{,}0\,\mathrm{m}} - 9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2}\right) = 11{,}4\,\mathrm{kN}.$$

Opg. 7.50

A 1500 kg rocket is to be launched with an initial upward speed of 50,0 m/s. In order to assist its engines, the engineers will start it from rest on a ramp that rises 53° above the horizontal (**Figur 4**). At the bottom, the ramp turns upward and launches the rocket vertically. The engines provide a constant forward thrust of 2000 N, and friction with the ramp surface is a constant 500 N. How far from the base of the ramp should the rocket start, as measured along the surface of the ramp?

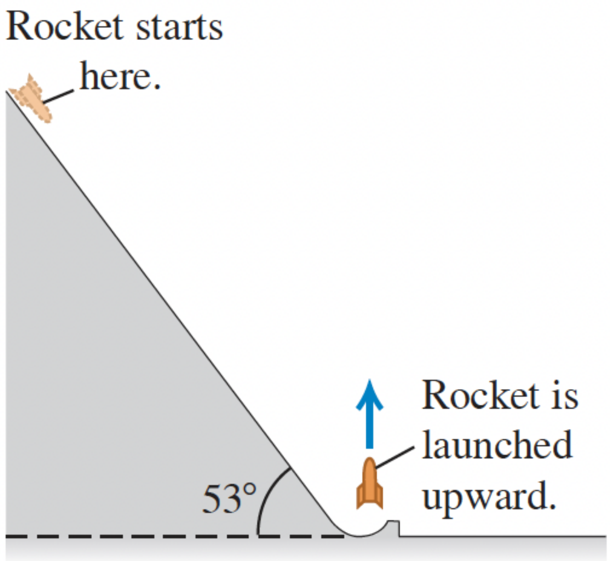
Først findes den kinetiske energi der er påkrævet for at opnå den ønskede affyringshastighed. Dette gøres som

$$k = \frac{1}{2} \cdot m \cdot v^2 = \frac{1}{2} \cdot 1500\,\mathrm{kg} \cdot \left(50{,}0\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2 = 1\,875\,000\,\mathrm{J}.$$

Denne kinetiske energi må være givet ved

$$k = W_t + W_{rocket} + W_\mu.$$

Figur 4:



Først kan tyngdekraftens arbejde findes som

$$U_t = m \cdot g \cdot h = m \cdot g \cdot \sin \alpha \implies W_t = m \cdot g \cdot \sin(\alpha) \cdot s.$$

Rakettens arbejde må være

$$W_{rocket} = F_{rocket} \cdot s.$$

Og friktionskraftens arbejde må være

$$W_{\mu} = F_{\mu} \cdot s.$$

Altså har vi at

$$k = m \cdot g \cdot s \cdot \sin(\alpha) + F_{rocket} \cdot s + F_{\mu} \cdot s = s\left(m \cdot g \cdot \sin(\alpha) + F_{rocket} + F_m g\right).$$

Strækningen kan isoleres

$$s = \frac{k}{m \cdot g \cdot \sin(\alpha) + F_{rocket} + F_{\mu}} = \frac{1\,875\,000\,\mathrm{J}}{1500\,\mathrm{kg} \cdot 9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot \sin(53^\circ) + 2000\,\mathrm{N} - 500\,\mathrm{N}} = 141\,\mathrm{m}.$$

Opg. 7.59

A certain spring found *not* to obey Hooke's law exerts a restoring force $F_x(x) = -\alpha x - \beta x^2$ if it is stretched or compressed, where $\alpha = 60{,}0\,\frac{\mathrm{N}}{\mathrm{m}}$ and $\beta = 18{,}0\,\frac{\mathrm{N}}{\mathrm{m}^2}$. The mass of the spring is negligible.

(a)

Calculate the potential-energy function $U(x)$ for this spring. Let $U = 0$ when $x = 0$.

Vi ved at

$$U(x) = \int F_x(x)\,\mathrm{d}x = \frac{1}{2}\alpha x^2 + \frac{1}{3}\beta x^3 + c.$$

Og

$$U(0) = 0 \implies c = 0.$$

Altså har vi at

$$U(x) = \frac{1}{2}\alpha x^2 + \frac{1}{3}\beta x^3.$$

(b)

An object with mass 0,900 kg on a frictionless, horizontal surface is attached to this spring, pulled a distance 1,00 m to the right (the $+x$-direction) to stretch the spring, and released. What is the speed of the object when it is 0,50 m to the right of the $x = 0$ equilibrium position?

Først findes den potentielle energi lagret i fjederen i yderpositionen som

$$U(1{,}00\,\mathrm{m}) = \frac{1}{2}\cdot 60{,}0\,\frac{\mathrm{N}}{\mathrm{m}}\cdot(1{,}00\,\mathrm{m})^2 + \frac{1}{3}\cdot 18{,}0\,\frac{\mathrm{N}}{\mathrm{m}^2}\cdot(1{,}00\,\mathrm{m})^3 = 36\,\mathrm{J}.$$

Og dernæst kan det samme gøres for $U(0{,}50\,\mathrm{m})$

$$U(0{,}50\,\mathrm{m}) = \frac{1}{2}\cdot 60{,}0\,\frac{\mathrm{N}}{\mathrm{m}}\cdot(0{,}50\,\mathrm{m})^2 + \frac{1}{3}\cdot 18{,}0\,\frac{\mathrm{N}}{\mathrm{m}^2}\cdot(1{,}00\,\mathrm{m})^3 = 8{,}25\,\mathrm{J}.$$

Altså må den samlede kinetiske energi af objektet være

$$k = U(1{,}00\,\mathrm{m}) - U(0{,}50\,\mathrm{m}) = 36\,\mathrm{J} - 8{,}25\,\mathrm{J} = 27{,}75\,\mathrm{J}.$$

Dette svarer til en hastighed på

$$v = \sqrt{\frac{2k}{m}} = \sqrt{\frac{2\cdot 27{,}75\,\mathrm{J}}{0{,}900\,\mathrm{kg}}} = 7{,}85\,\frac{\mathrm{m}}{\mathrm{s}}.$$