HW10

a) $J_\nu(T) = (h\nu/k)\left[e(h\nu/kT)-1\right]^{-1}$; $T_A = \frac{c^2}{2k\nu^2} I_\nu \Rightarrow I_\nu = \frac{2k\nu^2}{c^2} T_A$

$I_\nu = I_{\nu,bg}\, e^{-\tau_\nu} + S_\nu(1-e^{-\tau_\nu})$

Planck: $b_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \approx I_{\nu,bg}(T_{bg}) \approx S_\nu(T)$

$$\Rightarrow \frac{2k\nu^2}{c^2} T_A = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT_{bg}}-1} e^{-\tau_\nu} + \frac{2h\nu^3}{c^2}\cdot\frac{1}{e^{h\nu/kT}-1}(1-e^{-\tau_\nu})$$

$$\Rightarrow T_A = \frac{h\nu}{k}\left[\frac{e^{-\tau_\nu}}{e^{h\nu/kT_{bg}}-1} + \frac{1-e^{-\tau_\nu}}{e^{h\nu/kT}-1}\right]$$

$$\Rightarrow T_A = J_\nu(T_{bg})\, e^{-\tau_\nu} + J_\nu(T)(1-e^{-\tau_\nu})$$

b) Rayleigh-Jeans: $\frac{h\nu}{kT} \ll 1 \Rightarrow e^{h\nu/kT} \approx 1+\frac{h\nu}{kT}$

$$\Rightarrow T_A = \frac{\cancel{h\nu}}{\cancel{k}}\left[\frac{e^{-\tau_\nu}}{\cancel{1}+\cancel{h\nu}/\cancel{k}T_{bg}-\cancel{1}} + \frac{1-e^{-\tau_\nu}}{1+h\nu/kT-1}\right]$$

$\Rightarrow T_A = e^{-\tau_\nu} T_{bg} + (1-e^{-\tau_\nu})T$

$\Rightarrow T_{A_b} = e^{-\tau_b} T_c + (1-e^{-\tau_b})T_b$

$\Rightarrow T_{Ax} = e^{-\tau} T_{A_b} + (1-e^{-\tau})T_x$

$\Rightarrow T_{Af} = e^{-\tau_f} T_{Ax} + (1-e^{\tau_f})T_f$

$= e^{-\tau_f}\left[e^{-\tau} T_{A_b} + (1-e^{-\tau})T_x\right] + (1-e^{-\tau_f})T_f$

$= e^{-\tau_f}\{e^{-\tau}[e^{-\tau_b}T_c + (1-e^{-\tau_b})T_b] + (1-e^{-\tau})T_x\} + (1-e^{-\tau_f})T_f$

idk if I should write τ_b or τ_{ν_b} ; τ_x or τ_ν ; τ_f or τ_{ν_f}
please don't mark me wrong ^^