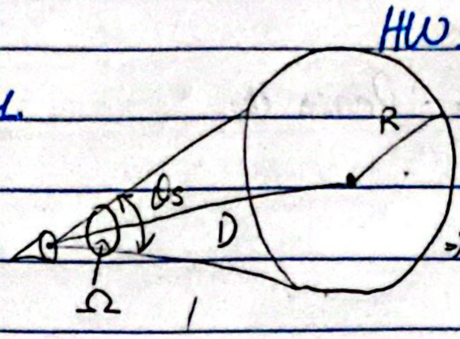
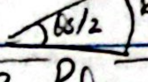
HW 1

1.



a) $\Omega = \frac{A}{D^2} = \pi \frac{\frac{\theta_s^2}{4} D^2}{D^2} = \frac{\pi \theta_s^2}{4}$ (1)



$\Rightarrow \boxed{\frac{R}{D} = \frac{\theta_s}{2} \Rightarrow R^2 = \frac{\theta_s^2 D^2}{4}}$

$= \frac{\pi R^2}{D^2}$ (2)

b) $1° = 3600$ arcsec , $1\,\text{rad}^2 = 1$ sterad

$\Rightarrow 1^2\text{arcsec} = \left(\frac{1°}{3600\,\text{arcsec}}\right)^2 \left(\frac{2\pi\,\text{rad}}{360°}\right)^2 \simeq 2.35 \cdot 10^{-11}$ sterad

2. when $P_n(\theta = 0) \Rightarrow P_n(\theta_{mb}/2) = 1/2$

a) $P_n(\theta) = \exp\left[-4\ln(2)\frac{\theta^2}{\theta^2_{mb}}\right]$

$\Omega = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\infty} P_n(\theta)\, \theta\, d\phi\, d\theta$ $\quad u = 4\ln 2 \frac{\theta^2}{\theta^2_{mb}} \Rightarrow du = \frac{8\ln 2\,\theta}{\theta^2_{mb}} d\theta$

$\theta = 0 \Rightarrow u = 0,\ \theta = \infty \Rightarrow u = \infty$

$= \int_0^{2\pi}\int_0^{\infty} e^{-4\ln(2)\frac{\theta^2}{\theta^2_{mb}}} d\phi\, d\theta = 2\pi \int_0^{\infty} \frac{1}{e^u} \frac{\theta^2_{mb}}{8\ln 2\,\theta} du$

$= \frac{\pi\theta^2_{mb}}{4\ln 2} (-e^{-u})\Big|_0^{\infty} = \frac{\pi\theta^2_{mb}}{4\ln 2}$

b) $P_n(\theta) = 4\frac{J_1^2\left(\frac{\pi D_t \theta}{\lambda}\right)}{\left(\frac{\pi D_t \theta}{\lambda}\right)^2}$, $\theta_{mb} = \frac{2u_{1/2}\lambda}{\pi D_t} \Rightarrow u_{1/2} = \frac{\theta_{mb}\pi D_t}{2\lambda}$

$\Rightarrow \frac{\pi D_t \theta}{\lambda} = \frac{2u_{1/2}\theta}{\theta_{mb}}$

$P_n(u_{1/2}) = 1/2$ when $u_{1/2} \sim 1{,}616$

$\Omega = 2\pi \int_0^{\infty} \frac{4J_1^2\left(\frac{\pi D_t\theta}{\lambda}\right)}{\left(\frac{\pi D_t \theta}{\lambda}\right)^2} \theta d\theta = 8\pi \int_0^{\infty} \frac{J_1^2 \cdot \left(\frac{2u_{1/2}\theta}{\theta_{mb}}\right)}{\left(\frac{2u_{1/2}\theta}{\theta_{mb}}\right)^2} \theta d\theta$

$V = \frac{2u_{1/2}\theta}{\theta_{mb}} \Rightarrow dV = \frac{2u_{1/2}}{\theta_{mb}} d\theta \Rightarrow \Omega = 8\pi \int_0^{\infty} \frac{J_1^2 \cdot V}{V\, 4u_{1/2}^2} \theta_{mb}^2\, dV$

$\theta = 0 \Rightarrow V = 0,\ \theta = \infty \Rightarrow V = \infty$

$\Rightarrow \Omega = \frac{2\pi\theta^2_{mb}}{u^2_{1/2}}$