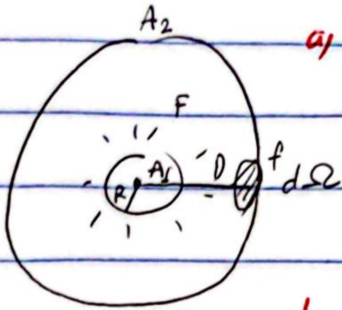
HW 3,4

3.



a) $F = \frac{P}{A} \Rightarrow FA = P$

No energy lost $\Rightarrow FA_1 = f A_2 \Rightarrow F 4\pi R^2 = f 4\pi D^2$

$\Rightarrow \frac{F}{f} = \frac{D^2}{R^2} \Rightarrow f = \frac{FR^2}{D^2}$

b) For $D >> R \Rightarrow \Omega = \pi \frac{R^2}{D^2}$ (HW1) $\Rightarrow B = \frac{f}{\Omega} = \frac{fD^2}{\pi R^2}$

c) Unit ~~of~~ $B = \underbrace{erg \cdot s^{-1} \cdot cm^{-2}}_{flux} \cdot \underbrace{str^{-1}}_{/\,\Omega} \Rightarrow$ specific intensity

B doesn't depend on D because: $B = \frac{fD^2}{\pi R^2} = \frac{F}{\pi}$

Another way to think about it is that intensity doesn't depend on D because: $\underbrace{F}_{1/D^2} = \int \underbrace{I}_{\text{No } D} \ldots \underbrace{d\Omega}_{1/D^2}$

4. a) $I_\nu = I = const$ (isotropic)

$$F = \int_0^{2\pi} \int_0^{\pi/2 \text{ (only view half the surface)}} I \underbrace{\cos\theta \sin\theta}_{1/2 \sin(2\theta)} d\theta \, d\phi \qquad \boxed{2\cos\theta \sin\theta = \sin(2\theta)}$$

$$= I 2\pi \int_0^{\pi/2} \frac{1}{2} \sin(2\theta) d\theta = I \frac{2\pi}{4} \cdot [-\cos(2\theta)]\Big|_0^{\pi/2}$$

$$= -I \frac{\pi}{2} (-1-1) = I\pi$$

b) $F = \int_0^{2\pi} \int_0^{\pi} I \cos\theta \sin\theta \, d\theta \, d\phi = -I \frac{2\pi}{4} [\cos(2\theta)]\Big|_0^{\pi} = 0$