1) $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix} \xrightarrow[\substack{R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - 6R_1}]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 2 \\ 0 & -4 & -8 & 3 \\ 0 & -4 & -11 & 5 \end{bmatrix}$

$\downarrow R_3 \to R_3 - R_4,\ R_2 \leftrightarrow R_4$

$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & -4 & -11 & 5 \\ 0 & 0 & 3 & -2 \\ 0 & 0 & -3 & 2 \end{bmatrix}$

$\rho(A) = 3$ (no. of non-zero rows).

2) vector space of 2×2 symmetric matrices $\to w$

$T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a-b) + (b-c)x + (c-a)x^2$

$\dim(w) = 3$

Rank of T :- 3

using rank-nullity theorem $\to$ nullity $= \dim(w) - \rho(T)$

nullity $= 3 - 3 = 0$

3) $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$, char. eq^n $(A) = \left|\begin{bmatrix} 2-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix}\right|$

Eigen value of $A = 1, 3$

" " " $A^{-1} = 1, 1/3$

" " " $A + 4I = 5, 7$

$(2-\lambda)^2 - 1 = 0$

$\lambda^2 + 4 - 4\lambda - 1 = 0$

$\lambda^2 - 4\lambda + 3 = 0$

$\lambda = 1, 3$

now for $\lambda = 1 \rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$x - y = 0$
$-x + y = 0$ $\rightarrow \begin{bmatrix} K \\ K \end{bmatrix}$

for $\lambda = 3$,

$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$

$-x - y = 0$
$-x - y = 0$ $\rightarrow \begin{bmatrix} K \\ -k \end{bmatrix}$

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4) Given $\rightarrow$ $3x - 0.1y - 0.2z = 7.85$
$0.1x + 7y - 0.3z = -19.3$
$0.3x - 0.2y + 10z = 71.4$

$y = \left(\frac{-19.3 - 0.1x + 0.3z}{7} \right)$ & $z = \left(\frac{71.4 - 0.3x - 0.2y}{10} \right)$

$x = \left(\frac{7.85 + 0.1y + 0.2z}{3} \right)$

for first term, $x = 2.6167$, $y = 19.5617$, $z = 6.67$

" second " , $x = 7.716$, $y = -2.36$, $z = 6.956$

" third " , $x = 3.002$, $y = -2.455$, $z = 7.099$

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(5) eq^n with sol^n $\rightarrow$ consistant eq^n
" no-sol^n $\rightarrow$ inconsistant eq^n

$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix} \xrightarrow[R_4 \rightarrow R_4 - R_1]{R_2 \rightarrow R_2 - 2R_1,\ R_3 \rightarrow R_3 - 3R_1} \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$

$$\xrightarrow[R_3 \to R_3 - 2R_2]{R_4 \to R_4 + R_3} \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = \rho(A|B) < n$

inconsistent eq^n

$3x - 6y + 2z = 23$, $-4x + y - z = -15$

$x - 3y + 7z = 16$

$x = \dfrac{23 + 6y - 2z}{3}$, $y = -15 + 4x + z$

$z = (16 - x + 3y)/7$

iter^n ① $\rightarrow$ ~~$x = 23+6$~~, $x = 0.67$, $y = -11.33$

$z = -6.22$

iter^n ② $\rightarrow$ $x = -10.85$, $y = -64.62$, $z = -37.77$

iter^n ③ $\rightarrow$ $x = -289.18$, $y = -1209.49$, $z = -3323.29$

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⑥ $T : W \to P$ $\quad T : P_2 \to P_2$

$T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$

for additivity $\rightarrow$ $T(u+v) = T(u) + T(v)$

$T(u) = (a+1) + (b+1)x + (c+1)x^2$

$T(v) = (\alpha+1) + (\beta+1)x + (\gamma+1)x^2$

$T(u+v) = (a+\alpha+2) + (b+\beta+2)x + (c+\gamma+2)x^2$

satisfyies additivity.

for homogenity $\rightarrow$

$T(du) = T(da + dbx + dcx^2)$

$dT(u) = d(a+1) + d(b+1)x + d(c+1)x^2$

satisfyies homogenity

Funⁿ T satisfyies the required conditions, so it is a valid linear transformatⁿ.

(9) To rotate an image, we would require some matrix transformations. Consider an angle of rotation 'θ';

so, $R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Now, to rotate an image represted by a matrix I, we need,

$I_{rotate} = R \cdot I$

(10) for rotation of an image, a linear transformation is applied to the original image matrix, this results in the changed values of each pixel, rendering a rotated image. By using linear transformatⁿ, we preserve the data of the image while only rotating it.