# CSE 215
# Homework 5

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**Question 1: Problem 1**

Write the first four terms of the following sequence:
$e_n = \left\lfloor \frac{n}{2} \right\rfloor * 2$, for all integers $n \geq 0$.
Answer: $0, 0, 2, 2$

**Question 2: Problem 2**

Write the first ten terms of the following sequence:
$g_n = \lfloor \log_2 n \rfloor$ for all integers $n \geq 1$.
Answer: $0, 1, 1, 2, 2, 2, 2, 3, 3, 3$

**Question 3: Problem 3**

Find an explicit formula to represent a sequence with the following initial terms: -1, 1, -1, 1, -1, 1.
Answer: $a_n = (-1)^n$ where $a_1 = -1$

**Question 4: Problem 4**

Find an explicit formula to represent a sequence with the following initial terms: 0, 1, -2, 3, -4, 5.
Answer: $a_n = (-1)^n(n-1)$ where $a_1 = 0$

**Question 5: Problem 5**

Find an explicit formula to represent a sequence with the following initial terms: $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}$.
Answer: $a_n = \frac{n}{n+1}$ where $a_1 = \frac{1}{2}$

**Question 6: Problem 6**

Compute the summation: $sum_{k=1}^{5}(k+1)$.
Answer: $2+3+4+5+6 = 20$

**Question 7: Problem 7**

Compute the product: $\Pi_{k=2}^{4}(k^2)$.
Answer: $4 \cdot 9 \cdot 16 = 36 \cdot 16 = 360 + 216 = 576$

**Question 8: Problem 8**

What is: $\frac{100!}{98!}$?
Answe: $100 \cdot 99 = 9900$

**Question 9: Problem 9**

Reduce the following so that it has no factorial: $\frac{(n-1)!}{(n+1)!}$.
Answer: $\frac{(n-1)!}{(n+1)!} = \frac{(n-1)!}{(n+1)(n)(n-1)!} = \frac{1}{n(n+1)}$

**Question 10: Problem 10**

Use mathematical induction to prove that shows the following: $1^2 + 2^2 + \cdots + n^2 \equiv \frac{n(n+1)(2n+1)}{6}$ for all integers $n \geq 1$.

Base case: $n = 1$

$$n^2 = 1^2 = \frac{1(1+1)(2(1)+1)}{6} = \frac{1(2)(3)}{6} \equiv 1$$

Inductive Hypothesis: Assume that the formula holds for $n = k$.
Inductive Step: Show that the formula holds for $n = k + 1$.

$$1^2 + 2^2 + \cdots + k^2 + (k+1)^2 = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$
$$\frac{k(k+1)(2k+1) + 6(k+1)^2}{6} = \frac{(k+1)(k(2k+1) + 6(k+1))}{6}$$
$$\frac{(k+1)(2k^2 + k + 6k + 6)}{6} = \frac{(k+1)(2k^2 + 7k + 6)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

**Question 11: Problem 11**

Use mathematical induction to prove that shows the following: $1 + 3 + 5 + \cdots + (2n-1) \equiv n^2$ for all integers $n \geq 1$.

Answer:
Base case: $n = 1$

$$1 = 1^2 \equiv 1$$

Inductive Hypothesis: Assume that the formula holds for $n = k$.
Inductive Step: Show that the formula holds for $n = k + 1$.

$$1 + 3 + 5 + \cdots + (2k-1) + (2(k+1)-1) = k^2 + 2k + 1$$
$$k^2 + 2k + 1 = (k+1)(k+1) = (k+1)^2$$

**Question 12: Problem 12**

Use mathematical induction to prove that: $2 + 4 + 6 + ... + 2n \equiv n^2 + n$ for all integers $n \geq 1$ .

Answer:
Base case: $n = 1$

$$2(1) = 1^2 + 1 \equiv 2$$

Inductive Hypothesis: Assume that the formula holds for $n = k$.
Inductive Step: Show that the formula holds for $n = k + 1$.

$$2 + 4 + 6 + \cdots + 2k + 2(k+1) = k^2 + k + 2(k+1)$$
$$k^2 + k + 2k + 2 = k^2 + 3k + 2 = (k^2 + 2k + 1) + 2k + 1 = (k+1)^2 + (k+1)$$

**Question 13: Problem 13**

Use mathematical induction to prove that: $4^n - 1$ is divisible by 3 for each integer $n \geq 1$.

Answer:

Base case: $n = 1$

$$(4^1 - 1) = 3 \text{ which is divisible by } 3$$

Inductive Hypothesis: Assume that the formula holds for $n = k$.
Inductive Step: Show that the formula holds for $n = k + 1$.

$$4^{k+1} - 1 = 4 \cdot 4^k - 1 = 4 \cdot 4^k - 4 + 3 = 4(4^k - 1) + 3$$

Since $4^k - 1$ is divisible by 3, $4(4^k - 1)$ is also divisible by 3. Therefore, $4^{k+1} - 1$ is divisible by 3.

**Question 14: Problem 14**

Use mathematical induction to prove that: $n^3 - n$ is divisible by 6 for all integers $n \geq 2$.

Answer:
Base case: $n = 2$

$$n^3 - n = 2^3 - 2 = 8 - 2 = 6 \text{ which is divisible by } 6$$

Inductive Hypothesis: Assume that the formula holds for $n = k$.
Inductive Step: Show that the formula holds for $n = k + 1$.

$$(k+1)^3 - (k+1) = (k+1)(k^2 + 2k + 1) - (k+1)$$
$$(k+1)(k^2 + 2k + 1 - 1) = (k+1)(k^2 + 2k) = (k+1)k(k+2)$$
$$(k+1)(k)(k+2) = (k^3 + 3k^2 + 2k) = (k^3 - k) + 3k^2 + 3k$$
$$= (k^3 - k) + 3k(k+1)$$

Since $k^3 - k$ is divisible by 6, and $3k(k+1)$ is divisible by 6, $(k+1)^3 - (k+1)$ is divisible by 6.