Dynamics and Vibrations – Excercises

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Lecture 1: Introduction — 26. August 2025

Exercise 1.1

We consider the rotation of a rigid body shown in **Figure 0.1** about the point O. The rod has mass m and the spring fastened to the rod at distance L from the point O has stiffness k. A force $F(t)$ is applied at a distance $L/2$ from the point O.

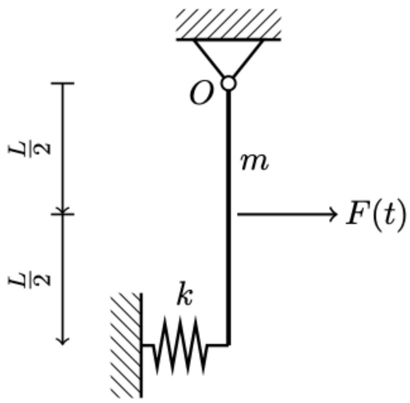

Figure 0.1:

a) Derive the non-linear equation of motion for the rotation of the rigid body.

We determine the resultant moment around O as:

$$I_O\ddot{\theta} = -kL\sin\theta L\cos\ \theta - mg\frac{L}{2}\sin\theta + F\frac{L}{2}\cos\theta$$

$$I_O\ddot{\theta} + \sin\theta\left(kL^2\cos\theta + mg\frac{L}{2}\right) = F\frac{L}{2}\cos\theta$$

.

The moment of inertia for a rod rotating about its end is given by $I_O = \frac{1}{3}mL^2$. Therefore we get:

$$\frac{1}{3}mL^2\ddot{\theta} + \sin\theta\left(kL^2\cos\theta + mg\frac{L}{2}\right) = F\frac{L}{2}\cos\theta.$$

b) Linearize the non-linear equation of motion using the assumption of small rotations.

By assuming $\cos\theta \approx 1$ and $\sin\theta \approx \theta$ we get:

$$\frac{1}{3}mL^2\ddot{\theta} + \theta\left(kL^2 + mg\frac{L}{2}\right) = F\frac{L}{2}.$$

Exercise 1.2

A person weighing 75 kg is standing on a scale in an elevator as shown on **Figure 0.2**. During the first 3 seconds of vertical motion upwards from rest, the force in the cable is 8300 N. Determine what the scale shows (in Newton) in this configuration. It is noted that the total mass of the elevator, person and scale is 750 kg.

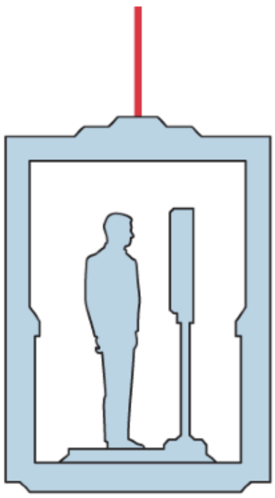

Figure 0.2:

Let $m_p = 75\,\text{kg}$ denote the mass of the person and $m = 750\,\text{kg}$ denote the entire mass of the system. Also let $F_c = 8300\,\text{N}$ denote the force in the cable. Newton's second law on the entire system gives

$$m\ddot{d} = F_c - mg \implies \ddot{d} = \frac{F_c - mg}{m}.$$

For the person inside the elevator using Newton's second law gives

$$m_p\ddot{d} = F_R - m_pg \implies F_R = m_p\ddot{d} + m_pg$$

where F_R is the reactant force the scale exerts on the person (i.e. the force the person exerts on the scale). Substituting in the previous expression for $\ddot{d}$ we get

$$\begin{aligned}
F_R &= m_p\left(\ddot{d} + g\right) \\
&= m_p\left(\frac{F_c - mg}{m} + g\right) \\
&= m_p\left(\frac{F_c}{m}\right) \\
&= \frac{m_p F_c}{m} \\
&= \frac{75\,\mathrm{kg} \cdot 8300\,\mathrm{N}}{750\,\mathrm{kg}} \\
&= 830\,\mathrm{N}.
\end{aligned}$$

Therefore the scale will show a reading of $830\,\mathrm{N}$.