Take-Home Assignment weeks 35–36

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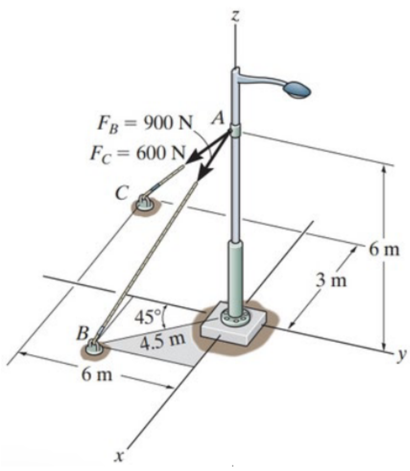
Exercise P1-1

Determine the magnitude and coordinate direction angles of the resultant force acting at A.



We start by adding a coordinate system such that the point A is located at the origin O. The resultant force here $\mathbf{F}_A$ is simply the sum of the two acting forces $F_B = 900\,\mathrm{N}$ and $F_C = 600\,\mathrm{N}$. We must therefore express these two in terms of Cartesian coordinates. First of all we start by determining the side length of the isoceles right triangle comprised by $\mathbf{r}_B$ and its components. These components can be found as:

$$\cos 45^\circ \cdot 4{,}5\,\mathrm{m} = \sin 45^\circ \cdot 4{,}5\,\mathrm{m} = 3{,}18\,\mathrm{m}.$$

This means that the position vector $\mathbf{r}_B$ can be written as:

$$\mathbf{r}_B = (3{,}18\,\mathrm{m}\mathbf{i} - 3{,}18\,\mathrm{m}\mathbf{j} - 6\,\mathrm{m}\mathbf{k})\,.$$

A unit vector along this direction can be found as:

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{|\mathbf{r}_B|} = \frac{(3{,}18\,\mathrm{m}\mathbf{i} - 3{,}18\,\mathrm{m}\mathbf{j} - 6\,\mathrm{m}\mathbf{k})}{\sqrt{(3{,}18\,\mathrm{m})^2 + (3{,}18\,\mathrm{m})^2 + (6\,\mathrm{m})^2}} = 0{,}4241\mathbf{i} - 0{,}4241\mathbf{j} - 0{,}8002\mathbf{k}.$$

Now if we just multiply this by the magnitude of the force F_B we get the force in the direction of B in cartestian coordinates:

$$\mathbf{F}_B = F_B \cdot \mathbf{u}_B = 900\,\mathrm{N} \cdot (0{,}4241\mathbf{i} - 0{,}4241\mathbf{j} - 0{,}8002\mathbf{k}) = 381{,}69\,\mathrm{N}\,\mathbf{i} - 381{,}69\,\mathrm{N}\,\mathbf{j} - 720{,}18\,\mathrm{N}\,\mathbf{k}.$$

We can also find the unit vector along $\mathbf{r}_C$ as:

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{|\mathbf{r}_C|} = \frac{(-3\,\mathrm{m}\,\mathbf{i} - 6\,\mathrm{m}\,\mathbf{j} - 6\,\mathrm{m}\,\mathbf{k})}{\sqrt{(3\,\mathrm{m})^2 + (6\,\mathrm{m})^2 + (6\,\mathrm{m})^2}} = -0{,}333\mathbf{i} - 0{,}666\mathbf{j} - 0{,}666\mathbf{k}.$$

Therefore we can now find $\mathbf{F}_C$ as:

$$\mathbf{F}_C = F_C \mathbf{u}_C = 600\,\mathrm{N} \cdot (-0{,}333\mathbf{i} - 0{,}666\mathbf{j} - 0{,}666\mathbf{k}) = -200\,\mathrm{N}\,\mathbf{i} - 400\,\mathrm{N}\,\mathbf{j} - 400\,\mathrm{N}\,\mathbf{k}.$$

The resultant force $\mathbf{F}_A$ can now be found as:

$$\mathbf{F}_A = \mathbf{F}_B + \mathbf{F}_C = 0{,}182\,\mathrm{kN}\,\mathbf{i} - 0{,}782\,\mathrm{kN}\,\mathbf{j} - 1{,}12\,\mathrm{kN}\,\mathbf{k}.$$

The magnitude of this can be found by the Pythagorean theorem as:

$$F_A = \sqrt{(0{,}182\,\mathrm{kN})^2 + (0{,}782\,\mathrm{kN})^2 + (1{,}12\,\mathrm{kN})^2} = 1{,}38\,\mathrm{kN}.$$

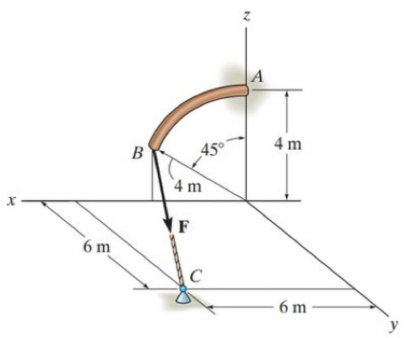
Now we can find the angles with each of the three axes as:

$$\begin{aligned}
\alpha &= \cos^{-1}\left(\frac{F_{Ax}}{F_A}\right) = \cos^{-1} 0{,}132 = 82{,}4^\circ \\
\beta &= \cos^{-1}\left(\frac{F_{Ay}}{F_A}\right) = \cos^{-1} -0{,}567 = 124{,}5^\circ \\
\gamma &= \cos^{-1}\left(\frac{F_{Az}}{F_A}\right) = \cos^{-1} -0{,}813 = 144{,}4^\circ.
\end{aligned}$$

Therefore the resultant force $\mathbf{F}_A$ is of magnitude 1,38 kN and makes angles of 82,4°, 124,5°, and 144,4° with the positive x-, y-, and z-axes respectively.

Exercise P2-1

Determine the moment of the force of $F = 600\,\mathrm{N}$ about point A.



The vector formulation of the moment is

$$\mathbf{M}_A = \mathbf{r} \times \mathbf{F}.$$

Where $\mathbf{r}$ is the position vector between the point A and any point on the line of action of $\mathbf{F}$, e.g. B. Therefore

$$\mathbf{r} = \mathbf{r}_B - \mathbf{R}_A = (4 \cdot \sin 45^\circ, 0, 4 \cdot \cos 45^\circ) - (0, 0, 4) = \left(2\sqrt{2}, 0, 2\sqrt{2} - 4\right)\,\mathrm{m}.$$

The unit vector in the direction of $\mathbf{F}$ is:

$$\mathbf{u}_F = \frac{\mathbf{C} - \mathbf{B}}{C - B} = \frac{\left(6 - 2\sqrt{2}, 6, -2\sqrt{2}\right)}{\sqrt{\left(6 - 2\sqrt{2}\right)^2 + 6^2 + \left(2\sqrt{2}\right)^2}} = (0{,}4314, 0{,}8161, -0{,}3847)\,.$$

And the force vector can thus be found as:

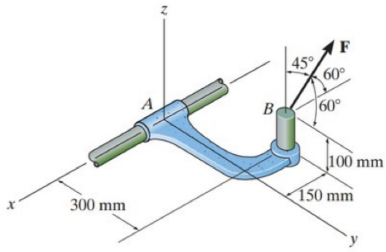
$$\mathbf{F} = F \cdot \mathbf{u}_F = 600\,\mathrm{N} \cdot (0{,}4314, 0{,}8161, -0{,}3847) = (258{,}82, 489{,}63, -230{,}81)\,\mathrm{N}.$$

And the moment about A is thus

$$\mathbf{M}_A = \mathbf{r} \times \mathbf{F} = \left(2\sqrt{2}, 0, 2\sqrt{2} - 4\right)\mathrm{m} \cdot (258{,}82, 489{,}63, -230{,}81)\,\mathrm{N} = (573{,}64, 349{,}62, 1384{,}89)\,\mathrm{N\,m}.$$

Exercise P2-2

The friction at sleeve A can provide a maximum resisting moment of $125\,\mathrm{N\,m}$ about the x-axis. Determine the largest magnitude of force $\mathbf{F}$ that can be applied to the bracked so that the bracket will not turn.



Here, the same kind of reasoning is applied as above. We first find a position vector from the point of rotation A to a point on the line of attack of the force $\mathbf{F}$ at B. This is done as:

$$\mathbf{r} = \mathbf{r}_B - \mathbf{r}_A = (-150, 300, 100)\,\mathrm{mm} - \mathbf{0} = (-150, 300, 100)\,\mathrm{mm}.$$

Now we must find a unit vector in the direction of $\mathbf{F}$, $\mathbf{u}_F$. The drawing is a bit hard to decipher but it is understood as the force $\mathbf{F}$ lying 60° from the $-x$-axis, 60° from the $+y$-axis, and 45° from the $+z$-axis. Using the direction cosines we therefore get:

$$\begin{aligned} \cos\alpha &= -\cos 60^\circ = -0{,}5 \\ \cos\beta &= \cos 60^\circ = 0{,}5 \\ \cos\gamma &= \cos 45^\circ = 0{,}7071. \end{aligned}$$

The unit vector of $\mathbf{F}$ is therefore:

$$\mathbf{u}_F = (-0{,}5; 0{,}5; 0{,}7071)\,.$$

We have the following condition for the largest force the resisting friction can withstand:

$$\mathbf{M}_A = \mathbf{r} \times \mathbf{F} = \mathbf{r} \times (\mathbf{u}_F \cdot F)\,.$$

We can solve this for the magnitude F of the force $\mathbf{F}$ as:

$$\begin{aligned} \mathbf{M}_A &= \mathbf{r} \times (\mathbf{u}_F \cdot F) \\ \mathbf{M}_A &= F\,(\mathbf{r} \times \mathbf{u}_F) \\ F &= \frac{\mathbf{M}_A}{\mathbf{r} \times \mathbf{u}_F}. \end{aligned}$$

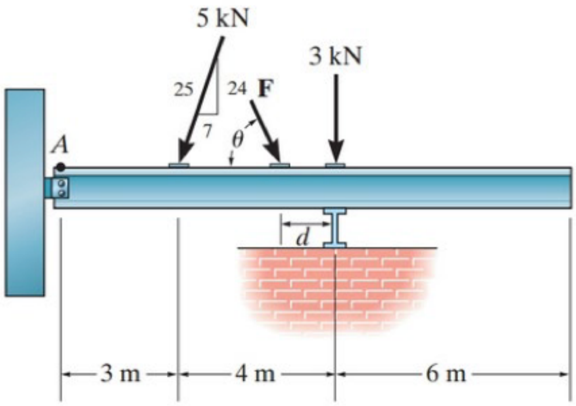
And plugging in known values we get:

$$F = \frac{(125, 0, 0)\,\mathrm{N\,m}}{(-0{,}15; 0{,}3; 0{,}1)\,\mathrm{m} \times (-0{,}5; 0{,}5; 0{,}7071)} = 771\,\mathrm{N}.$$

Therefore the force $\mathbf{F}$ should have a magnitude of at least $F = 771\,\mathrm{N}$ before the friction force would not be able to hold the sleeve still.

Exercise P3-1

Determine the magnitude and direction of θ of force **F** and its displacement d on the beam to that the loading system is equivalent to a resultant force of 12 kN acting vertically downward at point A and a clockwise couple moment of 50 kN m.



The forces in the x- and y-directions can be expressed independently as:

$$F_x = \cos\theta F - \frac{7}{25} \cdot 5\,\text{kN}$$
$$F_y = -\left(3\,\text{kN} + \frac{24}{25} \cdot 5\,\text{kN} + \sin\theta F\right).$$

The moments that each of the forces induce can also be expressed as (note that here clockwise is taken as the positive direction to simplify the calculations):

$$M_A = \left(3\,\text{m} \cdot \frac{24}{25} \cdot 5\,\text{kN}\right) + ((7\,\text{m} - d) \cdot \sin\theta F) + (7\,\text{m} \cdot 3\,\text{kN}).$$

We know that these, in static equilibrium will have the sizes $F_x = 0$, $F_y = -12\,\text{kN}$ and $M_A = 50\,\text{kNm}$. We can therefore write:

$$\cos\theta F = 1{,}4\,\text{kN}$$
$$\sin\theta F = 12\,\text{kN} - 3\,\text{kN} - 4{,}8\,\text{kN} = 4{,}2\,\text{kN}$$
$$(7\,\text{m} - d) \cdot \sin\theta F = 50\,\text{kN m} - (3\,\text{m} \cdot 4{,}8\,\text{kN}) - 21\,\text{kN m} = 14{,}6\,\text{kN m}.$$

By using the result for $\sin\theta F$ in the expression for the moment we can find d as:

$$(7\,\text{m} - d) = \frac{14{,}6\,\text{kN m}}{4{,}2\,\text{kN}}$$
$$7\,\text{m} - d = 3{,}47\,\text{m}$$
$$d = 7\,\text{m} - 3{,}47\,\text{m} = 3{,}52\,\text{m}.$$

From the expressions for $\sin\theta F$ and $\cos\theta F$ we see that $\sin\theta F$ is 3 times as big. I.e.

$$3 \cdot \cos\theta F = \sin\theta F$$
$$\cot\theta = \frac{1}{3}$$
$$\theta = \cot^{-1}\frac{1}{3} = 71{,}56°.$$

And now the size **F** is easily found as:

$$F = \frac{1{,}4\,\mathrm{kN}}{\cos 71{,}56^\circ} = 4{,}43\,\mathrm{kN}.$$

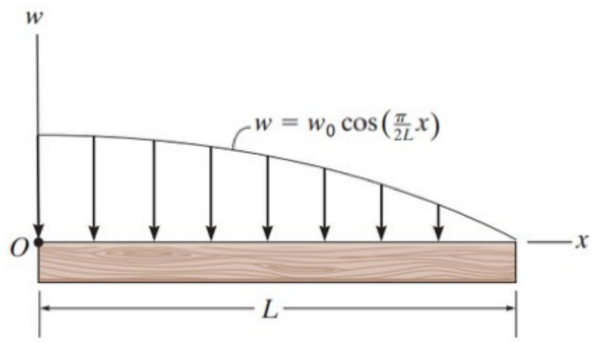
Therefore **F** is given as:

$$\mathbf{F} = 4{,}43\,\mathrm{kN}\angle 71{,}56^\circ$$

and it is applied at a distance $d = 3{,}52\,\mathrm{m}$ from the support.

Exercise P3-2

Replace the loading by an equivalent resultant force and couple moment acting at point O.



The magnitude of the resultant force F_R is obtained by integrating the distributed load over its entire length as:

$$F_R = \int_L w(x)\,\mathrm{d}x.$$

We therefore get:

$$\begin{aligned} F_R &= \int_0^L w_0 \cos\left(\frac{\pi}{2L}x\right)\,\mathrm{d}x \\ &= \left[w_0 \cdot \frac{2L}{\pi}\sin\left(\frac{\pi}{2L}x\right)\right]_0^L \\ &= w_0 \cdot \frac{2L}{\pi}\sin\left(\frac{\pi}{2}\right) - w_0\frac{2L}{\pi}\sin 0 \\ &= \frac{2Lw_0}{\pi}. \end{aligned}$$

The magnitude of the resultant force is therefore $F = \frac{2Lw_0}{\pi}$. This will act vertically downwards from the centroid, the x-coordinate $\overline{x}$ of which can be found as:

$$\overline{x} = \frac{\int_L xw(x)\,\mathrm{d}x}{\int_L w(x)\,\mathrm{d}x}.$$

We have already found the denominator of this so now we only need the numerator. For this we get:

$$w_0\int_0^L x \cdot \cos\left(\frac{\pi}{2L}x\right)\,\mathrm{d}x = \frac{2\,(\pi-2)\,L^2 w_0}{\pi^2}.$$

And therefore $\overline{x}$ can now be determined as:

$$\begin{aligned}\overline{x} &= \frac{2\left(\pi-2\right)L^2 w_0 \cdot \pi}{\pi^2 2 L w_0} \\ &= \frac{\left(\pi-2\right)L}{\pi}.\end{aligned}$$

And thus the resultant moment is:

$$M_O = \overline{x} F_R = \frac{(\pi-2)L}{\pi} \cdot \frac{2Lw_0}{\pi} = \frac{2\left(\pi-2\right)L^2 w_0}{\pi^2}.$$

The equivalent resultant force and couple moment is therefore (following the standard conventions of y being upwards and positive moment is counterclockwise):

$$\begin{aligned}F_R &= -\frac{2Lw_0}{\pi} \\ M_O &= -\frac{2\left(\pi-2\right)L^2 w_0}{\pi^2}.\end{aligned}$$