# ASTR 300B – Fall 2024
## <u>Due: Tues. Sept. 10</u>

5. Consider a blackbody radiating at a temperature T.

(a) The spectral index of a blackbody is defined as the logarithmic derivative $\alpha = \partial(\ln B_\nu)/\partial(\ln \nu)$ or $\alpha = \partial(\ln B_\lambda)/\partial(\ln \lambda)$. Calculate $\alpha$ for both $B_\nu$ and $B_\lambda$ **in the Rayleigh-Jeans limit** and show that $\alpha$ is equal to the power-law exponent of the frequency dependence of $B_\nu$ and the wavelength dependence of $B_\lambda$ , respectively, in the Rayleigh-Jean limit (i.e. show that $B_\nu \sim \nu^\alpha$ and $B \sim \lambda^\alpha$ ).

(b) Prove that the peaks of $B_\lambda$ and $B_\nu$ do ***not*** follow the relationship $\lambda_{pk}\nu_{pk} = c$. This is because $B_\lambda$ and $B_\nu$ are two different versions of the Planck function, one per unit wavelength and the other per unit frequency.

(c) As a result of the linear proportionality between monochromatic specific intensity and temperature in the Rayleigh-Jeans limit, radio astronomers sometimes convert their observed flux density into a "brightness temperature", $T_B$. Calculate the formula to convert from flux density, $F_\nu$ (Jy), to "brightness temperature" in K for a radio telescope with a (small on the sky) Gaussian Power Pattern with main beam FWHM of $\theta_{mb}$. Write your answer so that you have solved for $T_B$ (i.e. $T_B$ = stuff $\times$ $F_\nu$). Assume that the source intensity is constant and completely fills the solid angle of the telescope beam. HINT: Use results from prior homework.

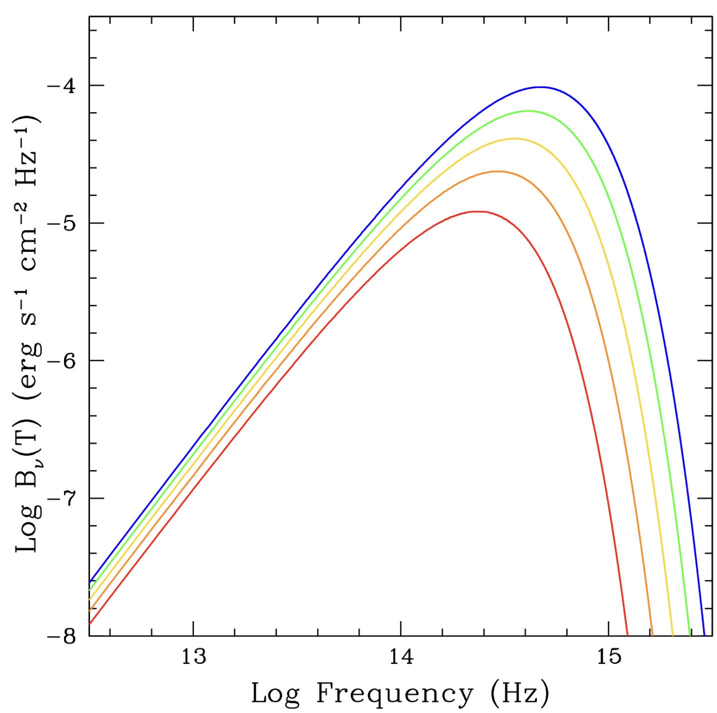


Fig. 1: The logarithm of $\pi$ x Planck function (per unit frequency) plotted vs. log frequency (note the y-axis is mis-labeled and is missing the factor of $\pi$). The different colors correspond to different T.