HW5: a) $\alpha = \frac{\partial(\ln B_\nu)}{\partial(\ln \nu)} = \frac{\partial(\ln B_\lambda)}{\partial(\ln \lambda)}$

Rayleigh-Jeans: • $B_\nu(T) \sim \frac{2h\nu^3}{c^2} \sim \frac{2k\nu^2 T}{c^2} \Rightarrow \ln(B_\nu) = \ln\left(\frac{2k\nu^2 T}{c^2}\right)$

$\Rightarrow \ln(B_\nu) = 2\ln\nu + \ln\left(\frac{2kT}{c^2}\right) \Rightarrow \frac{\partial \ln(B_\nu)}{\partial \ln(\nu)} = \frac{\partial}{\partial \ln(\nu)}\left[2\ln(\nu) + \ln\left(\frac{2kT}{c^2}\right)\right]$

$\Rightarrow \alpha = 2 \Rightarrow B_\nu \sim \nu^2$

• $B_\lambda(T) \sim \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} = \frac{2hc^{\cancel{2}}}{\lambda^{\cancel{5}4}} \frac{1}{1 + \frac{hc}{kt\lambda} - 1} = \frac{2ckT}{\lambda^4}$

$\Rightarrow \frac{\partial(\ln B_\lambda)}{\partial(\ln \lambda)} = \frac{\partial[\ln(2ckT) + \ln(\lambda^{-4})]}{\partial \ln(\lambda)} = \frac{\partial\,[\ln(2ckT) - 4\ln(\lambda)]}{\partial \ln(\lambda)} = -4 \Rightarrow B_\lambda \sim \frac{1}{\lambda^4}$

b) $\nu_{pk}\,\lambda_{pk} \overset{?}{=} c \Rightarrow \frac{0.2898}{T} \cdot 5.879 \cdot 10^{10}\, T \overset{?}{=} c \Rightarrow 1.704 \cdot 10^{10}\, \frac{cm}{s} \neq c$

c) $T_B = \text{stuff} \times F_\nu$; $F_\nu = \frac{2\nu^2 kT}{c^2} \cdot \frac{\pi}{4} \frac{\theta^2_{mb}}{\ln 2}$; $\Omega = \frac{\pi}{4} \frac{\theta^2_{mb}}{\ln 2}$ (HW2)

$\Rightarrow F_\nu = \frac{\pi}{2} \frac{\theta^2_{mb} kT\nu^2}{\ln 2\; c^2} \Rightarrow \boxed{T_B = \frac{2F_\nu c^2 \ln 2}{\pi \theta^2_{mb} k}}$