HW 15

a) $T_d \sim 10K$; $A_V \sim 100$ mag; $\frac{m_g}{m_d} = 100$; $R_V = 3.1$

Optically thin $\Rightarrow I_\nu = \int B_\nu[T_d(s)]\, \rho_d(s)\, K_\nu(s)\, ds$

$= B_\nu(T_d) K_\nu \Sigma_d = B_\nu K_\nu \rho_d \int ds$

Since optically thin $\Rightarrow$ $\boxed{F_\nu = \frac{j_\nu \cdot V}{D^2}}$ (previous HW)

$= \frac{\rho_d K_\nu B_\nu V}{D^2} = \boxed{\frac{M_d K_\nu B_\nu}{D^2} = F_\nu}$

$j_\nu = \alpha_\nu S_\nu$

$= \rho_d \cdot K_\nu$; $= B_\nu(T_d)$

b) $\lambda = ?$ $\tau = 1$ for $A_V = 1.20 \text{ mag}$

Optically thin $\Rightarrow \tau_\nu = \int \rho_d \kappa_\nu \, ds = \kappa_\nu \Sigma_d = 1$

$\frac{N(H)}{A_V} = 1.9 \cdot 10^{21}$ for $R_V = 3.1 \Rightarrow N(H) = 1.9 \cdot 10^{21} A_V$

$\Sigma_d = \mu m_H N(H) = 1.404 \cdot 1.67 \cdot 10^{-24} \cdot \frac{A_V}{100} \cdot 1.9 \cdot 10^{21}$

$\approx 0.445 \text{ cm}^2 \text{ g}$

$\Rightarrow \kappa_\nu = 1/\Sigma_d \approx 2.24472 \text{ cm}^2 \text{g}^{-1} \xrightarrow{\text{Table}} \lambda \approx 5.72 \cdot 10^2 \mu m$

$= 10^{-25} \text{ergs}^{-1} \text{cm}^{-2} \text{Hz}^{-1} \approx 1 \cdot 10^{-27} \text{W} \cdot \text{m}^{-2}$

c) $\lambda = 500 \mu m$; $d = 135 pc$; $F_\nu = 0.1 Jy$; $T = 10 K$

$F_\nu = \frac{M_d \kappa_\nu B_\nu}{D^2} \Rightarrow M_d = \frac{F_\nu D^2}{\kappa_\nu B_\nu}$ ← $135 pc \approx 4.166 \cdot 10^{18} m$

$B_\nu = \frac{2hc^2}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1} \approx 0.227 \text{W} \cdot \text{m}^{-2} \cdot \text{ster}^{-1}$

$\lambda = 500 \mu m \Rightarrow \kappa_\nu \approx 2.775 \text{ cm}^2 \text{g}^{-1}$ (Table)

$\approx 0.2775 \text{ m}^2 \text{kg}^{-1}$

$\Rightarrow M_d = \frac{1 \cdot 10^{-27} \text{W m}^{-2} \cdot (4.166 \cdot 10^{18} \text{m})^2}{0.2775 \text{ m}^2 \text{kg}^{-1} \cdot 0.227 \text{W} \cdot \text{m}^{-2} \cdot \text{ster}^{-1}} \approx 2.755 \cdot 10^{11} \text{ kg}$

$\frac{m_g}{m_d} = 100 \Rightarrow m_g = 2.755 \cdot 10^{13} \text{kg}$

$\Rightarrow m_g + m_d = 2.783 \cdot 10^{13} \text{ kg} = M$

Only 50% form ☆ $\Rightarrow M/2 \approx 1.39 \cdot 10^{13} \text{ kg}$

$0.08 M_\odot = 0.08 \cdot 1.99 \cdot 10^{30} \text{ kg} \approx 1.59 \cdot 10^{29} \text{ kg} \gg M$

$\Rightarrow$ This gas+dust thingy can't form brow drawf $\Rightarrow$ no detection.