HW 11, 12

optically

11. a) Optical thick $\Rightarrow \tau_\nu \gg 1 \Rightarrow I_\nu = S_\nu = B_\nu(T) = \frac{F_\nu}{\Omega}$

HW3: $\Omega = \frac{\pi R^2}{D^2}$

$\Rightarrow F_\nu = B_\nu(T)\,\pi \frac{R^2}{D^2}$

b) Optically thin $\Rightarrow \tau_\nu \ll 1 \Rightarrow I_\nu = I_{\nu,bg}(1-\tau_\nu) + S_\nu[1-(1-\tau_\nu)]$ ($I_{\nu,bg} \to 0$)

$= S_\nu \int \alpha_\nu ds = 2 j_\nu \sqrt{R^2 - D^2\theta^2}$ (HW9)

$\Rightarrow F_\nu = 2\pi \int_0^{R/D} 2 j_\nu \sqrt{R^2 - D^2\theta^2}\,\theta\, d\theta = \frac{-2\pi j_\nu}{D} \int_0^{R^2} \sqrt{u}\, du = \frac{-2\pi j_\nu}{D^2}\left[\frac{2}{3}u^{3/2}\right]_{R^2}^{0}$

$u = R^2 - D^2\theta^2 \Rightarrow du = -2D^2\theta\, d\theta$

$\Rightarrow F_\nu = \frac{4\pi R^3}{3D^2} j_\nu$

12. $n(h) = n_0 e^{-h/h_0}$, $h_0 = 9\,Km$; $\sigma_D = 5.1 \cdot 10^{-27}\,cm^2$; $\lambda = 532\,nm$

$n_0 = 2.7 \cdot 10^{19}\,cm^{-3} \cdot 78\% \approx 2.106 \cdot 10^{19}$ molecules/cm^2

$\Rightarrow \tau = \int n \sigma_D \, ds = \sigma_D \int_0^h n_0 e^{-h/h_0} \, dh = \sigma_D n_0 [-h_0 e^{-h/h_0}]_0^h$

$= \sigma_D n_0 h_0 (1 - e^{-h/h_0})$ $h \to \infty \Rightarrow \tau = \sigma_D n_0 h_0 = \boxed{0.10}$

$= 5.1 \cdot 10^{-27} \cdot 2.106 \cdot 10^{19} \cdot 9 \cdot 10^5$