Fluid Mechanics – Exercises

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Lecture 2: Basic equations of fluid statics and buoyancy, surface tension 28. August 2025

Exercise 3.7

Calculate the absolute pressure and gauge pressure in an open tank of crude oil 2,4 m below the liquid surface. If the tank is closed and pressurized to 130 kPa , that are the absolute pressure and gauge pressure at this location?

It is assumed that the oil is totally incompressible and that the gauge pressure is zero at the liquid surface before the tank is pressurized – i.e. $p_0 = 101{,}325\,\mathrm{kPa}$. Also the density of the oil is assumed to be $800\,\mathrm{kg/m^3}$. The formula for the pressure at a depth h in a liquid is:

$$p_{a_1} = p_0 + \rho g h = 101{,}325\,\mathrm{kPa} + 800\,\mathrm{kg/m^3} \cdot 9{,}81\,\mathrm{m/s^2} \cdot 2{,}4\,\mathrm{m} = 120\,\mathrm{kPa}.$$

The gauge pressure is simply this but without the atmospheric pressure added on:

$$p_{g_1} = \rho g h = 800\,\mathrm{kg/m^3} \cdot 9{,}81\,\mathrm{m/s^2} \cdot 2{,}4\,\mathrm{m} = 18{,}8\,\mathrm{kPa}.$$

After the tank is pressurized the gauge pressure will stay constant, whereas the absolute pressure will grow with $p_p = 130\,\mathrm{kPa}$ and become $p_{a_2} = 120\,\mathrm{kPa} + 130\,\mathrm{kPa} = 250\,\mathrm{kPa}$ and $p_{g_2} = 18{,}8\,\mathrm{kPa} + 130\,\mathrm{kPa} = 148{,}8\,\mathrm{kPa}$.

Exercise 3.5

A piston is placed on a tank filled with mercury at 20 °C as shown on **Figure 0.1**. A force is applied to the piston and the height of the mercury column rises. Determine the weight of the piston and the applied force.

For the system to be static the force exerted by the piston on the mercury must be equal to the force exerted by the column. When no force other than the weight of the piston itself is applied a mercury cylinder of height 25 mm and diameter 10 mm is produced. The height rise of this must give rise to a pressure increase given by:

$$\Delta p = \rho g \Delta H.$$

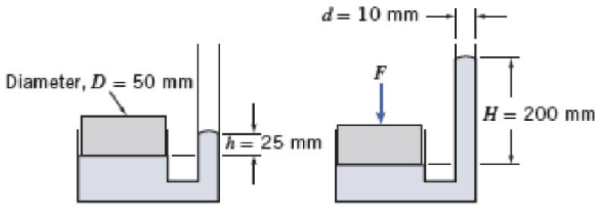


Figure 0.1:

The piston must therefore supply a weight big enough to bring about this pressure. I.e.

$$W = \Delta p A_p = \rho g \Delta H \cdot \pi \cdot r_p^2 = 13{,}5\,\mathrm{g/cm^3} \cdot 9{,}81\,\mathrm{m/s^2} \cdot 25\,\mathrm{mm} \cdot \pi \cdot (25\,\mathrm{mm})^2 = 0{,}26\,\mathrm{N}.$$

When the additional force is added the column rises an additional $\Delta H = 175\,\mathrm{mm}$. Therefore we here get:

$$F = \Delta p A_p = 13{,}5\,\mathrm{g/cm^3} \cdot 9{,}81\,\mathrm{m/s^2} \cdot 175\,\mathrm{mm} \cdot \pi \cdot (25\,\mathrm{mm})^2 = 45{,}5\,\mathrm{N}.$$

Exercise 3.6

A 125 mL cube of solid oak is held submerged by a tether as shown on **Figure 0.2**. Calculate the force of the water on the bottom surface of the cube and the tension in the tether.

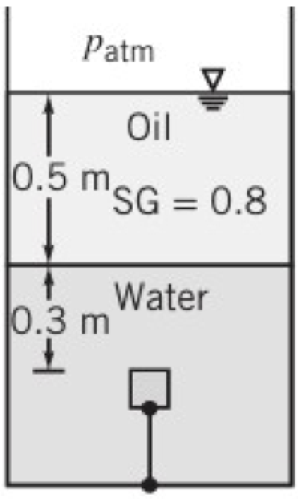


Figure 0.2:

First the side length of the cube is found as

$$s = \sqrt[3]{V_{\mathrm{oak}}} = \sqrt[3]{125\,\mathrm{cm^3}} = 5\,\mathrm{cm^3}.$$

Now we can determine its weight as the specific gravity of oak is 0,77 as:

$$W = V_{\mathrm{oak}} \rho \mathrm{SG}_{\mathrm{oak}} g = 125\,\mathrm{mL} \cdot 1000\,\mathrm{kg/m^3} \cdot 0{,}77 \cdot 9{,}81\,\mathrm{m/s^2} = 0{,}944\,\mathrm{N}.$$

The force on the bottom surface is equal to the pressure at this depth multiplied by the surface area of the bottom face. Firstly the pressure is found as:

$$\begin{aligned} p_L &= p_{\text{atm}} + \text{SG}_{\text{oil}} \cdot \rho \cdot g \cdot h_{\text{oil}} + \rho g h_L \\ &= p_{\text{atm}} + \rho g \left(\text{SG}_{\text{oil}} \cdot h_{\text{oil}} + h_L\right) \\ &= 101{,}325\,\text{kPa} + 1000\,\text{kg/cm}^3 \cdot 9{,}81\,\text{m/s}^2\,(0{,}8 \cdot 0{,}5\,\text{m} + 0{,}35\,\text{m}) \\ &= 108{,}6825\,\text{kPa}. \end{aligned}$$

Now this can be multiplied by the surface area of the bottom as:

$$F_L = p_L \cdot A = 108{,}6825\,\text{kPa} \cdot 25\,\text{cm}^2 = 271{,}7\,\text{N}.$$

Therefore the force exerted by the fluid on the bottom face is about 271,7 N.

To find the tension in the tether we realize that the force exerted by the fluid on the bottom face is counteracted by a force exerted by the fluid on the top face. We can find this using the same procedure as above.

$$F_U = \left(p_{\text{atm}} + \rho g \left(\text{SG}_{\text{oil}} \cdot h_{\text{oil}} + h_U\right)\right) \cdot A.$$

We can calculate the resultant force exerted by the liquid on the cube as:

$$\Delta F = F_L - F_U = \rho g \cdot (h_L - h_U) \cdot A = 1000\,\text{kg/m}^3 \cdot 9{,}81\,\text{m/s}^2 \cdot (5\,\text{cm}) \cdot 25\,\text{cm}^2 = 1{,}226\,25\,\text{N}.$$

This force (directed directly upwards) is counteracted by the weight of the cube. I.e.

$$T = \Delta F - W = 1{,}226\,25\,\text{N} - 0{,}944\,\text{N} = 0{,}282\,\text{N}.$$

Exercise 3.43

Determine the specific weight of the cube when one half is submerged as shown on **Figure 0.3**. Determine the position of the center of the cube relative to the water level when the weight is removed.

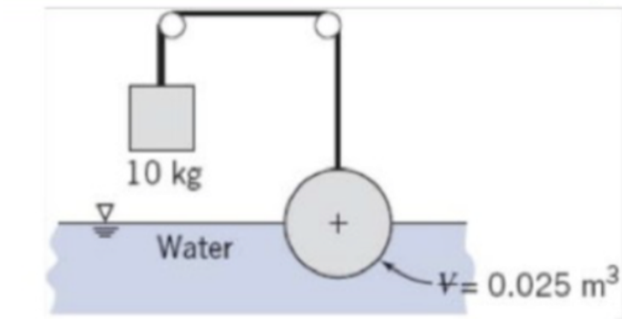


Figure 0.3: Caption

For static equilibrium the sum of the forces in the upwards/downwards direction must be 0. The only forces acting in this direction on the sphere is its weight (downwards) and the tension and the bouyancy forces (upwards). The tension in the cable must be exactly enough to keep the cube steady:

$$T = M \cdot g.$$

The buoyancy force on the sphere is given by Archimedes principle as:

$$F_B = \rho \cdot g \cdot \frac{V}{2}.$$

The weight of the sphere is given by:

$$W = \mathrm{SG} \cdot \rho \cdot g \cdot V.$$

These must, as mentioned, all sum to zero as:

$$Mg + \rho g \frac{V}{2} - \mathrm{SG}\rho g V = 0 \implies \mathrm{SG} = \frac{M}{\rho V} + \frac{1}{2}.$$

Now known values can be substituted in as:

$$\mathrm{SG} = 10\,\mathrm{kg} \cdot \frac{1}{1000\,\mathrm{kg/m^3} \cdot 0{,}025\,\mathrm{m^3}} + \frac{1}{2} = 0{,}9.$$

The specific weight is

$$\gamma = \frac{\text{Weight}}{\text{Volume}} = \frac{\mathrm{SG} \cdot \rho \cdot g \cdot V}{V} = \mathrm{SG}\rho \cdot g = 0{,}9 \cdot 1000\,\mathrm{kg/m^3} \cdot 9{,}81\,\mathrm{m/s^2} = 8829\,\mathrm{N/m^3}.$$

Now to find the equilibrium position when floating we repeat the force balance, but this time with $T = 0$ as

$$F_B - W = 0 \implies W = F_B = \rho g V_{\text{submerged}}.$$

Now we must realize that the submerged part of the sphere will be a spherical cap. The volume of such an object is:

$$V_{\text{submerged}} = \frac{\pi \cdot h_{\text{submerged}^2}}{3} \cdot (3 \cdot R - h_{\text{submerged}}).$$

Where R is the radius of the sphere. The radius of the sphere can be found as:

$$\begin{aligned} V &= \frac{4}{3}\pi R^3 \\ R &= \sqrt[3]{\frac{3V}{4\pi}} \\ &= \sqrt[3]{\frac{3}{4\pi} \cdot 0{,}025\,\mathrm{m^3}} \\ &= 0{,}181\,\mathrm{m}. \end{aligned}$$

Therefore:

$$\begin{aligned} W &= \rho g V_{\text{submerged}} \\ \mathrm{SG} \cdot \rho g V &= \rho g \frac{\pi h_{\text{submerged}^2}}{3} \cdot (3 \cdot R - h_{\text{submerged}}) \\ \mathrm{SG} \cdot V &= \frac{\pi h_{\text{submerged}^2}}{3} (3R - h_{\text{submerged}}) \\ \frac{3\mathrm{SG} \cdot V}{\pi} &= h_{\text{submerged}}^2 (3R - h_{\text{submerged}}) \\ h_{\text{submerged}}^2 (3 \cdot 0{,}181\,\mathrm{m} - h_{\text{submerged}}) &= \frac{3 \cdot 0{,}9 \cdot 0{,}025\,\mathrm{m^3}}{\pi} \\ h_{\text{submerged}}^2 (0{,}544\,\mathrm{m} - h_{\text{submerged}}) &= 0{,}0215\,\mathrm{m^3} \\ \implies h_{\text{submerged}} &\in \{-0{,}173\,\mathrm{m}, 0{,}292\,\mathrm{m}, 0{,}425\,\mathrm{m}\}. \end{aligned}$$

Here the negative solution can be rejected as it is unphysical. The largest solution can also be rejected as $0{,}425\,\mathrm{m} > 2 \cdot R = 0{,}362\,\mathrm{m}$ and this would require the sphere to be floating beneath the water surface, which is also unphysical. Therefore the sphere must be floating at a height of $h = 0{,}292\,\mathrm{m}$.

Lecture 3: Control volume analysis I: Basic laws and mass conservation 1. September 2025

Exercise 4.5

A 0,3 m by 0,5 m rectangular air duct carries a flow of 0,45 m^3/s at a density of 2 kg/m^3, Calculate the mean velocity in the duct. If the duct tapers to 0,15 m by 0,5 m size, determine the mean velocity in this section if the density is 1,5 kg/m^3 in this section.

In the initial state 0,45 m^3/s of air is passing by an area 0,3 m by 0,5 m. This area is:

$$A = 0{,}3\,\mathrm{m} \cdot 0{,}5\,\mathrm{m} = 0{,}15\,\mathrm{m}^2.$$

Now if we divide the flow by the area through which it passes we will obtain the velocity of the air as:

$$V_1 = \frac{0{,}45\,\frac{\mathrm{m}^3}{\mathrm{s}}}{0{,}15\,\mathrm{m}^2} = 3\,\frac{\mathrm{m}}{\mathrm{s}}.$$

Therefore the air will initially be moving at a velocity of $V_1 = 3\,\frac{\mathrm{m}}{\mathrm{s}}$.

As mass if a conserved property the mass flow in the initial section must equal the mass flow of the tapered section. Using the continuity equation we can calculate the mass flow rate initially as:

$$\dot{m}_1 = \rho_1 V_1 A_1 = 2\,\frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 3\,\frac{\mathrm{m}}{\mathrm{s}} \cdot 0{,}15\,\mathrm{m}^2 = 0{,}9\,\frac{\mathrm{kg}}{\mathrm{s}}.$$

This must equal the mass flow rate after the taper, thus:

$$\begin{aligned}
\dot{m}_1 &= \dot{m}_2 \\
\dot{m}_1 &= \rho_2 V_2 A_2 \\
V_2 &= \frac{\dot{m}_1}{\rho_2 A_2} \\
V_2 &= \frac{0{,}9\,\frac{\mathrm{kg}}{\mathrm{s}}}{1{,}5\,\frac{\mathrm{kg}}{\mathrm{m}^3} \cdot 0{,}15\,\mathrm{m} \cdot 0{,}5\,\mathrm{m}} \\
V_2 &= 8\,\frac{\mathrm{m}}{\mathrm{s}}.
\end{aligned}$$

Therefore the mean velocity of the tapered section is 8 m/s.

Exercise 4.9

A pipeline 0,3 m in diameter divides at a Y into two branches 200 mm and 150 mm in diameter. If the flow rate in the main line is 0,3 m^3/s and the mean velocity in the 200 mm pipe is 2,5 m/s, determine the flow rate in the 150 mm pipe.

We assume that the water flow can be modelled as steady and incompressible through a fixed control volume. In this case the continuity equation simplifies to:

$$\int_{\mathrm{CS}} \mathbf{V} \cdot \mathrm{d}\mathbf{A} = 0$$

or

$$V_1A_1 = V_2A_2 + V_3A_3.$$

Using the flow rates $Q_1 = V_1A_1$, $Q_2 = V_2A_2$, and $Q_3 = V_3A_3$ we can write:

$$Q_1 = Q_2 + Q_3.$$

Let the 150 mm pipe be number 3 and we can then isolate the flow rate we wish to find Q_3 in this as:

$$Q_3 = Q_1 - Q_2.$$

We can now find the flow rate of Q_2 as

$$Q_2 = V_2A_2 = 2{,}5\,\frac{\mathrm{m}}{\mathrm{s}} \cdot \pi \cdot (100\,\mathrm{mm})^2 = 0{,}0785\,\frac{\mathrm{m}^3}{\mathrm{s}}.$$

And thus the flow rate in the 150 mm pipe Q_3 can be found as:

$$Q_3 = 0{,}3\,\frac{\mathrm{m}^3}{\mathrm{s}} - 0{,}0785\,\frac{\mathrm{m}^3}{\mathrm{s}} = 0{,}221\,\frac{\mathrm{m}^3}{\mathrm{s}}.$$

Exercise 4.12

A cylindrical tank, of diameter $D = 150\,\mathrm{mm}$, drains through an opening, $d = 5\,\mathrm{mm}$, in the bottom of the tank. The speed of the liquid leaving the tank is approximately $V = \sqrt{2gy}$, where y is the height from the tank bottom to the free surface. If the tank is initially filled with water to $y_0 = 0{,}4\,\mathrm{m}$, determine the water depths at $t = 60\,\mathrm{s}$, $t = 120\,\mathrm{s}$, and $t = 180\,\mathrm{s}$. Plot $y(m)$ versus t for the first 180 s.

We assume uniform incompressible flow. The continuity equation states:

$$\frac{\partial}{\partial t}\int_{\mathrm{CV}} \rho\,\mathrm{d}V + \int_{\mathrm{CS}} \rho\mathbf{V}\cdot\mathrm{d}\mathbf{A} = 0.$$

In other words, the rate increase of mass in the control volume plus the mass outflow from the control volume equals zero. If we treat the tank as out control volume we get:

$$\frac{\partial}{\partial t}\int_0^y \rho\cdot A_{\mathrm{tank}}\,\mathrm{d}y + \rho\cdot V\cdot A_{\mathrm{opening}} = 0$$
$$\rho\cdot\pi\cdot R^2\cdot\frac{\mathrm{d}y}{\mathrm{d}t} + \rho\cdot\pi\cdot r^2\cdot V = 0.$$

We can now use $V = \sqrt{2gy}$ and get:

$$\rho\cdot\pi\cdot R^2\cdot\frac{\mathrm{d}y}{\mathrm{d}t} + \rho\cdot\pi\cdot r^2\cdot\sqrt{2gy} = 0$$
$$R^2\,\frac{\mathrm{d}y}{\mathrm{d}t} + r^2\cdot\sqrt{2gy} = 0$$
$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{r^2}{R^2}\cdot\sqrt{2gy}.$$

Separating variables we get:

$$\frac{1}{y^{\frac{1}{2}}}\,\mathrm{d}y = -\frac{r^2}{R^2}\cdot\sqrt{2g}\,\mathrm{d}t.$$

Integrating we get:

$$\begin{aligned} 2\left(y^{\frac{1}{2}} - y_0^{\frac{1}{2}}\right) &= -\frac{r^2}{R^2}\sqrt{2g}t \\ y^{\frac{1}{2}} &= -\frac{r^2}{R^2}\sqrt{\frac{g}{2}}t + y_0^{\frac{1}{2}} \\ y^{\frac{1}{2}} &= y_0^{\frac{1}{2}}\left(1 - \frac{r^2}{R^2}\sqrt{\frac{g}{2y_0}}t\right) \\ y &= y_0\left(1 - \left(\frac{r}{R}\right)^2\sqrt{\frac{g}{2y_0}}t\right)^2. \end{aligned}$$

We have that $y_0 = 0{,}4\,\mathrm{m}$ and $\frac{r}{R} = \frac{2{,}5\,\mathrm{mm}}{75\,\mathrm{mm}} = \frac{1}{30}$. At $t = 60\,\mathrm{s}$ we therefore get:

$$y_{60\,\mathrm{s}} = 0{,}4\,\mathrm{m} \cdot \left(1 - \left(\frac{1}{30}\right)^2\sqrt{\frac{9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2}}{2 \cdot 0{,}4\,\mathrm{m}}} \cdot 60\,\mathrm{s}\right)^2 = 0{,}235\,\mathrm{m}.$$

At $t = 120\,\mathrm{s}$ we get

$$y_{120\,\mathrm{s}} = 0{,}4\,\mathrm{m} \cdot \left(1 - \left(\frac{1}{30}\right)^2\sqrt{\frac{9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2}}{2 \cdot 0{,}4\,\mathrm{m}}} \cdot 120\,\mathrm{s}\right)^2 = 0{,}114\,\mathrm{m}.$$

And at $t = 180\,\mathrm{s}$ we get:

$$y_{180\,\mathrm{s}} = 0{,}4\,\mathrm{m} \cdot \left(1 - \left(\frac{1}{30}\right)^2\sqrt{\frac{9{,}81\,\frac{\mathrm{m}}{\mathrm{s}^2}}{2 \cdot 0{,}4\,\mathrm{m}}} \cdot 180\,\mathrm{s}\right)^2 = 0{,}036\,\mathrm{m}.$$

$y(m)$ versus t has been plotted in GeoGebra on Figure 0.4

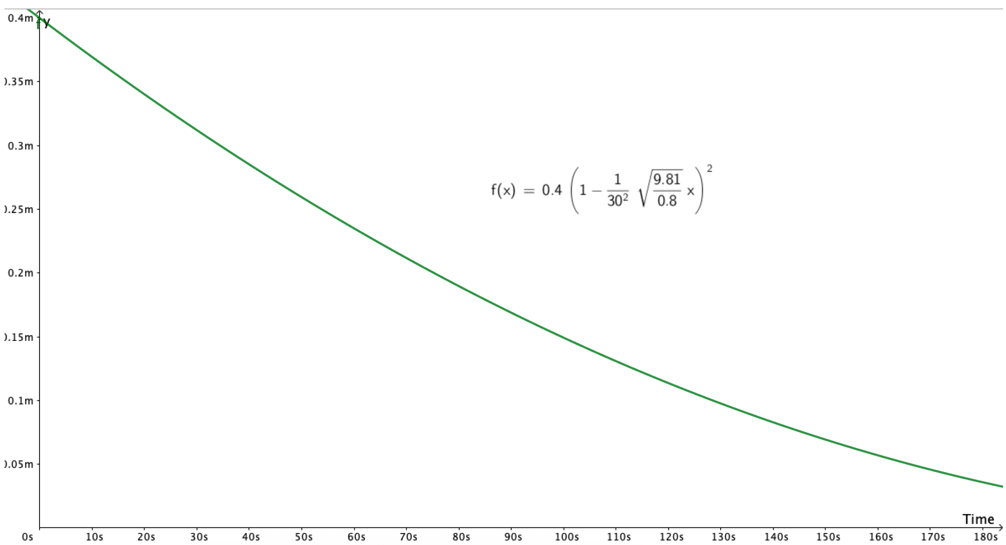
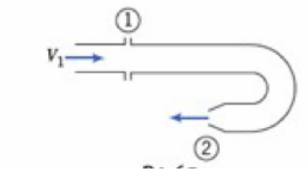


Figure 0.4: Plot of $y(m)$ versus t.

Lecture 4: Control volume analysis II: Momentum & energy equation 3. September 2025

Exercise 4.17

Water is flowing steadily through the 180° elbow shown. At the inlet to the elbow the gage pressure is 103 kPa. The water discharges to atmospheric pressure. Assume properties are uniform over the inlet and outlet areas: $A_1 = 2500\,\mathrm{mm}^2$, $A_2 = 650\,\mathrm{mm}^2$, and $V_1 = 3\,\frac{\mathrm{m}}{\mathrm{s}}$. Find the horizontal component of force required to hold the elbow in place.



The momentum equation in the x-direction is:

$$R_x + F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{\mathrm{CV}} u\rho\,\mathrm{d}V + \int_{\mathrm{CS}} u\rho\mathbf{V}\cdot\mathrm{d}\mathbf{A}.$$

As we are seeking the horizontal force on the elbow we can neglect the gravitational force (i.e. $F_{B_x} = 0$). The pressure force is simply:

$$F_{S_x} = p_1 A_1.$$

As we assume steady flow we simply get:

$$\begin{aligned} R_x + p_1 A_1 &= V_1\left(-\rho V_1 A_1\right) - V_2\left(\rho V_2 A_2\right) \\ R_x &= -p_1 A_1 - \rho\left(V_1^2 A_1 + V_2^2 A_2\right). \end{aligned}$$

From the continuity equation we can calculate the outlet velocity V_2 as:

$$\begin{aligned} 0 &= -\rho V_1 A_1 + \rho V_2 A_2 \\ V_2 &= \frac{V_1 A_1}{A_2} \\ V_2 &= 3\,\frac{\mathrm{m}}{\mathrm{s}}\cdot\frac{2500\,\mathrm{mm}^2}{650\,\mathrm{mm}^2} \\ &= 11{,}54\,\frac{\mathrm{m}}{\mathrm{s}}. \end{aligned}$$

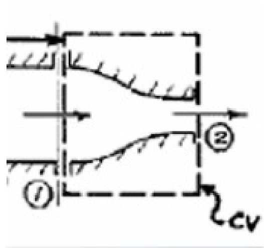
Now all parameters in the equation for R_x are known and it can be found as:

$$\begin{aligned} R_x &= -103\,\mathrm{kPa}\cdot 2500\,\mathrm{mm}^2 - 1000\,\frac{\mathrm{kg}}{\mathrm{m}^3}\left(\left(3\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2\cdot 2500\,\mathrm{mm}^2 + \left(11{,}54\,\frac{\mathrm{m}}{\mathrm{s}}\right)^2\cdot 650\,\mathrm{mm}^2\right) \\ &= -366{,}56\,\mathrm{N}. \end{aligned}$$

Therefore a force of 366,56 N directed against the wall is needed to hold the elbow in place.

Exercise 4.16

Water flows steadily through a fire hose and nozzle. The hose is 35 mm in diameter and the nozzle tip is 25 mm in diameter; water gage pressure in the hose is 510 kPa, and the stream leaving the nozzle is uniform. The exit speed and pressure are 32 m/s and atmospheric, respectively. Find the force transmitted by the coupling between the nozzle and hose. Indicate whether the coupling is in tension or compression.



Here the same formula as above (momentum in a control volume) can be used as:

$$R_x + F_{S_x} + F_{B_x} = \frac{\partial}{\partial t}\int_{\mathrm{CV}} u\rho\,\mathrm{d}V + \int_{\mathrm{CS}} u\rho\mathbf{V}\cdot\mathrm{d}\mathbf{A}.$$

With the same assumption of steady flow as above we get:

$$R_x + p_1 A_1 = V_2\left(\rho V_2 A_2\right) - V_1\left(\rho V_1 A_1\right).$$

For incompressible steady flow we have:

$$0 = -\rho_1 V_1 A_1 + \rho_2 V_2 A_2 \implies V_1 = V_2\frac{A_2}{A_1} = V_2\left(\frac{D_2}{D_1}\right)^2 = 32\,\frac{\mathrm{m}}{\mathrm{s}}\cdot\left(\frac{25\,\mathrm{mm}}{35\,\mathrm{mm}}\right)^2 = 16{,}33\,\frac{\mathrm{m}}{\mathrm{s}}.$$

The condition from above can also be used to rewrite the expression from before as:

$$\begin{aligned} R_x + p_1 A_1 &= V_2\left(\rho V_2 A_2\right) - V_1\left(\rho V_1 A_1\right)\\ R_x &= V_2\left(\rho V_2 A_2\right) - V_1\left(\rho V_1 A_1\right) - p_1 A_1\\ &= V_2\left(\rho V_2 A_2\right) - V_1\left(\rho V_2 A_2\right) - p_1 A_1\\ &= \left(\rho V_2 A_2\right)\left(V_2 - V_1\right) - p_1 A_1. \end{aligned}$$

Plugging in all the known values we get:

$$R_x = \left(1000\,\frac{\mathrm{kg}}{\mathrm{m}^3}\cdot 32\,\frac{\mathrm{m}}{\mathrm{s}}\cdot\pi\cdot(12{,}5\,\mathrm{mm})^2\right)\left(32\,\frac{\mathrm{m}}{\mathrm{s}} - 16{,}33\,\frac{\mathrm{m}}{\mathrm{s}}\right) - 510\,\mathrm{kPa}\cdot\pi\cdot(17{,}5\,\mathrm{mm})^2 = -245\,\mathrm{N}.$$

Therefore a force of $R_x = -245\,\mathrm{N}$ would be applied to the nozzle. The sign tells us that the force acts against the control volume to the left meaning the nozzle must itself be in tension.

Exercise 4.14

The radial-flow turbine in an automotive turbocharger has a diameter of 60 mm and rotates at 100 000 RPM. An air flow of 0,016 kg/s enters the turbine wheel at 60 m/s and an angle of 25° as shown in Figure 0.5. The air flow leaving has negligible angular momentum. Determine the power (W and hp) the turbine produces.

In this exercise the angular-momentum principle for an inertial reference system will be used. It is:

$$\mathbf{r}\times\mathbf{F}_s + \int_{\mathrm{CV}}\mathbf{r}\times\mathbf{g}\rho\,\mathrm{d}V + \mathbf{T}_{\mathrm{shaft}} = \frac{\partial}{\partial t}\int_{\mathrm{CV}}\mathbf{r}\times\mathbf{V}\rho\,\mathrm{d}V + \int_{\mathrm{CS}}\mathbf{r}\times\mathbf{V}\rho\mathbf{V}\cdot\mathrm{d}\mathbf{A}.$$

As we assume that there is no body or surface forces acting on the system and that the flow is steady this reduces to:

$$\mathbf{T}_{\mathrm{shaft}} = \int_{\mathrm{CS}}\mathbf{r}\times\mathbf{V}\rho\mathbf{V}\cdot\mathrm{d}\mathbf{A}.$$

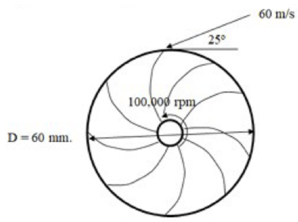


Figure 0.5:

In scalar form this becomes

$$T_{\text{shaft}} = r_o V_{\tan} \rho Q = r_0 V_{\tan} \dot{m}.$$

The tangential velocity is the component of the velocity tangent to the impeller as

$$V_{\tan} = 60\,\frac{\text{m}}{\text{s}} \cdot \cos 25^\circ = 54{,}4\,\frac{\text{m}}{\text{s}}.$$

And therefore the torque produced by the rotor is:

$$T_{\text{shaft}} = r_o V_{\tan} \dot{m} = \frac{60\,\text{mm}}{2} \cdot 54{,}4\,\frac{\text{m}}{\text{s}} \cdot 0{,}016\,\frac{\text{kg}}{\text{s}} = 0{,}0261\,\text{N}\,\text{m}.$$

The power produced is given as:

$$\dot{W} = T_{\text{shaft}} \omega$$

where ω is the rotational frequency. The speed of 1600 RPM is therefore converted to frequency as:

$$\omega = 100\,000\,\text{RPM} \cdot 2\pi \text{rev}^{-1} \cdot \frac{\text{min}}{60\,\text{s}} = 10\,470\,\text{s}^{-1}.$$

Therefore the power is:

$$\dot{W} = T_{\text{shaft}} \cdot \omega = 0{,}0261\,\text{N}\,\text{m} \cdot 10\,470\,\text{s}^{-1} = 273\,\text{W} = 0{,}367\,\text{hp}.$$