HW(5,6)+1 = 6,7

6. a) $T = 2.7255\,K$

$$u_\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{h\nu/k_B T}-1} \Rightarrow N_\nu = \frac{u_\nu}{h\nu} = \frac{8\pi\nu^2}{c^3}\frac{1}{e^{h\nu/k_B T}-1}$$

$$\Rightarrow N = \int_0^\infty N_\nu\, d\nu = \frac{8\pi}{c^3}\int_0^\infty \frac{\nu^2}{e^{h\nu/k_B T}-1}\, d\nu = \frac{8\pi}{c^3}\int_0^\infty \left(\frac{h\nu}{k_B T}\cdot\frac{k_B T}{h}\right)^2 \frac{d\left(\frac{h\nu}{k_B T}\right)\frac{k_B T}{h}}{e^{h\nu - k_B T}-1}$$

$$= \frac{8\pi}{c^3}\int_0^\infty \frac{k_B^3 T^3}{h^3}\cdot\frac{(h\nu/k_B T)^2}{e^{h\nu/k_B T}-1}\, d\left(\frac{h\nu}{k_B T}\right) \quad \boxed{\alpha = 2}$$

$$\Rightarrow N = \frac{8\pi k_B^3 T^3}{c^3 h^3}\,\Gamma(3)\,\zeta(3) \approx 410 \text{ photons}/cm^3$$

b) $E_{avg} = \frac{E_{tot}}{N_{tot}} = \frac{U_{tot}}{\#\,Density_{tot}} = \frac{8\pi^5 k_B^4 T^4}{15 h^3 c^3} \cdot \frac{c^3 h^3}{8\pi k_B^3 T^3 \cdot \Gamma(3)\zeta(3)}$

$= \frac{\pi^4 k_B T}{15\Gamma(3)\zeta(3)} \approx 2.7\, k_B T \Rightarrow \nu = \frac{E}{h} = 2.7\frac{k_B T}{h} \approx 153.41\,GHz$

7. a) $u_{ISRF} \sim 1.05 \cdot 10^{-12}\, erg\, cm^{-3}$; isotropic

$\Rightarrow u = \frac{4\pi I}{c}$, $F = I\pi$ (HW4) $\Rightarrow u = \frac{4\pi F}{c\pi} \Rightarrow F = \frac{uc}{4} \approx 0.007875$ $(erg \cdot s^{-1} \cdot cm^{-2})$

b) $u = \frac{4\sigma_{SB} T^4}{c} \Rightarrow T = \sqrt[4]{\frac{cu}{4\sigma_{SB}}} \approx 3.41K$