HW20

20\. a) $\alpha_\nu = \frac{j_\nu}{S_\nu} = \frac{j_\nu}{\frac{2\pi\nu^3}{c^2} \cdot \frac{1}{e^{h\nu/kT}-1}}$; $g_{ff} \approx 6.155 (Z_i \nu_g)^{-0.118} T_4^{0.177}$

q^6: (charge of e^-)

$\tau_\nu = \int d\nu \, ds = \int \frac{j_\nu}{S_\nu} ds = \frac{8}{3}\left(\frac{2\pi}{3}\right)^{1/2} g_{ff} \cancel{\frac{e^6}{m_e^2 c^3}} \left(\frac{m_e}{kT}\right)^{1/2} e^{-h\nu/kT} \cancel{Z_i^2}$

j_ν: $\int n_e \underbrace{n_i \, ds}_{EM}$ (substitute & ignore all const) ; $T_4 \propto T$; $\nu_g \propto \nu$

$EM \Rightarrow \tau_\nu \propto EM \cdot T^{-1/2} \cdot e^{-h\nu/kT} (e^{h\nu/kT} - 1) \nu^{-3} \cdot T_4^{0.177} \cdot \nu_g^{-0.118}$

$\Rightarrow \tau_\nu \propto EM \cdot T^{-1/2} \cdot \nu^{-3} \cdot \nu \cdot T^{-1} \cdot T^{0.177} \cdot \nu^{-0.118}$ | $= 1 - e^{-h\nu/kT} = 1 - \left(1 - \frac{h\nu}{kT}\right) = \frac{h\nu}{kT}$

$\Rightarrow \boxed{\tau_\nu \propto EM \cdot T^{-1.323} \cdot \nu^{-2.118}}$ (Rayleigh-Jeans)

b) • $F_{thin} = \frac{j_\nu V}{D^2} \propto \nu^{-0.118} e^{-h\nu/kT} \propto \nu^{-0.118} \left(1 - \frac{h\nu}{kT}\right)$ (optically thin)

$\propto \nu^{0.882}$

We have 2 terms: $F_{thin} \propto \nu^{-0.118} \cdot \nu^1$. But since Rayleigh-Jeans $\Rightarrow \frac{h\nu}{kT} \ll 1 \Rightarrow \left(1 - \frac{h\nu}{kT}\right)$ is neglectable

$\Rightarrow \nu^{0.118}$ dominates. $\Rightarrow \boxed{F_{thin} \sim \nu^{-0.118}}$

• $F_{thick} = \frac{B_\nu(T)}{D^2} \pi R^2 \propto \frac{\nu^3}{e^{h\nu/kT}} \propto \nu^2$

$\Rightarrow \boxed{F_{thick} \sim \nu^2}$ Rayleigh-Jeans $= 1 + \frac{h\nu}{kT}$