Dynamics and Vibrations – Excercises

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Lecture 1: Introduction

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Exercise 1.1

We consider the rotation of a rigid body shown in **Figure 0.1** about the point O. The rod has mass m and the spring fastened to the rod at distance L from the point O has stiffness k. A force $F(t)$ is applied at a distance $L/2$ from the point O.

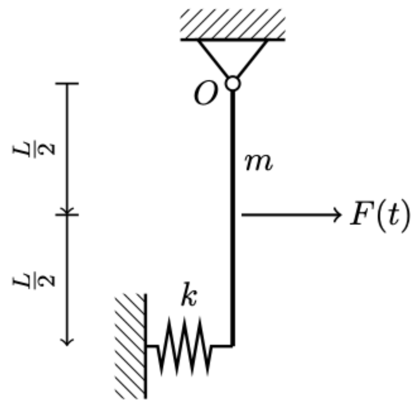


Figure 0.1:

a) Derive the non-linear equation of motion for the rotation of the rigid body.

We determine the resultant moment around O as:

$$I_O\ddot{\theta} = -kL\sin\theta L\cos\ \theta - mg\frac{L}{2}\sin\theta + F\frac{L}{2}\cos\theta$$

$$I_O\ddot{\theta} + \sin\theta\left(kL^2\cos\theta + mg\frac{L}{2}\right) = F\frac{L}{2}\cos\theta$$

.

The moment of inertia for a rod rotating about its end is given by $I_O = \frac{1}{3}mL^2$. Therefore we get:

$$\frac{1}{3}mL^2\ddot{\theta} + \sin\theta\left(kL^2\cos\theta + mg\frac{L}{2}\right) = F\frac{L}{2}\cos\theta.$$

b) Linearize the non-linear equation of motion using the assumption of small rotations.

By assuming $\cos\theta \approx 1$ and $\sin\theta \approx \theta$ we get:

$$\frac{1}{3}mL^2\ddot{\theta} + \theta\left(kL^2 + mg\frac{L}{2}\right) = F\frac{L}{2}.$$

Exercise 1.2

A person weighing 75 kg is standing on a scale in an elevator as shown on **Figure 0.2**. During the first 3 seconds of vertical motion upwards from rest, the force in the cable is 8300 N. Determine what the scale shows (in Newton) in this configuration. It is noted that the total mass of the elevator, person and scale is 750 kg.

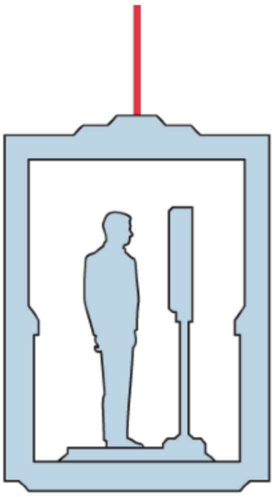


Figure 0.2:

Let $m_p = 75\,\mathrm{kg}$ denote the mass of the person and $m = 750\,\mathrm{kg}$ denote the entire mass of the system. Also let $F_c = 8300\,\mathrm{N}$ denote the force in the cable. Newton's second law on the entire system gives

$$m\ddot{d} = F_c - mg \implies \ddot{d} = \frac{F_c - mg}{m}.$$

For the person inside the elevator using Newton's second law gives

$$m_p\ddot{d} = F_R - m_pg \implies F_R = m_p\ddot{d} + m_pg$$

where F_R is the reactant force the scale exerts on the person (i.e. the force the person exerts on the scale). Substituting in the previous expression for $\ddot{d}$ we get

$$\begin{aligned} F_R &= m_p\left(\ddot{d}+g\right) \\ &= m_p\left(\frac{F_c - mg}{m}+g\right) \\ &= m_p\left(\frac{F_c}{m}\right) \\ &= \frac{m_p F_c}{m} \\ &= \frac{75\,\mathrm{kg}\cdot 8300\,\mathrm{N}}{750\,\mathrm{kg}} \\ &= 830\,\mathrm{N}. \end{aligned}$$

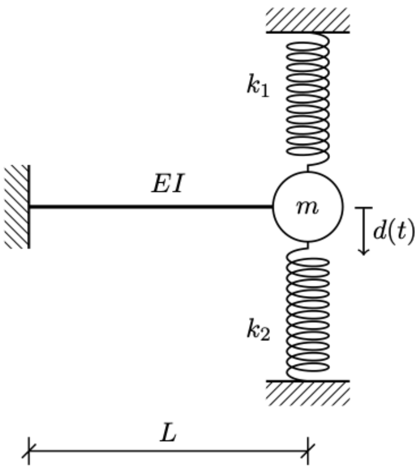
Therefore the scale will show a reading of $830\,\mathrm{N}$.

Lecture 2: Free motion/eigenmotion of SDOF systems

2. September 2025

Exercise 2.1

We consider the shown system, which is to be analysed as an equivalent single-degree of freedom system (SDOF-system) with the transverse motion of the point mass m as the degree of freedom. It is noted, that the stiffness of the beam (without taking the discrete springs into account) against transverse direction in the free end due to a transverse force acting in this point is $k_{\text{beam}} = \frac{3EI}{L^3}$, where L, I, and E is the length, area-moment of inertia and the elastic modulus of the beam. Assume, that the beam and the discrete springs are mass-less.



a) Determine (symbolically) the undampened angular eigenfrequency ω_u of the equivalent SDOF-system.

The undampened angular eigenfrequency ω_u is given as:

$$\omega_u = \sqrt{\frac{k}{m}}.$$

In this case the three springs are connected in parallel and so:

$$k_{eq} = k_{\text{beam}} + k_1 + k_2 = k_1 + k_2 + \frac{3EI}{L^3}.$$

And therefore the undampened angular eigenfrequency of the system is

$$\omega_u = \sqrt{\frac{k_1 + k_2 + \frac{3EI}{L^3}}{m}}.$$

b) It is now noted that (in SI-units) $EI = 100$, $L = 1{,}3$, $m = 0{,}5$, $k_1 = k_2 = 25$, $d_0 = 0{,}1$ and $\dot{d}_0 = 0$. Determine the undampened eigenperiod of the equivalent SDOF-system.

Note that the connection between frequency as determined in a) and period is:

$$T = \frac{2\pi}{\omega_u} = \frac{2\pi}{\sqrt{\frac{2\cdot 25 + \frac{300}{1{,}3^3}}{0{,}5}}} = 0{,}33\,\text{s}.$$

c) Determine the maximal speed, that the equivalent SDOF-system experiences in its eigenmotion.

The displacement $d(t)$ is given as:

$$d(t) = A\cos\left(\omega_u t - \phi\right).$$

And the speed $\dot{d}(t)$ can be found simply vie differentiation as

$$\dot{d}(t) = -\omega_u A \sin\left(\omega_u t - \phi\right).$$

In its maximum this is

$$\max(\dot{d}(t)) = \omega_u A.$$

We also know that:

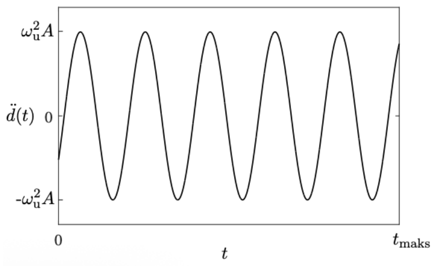
$$A = \sqrt{A_1^2 + A_2^2} = \sqrt{d_0^2 + \frac{\dot{d}_0^2}{\omega_u^2}} = d_0.$$

Therefore the maximum velocity is:

$$\max\left(\dot{d}(t)\right) = \omega_u A = \sqrt{\frac{k_1 + k_2 + \frac{3EI}{L^3}}{m}} \cdot d_0 = 1{,}73\,\frac{\text{m}}{\text{s}}.$$

Exercise 2.2

With reference to the following harmonic response choose the correct starting conditions.

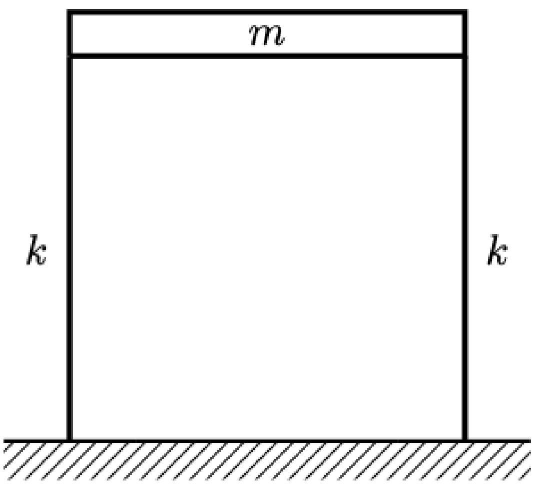


1. $d_0 < 0 \wedge \dot{d}_0 = 0$
2. $d_0 > 0 \wedge \dot{d}_0 = 0$
3. $d_0 < 0 \wedge \dot{d}_0 > 0$
4. $d_0 > 0 \wedge \dot{d}_0 < 0$
5. $d_0 = 0 \wedge \dot{d}_0 > 0$
6. $d_0 = 0 \wedge \dot{d}_0 < 0$

The plot looks to be showing sinusoidal acceleration. In the initial case the acceleration is negative, this means that the speed here is necessarily also negative, a negative speed means that the initial displacement must have been positive, hence the correct answer is 4

Exercise 2.3

We consider the shown system, which is to be analysed as an equivalent SDOF-system, wherein the degree of freedom is horizontal movement of the horizontal beam. It is noted that the horizontal beam has mass m and is assumed to be infinitely stiff, whilst the two vertical beams are assumed massless and with stiffness k.



a) Determine (symbolically) what the undampened angular eigenfrequency ω_u of the equivalent SDOF-system is.

The two beams are connected in parallel, hence:

$$k_{eq} = 2k \implies \omega_u = \sqrt{\frac{2k}{m}}.$$

b) It is noted that (in SI-units) $m = 3$, $k = 30$, $d_0 = -0{,}1$, and $\dot{d}_0 = 0{,}2$. Determine the initial acceleration of the equivalent SDOF-system

From Newton's second law we get:

$$m\ddot{d}(t) + kd(t) = 0 \implies m\ddot{d}_0 + k_{eq}d_0 = 0.$$

And now we can solve for $\ddot{d}_0$ as:

$$\ddot{d}_0 = \frac{-k_{eq}d_0}{m} = \frac{-60 \cdot (-0{,}1)}{3} = 2\,\frac{\mathrm{m}}{\mathrm{s}}.$$