

AP Physics 1 Notes

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Contents

Chapter 1	AP PHYSICS 1 EXAM TOPICS AND DISTRIBUTION	Page 3
Chapter 2	Kinematics	Page 4
2.1	Motion	4
2.2	Changes in position	4
2.3	Velocity	4
2.4	Speed	4
2.5	Vector vs. Scalar	5
2.6	Describing motion	5
2.7	Acceleration	5
2.8	The five equations of motion	6
2.9	Free fall	6
2.10	Motion on a Ramp	6
2.11	Projectile motion	6
2.12	Relative motion	7
Chapter 3	Dynamics	Page 8
3.1	Forces and Newton's Laws of Motion	8
3.2	Types of Forces	8
3.3	Free Body Diagrams	10
3.4	Equilibrium	10
Chapter 4	Circular Motion/Gravity	Page 11
4.1	Circular Motion	11
4.2	Period, Frequency, Speed, Acceleration, and Force for Circular Motion	11
	Example questions —	12
4.3	Apparant forces in circular motion	12
4.4	Orbital motion	13
4.5	Newton's law of gravity	13
Chapter 5	Energy	Page 15
5.1	Energy and Work	15
5.2	Power	16

Chapter 6	Momentum	Page 17
6.1	Impulse, collisions, and momentum	17
6.2	Angular momentum	18
Chapter 7	Simple Harmonic Motion	Page 19
7.1	The spring force	19
7.2	Equilibrium and Oscillation	19
7.3	Frequency and Period	19
7.4	Pendulum	20
7.5	Describing SHM	20
7.6	Connecting SMH to circular motion	20
7.7	Energy in SHM	20
Chapter 8	Torque + Rotational Motion	Page 22
8.1	Rotational Kinematics	22
8.2	Relating angular and linear motion	23
8.3	Tangenetial Acceleration	23
8.4	Torque	23
8.5	Gravitational Torque and the Center of Gravity	24
8.6	Rotational Dynamics and Rotational Inertia	24
8.7	Rolling Motion	25
8.8	Torque and equilibrium	25
8.9	Stability and Balance	25
Chapter A	Scientific Notation, Significant Figures, and Unit Conversions	Page 26
A.1	Scientific Notation	26
A.2	Scientific Figures	26
A.3	SI Units	27
A.4	Unit Prefixes	27
Chapter B	Important Overall Concepts	Page 28
B.1	Closed and Open Systems	28
B.2	The analysis and use of graphs	28

Chapter 1

AP PHYSICS 1 EXAM TOPICS AND DISTRIBUTION

The AP test is made of two 90 minute sections. The first is multiple choice and the second is free response. The AP test is made up of 7 topics shown below:

1. Kinematics - 12-18%
2. Dynamics - 16-20%
3. Circular Motion/Gravity - 6-8%
4. Energy - 20-28%
5. Momentum - 12-18%
6. Simple Harmonic Motion - 4-6%
7. Torque/Rotation - 12-18%

Chapter 2

Kinematics

2.1 Motion

Definition 2.1.1: Motion

Motion is the change of an object's position or orientation over time. The four basic types of motion are straight-line, circular, projectile, and rotational.

2.2 Changes in position

Definition 2.2.1: Displacement

Displacement is the change in position of an object. It is a vector quantity. Displacement is labeled Δx .

Definition 2.2.2: Distance

Distance is the total length of the path traveled by an object. It is a scalar quantity. Distance is often labeled d .

2.3 Velocity

Definition 2.3.1: Velocity

Velocity is the rate of change of an object's position. It is a vector quantity. Velocity is labeled v and measured in $\frac{m}{s}$ or meters per second. It is a vector quantity as it is calculated through the following equation:

$$v = \frac{\Delta x}{t}$$

2.4 Speed

Definition 2.4.1: Speed

Speed is the magnitude of the change of an object's position. It is a scalar quantity. Speed is labeled s and measured in $\frac{m}{s}$ or meters per second. It is a scalar quantity as it is calculated through the following equation:

$$s = \frac{d}{t}$$

2.5 Vector vs. Scalar

Definition 2.5.1: Vector

A vector is a quantity that has both magnitude and direction. We graphically represent vectors with an *arrow*!

Theorem 2.5.1 Adding vectors

To add vectors, we use a vectors magnitude and direction to find the x and y components of each vector and then add the components to get a x_{net} and y_{net} and then gets the angle of the vector from the formula:
 $\theta = \arctan(\frac{y}{x})$

You can also add vectors visually via the head to tail method.

Definition 2.5.2: Scaler

A scaler is a quantity that has only magnitude.

2.6 Describing motion

Definition 2.6.1: Uniform motion

Uniform motion is motion at a constant velocity.

Definition 2.6.2: Instantaneous Velocity

Instantaneous velocity is the velocity of an object at a specific point in time.

2.7 Acceleration

Definition 2.7.1: Acceleration

Acceleration is the rate of change of an object's velocity. It is a vector quantity. Acceleration is labeled a and measured in $\frac{m}{s^2}$ or meters per second squared. It is a vector quantity as it is calculated through the following equation:

$$a = \frac{\Delta v}{t}$$

an object's acceleration is the slope of its velocity vs. time graph.

2.8 The five equations of motion

Definition 2.8.1: The five equations of motion

1.

$$v_f = v_i + at$$

2.

$$\Delta x = v_i t + \frac{1}{2}at^2$$

3.

$$v_f^2 = v_i^2 + 2a\Delta x$$

4.

$$\Delta x = \frac{1}{2}(v_i + v_f)t$$

5.

$$\Delta x = vt$$

2.9 Free fall

Definition 2.9.1: Free fall

Free fall is the motion of an object under the influence of gravity only.

The acceleration due to gravity is

$$g = 9.8 \frac{m}{s^2}$$

The acceleration due to gravity is always negative as it is in the opposite direction of the velocity.

g by definition is **always positive!** $\rightarrow a_y = -g$ or $\rightarrow a_y = -9.8 \frac{m}{s^2}$

The acceleration in terms of g will always equal:

$$\text{acceleration in terms of g's} = \frac{a}{9.8m/s^2}$$

2.10 Motion on a Ramp

Definition 2.10.1: Motion on a Ramp

Motion on a ramp is the motion of an object on a ramp.

The acceleration of an object due to gravity on a ramp is

$$a = g \sin \theta$$

2.11 Projectile motion

Definition 2.11.1: Projectile motion

A projectile is an object that moves in two dimensions under the influence of gravity and nothing else.

The horizontal and vertical components of projectile motion are independent of each other.

The horizontal component of projectile motion is constant velocity motion.

The time in the air is dependent on the vertical component of the motion.

Example 2.11.1 (Projectile motion)

In a soccer free kick, a player kicks a stationary ball toward the goal that is 18 m away. He kicks the ball at an angle of 22° from the horizontal at a speed of 23 m/s. How long does the ball take to reach the goal? And how far off the ground is the ball when it reaches the goal?

Solution:

$$v_{ix} = 23 \cos 22 = 21.3 \frac{m}{s}$$

$$v_{iy} = 23 \sin 22 = 8.6 \frac{m}{s}$$

$$\Delta x = v_{ix} t$$

$$\Delta t = \frac{\Delta x}{v_{ix}} = \frac{18m}{21.3 \frac{m}{s}} = 0.845s$$

$$y_f = v_{iy} 0.845s - \frac{1}{2} (9.8 \frac{m}{s^2}) (0.845s)^2 = 3.8m$$

2.12 Relative motion

Definition 2.12.1: Relative motion

Relative motion is the motion of an object relative to another object.

The velocity of an object relative to another object is the difference between the velocities of the two objects.

The velocity of an object relative to another object is the sum of the relative velocities of the two objects.

Chapter 3

Dynamics

Definition 3.0.1: Dynamics

Dynamics is the study of the causes of motion, joining kinematics to form mechanics.

3.1 Forces and Newton's Laws of Motion

Definition 3.1.1: Forces

A force is a push or a pull. It is an interaction between two objects (ex: a human and a box). Forces cause objects to accelerate and are vectors. Forces can either be contact forces or long-range forces. Force is measured in Newtons or N which is equal to mass(kg) times acceleration(m/s^2)

Definition 3.1.2: Newton's First Law of Motion

An object at rest will remain at rest and an object in motion will remain in motion at a constant velocity unless acted upon by an unbalanced force.

Definition 3.1.3: Newton's Second Law of Motion

The acceleration of an object is directly proportional to the net force acting on it and inversely proportional to its mass.

The equation for Newton's Second Law of Motion is

$$\vec{F}_{net} = m\vec{a}$$

Definition 3.1.4: Newton's Third Law of Motion

For every action there is an equal and opposite reaction. This means that if I push John with 4 Newtons of Force, then I will also be pushed with a reacting force of 4 Newtons.

3.2 Types of Forces

- Applied Force(\vec{F}_A or \vec{F})
- Normal Force(\vec{F}_N or \vec{N} or \vec{n})
- Tension Force(\vec{F}_T or \vec{T})
- Spring Force(\vec{F}_{sp})

- Weight or Force of gravity(\vec{F}_W or \vec{W} or \vec{w} or \vec{F}_g)
- Friction(\vec{f}_k or \vec{f}_s)
- Drag(\vec{F}_D or \vec{D})
- Thrust(\vec{F}_{thrust})

Definition 3.2.1: Weight

Weight is the force due to gravity on an object. Weight is not the same thing as mass!

Definition 3.2.2: The spring force

The spring force is either a push(when compressed) or a pull (when stretched) that is exerted by a spring on any object that is attached to it.

Definition 3.2.3: Tension

Tension is the force of a rope or stringlike object pulling another object in the direction of the rope or string.

Definition 3.2.4: Normal force

The normal force is the force that a surface exerts on an object that is pressing on it. The normal force is always perpendicular to the surface and is equal to the force of gravity acting on the surface.

Definition 3.2.5: Friction

Friction is the force that opposes the motion of an object. Friction is parallel to the surface and is always in the opposite direction of the motion of the object.

There are two types of friction: kinetic friction and static friction. Kinetic friction is the friction between two objects that are moving relative to each other. Static friction is the friction between two objects that are not moving relative to each other.

The equation for kinetic friction is

$$f_k = \mu_k N$$

where μ_k is the coefficient of kinetic friction and N is the normal force.

The equation for static friction is

$$f_s \leq \mu_s N$$

where μ_s is the coefficient of static friction and N is the normal force.

Definition 3.2.6: Drag

Drag is the force that opposes the motion of an object through a fluid. Drag is parallel to the velocity of the object and is always in the opposite direction of the motion of the object.

Definition 3.2.7: Thrust

Thrust is the force that propels an object forward. Thrust is parallel to the velocity of the object and is always in the same direction as the motion of the object. Thrust is caused by the expulsion of gas particles at high speeds(burning jet fuel) with those gas particles applying an equal force back towards the object being thrust upwards.

3.3 Free Body Diagrams

Definition 3.3.1: Free Body Diagrams

A free body diagram is a diagram that shows all of the forces acting on an object. When making free body diagrams, you should:

1. Identify all forces acting on the object.
2. Draw a dot or box to represent the object.
3. Draw vectors representing each of the identified forces.
4. Draw and label the net force vector. Or write $\vec{F}_{net} = 0N$

3.4 Equilibrium

Definition 3.4.1: Equilibrium

Equilibrium is when the net force on an object is zero.

There are two types of equilibrium: static equilibrium and dynamic equilibrium. Static equilibrium is when an object is at rest. Dynamic equilibrium is when an object is moving at a constant velocity.

Chapter 4

Circular Motion/Gravity

4.1 Circular Motion

Definition 4.1.1: Uniform circular motion

Uniform circular motion is motion at a constant speed around a circle.

The acceleration of an object in circular motion is towards the center of a circle and this acceleration towards the center of the circle is called the centripetal acceleration.

The equation for the centripetal acceleration is

$$\vec{a} = \frac{v^2}{r}$$

4.2 Period, Frequency, Speed, Acceleration, and Force for Circular Motion

Definition 4.2.1: Period

The period (T) of an object in circular motion is the time it takes for the object to complete one revolution around the circle.

$$1 \text{ rev} = 2\pi$$

Definition 4.2.2: Frequency

The frequency (f) of an object in circular motion is the number of revolutions per second.

$$f = \frac{1}{T}$$

Definition 4.2.3: Speed

The speed of an object in circular motion is the distance traveled per unit time.

$$v = \frac{2\pi r}{T} = 2\pi r f$$

Definition 4.2.4: Acceleration

The acceleration of an object in circular motion is the change in velocity per unit time.

$$a = \frac{v^2}{r} = (2\pi f)^2 r = \left(\frac{2\pi}{T}\right)^2 r$$

Definition 4.2.5: Force

The force of an object in circular motion is the force that is required to keep the object moving in a circle.

$$F = ma = m \frac{v^2}{r} = m(2\pi f)^2 r = m\left(\frac{2\pi}{T}\right)^2 r$$

4.2.1 Example questions

Example 4.2.1 (Maximum speed of a car on a corner)

For a car with a given radius, coefficient of static friction (typically $\mu_s = 1.0$), and mass, you will be asked to find the maximum velocity of a car on a corner. To solve this question, you must get the force due to friction which will be equal to the net force and then with this force you will be able to find the maximum velocity via the following equation:

$$v_{max}^2 = \mu_s \times gr$$

or,

$$v_{max}^2 = \frac{F_x}{m} r$$

Question 1: Finding a car's speed on a banked turn

A curve on a racetrack of radius 70 m is banked at a 15° angle. At what speed can a car take this curve without assistance from friction?

Solution: With no friction acting on the car, the car's centripetal force will be equal to the horizontal component of the normal force acting on the object.

$$\begin{aligned} \sum F_x &= n \sin \theta = \frac{mv^2}{r} \\ \sum F_y &= n \cos \theta - mg = 0 \\ n &= \frac{mg}{\cos \theta} \\ \left(\frac{mg}{\cos \theta}\right) \sin \theta &= \frac{mv^2}{r} \\ v &= \sqrt{rg \tan \theta} \\ v &= \sqrt{70m \times 9.8 \frac{m}{s^2} \times \tan 15^\circ} = 14 \frac{m}{s} \end{aligned}$$

4.3 Apparant forces in circular motion

Definition 4.3.1: Centrifugal force

Centrifugal force is the apparent force that acts outwards on a body moving around a center, arising from the body's inertia.

Definition 4.3.2: Apparant Weight

The apparant weight of an object is equal to the normal force and because of this, the apparant weight of an object in vertical circular motion is the least at the top of a loop and the most at the bottom of a loop. This is why roller coaster's can go in loops and why water stays in a bucket that is spun quickly.

$$w_{app} = n = mg + \frac{mv^2}{r}$$

4.4 Orbital motion

Definition 4.4.1: Orbital motion

Orbital motion is the motion of an object around another object due to the gravitational force between the two objects. An orbiting object is in free fall and the circular motion of the object orbiting in free fall is under the influence of gravity and with a centripetal acceleration equal to the acceleration due to gravity.

$$a = \frac{F_{net}}{m} = \frac{w}{m} = \frac{mg}{m} = g$$

$$a = \frac{v^2}{r} = g$$

$$v_{orbit} = \sqrt{gr}$$

$$T = \frac{2\pi r}{v_{orbit}} = 2\pi\sqrt{\frac{r}{g}}$$

Note:-

Weightlessness in orbit:

When an object is in orbit, it is in free fall and because of this, the object is weightless. This weightlessness is not from an absense of gravity but is more similar to the weightlessness while in a free falling elevator.

4.5 Newton's law of gravity

Definition 4.5.1: Newton's law of gravity

Gravity is a universal force that affects all objects in the universe. The force of gravity is proportional to the product of the masses of the two objects and inversely proportional to the square of the distance between the two objects.

$$F = G \frac{m_1 m_2}{r^2}$$

where G is the universal gravitational constant.

$$G = 6.67 \times 10^{-11} \frac{N \cdot m^2}{kg^2}$$

Theorem 4.5.1 The acceleration on the surface of a planet

$$a = g_{planet} = \frac{GM}{r^2}$$

where M is the mass of the planet and r is the radius of the planet.

Theorem 4.5.2 The speed of a satellite in orbit

$$v = \sqrt{\frac{GM}{r}}$$

where M is the mass of the planet and r is the radius of the planet.

Theorem 4.5.3 The period of a satellite or the time for a satellite to complete an orbit

$$T^2 = \frac{4\pi^2}{GM} r^3$$

where M is the mass of the planet and r is the radius of the planet.

Chapter 5

Energy

5.1 Energy and Work

Definition 5.1.1: Energy

Energy is the ability to do work. Energy is neither created nor destroyed. It is universally conserved. Types of Energy include:

- Kinetic Energy - K
- Gravitational potential energy - U_g
- Elastic potential energy - U_s
- Thermal energy - E_{th}
- Chemical energy - E_{chem}
- Nuclear energy - $E_{nuclear}$

Definition 5.1.2: Work

Work is equal to the displacement caused by a force. $W = \vec{F}d$ and work is measured in the Joule which is equal to 1 Newton meter and is the unit of all energy. Work is equal to the force parallel to the displacement times the displacement as that component of the force is the cause of work not the entirety of the force.

$$W = Fd \cos \theta$$

This means that the sign of work is dependent on the direction of a force in comparison to its actual displacement.

Work is also the change in kinetic energy or the transfer of energy.

$$\Delta K = W$$

The forces that do no work either produce no displacement or are perpendicular to the displacement.

Definition 5.1.3: Kenetic Energy

Kenetic energy is the energy of movement.

$$K = \frac{1}{2}mv^2$$

Rotational energy is the energy of rotational movement.

$$K_{rot} = \frac{1}{2}I\omega^2$$

Definition 5.1.4: Potential energy

Potential energy is stored energy.

Gravitational potential energy: $U_g = mg\Delta y$

Elastic potential energy: $U_s = \frac{1}{2}kx^2$ where k is the spring constant and x being the displacement of a spring.

Definition 5.1.5: Thermal energy

Thermal energy is the kenetic energy lost through the transfer of heat typically due to friction.

Definition 5.1.6: Energy in collisions

Perfectly elastic collisions conserve mechanical energy(kenetic and potential, not thermal energy) whilst inelastic collisions lose mechanical energy to thermal energy.

5.2 Power

Definition 5.2.1: Power

Power is the rate at which energy is transformed.

$$P = \frac{\Delta E}{\Delta t} = \frac{W}{\Delta t} = F \times v$$

The unit of power is the watt: $W = 1 \text{ J} / \text{ s}$

Chapter 6

Momentum

6.1 Impulse, collisions, and momentum

Definition 6.1.1: Collisions

A collision is a short-duration interaction between two objects.

Definition 6.1.2: Impulse

Impulse is the area under the curve of a Force vs time graph. $\text{Impulse}(J) = F_{avg}\Delta t = N/s$.

$$\vec{J} = \vec{F}_{avg}\Delta t$$

Definition 6.1.3: Momentum

Momentum is the product of an object's mass and velocity.

$$\vec{p} = m\vec{v}$$

Definition 6.1.4: Impulse-Momentum Theorem

An impulse delivered to an object causes the object's momentum to change.

$$\vec{J} = \vec{p}_f - \vec{p}_i = \Delta\vec{p}$$

Definition 6.1.5: Total Momentum

The total momentum \vec{P} is the vector sum of the momentum of every object.

Definition 6.1.6: Conservation of momentum

The Conservation of Momentum states that momentum is conserved during a collision in an isolated system. If $\vec{F}_{net} = 0$, then the total momentum doesn't change.

Definition 6.1.7: Inelastic collisions

A collision in which the two objects stick together and move with a common final velocity is called a perfectly inelastic collision.

6.2 Angular momentum

Definition 6.2.1: Angular momentum

Angular momentum(L) is the rotational equivalent to linear momentum using the moment of inertia and angular velocity instead.

$$L = I\omega$$

Definition 6.2.2: The Law of conservation of angular momentum

The angular momentum of a rotating object subject to no net external torque is a constant. The final angular momentum L_f is equal to the initial angular momentum L_i . This means that for an object with an angular momentum with now a change in its moment of inertia can experience a change in its angular velocity.

Chapter 7

Simple Harmonic Motion

7.1 The spring force

Definition 7.1.1: Hooke's Law

$$F_{sp} = -k\Delta x$$

where k is the spring constant.

7.2 Equilibrium and Oscillation

Definition 7.2.1: Equilibrium position

The equilibrium position of an object in a bowl is at the bottom where $F_{net} = 0$.

Definition 7.2.2: The restoring force

The restoring force is the force that brings an object in a bowl back to the equilibrium position.

Definition 7.2.3: Oscillation

The repetitive motion in a bowl from side to side.

7.3 Frequency and Period

Definition 7.3.1: Frequency

The frequency(f) of an oscillation is the cycles of an oscillation per unit time(typically per second) and is measured in Hz or (s^{-1}).

Definition 7.3.2: Period

The period(T) of an oscillation is the time it takes for an oscillation to occur.

7.4 Pendulum

Definition 7.4.1: Pendulum

A pendulum is a weight in simple harmonic motion having a force of $F = -mg \sin \theta$.

7.5 Describing SHM

Definition 7.5.1: Describing SHM

$$\begin{aligned}x(t) &= A \cos\left(\frac{2\pi t}{T}\right) = 2 \cos(2\pi f t) \\v(t) &= -v_{\max} \sin\left(\frac{2\pi t}{T}\right) = -v_{\max} \sin\left(\frac{2\pi t}{T}\right) \\a_x = \frac{F}{m} &= -a_{\max} \cos\left(\frac{2\pi t}{T}\right) = -a_{\max} \cos(2\pi f t)\end{aligned}$$

7.6 Connecting SMH to circular motion

Definition 7.6.1: Connecting SMH to circular motion

$$\begin{aligned}x &= A \cos \phi \\v &= 2\pi f A \\a &= (2\pi f)^2 A \\\omega &= 2\pi f\end{aligned}$$

Definition 7.6.2: The maximum velocity and acceleration of SHM

$$\begin{aligned}v_{\max} &= 2\pi f A \\a_{\max} &= (2\pi f)^2 A\end{aligned}$$

7.7 Energy in SHM

Claim 7.7.1 Energy remains constant

Theorem 7.7.1 Finding Frequency in SHM

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

Theorem 7.7.2 Finding Period in SHM

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Theorem 7.7.3 Frequency of a pendulum

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

Theorem 7.7.4 Period of a pendulum

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Definition 7.7.1: The maximum velocity of pendulums

$$v_{max} = \sqrt{\frac{g}{L}} A = \sqrt{gL\theta_{max}}$$

Chapter 8

Torque + Rotational Motion

8.1 Rotational Kinematics

Definition 8.1.1: Rotational Motion

Rotational motion is the motion of a body that spins about an axis.

Definition 8.1.2: Angular Position

Angular position is the angle that is formed between a reference line and a line that connects the reference line to a point on a rotating object. Angular position is defined by the angle θ in radians measured clockwise from the x-axis, the arc length s , and the radius from the center of circular motion. $\theta(\text{radians}) = \frac{s}{r}$

Definition 8.1.3: Angular Displacement and Velocity

Angular displacement is the change in angular position and is measured in radians ($\Delta\theta$). Angular velocity is the rate of change of angular displacement and is measured in radians per second.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

$$\theta_f - \theta_i = \Delta\theta = \omega\Delta t$$

Definition 8.1.4: Angular Acceleration

Angular acceleration is the rate of change of angular velocity and is measured in radians per second squared.

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

$$\omega_f - \omega_i = \Delta\omega = \alpha\Delta t$$

8.2 Relating angular and linear motion

The relationship between angular motion and linear is that linear motion is equal to the radius multiplied by angular motion.

Note:-

Relationship between linear and angular motion:

Linear Variable	Linear Symbol	Angular Variable	Angular Symbol
Position(m)	x	Angle(rad)	θ
Velocity($\frac{m}{s}$)	v	Angular Velocity($\frac{rad}{s}$)	ω
Acceleration($\frac{m}{s^2}$)	a	Angular Acceleration($\frac{rad}{s^2}$)	α
Constant Velocity	$\Delta x = v\Delta t$	Constant Angular Velocity	$\Delta\theta = \omega\Delta t$
Constant Acceleration	$\Delta v = a\Delta t$	Constant Angular Acceleration	$\Delta\omega = \alpha\Delta t$

8.3 Tangential Acceleration

Definition 8.3.1: Tangential Acceleration

Tangential acceleration is the component of acceleration that is tangent to the circular path of an object in circular motion. This is the linear acceleration of non-uniform circular motion!

$$a_t = \alpha r$$

8.4 Torque

Note:-

The ability of a force to cause rotation depends on 3 factors:

1. The magnitude of the force
2. The direction r from the axis of rotation to the point of application of the force
3. The angle at which the force is applied

These factors are combined into a single vector quantity called torque.

Definition 8.4.1: Torque

Torque is the rotational analog of force. Torque($\tau \rightarrow$ Greek letter tau) is the product of the perpendicular component of the force acting at a distance r from the pivot.

$$\tau = rF_{\perp} = r_{\perp}F$$

Torque is measured in Newton-meters(Nm). Net torque is the sum of all torques acting on an object.

8.5 Gravitational Torque and the Center of Gravity

Definition 8.5.1: Gravitational Torque and the Center of Gravity

The gravitational torque is equal to the weight of the object multiplied by the distance from the pivot to the center of gravity. And the center of gravity is the point at which the entire weight of the object can be considered to act. An object that is free to rotate about a pivot will come to rest with the center of gravity below the pivot point.

Theorem 8.5.1 Finding the center of gravity

1. Choose an origin for your coordinate system.
2. Determine the coordinates of the center of gravity of each object ex: (x_1, y_1) , (x_2, y_2) , (x_3, y_3) for the masses.
3. The x coordinate of the center of gravity is the sum of the x coordinates of the masses multiplied by their respective masses divided by the total mass.

$$x_{cg} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

4. The y coordinate of the center of gravity is the sum of the y coordinates of the masses multiplied by their respective masses divided by the total mass.

$$y_{cg} = \frac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$$

8.6 Rotational Dynamics and Rotational Inertia

Definition 8.6.1: Newton's Second Law for Rotation

Just as $F = ma$, there is a similar equation involving mass for rotational motion. Because $\tau = F \times r$, and $F = ma$ and $\alpha = \frac{F}{mr}$, $\tau_1 = m_1r_1^2\alpha$ and so

$$\tau_{net} = \alpha(m_1r_1^2 + m_2r_2^2 + \dots) = \alpha \sum m_i r_i^2$$

The moment of inertia is the sum of the products of the masses of the particles in a system and the square of their perpendicular distances from a specified axis.

$$I = m_1r_1^2 + \dots = \sum m_i r_i^2$$

$$\tau_{net} = I\alpha$$

The moment of inertia is the rotational equivalent of mass.

Note:-**The moment of inertia for common objects:**

Object	Moment of Inertia
Solid sphere	$\frac{2}{5}mr^2$
Hollow sphere	$\frac{2}{3}mr^2$
Solid cylinder	$\frac{1}{2}mr^2$
Hollow cylinder	mr^2
Rod about center	$\frac{1}{12}ml^2$
Rod about end	$\frac{1}{3}ml^2$

8.7 Rolling Motion

Definition 8.7.1: Rolling Motion

An object that is rolling is both rotating and translating. A rolling object's center moves at velocity ($v = \omega r$) while the object has an angular velocity of $2v = 2\omega r$.

$$v = \frac{\Delta x}{T} = \frac{2\pi r}{T} = \omega r$$

8.8 Torque and equilibrium

Definition 8.8.1: Conditions for static equilibrium of an extended object

1. The net force acting on the object must be zero.
2. The net torque acting on the object must be zero.

8.9 Stability and Balance

Definition 8.9.1: Stability and Balance

An object is considered to be stable when its center of gravity remains above its base of support. Meaning as long as the center of gravity is above the base of support, the object will not topple over because the torque due to gravity will rotate the object back toward its stable equilibrium position. An object is unstable if its gravitational torque is in the direction away from its base of support or pivot point making a hypothetical object turn over.

An object is stable if $\theta \leq \theta_c$ and unstable if $\theta > \theta_c$ with θ_c being the critical angle.

The equation for the critical angle is:

$$\theta_c = \arctan \frac{\frac{1}{2}w}{h}$$

with w being the width of the base of support and h being the height of the center of gravity.

Appendix A

Scientific Notation, Significant Figures, and Unit Conversions

A.1 Scientific Notation

Definition A.1.1: Scientific Notation

Scientific notation is a way of writing numbers that are too big or too small to be conveniently written in decimal form by using multiples of 10 to simplify numbers.

Example A.1.1 (Scientific notation)

$$6,370,000m = 6.37 \times 10^6 m$$

A.2 Scientific Figures

Definition A.2.1: Significant Figures

Think of significant figures or sig figs as the number of digits that are reliably known.

Rules:

1. When you multiple/divide numbers, the number of sig figs in the answer should be equal to the sig figs of the least precise number used in your calculation.
2. When you add/subtract numbers, the number of decimal places in the answer should be equal to the number of decimal places in the least precise number used in your calculation.
3. Exact numbers have an infinite number of sig figs. Do not consider sig figs at all!
4. EXCEPTION!! It is acceptable to keep one or two extra digits during the *intermediate steps* of a calculation. HOWEVER, the final answer must have the right sig figs.

Sig figs aren't that important for AP Physics 1, but don't just dump every digit!

A.3 SI Units

Measurements of quantities require numerical values and a *unit* to represent themselves.

Definition A.3.1: The three basic SI units

1. **Length** is measured in meters (m)
2. **Mass** is measured in kilograms (kg)
3. **Time** is measured in seconds (s)

A.4 Unit Prefixes

Table A.1: Metric Prefixes

Prefix	Symbol	Power of 10
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
hecto	h	10^2
deka	da	10^1
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

Appendix B

Important Overall Concepts

B.1 Closed and Open Systems

B.2 The analysis and use of graphs