Introduction to Scientific Computing with Python

Assignment 3

Saturday January 25, 2020

What to turn in: Copy the text from your scripts and paste it into a document. If a question asks you to plot or display something to the screen, also include the plot and screen output your code generates. Submit a file named **`Myname.pdf`** either by upload or email.

1. Consider the numerical solution of the equation $y'' + y = 0$ as we have done the with the explicit Euler discretization for $[0, T]$ and $T = 4\pi$. Recall that the ODE has to be rewritten as a system with $u = y$ and $v = y'$. Compar the numerical results of the Euler method with the so called Euler-Cromer method which is defined by

$$v_{n+1} = v_n - hu_n \tag{1}$$

$$u_{n+1} = u_n + hv_{n+1} \tag{2}$$

   for $n \geq 0$ and $h = t_{n+1} - t_n$ being the stepsize. The intial conditions are supplied by $u_0 = y(0)$ and $v_0 = 0$. The comparison of the numerical results should be done both by the numerical values and graphically for the two methods.

2. Solve the boundary value problem $u'' = -6x$ for $-1 < x < 1$ with boundary conditions $u(-1) = 2$ and $u(1) = 0$ analytically. Further implement a Python program which verifies your calculations numerically. Hint: Use the code **`BVP.py`** provided with the lecture material.

3. Show that the numerical solution of the Laplace equation with the boundary conditions

$$\begin{aligned} u(x,0) &= 1 \\ u(x,c) &= 1 + c \\ u(0,y) &= 1 + y \\ u(b,y) &= 1 + y \end{aligned}$$

   on the domain $\Omega = [0, b] \times [0, c]$ with $0 \leq x \leq b$ and $0 \leq y \leq c$ for values of $b > 0$ and $c > 0$ by the discretization we have considered in class gives the exact solution. Note: This can be shown by verifying the the discretization error is identically zero.

   Verify your results numerically for $b = c = 2$ by applying the code **`laplace.py`** provided in **`Laplace_v1`**.