

BC Calculus - Middlesex School
Lab - Approximating functions by Polynomials

Ben Feuer & Joey Casper

Contents

Chapter 1

Page 2

1.1	Introduction	2
1.2	Finding our approximation for e^x	2
1.3	Testing our approximation's accuracy	3
1.4	Applying this approximation to compute an integrand that has no antiderivative	4
1.5	Using the analytical method that is also used in finding Taylor polynomials	4

Chapter 1

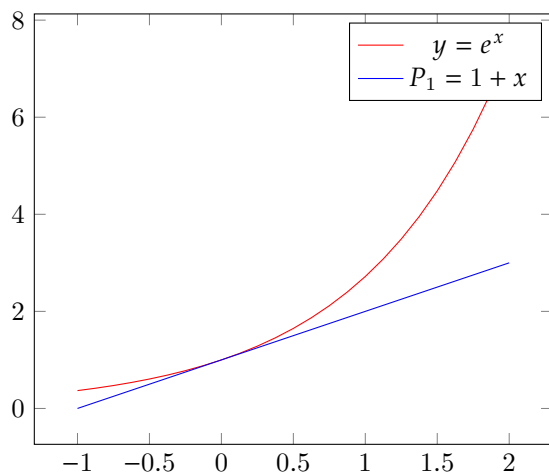
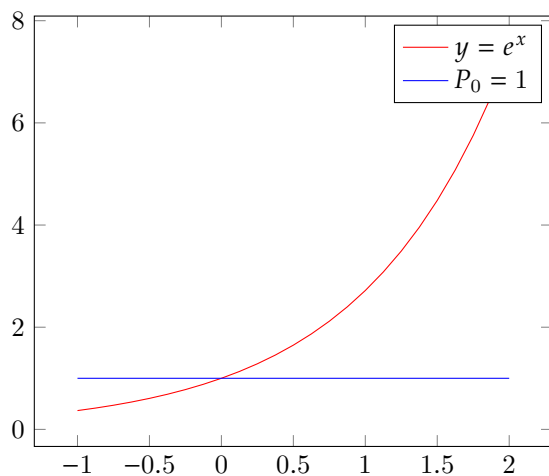
1.1 Introduction

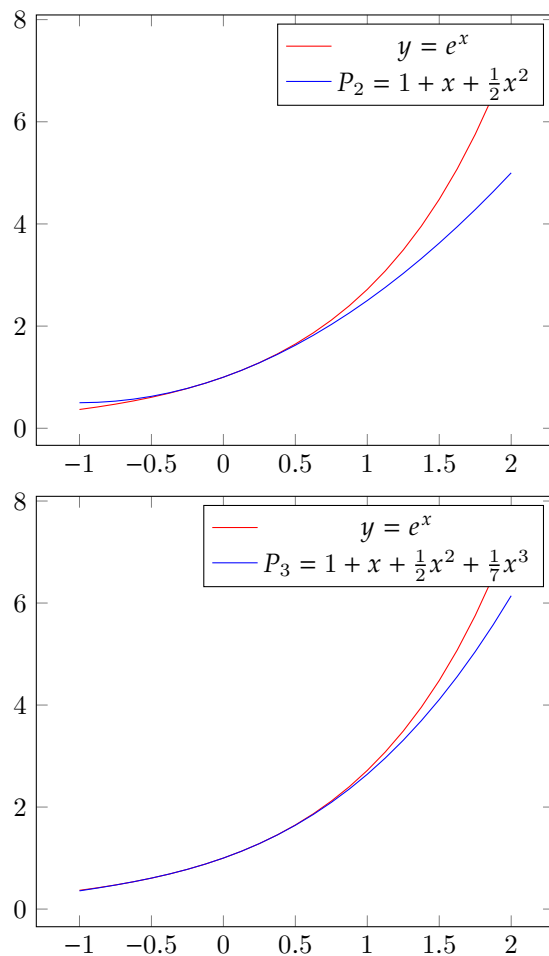
In this lab, we approximate e^x using polynomials in order to use our approximation for e^x to approximate the integral $\frac{1}{\sqrt{2\pi}} \int_0^{\frac{1}{2}} e^{-x^2}$.

1.2 Finding our approximation for e^x

To find our approximation, we used DESMOS in which we created a polynomial $y = 1 + ax + bx^2 + c^3$ where we manipulated the parameters, a, b, c in order to get the best fit polynomial to e^x from -1 to 1. Through this method we found that the parameters to be $a = 1$, $b = .5$, and $c = .25$.

Below are the steps we took in making our polynomial(adding another degree in each step):

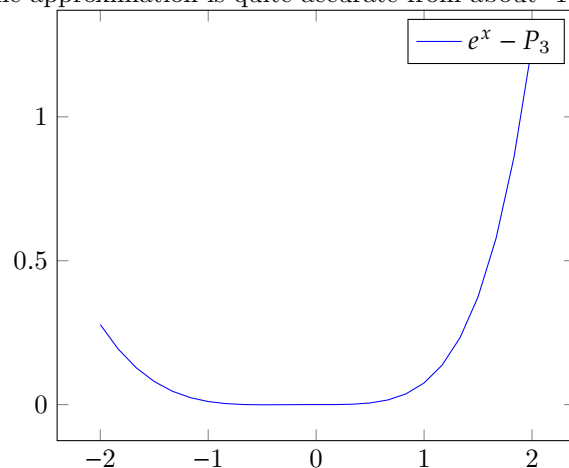




As shown above, our final approximation for e^x is $P_3 = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$.

1.3 Testing our approximation's accuracy

To test our approximation of e^x , we compare the function $P_3(0.5)$ with $e^{0.5}$. The difference between $P_3(0.5)$ and $e^{0.5}$ is 0.0058 with our approximation underapproximating the true value of e^x . This marginal underapproximation is less on the other points of the interval $[0, 0.5]$, and you can visualize this difference below where you can see how the approximation is quite accurate from about -1 to 1.



$f(x) = 1 + x + 1/2x^2 + 1/7x^3$			
nth derivative	$f(0)$	e^x	$d^n/dx^n [e^x] _{x=0}$
0	1		1
1	1		1
2	1		1
3	0.857142857143		1

Figure 1.1: Table that determines a,b, and c

1.4 Applying this approximation to compute an integrand that has no antiderivative

With this approximation for e^x , we can now evaluate $\int_0^{0.5} e^{-x^2} dx$ by using $P_3(-x^2)$. This gives us the new approximated function for e^{-x^2} which equals $P_3(-x^2) = 1 - x^2 + \frac{1}{2}x^4 - \frac{1}{7}x^6$. With this polynomial function we can now solve for the integral which is equal to:

$$\int_0^{0.5} 1 - x^2 + \frac{1}{2}x^4 - \frac{1}{7}x^6 dx = 0.461298894558$$

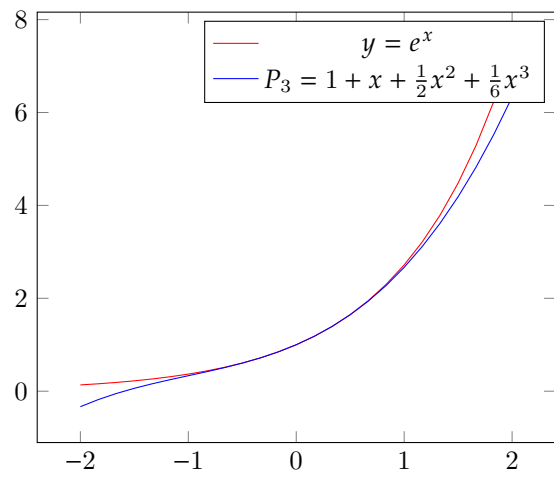
This is very similar to the answer the calculator gives for the actual function(0.461281006413) as it is equal up until the fifth decimal place.

1.5 Using the analytical method that is also used in finding Taylor polynomials

- $\frac{f'}{1} = 1$
- $\frac{f''}{\frac{1}{2}} = 2$
- $\frac{f'''}{\frac{1}{7}} = 6$

Now with these values for each degree (from 1 to 3) of the polynomial, we can now determine the cubic Taylor polynomial for the exponential function by adjusting the four coefficients so the values of the Taylor polynomial and its first three derivatives match those of e^x at $x = 0$.

Now we have a new value for the cubic Taylor polynomial ($C = \frac{1}{6}$). And thus now our new polynomial is



$y = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3$. This new polynomial can be shown below:
 where it is shown how close the two functions are to each other at $x = 0$