CSE 215 Homework 8

Ben Feuer

June 21, 2024

Question 1: Problem 1

Let A be the set of all strings of length 6 consisting of x's and y's. Then A is denoted $\sum^6$ where $\sum = \{x, y\}$. Define a binary relation R from A to A as follows:
For all strings s and t in A, $sRt \iff$ the first four characters of s equal the first four characters of t.

a) Is xxyxyx R xxxyxy ?
Answer: No, because the first four characters of xxyxyx are xxyx, and the first four characters of xxxyxy are xxxy, which are not equal.

b) Is yxyyyx R yxyyxy?
Answer: Yes, because the first four characters of yxyyyx are yxyy, and the first four characters of yxyyxy are yxyy, which are equal.

c) Is xyxxxx R yxxxxx?
Answer: No, because the first four characters of xyxxxx are xyxx, and the first four characters of yxxxxx are yxxx, which are not equal.

Question 2: Problem 2

Let A = 2, 3, 4 and B = 6, 8, 10 and define a binary relation R from A to B as follows:
for all $(x, y) \in A \times B, (x, y) \in R \iff x|y$.

a) Is 4 R 6?
Answer: No, because 4 does not divide 6.

b) Is 4 R 8?
Answer: Yes, because 4 divides 8, and 4 is in A and 8 is in B.

c) Is $(3, 8) \in R$? Is $(2, 10) \in R$?
Answer: No and Yes, respectively. 3 does not divide 8, but 2 divides 10.

d) Write R as a set of ordered pairs.
Answer: $R = \{(2, 6), (2, 8), (2, 10), (3, 6), (4, 8)\}$
Equal to the set of ordered pairs where the first element divides the second element.

Question 3: Problem 3

Prove that for all integers m and n, m - n is even if and only if both m and n are even or both m and n are odd.

$$m - n = 2k \iff (m = 2i \wedge n = 2j) \vee (m = 2i + 1 \wedge n = 2j + 1)$$

$$\begin{aligned} &\equiv [(m - n) = 2k \implies (m = 2i \wedge n = 2j) \vee (m = 2i + 1 \wedge n = 2j + 1)] \\ &\wedge[(m = 2i \wedge n = 2j) \vee (m = 2i + 1 \vee n = 2j + 1) \implies (m - n) = 2k] \end{aligned}$$

Proof:
Case 1: m and n are both even.
Let $m = 2i$ and $n = 2j$.
Then $m - n = 2i - 2j = 2(i - j) = 2k$ where $k = i - j$.
Thus, if m and n are both even, then m - n is even.

Case 2: m and n are both odd.
Let $m = 2i + 1$ and $n = 2j + 1$.

Then $m - n = 2i + 1 - 2j - 1 = 2(i - j) = 2k$ where $k = i - j$.
Thus, if m and n are both odd, then m - n is even.

Case 3: m is even and n is odd.
Let $m = 2i$ and $n = 2j + 1$.
Then $m - n = 2i - 2j - 1 = 2(i - j) - 1 = 2k - 1$ where $k = i - j$.
Thus, if m is even and n is odd, then m - n is odd.

Case 4: m is odd and n is even.
Let $m = 2i + 1$ and $n = 2j$.
Then $m - n = 2i + 1 - 2j = 2(i - j) + 1 = 2k + 1$ where $k = i - j$.
Thus, if m is odd and n is even, then m - n is odd.

Therefore, m - n is even if and only if both m and n are even or both m and n are odd.

Question 4: Problem 4

Declare a binary relation S from R to R as:
For all $(x, y) \in R \times R$, $xSy \iff x \geq y$.
Draw the graph of S in the Cartesian plane.
Graph below where the area below the line $y = x$ is shaded in red, including the line itself.

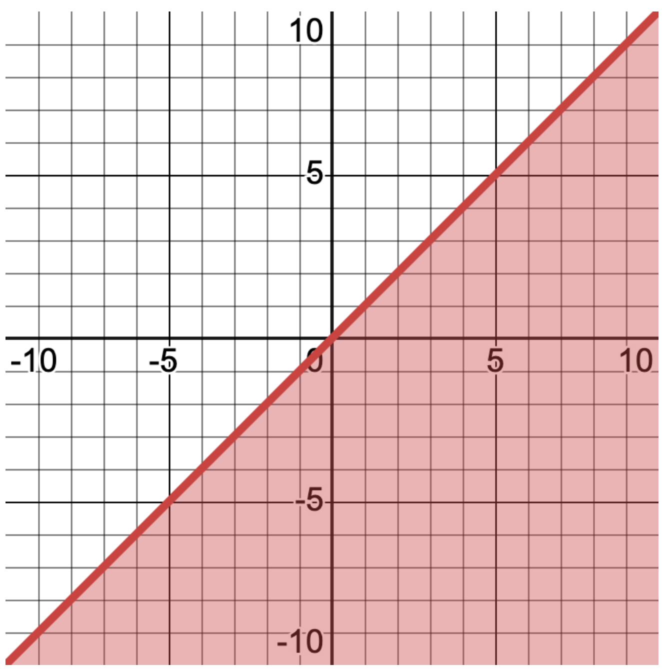

Figure 1: Graph for problem 4

Question 5: Problem 5

Define a binary relation T from R to R as follows:
For all $(x, y) \in R \times R, xTy \iff y = x^2$.
Draw the graph of T in the Cartesian plane.
Answer: The graph of T is the line of the parabola $y = x^2$ which is shown below.

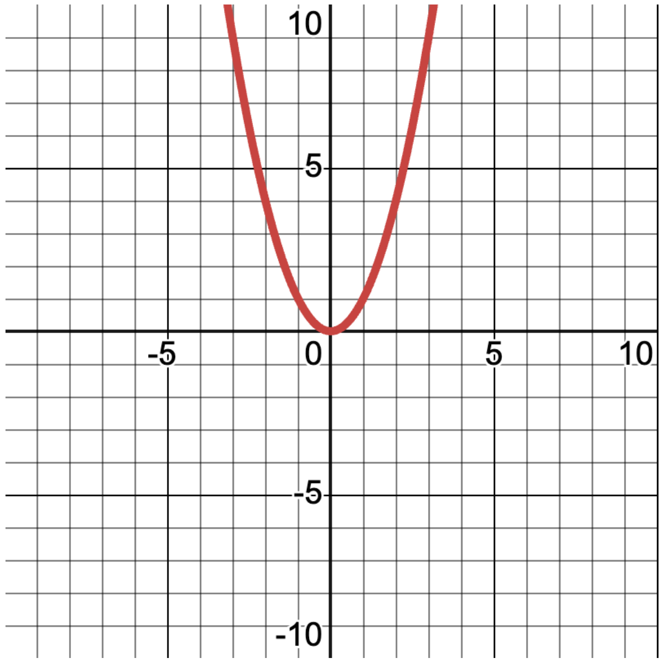

Figure 2: Graph for problem 5

Question 6: Problem 6

For the following relations, 1) draw the directed graph, 2) determine whether it is reflexive, symmetric, and transitive. Give a counterexample in each case in which the relation does not satisfy one of these properties.
All graphs are shown below.
a) $R_1 = \{(0,0), (0,1), (0,3), (1,1), (1,0), (2,3), (3,3)\}$
Answer: not reflexive, not symmetric, not transitive.
Reflexivity: (2, 2) is not in the relation.
Symmetry: (0, 3) is in the relation, but (3, 0) is not.
Transitivity: (1,3) is missing but (1, 0) and (0, 3) are present.

b) $R_2 = \{(2,3), (3,2)\}$
Answer: not reflexive, symmetric, transitive.
Reflexivity: (2, 2) and (3, 3) are not in the relation.
Symmetry: No counterexample.
Transitivity: Vacuously transitive.

c) $R_3 = \{(0,1), (0,2)\}$
Answer: not reflexive, not symmetric, transitive.
Reflexivity: (0, 0) is not in the relation.
Symmetry: (0, 1) is in the relation, but (1, 0) is not.
Transitivity: Vacuously transitive.

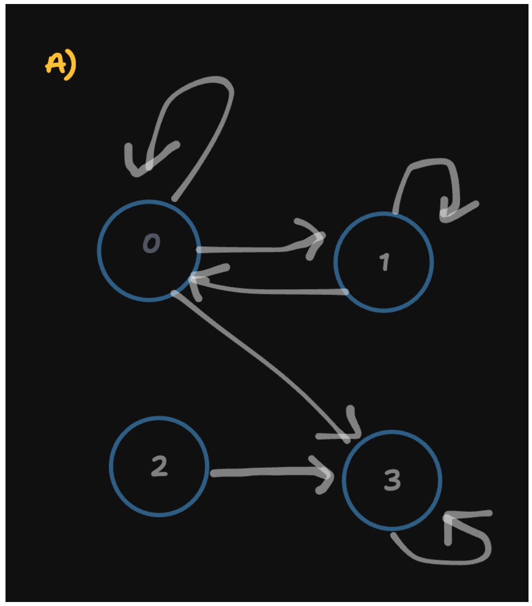

Figure 3: Graph for problem 6 A

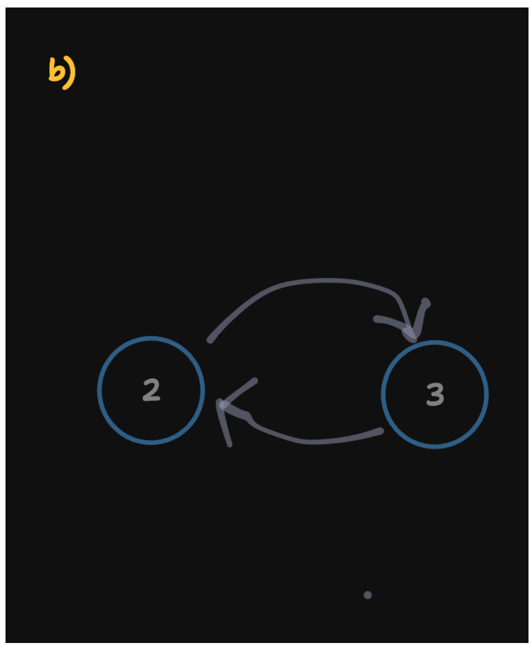

Figure 4: Graph for problem 6 B

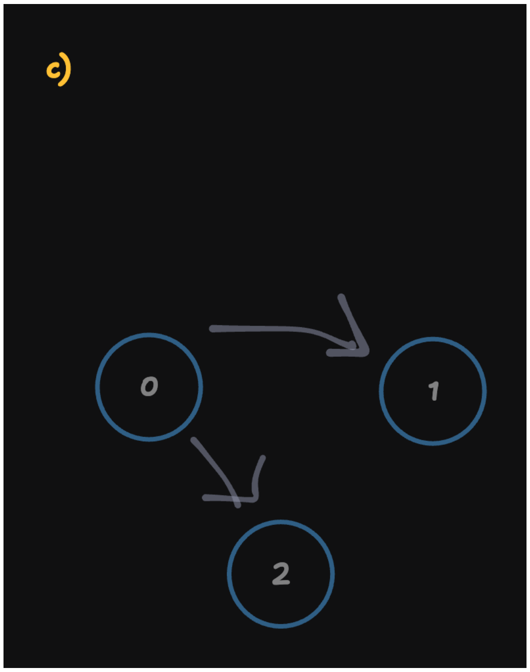

Figure 5: Graph for problem 6 C

Question 7: Problem 7

D is the binary relation defined on R as follows:
For all $x, y \in R, xDy \iff xy \geq 0$.

a) Draw a Cartesian Graph of the relation.
Answer: The graph is shown below.
b) Is it Reflexive?
Answer: Yes, because all real numbers multiplied by themselves are greater than or equal to 0, meaning that all real numbers are related to themselves.
c) Is it Symmetric?
Answer: Yes, because if $xy \geq 0$, then $yx \geq 0$, so the relation is symmetric.
d) Is it Transitive?
Answer: Yes, because if $xy \geq 0$ and $yz \geq 0$, then $xz \geq 0$, so the relation is transitive as x and z must have the same sign, meaning that their product is greater than or equal to 0.

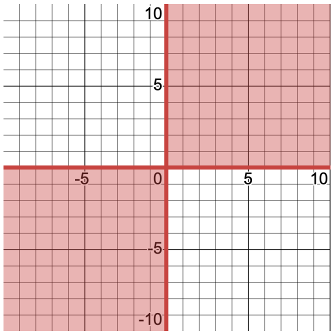

Figure 6: Graph for problem 7