ASTR 300B – Fall 2024
Due: Tuesday Sept. 3

1. Imagine that you have a spherical source with a radius R at a distance D that is far away from the observer (D >> R).

    a. What is the solid angle of the source from the point of view of the observer? Write your answer in two different ways: (1) in terms of the angular ***diameter*** θ_s and (2) in terms of R and D. [2 pts]

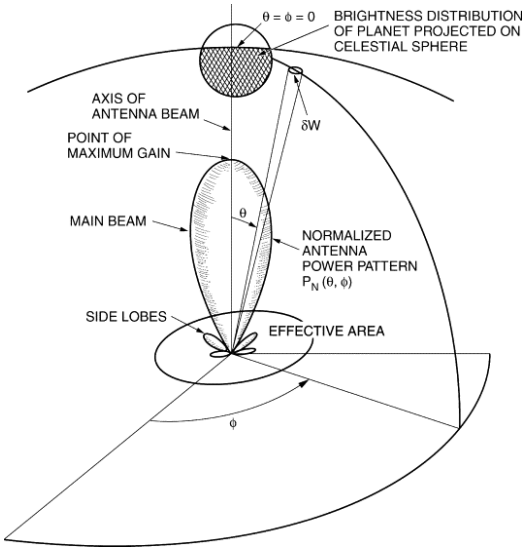
    b. If the source subtends a solid angle of 1 square arcseconds, then how many steradians does it subtend? Quote to 5 decimals. [2 pts]

2. Every telescope has a power pattern (also called the diffraction pattern or beam pattern), $P_n(\theta)$, that describes the sensitivity of the telescope to radiation coming from different angles. $P_n(\theta = 0) = 1$ in the telescope pointing direction. The width of the power pattern is characterized by the full-width half-maximum (FWHM) angle such that $P_n(\theta_{mb}/2) = ½$. In the two cases below, calculate the solid angle of the telescope power pattern assuming that the FWHM of the power pattern is a small angle.

    a. For a radio telescope, the power pattern is often a Gaussian function:

$$P_n(\theta) = \exp\left[-4\ln(2)\frac{\theta^2}{\theta_{mb}^2}\right]$$

    where θ_{mb} is the "main beam" FWHM. [2 pts]



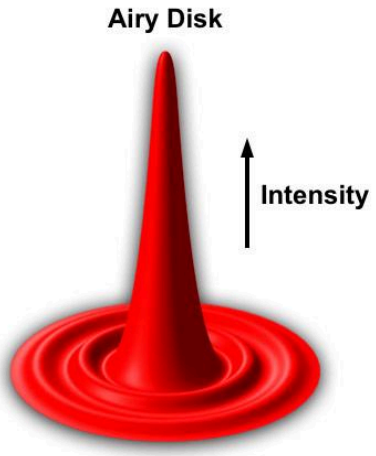
b. For an optical telescope, the power pattern is often an Airy function:

$$P_n(\theta) = 4\frac{J_1^2\left(\frac{\pi D_t \theta}{\lambda}\right)}{\left(\frac{\pi D_t \theta}{\lambda}\right)^2}$$

where $J_1(\ )$ is a Bessel Function of the First Kind and D_t is the telescope diameter. The Airy function FWHM is

$$\theta_{mb} = \frac{2u_{1/2}\lambda}{\pi D_t}$$

with $P_n(u_{1/2}) = ½$ occurring when $u_{1/2} \sim 1.616$. Simplify your answer to only include constants, θ_{mb}, and $u_{1/2}$. [2 pts]



The following integral is useful: $\int_0^\infty [4J_1^2(u)/u]\, du = 2$