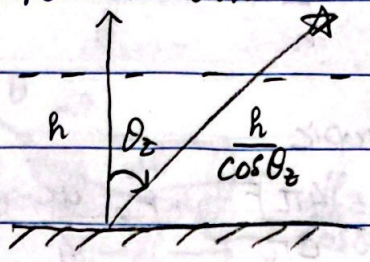
HW 8, 9: $\tau_0 = \alpha_\nu h$ $\quad \tau = \alpha_\nu \frac{h}{\cos\theta_z} = \tau_0/\cos\theta_z$ $\quad h = \tau_0/\alpha_\nu$

8.



$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu$

↳ = 0 (no emission)

$\Rightarrow \int \frac{dI_\nu}{I_\nu} = \int -\alpha_\nu ds \Rightarrow \ln(I_\nu) = -\alpha_\nu s + C$

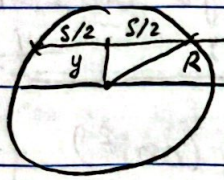
$\Rightarrow I_\nu = Ce^{-\alpha_\nu s} = Ce^{-\tau}$

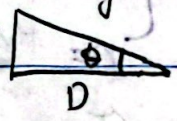
When $\tau = 0$ (no optical depth), $I_\nu = I_{\nu,0}$ (original intensity)

$\Rightarrow I_{\nu,0} = Ce^0 = C \Rightarrow I_\nu = I_{\nu_0} e^{-\alpha_\nu s}$

$\Rightarrow \boxed{I_\nu = I_{\nu_0} e^{-\tau_0/\cos\theta_z}}$

9.





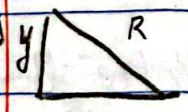
$\Rightarrow y = D\tan\theta \approx D\theta$ (small angle approx)

$\frac{dI_\nu}{ds} = j_\nu - \alpha_\nu I_\nu$

↳ = 0 (no absorption)

$\Rightarrow \int_0^{I_\nu} dI_\nu = \int_0^{\text{total path length} = s} j_\nu ds$

$\Rightarrow I_\nu = j_\nu s$ ↩



$\Rightarrow R^2 = \frac{s^2}{4} + y^2 = \frac{s^2}{4} + D^2\theta^2 \Rightarrow s = 2\sqrt{R^2 - D^2\theta^2}$

$\Rightarrow \boxed{I_\nu = j_\nu 2\sqrt{R^2 - D^2\theta^2}}$