AP Physics C - Electricity and Magnetism

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Chapter 1

Electrostatics

1.1 Electric Charge, Electric Force, and Electric Field

1.1.1 Electric Charge

Definition 1.1.1: Electric Charge

On the macro sale, an object's charge is the sum of the charges of its constituent particles.
Electric charge is a fundamental property of matter. It is quantized, meaning that it comes in discrete units. The unit of charge is the **coulomb** (C).

1. Charge is quantized.
2. Charge comes in two flavors: positive and negative.
3. Charges experience a force at a distance.
4. Charge is conserved.
5. Most mobile charge carriers are electrons.
6. The Coulomb is the SI unit of charge.

Note:-

The charge of an electron is $-e = -1.6 \times 10^{-19}$ C, and the charge of a proton is $+e = 1.6 \times 10^{-19}$ C.
The mass of an electron is 9.11×10^{-31} kg, and the mass of a proton is 1.67×10^{-27} kg.

Definition 1.1.2: Change in charge

Charge of an object can change by adding or removing electrons.
The ways in which an object can be charged are:

- Friction $\longrightarrow$ rubbing
- Conduction $\longrightarrow$ contact
- Induction $\longrightarrow$ no contact, except for grounding - polarizing, ground, remove ground, remove polarizing object
- Grounding $\longrightarrow$ contact with the earth - neutralizes charge

Claim 1.1.1 Conservation of Charge

The total charge of an isolated system is constant.

$$\sum_{i=1}^{n} q_i = \text{constant}$$

Charge is neither created nor destroyed. It is quantized. The number of protons and electrons in the universe is constant.

1.1.2 Conductors and Insulators

Definition 1.1.3: Insulators

An insulator is a material in which electrons are not free to move.
Examples: Rubber, glass, plastic, wood, air, etc.

Definition 1.1.4: Conductors

A conductor is a material in which electrons are free to move.
Examples: Metals, salt water, etc.

Definition 1.1.5: Superconductors

A superconductor is a material that has zero resistance to the flow of electric charge. Perfect conductors
Examples: Mercury, lead, etc.

Definition 1.1.6: Semiconductors

A semiconductor is a material that has a conductivity between that of an insulator and a conductor.
Examples: Silicon, germanium, etc.

1.1.3 Polarization

Definition 1.1.7: Polarization

Polarization is the separation of charges within an object.
This occurs when a charged object is brought near a neutral object that is a conductor.
No net charge is transferred.

1.1.4 Coulomb's Law / Electric Force

Definition 1.1.8: Coulomb's Law

The electrical force between two charged objects is directly proportional to the product of the quantity of charge on the objects and inversely proportional to the square of the separation distance between the two objects. The direction is determined by charges.

$$F = k\frac{|q_1 q_2|}{r^2}$$

$$k = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \approx 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

$$F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}$$

$$\varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$$

Note:-

The electrical force is a conservative force. It is also a field force/ action-at-a-distance force / non-contact force. Therefore, work doesn't depend on the path taken.

Note:-

The four fundamental forces are:

1. Gravitational Force
2. Electromagnetic Force
3. Strong Nuclear Force
4. Weak Nuclear Force

Columb's Law is a special case of the electromagnetic force.

Question 1: Calculate F_e and F_g between an electron and proton

$r = 5.3 \times 10^{-11} m$
$m_e = 9.11 \times 10^{-31} kg$
$m_p = 1.67 \times 10^{-27} kg$

$$F_e = k\frac{|q_1 q_2|}{r^2}$$

$$F_e = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(5.3 \times 10^{-11})^2} = 8.2 \times 10^{-8} N$$

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$F_g = \frac{6.67 \times 10^{-11} \times 9.11 \times 10^{-31} \times 1.67 \times 10^{-27}}{(5.3 \times 10^{-11})^2} = 3.6 \times 10^{-47} N$$

Question 2: Hanging Charged Spheres

2 25 gram spheres hang from light strings that are 35 cm long. They repel each other and carry the same negative charge. The two strings are seperated by 10 degrees.
Find the magnitude of the charge on each sphere.

$$F_e = k\frac{|q_1 q_2|}{r^2}$$

$$F_g = 0.025kg \times 9.8m/s^2 = 0.245N$$

$$\theta = \frac{10}{2} = 5$$

$$T\cos(\theta) = F_g = 0.245N$$

$$T = \frac{T}{\cos(\theta)} = \frac{0.245N}{\cos(5)} = 0.245N/0.9962 = 0.246N$$

$$T_x = T\sin(\theta) = 0.246N\sin(5) = 0.0214N$$

$$F_e = T_x = k\frac{|q_1 q_2|}{r^2}$$

$$r = 0.35m\sin(\theta) \times 2 = 0.061m$$

$$F_e = T_x = 0.0214N = 9 \times 10^9 \frac{q^2}{(0.061m)^2}$$

$$q = \sqrt{\frac{0.0214N \times (0.061m)^2}{9 \times 10^9}} = 9.37 \times 10^{-8}C$$

1.1.5 Electric Field

Definition 1.1.9: Electric Field

The electric field is a vector field that associates to each point in space the force experienced by a small positive test charge placed at that point.
The electric field is the ratio of force to charge.

$$E = \frac{\vec{F}_{net}}{q}$$

"Generalized description of electric force that is independent of the test charge."
The electric field created by a single point particle of charge Q is given by:

$$E = \frac{kQ}{r^2}\hat{r} = \frac{kQ}{r^2}$$

$\hat{r}$ is the unit vector pointing from the charge to the point in space where the electric field is being calculated.

Question 3: Finding electric field strength and direction

$Q = -8\mu C$
$r = 0.1m$
$q_0 = 0.02\mu C$
A) What is the electric field strength and direction q_0 experiences at r?

$$E = \frac{kQ}{r^2} = \frac{9 \times 10^9 \times -8 \times 10^{-6}}{0.1^2} = -7.2 \times 10^3 N/C$$

B) How would $\vec{E}$ change if you doubled the charge of q_0?
Answer: The electric field strength would double.

1.1.6 Calculating Electric Field

Definition 1.1.10: Continous charge distributions

$$\vec{E} = \sum_i k \frac{q_i}{r_i^2} \hat{r}_i$$

summation becomes an integral

$$\vec{E} = \int k \frac{dq}{r^2} \hat{r}$$

What does this mean?
Integrate over all charges (dq) in the distribution.
r is the vector from dq to the point at which E is defined.
Charge Density:

$$\lambda = \frac{Q}{L} \text{ Coulombs/meter - linear}$$

$$\sigma = \frac{Q}{A} \text{ Coulombs/meter}^2 \text{ - surface}$$

$$\rho = \frac{Q}{V} \text{ Coulombs/meter}^3 \text{ - volume}$$

GEOMETRY:

$$A_{sphere} = 4\pi r^2$$

$$V_{sphere} = \frac{4}{3}\pi r^3$$

$$A_{cylinder} = 2\pi r^2 + 2\pi r h$$

$$V_{cylinder} = \pi r^2 h$$

What has more net charge? a) a sphere w/ radius 2m and volume charge density $\rho = 2\frac{C}{m^3}$.
b) a sphere with radius 2m and a surface charge density $\sigma = 2\frac{C}{m^2}$.
c) both A) and B) have the same net charge.
Answer:

$$Q_a = \rho V = \rho \frac{4}{3}\pi R^3$$

$$Q_b = \sigma A = \sigma 4\pi R^2$$

$$\frac{Q_a}{Q_b} = \frac{\rho \frac{4}{3}\pi R^3}{\sigma 4\pi R^2} = \frac{\rho R}{3\sigma} = \frac{2R}{3}$$

Note:-

Procedure of finding the electric field from a continuous charge distribution:

1. Identify an arbitrary charge element dq of the distribution. Label it with appropriate parameters that will depend (in general) on the element's position in the distribution.

2. Determine the "tiny" contribution dE this element makes to the field a tthe point you wish to calculate the field.

3. Apply symmetry considerations. Because the electric field is vector, the direction of the field contributed by an element will depend on the element's position. Look for a symmetrically placed element that might produce canceling effects. From these considerations, identify the "effective" contribution dE_{eff} from the element.

4. Express dE_{eff} in terms of just one variable. Determine the limits of this variable.

5. Perform the integration.

Question 4: Calculate the electric field at the center of a uniformly charged semi-circle.

Given a semi-circle with radius R and charge density λ.

$$\lambda = \frac{Q}{L} = \frac{Q}{\pi R}$$

$$dq = \lambda dL = \lambda R d\theta = \frac{Q}{\pi} d\theta$$

$$dE = \frac{kdq}{r^2} = \frac{kQ}{\pi r^2} d\theta$$

$$dE_{eff} = dE \sin\theta = \frac{kQ}{\pi r^2} \sin\theta d\theta$$

$$E_{eff} = \frac{kQ}{\pi r^2} \int_0^{\pi} \sin\theta d\theta = \frac{kQ}{\pi r^2} (-\cos\theta|_0^{\pi}) = \frac{2kQ}{\pi r^2} = \frac{Q}{2\pi\varepsilon_0 r^2}$$

Question 5: Now do this for a three quarters circles.

$$E_{eff} = \frac{kQ}{\pi r^2} \int_0^{\frac{3\pi}{2}} \sin\theta d\theta = \frac{kQ}{\pi r^2} (-\cos\theta|_0^{\frac{3\pi}{2}}) = \frac{kQ}{\pi r^2} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

1.1.7 Electric Field from Electric Dipole

Definition 1.1.11: Electric Dipole

An electric dipole is a pair of equal and opposite point charges separated by a distance.
The electric dipole moment is a measure of the separation of positive and negative charges in the dipole.
The electric dipole moment is a vector pointing from the negative charge to the positive charge and has a magnitude equal to the product of the charge and the separation distance: $p = qd$. **Calculating the electric field from a dipole:**
The distance from the dipole to the point in space where the electric field is being calculated is r and the distance between the charges is d.

$$E = \frac{kq}{(z+d/2)^2} - \frac{kq}{(z-d/2)^2}$$

$$E = \frac{kq}{z^2}\left[\left(1 - \frac{d}{2z}\right)^{-2} - \left(1 + \frac{d}{2z}\right)^{-2}\right]$$

$$E = \frac{kq}{z^2}\left[\left(1 + \frac{d}{z}\right) - \left(1 - \frac{d}{z}\right)\right]$$

$$E = \frac{kq}{z^2}\left[2\frac{d}{z}\right] = \frac{2kqd}{z^3} = \frac{p}{2\pi\varepsilon_0 z^3} \text{ where } p = qd$$

1.1.8 Electric Field Lines

Definition 1.1.12: Electric Field Lines

Lines of force on a test q. Show the direction of the force on a positive test charge.
Negative charges would have field lines pointing towards them.
Positive charges would have field lines pointing away from them.
Rules:

1. Lines are perpendiucular to the surface of a conductor.
2. Lines represent direction a positive test charge would b eforced in a region around Q.
3. Lines never cross.
4. Line density is proprotional to field strength.
5. Electric field lines have arrows to show direction unlike equipoential lines.

1.1.9 Electric Flux

Definition 1.1.13: Electric Flux

The electric flux through a surface is the product of the electric field and the component of the area perpendicular to the field.

$$\Phi = \vec{E} \cdot \vec{A} = EA\cos\theta$$

Electric flux is a measure of the number of electric field lines passing through a surface.

Definition 1.1.14: Gauss's Law

The electric flux through a closed surface is equal to the net charge enclosed by the surface divided by the permittivity of free space.

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}$$

Gauss's Law is a powerful tool for calculating electric fields.

Note:-
Gauss's Law is a powerful tool for calculating electric fields.

1.1.10 Parallel Plate Capacitors

Definition 1.1.15: Parallel Plate Capacitors

Two plates of charge +Q and -Q evenly distributed across either surface.
Inside the capacitor, $\vec{E}$ is uniform (lines parallel to each other, strenght is constant).
There are bendy edge cases; however, we will assume that the electric field is uniform.
The electric field is uniform between the plates and zero outside the plates.
The Electric Field in a parallel plate capacitor is equal to: $\vec{E} = \frac{Q}{\varepsilon_0 A}$
Kinematic equations are valid in a parallel plate capacitor as there is constant acceleration!

1.2 Electric Potential Energy

Definition 1.2.1: Potential Energy

Energy due to position in a *field*.

Note:-

A comparison between the Gravitational Field and Electric field.
Gravitational Field: $F_g = GmM/r^2 = mg$ where g is the field strength
Electric Field: $F_e = kQ/r^2 = qE$ where E is the field strength
Both are conservative forces, meaning that the work done by the force is independent of the path taken.

Note:-

Kinematics Recall:

$$W = \int F \cdot dr = \int F dr \cos\theta = \Delta KE$$

$$\Delta U = -W_{conservative} = -\Delta KE$$

Definition 1.2.2: Electric Potential Energy

$$W = \int F \cdot dr$$

$$F = k\frac{q_1 q_2}{r^2}$$

$$W = -k\frac{q_1 q_2}{r}\Big|_b^a$$

$$U_e = \frac{kq_1 q_2}{r}$$

Question 6: Total Energy to bring identical 3 charges from infinity to an equilateral triangle

$$W_{q_1} = 0$$

$$W_{q_2} = -k\frac{Q^2}{r}$$

$$W_{q_3} = -k\frac{Q^2}{r} - k\frac{Q^2}{r} = -2k\frac{Q^2}{r}$$

$$W_{total} = -k\frac{Q^2}{r} - k\frac{Q^2}{r} - 2k\frac{Q^2}{r} = -3k\frac{Q^2}{r}$$

$$\Delta U = 3k\frac{Q^2}{r}$$

Question 7: Now do the same thing if one charge is negative

Let's say $q_3 = -Q$ and $q_1 = q_2 = Q$

$$W_{q_1} = 0$$

$$W_{q_2} = -k\frac{Q^2}{r}$$

$$W_{q_3} = +k\frac{Q^2}{r} + k\frac{Q^2}{r} = 2k\frac{Q^2}{r}$$

$$W_{total} = k\frac{Q^2}{r}$$

$$\Delta U = -k\frac{Q^2}{r}$$

Now let's say $q_1 = -Q$ and $q_2 = q_3 = Q$

$$W_{q_1} = 0$$

$$W_{q_2} = +k\frac{Q^2}{r}$$

$$W_{q_3} = 0$$

$$W_{total} = k\frac{Q^2}{r}$$

$$\Delta U = -k\frac{Q^2}{r}$$

Question 8: Find the work to move a particle of charge +Q to a very far away position

This charge is originally near a charge of +Q, seperated by a distance -d and a charge of -2Q, seperated by a distance d.

$$E_i = E_1 + E_2 = k\frac{Q \times +Q}{d} + k\frac{Q \times -2Q}{d} = -k\frac{Q^2}{d}$$

$$E_f = 0$$

$$W = \Delta U = E_f - E_i = k\frac{Q^2}{d}$$

1.3 Electric Potential

Note:-

Recall:
Electric Fields: $\vec{E} = \frac{\vec{F}}{q}$ is a property of space, a force per unit charge, generalized description of electric force independent of the test charge.
Goal: "Energy per charge" property of space, generalized description of energy.

Definition 1.3.1: Electric Potential

Potential: $V = \frac{U}{q}$
Electric Potential is measured in Volts (V), which is equivalent to Joules per Coulomb. It is a scalar.

$$\Delta U_{A\to B} = -\int_A^B \vec{F} \cdot d\vec{l} = -q\int_A^B \vec{E} \cdot d\vec{l}$$

$$\Delta V_{A\to B} = \frac{-q\int_A^B \vec{E} \cdot d\vec{l}}{q} = -\int_A^B E d\vec{l} = -\int_A^B k\frac{q}{r^2} d\vec{l}$$

The change in electric potential between two points$(r_a \to r_b)$ **is:** $\Delta V_{AB} = k\frac{q}{r_b} - k\frac{q}{r_a}$

Question 9: Find where potential is zero

A charge of +2q is at the origin and a charge of -q is 10 cm away from the first charge on the x-axis. $q = 2\mu C$

$$V = k\frac{4\times10^{-6}}{r+.1m} + k\frac{-2\times10^{-6}}{r} = 0$$
$$\frac{2}{r+.1} - \frac{1}{r} = 0$$
$$2r = r + .1$$
$$r = .1m$$

The potential is zero at 20 cm from the origin.

But there is also a point between the two charges where the potential is zero.

$$V = k\frac{4\times10^{-6}}{.1m-r} + k\frac{-2\times10^{-6}}{r} = 0$$
$$\frac{2}{.1-r} - \frac{1}{r} = 0$$
$$2r = .1 - r \longrightarrow 3r = .1 \longrightarrow r = .0333m$$

The potential is zero at 6.67 cm from the origin.

Could there be a point where the potential is zero in the negative x direction?

$$V = k\frac{4\times10^{-6}}{r} + k\frac{-2\times10^{-6}}{r+.1m} = 0$$
$$\frac{2}{r} - \frac{1}{r+.1} = 0 \longrightarrow \frac{2}{r} = \frac{1}{r+.1}$$
$$2r + .2 = r \longrightarrow r = -.2m$$

Answer: No, there is no point in the negative x direction where the potential is zero, because the value above is negative in the negative x-direction (aka positive) and therefore gives the same values as our first part.

1.3.1 Voltage

Definition 1.3.2: Voltage

The change in electric potential.

Example 1.3.1 (A 12 Volt Battery)
12 Volts is the difference in electric potential between the positive and negative terminals of the battery.

Theorem 1.3.1 Electric field by differentiating the potential

$$\vec{E} = -\vec{\nabla}V$$
$$E_x = -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{\partial V}{\partial y}$$

1.3.2 Equipotential Surfaces & Lines

Definition 1.3.3: Equipotential Surfaces

A surface on which the electric potential is the same at every point.
Properties:

- Electric field lines are perpendicular to equipotential surfaces.
- No work is done in moving a charge along an equipotential surface.
- Equipotential surfaces are always perpendicular to electric field lines.

Definition 1.3.4: Equipotential Lines

A line on which the electric potential is the same at every point.
Properties:

- Electric field lines are perpendicular to equipotential lines.
- No work is done in moving a charge along an equipotential line.
- Equipotential lines are always perpendicular to electric field lines.

The change in electric potential between equipotential lines is constant.

1.3.3 Conductors and Equipotential Surfaces

Note:-

Conductors are equipotential surfaces.

1.3.4 Electric Potential on and in a conducting sphere

Question 10: Find the electric potential at radius r of a conducting sphere with charge(+Q) and radius(R)

Inside the conductor when $r < R$, the electric potential is constant, because the electric field is zero and the electric potential is therefore zero.

$$V_{in} = 0$$

Outside the conductor when $r > R$, the electric potential is the same as that of a point charge.

$$V_{out} = k\frac{Q}{r}$$

1.3.5 Electric Potential on and in a non-conducting sphere

Question 11: Find the electric potential at radius r of a non-conducting sphere with charge(+Q) and radius(

When $r > R$, the electric potential is the same as that of a point charge.

$$V_{out} = k\frac{Q}{r}$$

When $r < R$, charge is distributed uniformly throughout the sphere.

$$\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$EA = \frac{Q}{\epsilon_0}$$

$$Q = \rho V = \rho\frac{4}{3}\pi R^3$$

$$\rho = \frac{Q}{\frac{4}{3}\pi R^3}$$

$$A = 4\pi r^2$$

$$E = \frac{\rho\frac{4}{3}\pi r^3}{\epsilon_0 4\pi r^2} = \frac{\rho r}{3\epsilon_0}$$

$$V_{in} = \int_R^r Edr = \int_R^r \frac{\rho r}{3\epsilon_0}dr = \frac{\rho}{6\epsilon_0}r^2|_r^R = \frac{\rho}{6\epsilon_0}(r^2 - R^2)$$

1.4 Capacitance

Definition 1.4.1: Capacitance

Because each conductor is an equipotential surface, there is a potential difference(voltage) between the two conductors.
The ratio of the charge seperated to the potential difference created is called the capacitance.
It is a measure of the capacity of a capacitor to store charge.

$$C \equiv \frac{Q}{V}$$

$$\text{Units:}\frac{Coulombs}{Volt} = Farad(F)$$

Capacitance is a scalar.
Capacitance only depends on the geometry of the conductors and the permittivity of the medium between the conductors.

Theorem 1.4.1 Calclating Capacitance

- Assume the two conductors carry +Q and -Q respectively.
- Determin the electric field in the region between the conductors. This will often involve using Gauss's Law.

- Determine the potential difference between the conductors using the definition of potential difference.

$$V = \int_a^b \vec{E} \cdot d\vec{l}$$

- Use the definition of capacitance to find the ratio of Q to V.
- Q will always cancel out of the ratio.
- You can be careless with signs.

Example 1.4.1 (Calculating the capacitance of a parallel plate capacitor)
The two plates are seperated by distance d and have a charge of +Q and -Q.

$$E = \frac{\sigma}{\varepsilon_0}$$

$$\sigma = \frac{Q}{A}$$

$$\Delta V = -\int_0^d -\vec{E} \cdot d\vec{y} = \int_0^d \vec{E} \cdot d\vec{y}$$

$$\Delta V = \int_0^d \frac{\sigma}{\varepsilon_0} dy = \frac{\sigma}{\varepsilon_0} y|_0^d = \frac{\sigma d}{\varepsilon_0} = \frac{Qd}{A\varepsilon_0}$$

$$C = \frac{Q}{\Delta V} = \frac{A\varepsilon_0}{d}$$

1.4.1 Parallel Plate Capacitors

Definition 1.4.2: Parallel Plate Capacitors

Electrode = positive or negative conductor, usually used in circuits.
Notice that the difference (not finished)

Question 12: Derive the capacitance of a spherical capacitor

1. Gauss

$$EA = \frac{Q_e nc}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

2. Integrate

$$\delta V = -\int \vec{E} \cdot d\vec{l} \text{ dot product is negative} = +\int_R^r \frac{Q}{4r\pi l^2 \varepsilon_0} d\vec{l}$$

$$= \frac{Q}{4\pi\varepsilon_0} \left[-\frac{1}{l}\right]_R^r = \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{R} - \frac{1}{r}\right] = \frac{Q}{4\pi\varepsilon_0} \left[\frac{r-R}{rR}\right]$$

3. Capacitance

$$C = \frac{Q}{\Delta V}$$

$$C = \frac{Q}{\frac{Q}{4\pi\varepsilon_0}\left[\frac{r-R}{rR}\right]} = 4\pi\varepsilon_0 \frac{rR}{r-R}$$

1.4.2 Energy Stored in a Capacitor

Definition 1.4.3: Energy Stored in a Capacitor

The energy stored in a capacitor is equal to the work done to charge the capacitor.

$$dU = dq \cdot V$$

$$U = \int_0^Q \Delta V dq = \int_0^Q \frac{q}{C} dq = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CV^2$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}QV = \frac{1}{2}CV^2$$

Energy Density(Energy per unit volume): $u = \frac{1}{2}\varepsilon_0 E^2$j

Question 13: How much potential should you charge a $1.0\mu F$ capacitor to store 1J?

$$U = \frac{1}{2}CV^2$$

$$1J = \frac{1}{2}(1\mu F)V^2$$

$$2 * 10^6 J = V^2$$

$$V = \sqrt{2 * 10^6 \frac{J}{F}} = 1414.2V$$

Question 14: A 2.0 cm diameter capacitor with a 0.5mm distance is charged to 200V.

What is the total energy stored in the electric field and energy density?
a)

$$U = \frac{1}{2}C(\Delta V)^2$$

$$r = 1.0cm = 0.01m$$

$$d = 0.5mm = 0.0005m$$

$$A = \pi r^2 = \pi * 10^{-4}$$

$$C = \frac{\varepsilon_0 A}{d}$$

$$V = 200V$$

$$U = \frac{1}{2}\frac{\varepsilon_0 A}{d}V^2 = 1.1 * 10^{-7}$$

b) Double check

$$u_E = \frac{U}{volume}$$

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(0.01)^3 = 4.19 * 10^{-6} m^3$$

$$u_E = \frac{1.1 * 10^{-7}}{4.19 * 10^{-6}} = 0.026 J/m^3$$

Question 15: 60pJ of energy is stored in a 2cm cube. What is the electric field strength?

$$U = 60pJ = 60 \cdot 10^{-12} J$$

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{60 \cdot 10^{-12} J}{(0.02m)^3}$$

$$E = \sqrt{2\frac{60 \cdot 10^{-12} J}{(0.02m)^3 \varepsilon_0}} = 1302\frac{N}{C} = 1302 V/m$$

1.4.3 Dielectrics

Definition 1.4.4: Dielectrics

A dielectric is a non-conducting material.
When a dielectric is placed between the conductors of a capacitor it:

1. ensures that the plates do not "short out"
2. increases the capacitance of the capacitor

The dielectric material will distort at the molecular level and become an induced dipole.
At the surfaces of the dielectric, an induced surface charge density appears, with polarity opposite the neighboring plate.

From positive to negative of the entire capacitor their is an E_{applied} and $E_{\text{induced}}$from positive to negative of the capacitor in the capaitor that goes the opposite way of the E_{applied}. Thus, $E_{\text{total}} = E_{\text{applied}} - E_{\text{induced}}$

$$C_{\text{dielectric}} = \frac{Q}{\Delta V} = \frac{Q}{E_{\text{applied}} - E_{\text{induced}}} > \frac{Q}{E_{\text{applied}}}$$

$$C_{\text{dielectric}} = \kappa C_{\text{without dielectric}}$$

Note:-

Change induced from a dielectric:

$$C' = \kappa C = \kappa\frac{Q}{V} = \frac{Q}{V'}$$

$$V' = \frac{V}{\kappa}$$

$$E' = \frac{E}{\kappa}$$

Question 16: A capacitor uses 0.6mm paper as a dielectric to its max sustainable voltage

Max sustainable Voltage: $E_{\max} = 16 \cdot 10^6 V/m$

$$\kappa = 3.7 \text{ for paper}$$

a) What is the max voltage the capacitor can hold?

$$\Delta V = E \cdot d = 16 \cdot 10^6 V/m \cdot 0.6 \cdot 10^{-3} m = 9600V = 9.6kV$$

b) what is the strength of the induced field?

$$E = E_{\text{applied/without dielectric}} - E_{\text{induced}} = 16 \cdot 10^6 V/m$$
$$E = 16 \cdot 10^6 V/m - 16 \cdot 10^6 V/m \cdot 3.7 = -43.2 \cdot 10^6 V/m = -E_{\text{induced}}$$
$$E_{\text{induced}} = 43.2 \cdot 10^6 V/m = 4.32 \cdot 10^7$$

Question 17: Energy of a capacitor with vs. without a dielectric with the same voltage

$$U_0 = \frac{1}{2} C V^2$$
$$U_1 = \frac{1}{2} \kappa C' V^2$$
$$\kappa > 1$$
$$U_0 < U_1$$

Question 18: Capacitor with a dielectric of thickness d/2

$$C_0 = \frac{\varepsilon_0 A}{d} = C_{\text{without dielectric}}$$
$$\frac{1}{C} = \frac{1}{C_1} + C_2 = \frac{1}{C_{\text{with half dielectric}}}$$
$$C_1 = \frac{\varepsilon_0 A}{d/2} = 2C_0$$
$$C_2 = \kappa \frac{\varepsilon_0 A}{d/2} = \kappa 2 C_0$$
$$\frac{1}{C} = \frac{1}{2C_0} + \frac{1}{\kappa 2 C_0}$$
$$C = \frac{\kappa 2 C_0 + 2C_0}{\kappa + 1}$$

Chapter 2

Circuits

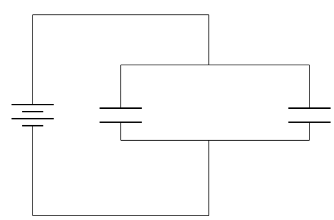
2.1 Circuits with Capacitors

2.1.1 Parallel Capacitor Circuits

Definition 2.1.1: Parallel Capacitor Circuits

- The voltage across each capacitor is the same.
- The charge on each capacitor is different.
- The total charge is the sum of the charges on each capacitor.
- The total capacitance is the sum of the capacitances of each capacitor.

$$C_{\text{total}} = C_1 + C_2 + C_3 + \dots$$



2.2 Electric Current

Definition 2.2.1: Electric Current

The flow of charge.

$$I = \frac{Q}{t}$$

$$i = \frac{dq}{dt}$$

Units: Ampere(A) = Coulombs/second
Conventional Current: The flow of positive charge.

2.2.1 Thermal Effects of Current

Definition 2.2.2: Thermal Motion

Thermal motion: electrons move rapidly but randomly.

$$v_{\text{thermal}} \approx 10^6 m/s$$

2.2.2 Drift Motion

Definition 2.2.3: Drift Motion

Drift motion: electrons move slowly in the direction of the electric field.

$$v_{\text{drift}} \approx 10^{-4} m/s \text{ DEPENDS ON MATERIAL!}$$

$$I = nAeV_d$$

$$\mathbf{V_d} = \frac{\mathbf{I}}{\mathbf{nAe}} \text{ n is the electron density}$$

Definition 2.2.4: Drift velocity and current density

$$Q = n_e \cdot A \cdot L$$

$$i = \frac{Q}{t} = \frac{n_e \cdot A \cdot L}{\frac{L}{v_d}} = n_e \cdot A \cdot v_d$$

$$J = \frac{i}{A} = n_e \cdot v_d$$

$$\mathbf{J} = \mathbf{n_e} \cdot \mathbf{v_d} = \rho \mathbf{v_d} \text{ where } \rho \text{ is the number of electrons and e is the charge of an electron}$$

2.2.3 Current Density

Definition 2.2.5: Current Density

The current per unit area.

$$J = \frac{i}{A}$$

$$di = \vec{J} \cdot d\vec{A}$$

$$i = \int di = \int \vec{J} \cdot d\vec{A}$$

2.2.4 Direct Current vs. Alternating Current

Definition 2.2.6: Direct Current

Current that flows in one direction.

Definition 2.2.7: Alternating Current

Current that changes direction periodically.

2.3 Resistance and Resistivity

Definition 2.3.1: Resistance

The opposition to the flow of charge.

$$R = \frac{V}{I} \leftarrow I = \frac{V}{R}$$

$$R = \rho\frac{L}{A}$$

Units: Ohm(Ω) = Volt/Ampere
When a potential difference is maintained across a conductor, a current will be established within the conductor.
We could plot V vs. I, and if the graph is linear, the resistance is constant and the material/resistor obeys Ohm's Law as $V = IR$.

2.3.1 Ohm's Law

Definition 2.3.2: Ohm's Law

The current in a conductor is directly proportional to the potential difference across the conductor.

$$V = IR$$

$$I = \frac{V}{R}$$

Ohmic Materials: Materials that obey Ohm's Law.

$$\mathbf{J} = \sigma\mathbf{E}$$

$$\sigma = \text{conductivity} = \frac{1}{\rho}$$

$$R = \text{resistance}$$

$$\rho = \text{resistivity} = \frac{1}{\sigma}$$

Note:-

Because,

$$V = EL$$

$$I = JA$$

$$\rightarrow \frac{I}{A} = \sigma\frac{V}{L} \rightarrow I = \frac{V}{\frac{L}{\sigma A}}$$

Therefore,

$$\frac{L}{\sigma A} = R$$

Question 19: Two 5cm diameter disks seperated by pyrex glass are charged to 1000V

$$r = 2.5cm = 0.025m$$

$$d = 0.61mm = 0.00061m$$

$$V = 1000V$$

$$A = \pi r^2 = \pi(0.025)^2 = 0.00196m^2$$

What is the charge density on the disks?

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \cdot 10^{-12} \cdot 0.00196}{0.00061} = 2.85 \cdot 10^{-11} F$$
$$Q = CV = 2.85 \cdot 10^{-11} \cdot 1000 = 2.85 \cdot 10^{-8} C$$
$$\rho = \frac{Q}{V_{ol}} = \frac{Q}{A \cdot d} = 0.2379 \frac{C}{m^x}$$

Question 20: Comparing voltages across resistors

$$R \propto \frac{L}{A}$$
$$V = IR \propto \frac{L}{A}$$

A cylindrical resistor with radius d vs 2d:
Because the area of the resistor with radius 2d is 4 times the area of the resistor with radius d, the resistance of the resistor with radius 2d is 4 times less the resistance of the resistor with radius d. $R_{2d} < R_d$

A cylindrical resistor with length 2L vs L:
Because the length of the resistor with length 2L is 2 times the length of the resistor with length L, the resistance of the resistor with length 2L is 2 times the resistance of the resistor with length L. $R_{2L} > R_L$

Definition 2.3.3: Electrical Power

The rate at which electrical energy is converted to another form of energy.

$$P_{bat} = \frac{dU}{dt} = \frac{d(q\Delta V)}{dt} = \frac{dq}{dt}\Delta V = IV$$
$$P = IV$$
$$P = I^2 R$$
$$P = \frac{V^2}{R}$$

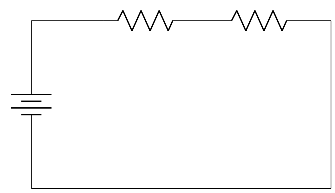
Units: Watt(W) = Joules/second

2.3.2 Resistors in Series

Definition 2.3.4: Resistors in Series

- The current is the same through each resistor.
- The voltage across each resistor is different.
- The total voltage is the sum of the voltages across each resistor.
- The total resistance is the sum of the resistances of each resistor.

$$R_{\text{total}} = R_1 + R_2 + R_3 + \dots$$

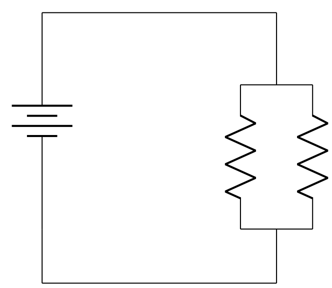


2.3.3 Resistors in Parallel

Definition 2.3.5: Resistors in Parallel

- The voltage is the same across each resistor.
- The current through each resistor is different.
- The total current is the sum of the currents through each resistor.
- The total resistance is the reciprocal of the sum of the reciprocals of the resistances of each resistor.

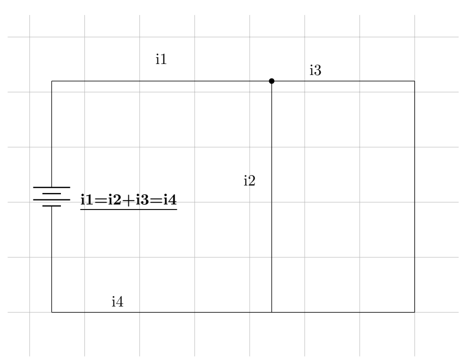
$$\frac{1}{R_{\text{total}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$



2.3.4 Kirchhoff's Rules

Definition 2.3.6: Kirchhoff's Rules

- **Junction Law:** The sum of the currents entering a junction is equal to the sum of the currents leaving the junction.
- **Loop Law:** The sum of the potential differences around any closed loop in a circuit is zero.



2.3.5 Terminal Voltage

Definition 2.3.7: Terminal Voltage

When charges move through a battery, they experience an "internal resistance" that ordinarily reduces the potential difference between the two terminals of the battery.
One can model a real battery as an ideal battery (with no internal resistance) in series with a resistor equal to the internal resistance.

$$V_{\text{terminal}} = \epsilon - Ir$$

2.3.6 RC Circuits

Definition 2.3.8: RC Circuits

Circuits that contain resistors and capacitors.
Charging: The capacitor charges up to the battery voltage.
Discharging: The capacitor discharges through the resistor.
Time Constant: The time it takes for the charge on the capacitor to reach 63.2% of its final value.

RC Time Constant

Definition 2.3.9: RC Time Constant

The time it takes for the charge on the capacitor to reach 63.2% of its final value.

$$\tau = RC$$

$$1 - \frac{1}{e} = 0.632$$

Charging a Capacitor

Definition 2.3.10: Charging a Capacitor

The capacitor charges up to the battery voltage.

$$\epsilon - \frac{q}{c} - iR = 0$$

$$i = \frac{dq}{dt}$$

$$\frac{dq}{dt} = \frac{\epsilon - q/c}{R} = \frac{c\epsilon - q}{Rc}$$

$$\frac{dq}{c\epsilon - q} = \frac{dt}{Rc}$$

$$\int_0^q \frac{dq}{c\epsilon - q} = \int_0^t \frac{dt}{Rc}$$

$$\int_0^q \frac{dq}{q - c\epsilon} = -\frac{1}{Rc}\int_0^t dt$$

$$\ln(q - c\epsilon)\big|_0^q = -\frac{t}{Rc}$$

$$\ln\left(\frac{q - c\epsilon}{-c\epsilon}\right) = -\frac{t}{Rc}$$

$$\frac{q - c\epsilon}{-c\epsilon} = e^{-\frac{t}{Rc}}$$

$$q = c\epsilon(1 - e^{-\frac{t}{Rc}}) = Q(1 - e^{-\frac{r}{RC}})$$

Charge, Current, and Voltage in a Charging Capacitor

Definition 2.3.11: Change in C, A, and V

$$q = Q(1 - e^{-\frac{r}{RC}})$$

$$I = \frac{\epsilon}{R}e^{-\frac{t}{RC}}$$

$$V = \epsilon(1 - e^{-\frac{t}{RC}})$$

Discharging a Capacitor

Definition 2.3.12: Discharging a Capacitor

The capacitor discharges through the resistor.

$$\frac{q}{c} + iR = 0 \longrightarrow \frac{q}{c} = -iR \longrightarrow -\frac{q}{RC} = i$$

$$i = \frac{dq}{dt} = -\frac{q}{RC}$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

$$\int_{Q_\infty}^{Q} \frac{dq}{q} = -\int_0^t \frac{dt}{RC}$$

$$\ln(q)|_{Q_0}^{Q} = -\frac{t}{RC}$$

$$\ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC}$$

$$\frac{Q}{Q_0} = e^{-\frac{t}{RC}}$$

$$Q = Q_0 e^{-\frac{t}{RC}}$$

Charge, Current, and Voltage in a Discharging Capacitor

Definition 2.3.13: Change in C, A, and V

$$q = Q_0 e^{-\frac{t}{RC}}$$

$$I = -\frac{Q_0}{RC} e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$

$$V = -Q_0 e^{-\frac{t}{RC}}$$

Chapter 3

Magnetism

3.1 Magnetic Fields and Forces

3.1.1 Magnetic Fields

Definition 3.1.1: Magnetic Fields

Properties:

- Magnetic fields are created by moving charges.
- Magnetic fields are vectors.
- Magnetic fields are measured in Tesla(T) or Gauss(G).
- Magnetic fields are created by moving charges.

3.1.2 Magnetic Force on a Moving Charge

Definition 3.1.2: Magnetic Force on a Moving Charge

The force on a moving charge in a magnetic field is perpendicular to both the velocity of the charge and the magnetic field.

$$F = q\vec{v}x\vec{B}$$

$$F = qvB$$

$$F = qvB\sin(90) = qvB$$

$$F = qvB\sin(180) = 0$$

3.1.3 Trajectories of Charged Particles

Definition 3.1.3: Trajectories of Charged Particles

When moving in a uniform field

3.1.4 Velocity Selector

3.1.5 Lorentz Force

Definition 3.1.4: Lorentz Force Law

The force on a charge moving in both an electric and magnetic field.

$$\vec{F} = q\vec{E} + q\vec{v}x\vec{B}$$

3.1.6 Hall Effect

Definition 3.1.5: Charges in a conductor - The Hall Effect

1. A current is introduced through a conductor, and a magnetic field is applied perpendicular to the current.
2. Charge carriers deflected. The direction of the deflection independent on the charge.
3. As charges deflect, an electric field is created acrosss the conductor, and the direction of the electric field is dependent on the charge of the charge carriers. The elctric field is perpendicular to both the current and the magnetic field.
4. Charges are deflected until $F_{\text{magnetic}} = F_{\text{electric}} \longrightarrow F_e = F_b$
5. The "Hall Voltage" can be measured to determine the sign and magnitude of the density of the charge carriers in the conductor.

$$V_H = Ed$$

$$qE = qv_dB$$

$$v_d = \frac{J}{ne} = \frac{i}{neA}$$

$$V_H = dv_dB$$

$$\text{Charge Density: } n = \frac{Bi}{V_Hde}$$

3.1.7 Compass Needles

Definition 3.1.6: Compass Needles

Compass needles align with the magnetic field.

3.2 Magnetic Materials

Definition 3.2.1: Why are some materials magnetic?

- Magnetic materials have magnetic domains.
- Magnetic domains are regions of aligned magnetic fields.
- When a magnetic field is applied, the domains align.
- When the magnetic field is removed, the domains remain aligned.

Question 21: Calculate B

A particle of charge q and mass m is accelertaed from rest by an electric field E through a distance d and enters and exists a region containing a constant magnetic field B at points shown.
Assume q, m, E, d, and x_0 are known.
The particle exits the magnetic field at $x = x_0/2$ and $y = x_0/2$.
First we find the velocity of the particle when it enters the magnetic field.

$$U = qV = qEd = \frac{1}{2}mv_0^2 \rightarrow v_0 = \sqrt{\frac{2qED}{m}}$$

Now, we need to find the radius of the particle's trajectory in the magnetic field.

$$R = \frac{1}{2}x_0$$

$$F = m\frac{v_0^2}{R} = qv_0B$$

$$B = m\frac{v_0^2}{Rv_0q} = m\frac{v_0}{Rq} = \frac{m}{Rq}\sqrt{\frac{2qED}{m}} = \frac{2}{x_0}$$

3.3 Magnetic Force on Currents

3.3.1 Magnetic Force on a Current-Carrying Wire

Definition 3.3.1: Magnetic Force on a Current-Carrying Wire

The force on a current-carrying wire in a magnetic field is perpendicular to both the current and the magnetic field.

$$F = I\vec{L}x\vec{B}$$

$$F = ILB\sin(90) = ILB$$

$$F = ILB\sin(180) = 0$$

3.3.2 Torque on a Current Loop

Definition 3.3.2: Torque on a Current Loop

The torque on a current loop in a magnetic field is perpendicular to both the current and the magnetic field.

$$\vec{\tau} = \vec{R}x\vec{F} \text{ where R is the distance from the axis of rotation}$$

$$\vec{\tau} = \vec{R}x\vec{F} = \vec{R}xI\vec{L}x\vec{B} = I\vec{A}x\vec{B}$$

$$\vec{\tau} = I\vec{A}x\vec{B}$$

$$\vec{\tau} = IAB\sin(90) = IAB$$

$$\vec{\tau} = IAB\sin(180) = 0$$

Note:-

How to find the direction of the torque:
It should be up the axis of rotation if the current is going counter-clockwise.
It should be down the axis of rotation if the current is going clockwise.

3.3.3 Magnetic Dipole Moment

Definition 3.3.3: Magnetic Dipole Moment

$$\mu \equiv NI\vec{A}$$

$$\vec{\tau} = NI\vec{A}x\vec{B} = \vec{\mu}x\vec{B}$$

$$\vec{\tau} = \vec{\mu}x\vec{B}$$

Note:-

μ makes torque easy!
The torque always wants to line μ up with B. It turns towards B.
Torque is greatest when μ is perpendicular to B.

3.3.4 Work Done By a Magnetic Field on a Current Loop

Definition 3.3.4: Work Done By a Magnetic Field on a Current Loop

$$W = \int \tau d\theta$$

$$\vec{\tau} = \vec{\mu}x\vec{B} = \mu B\sin(\theta)$$

$$W = \int \mu B\sin(\theta)d\theta = -\mu \cdot B\cos(\theta)|_{\theta_1}^{\theta_2} = \vec{\mu} \cdot \vec{B}(\cos(\theta_1) - \cos(\theta_2))$$

$$\Delta U = -W$$

$$U \equiv = -\vec{\mu} \cdot \vec{B}$$