Take-Home Assignment weeks 35–36

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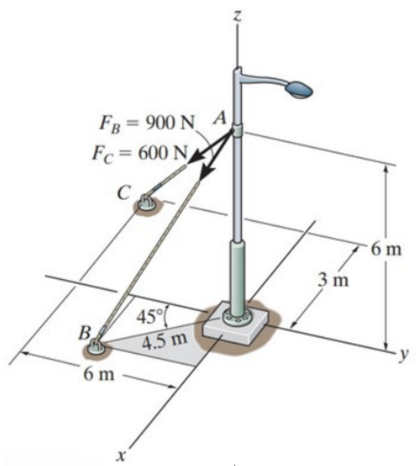
Exercise P1-1

Determine the magnitude and coordinate direction angles of the resultant force acting at A.



We start by adding a coordinate system such that the point A is located at the origin O. The resultant force here $\mathbf{F}_A$ is simply the sum of the two acting forces $F_B = 900\,\text{N}$ and $F_C = 600\,\text{N}$. We must therefore express these two in terms of Cartesian coordinates. First of all we start by determining the side length of the isoceles right triangle comprised by $\mathbf{r}_B$ and its components. These components can be found as:

$$\cos 45^\circ \cdot 4{,}5\,\text{m} = \sin 45^\circ \cdot 4{,}5\,\text{m} = 3{,}18\,\text{m}.$$

This means that the position vector $\mathbf{r}_B$ can be written as:

$$\mathbf{r}_B = (3{,}18\,\text{m}\mathbf{i} - 3{,}18\,\text{m}\mathbf{j} - 6\,\text{m}\mathbf{k})\,.$$

A unit vector along this direction can be found as:

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{|\mathbf{r}_B|} = \frac{(3{,}18\,\text{m}\mathbf{i} - 3{,}18\,\text{m}\mathbf{j} - 6\,\text{m}\mathbf{k})}{\sqrt{(3{,}18\,\text{m})^2 + (3{,}18\,\text{m})^2 + (6\,\text{m})^2}} = 0{,}4241\mathbf{i} - 0{,}4241\mathbf{j} - 0{,}8002\mathbf{k}.$$

Now if we just multiply this by the magnitude of the force F_B we get the force in the direction of B in cartestian coordinates:

$$\mathbf{F}_B = F_B \cdot \mathbf{u}_B = 900\,\text{N} \cdot (0{,}4241\mathbf{i} - 0{,}4241\mathbf{j} - 0{,}8002\mathbf{k}) = 381{,}69\,\text{N}\,\mathbf{i} - 381{,}69\,\text{N}\,\mathbf{j} - 720{,}18\,\text{N}\,\mathbf{k}.$$

We can also find the unit vector along $\mathbf{r}_C$ as:

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{|\mathbf{r}_C|} = \frac{(-3\,\text{m}\,\mathbf{i} - 6\,\text{m}\,\mathbf{j} - 6\,\text{m}\,\mathbf{k})}{\sqrt{(3\,\text{m})^2 + (6\,\text{m})^2 + (6\,\text{m})^2}} = -0{,}333\mathbf{i} - 0{,}666\mathbf{j} - 0{,}666\mathbf{k}.$$

Therefore we can now find $\mathbf{F}_C$ as:

$$\mathbf{F}_C = F_C \mathbf{u}_C = 600\,\mathrm{N} \cdot (-0{,}333\mathbf{i} - 0{,}666\mathbf{j} - 0{,}666\mathbf{k}) = -200\,\mathrm{N}\,\mathbf{i} - 400\,\mathrm{N}\,\mathbf{j} - 400\,\mathrm{N}\,\mathbf{k}.$$

The resultant force $\mathbf{F}_A$ can now be found as:

$$\mathbf{F}_A = \mathbf{F}_B + \mathbf{F}_C = 0{,}182\,\mathrm{kN}\,\mathbf{i} - 0{,}782\,\mathrm{kN}\,\mathbf{j} - 1{,}12\,\mathrm{kN}\,\mathbf{k}.$$

The magnitude of this can be found by the Pythagorean theorem as:

$$F_A = \sqrt{(0{,}182\,\mathrm{kN})^2 + (0{,}782\,\mathrm{kN})^2 + (1{,}12\,\mathrm{kN})^2} = 1{,}38\,\mathrm{kN}.$$

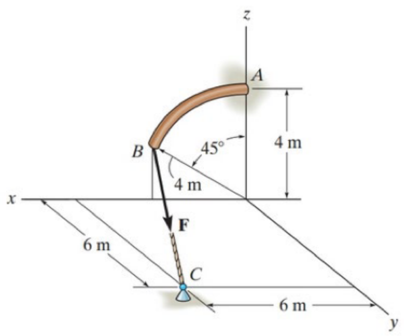
Now we can find the angles with each of the three axes as:

$$\begin{aligned}
\alpha &= \cos^{-1}\left(\frac{F_{Ax}}{F_A}\right) = \cos^{-1} 0{,}132 = 82{,}4^\circ \\
\beta &= \cos^{-1}\left(\frac{F_{Ay}}{F_A}\right) = \cos^{-1} -0{,}567 = 124{,}5^\circ \\
\gamma &= \cos^{-1}\left(\frac{F_{Az}}{F_A}\right) = \cos^{-1} -0{,}813 = 144{,}4^\circ.
\end{aligned}$$

Therefore the resultant force $\mathbf{F}_A$ is of magnitude 1,38 kN and makes angles of 82,4°, 124,5°, and 144,4° with the positive x-, y-, and z-axes respectively.

Exercise P2-1

Determine the moment of the force of $F = 600\,\mathrm{N}$ about point A.



The vector formulation of the moment is

$$\mathbf{M}_A = \mathbf{r} \times \mathbf{F}.$$

Where $\mathbf{r}$ is the position vector between the point A and any point on the line of action of $\mathbf{F}$, e.g. B. Therefore

$$\mathbf{r} = \mathbf{r}_B - \mathbf{R}_A = (4 \cdot \sin 45^\circ, 0, 4 \cdot \cos 45^\circ) - (0, 0, 4) = \left(2\sqrt{2}, 0, 2\sqrt{2} - 4\right)\,\mathrm{m}.$$

The unit vector in the direction of $\mathbf{F}$ is:

$$\mathbf{u}_F = \frac{\mathbf{C} - \mathbf{B}}{C - B} = \frac{\left(6 - 2\sqrt{2}, 6, -2\sqrt{2}\right)}{\sqrt{\left(6 - 2\sqrt{2}\right)^2 + 6^2 + \left(2\sqrt{2}\right)^2}} = (0{,}4314, 0{,}8161, -0{,}3847)\,.$$

And the force vector can thus be found as:

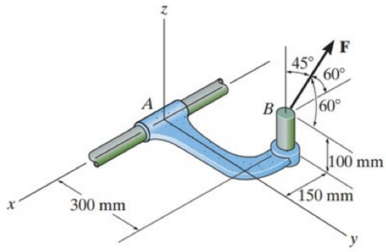
$$\mathbf{F} = F \cdot \mathbf{u}_F = 600\,\mathrm{N} \cdot (0{,}4314, 0{,}8161, -0{,}3847) = (258{,}82, 489{,}63, -230{,}81)\,\mathrm{N}.$$

And the moment about A is thus

$$\mathbf{M}_A = \mathbf{r} \times \mathbf{F} = \left(2\sqrt{2}, 0, 2\sqrt{2} - 4\right)\mathrm{m} \cdot (258{,}82, 489{,}63, -230{,}81)\,\mathrm{N} = (573{,}64, 349{,}62, 1384{,}89)\,\mathrm{N\,m}.$$

Exercise P2-2

The friction at sleeve A can provide a maximum resisting moment of $125\,\mathrm{N\,m}$ about the x-axis. Determine the largest magnitude of force $\mathbf{F}$ that can be applied to the bracked so that the bracket will not turn.



Here, the same kind of reasoning is applied as above. We first find a position vector from the point of rotation A to a point on the line of attack of the force $\mathbf{F}$ at B. This is done as:

$$\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = (-150, 300, 100)\,\mathrm{mm} - \mathbf{0} = (-150, 300, 100)\,\mathrm{mm}.$$

Now we must find a unit vector in the direction of $\mathbf{F}$, $\mathbf{u}_F$. The drawing is a bit hard to decipher but it is understood as the force $\mathbf{F}$ lying $60°$ from the $-x$-axis, $60°$ from the $+y$-axis, and $45°$ from the $+z$-axis. Using the direction cosines we therefore get:

$$\begin{aligned}\cos\alpha &= -\cos 60° = -0{,}5\\ \cos\beta &= \cos 60° = 0{,}5\\ \cos\gamma &= \cos 45° = 0{,}7071.\end{aligned}$$

The unit vector of $\mathbf{F}$ is therefore:

$$\mathbf{u}_F = (-0{,}5; 0{,}5; 0{,}7071)\,.$$

We have the following condition for the largest force the resisting friction can withstand:

$$\mathbf{M}_A = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{u}_F \cdot F)\,.$$

We can solve this for the magnitude F of the force $\mathbf{F}$ as:

$$\begin{aligned}\mathbf{M}_A &= \mathbf{r} \times (\mathbf{u}_F \cdot F)\\ \mathbf{M}_A &= F\,(\mathbf{r} \times \mathbf{u}_F)\\ F &= \frac{\mathbf{M}_A}{\mathbf{r} \times \mathbf{u}_F}.\end{aligned}$$

And plugging in known values we get:

$$F = \frac{(125, 0, 0)\,\mathrm{N\,m}}{(-0{,}15; 0{,}3; 0{,}1)\,\mathrm{m} \times (-0{,}5; 0{,}5; 0{,}7071)} = 771\,\mathrm{N}.$$

Therefore the force $\mathbf{F}$ should have a magnitude of at least $F = 771\,\mathrm{N}$ before the friction force would not be able to hold the sleeve still.