CSE 215
Homework 7

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June 14, 2024

Question 1: Question 1

Define $f : R- > R$ and $g : R- > R$ by the following formulas: $f(x) = |x|$ for all $x \in R, g(x) = (x^2)^(1/2)$ for all $x \in R$.
Does $f == g$?
Answer: Yes, $f == g$ because $|x| = (x^2)^(1/2)$ for all $x \in R$.
$|x| = (x^2)^(1/2)$ for all $x \in R$ because the absolute value of x is the square root of x squared as the square root of x squared is always positive.

Question 2: Question 2

How many functions are there from a set with three elements to a set with four elements? Answer: $4^3 = 64$ There are 4 choices for each of the 3 elements in the domain, so there are $4^3 = 64$ functions, and there are no restrictions on the function(i.e. injective, surjective).

Question 3: Question 3

Define functions f and g from R to R by the formulas: for all $x \in R$,

$$f(x) = 2x$$

$$g(x) = (2x^3 + 2x)/(x^2 + 1)$$

Show that $f == g$.

$$g(x) = (2x^3 + 2x)/(x^2 + 1) = 2x(x^2 + 1)/(x^2 + 1) = 2x = f(x)$$

Question 4: Question 4

For $f(x) = (x + 1)/x$ for all real numbers where $x \neq 0$,
a) Is this function onto? or b) Is this function one-to-one?
Answer: This function is one-to-one. Suppose $f(x_1) = f(x_2)$ for $x_1, x_2 \in R$.

$$\frac{x_1 + 1}{x_1} = \frac{x_2 + 1}{x_2}$$

$$x_1 x_2 + x_1 = x_1 x_2 + x_2 \text{ cross multiplication}$$

$$x_1 = x_2 \text{ subtraction, yielding equality}$$

Since $f(x_1) = f(x_2)$ implies $x_1 = x_2$, the function is one-to-one.

Question 5: Question 5

For $f(x) = x/(x^2 + 1)$ for all real numbers x
a) Is this function onto? or b) Is this function one-to-one?
Answer: This function is one-to-one.
Suppose $f(x_1) = f(x_2)$ for $x_1, x_2 \in R$.

$$\frac{x_1}{x_1^2 + 1} = \frac{x_2}{x_2^2 + 1}$$

$$x_1 x_2^2 + x_1 = x_1^2 x_2 + x_2 \text{ cross multiplication}$$

$$x_1 = x_2 \text{ subtraction, yielding equality}$$

Since $f(x_1) = f(x_2)$ implies $x_1 = x_2$, the function is one-to-one.

Question 6: Question 6

How many one-to-one functions are there from a set with three elements to a set with four elements?
Answer: 24
There are 4 choices for the first element in the codomain, 3 choices for the second element in the codomain, and 2 choices for the third element in the codomain.
$4 * 3 * 2 = 24$ one-to-one functions.

Question 7: Question 7

How many onto functions are there from a set with three elements to a set with two elements?
Answer: 6
$1 : x_1 \to y_1, x_2 \to y_1, x_3 \to y_2$
$2 : x_1 \to y_1, x_2 \to y_2, x_3 \to y_1$
$3 : x_1 \to y_1, x_2 \to y_2, x_3 \to y_2$
$4 : x_1 \to y_2, x_2 \to y_1, x_3 \to y_1$
$5 : x_1 \to y_2, x_2 \to y_1, x_3 \to y_2$
$6 : x_1 \to y_2, x_2 \to y_2, x_3 \to y_1$
This is also equivalent to the number of functions from a set with three elements to a set with two elements, which is $2^3 = 8$, minus the number of functions that are not onto, which is $2 * 1 = 2$ when all y_1 is mapped to all elements of X and y_2 is mapped to all elements of X.

Question 8: Question 8

In a group of 30 people, at least how many must have been born in the same month?
Answer: 3
There are 12 months in a year, so if there are 30 people, there must be at least 3 people born in the same month by the Pigeonhole Principle as the number of people per month codomain is 3 for 6 months and 2 for the other 6 months. From 25 people onwards, there must be at least 3 people born in the same month by the Pigeonhole Principle for one of the months.