# HW 20+1=21

21 $D = 3.6\,Mpc = 3.6\,10^6\,pc$ ; $F_\nu = 75\,mJy = 75\cdot10^{-3}\cdot10^{-23}\,erg\,s^{-1}\,cm^{-2}\,Hz^{-1}$

$R = 0.0123\,pc$ ; $\boxed{|\vec{B}_\perp| = ?}$ ; $|\vec{B}_{earth}| = 0.5G$

$\nu = 1.43\,GHz = 1.43\cdot10^9\,Hz$ $\sim \nu^{2.5}$ (optically thick)

$\nu = 23GHz/30GHz \sim \nu^{-1}$ (optically thin)

Optically thick: $F_\nu = \underbrace{S_\nu}\cdot\frac{\pi R^2}{D^2}$ $\quad c_1 = 6.27\cdot10^{18}$

$\hookrightarrow = \frac{j_\nu}{\alpha_\nu} = \frac{c_5}{c_6} B_\perp^{-1/2}\left(\frac{\nu}{2c_1}\right)^{5/2}$

$\Rightarrow F_\nu = \frac{c_5(\Gamma)}{c_6(\Gamma)} B_\perp^{-1/2}\left(\frac{\nu}{2c_1}\right)^{5/2}\cdot\frac{\pi R^2}{D^2}$ (need to find $\Gamma$)

$\hookrightarrow$ get $c_5$ & $c_6$

$\Rightarrow B_\perp = \left[\frac{1}{F_\nu}\frac{c_5(\Gamma)}{c_6(\Gamma)}\left(\frac{\nu}{2c_1}\right)^{5/2}\frac{\pi R^2}{D^2}\right]^2$ (1)

To find $\Gamma$: Optically thin: $F_\nu = j_\nu\frac{V}{D^2} \Rightarrow F_\nu \sim j_\nu \sim \nu^{-\frac{\Gamma-1}{2}} \sim \nu^{-1}$

$\Rightarrow -\frac{\Gamma-1}{2} = -1 \Rightarrow \boxed{\Gamma = 3}$ (Plug back in (1))

$\Rightarrow c_5(3) = 7.52\cdot10^{-24}$

$c_6(3) = 7.97\cdot10^{-41}$

$\Rightarrow B_\perp = \left[\frac{1}{75\cdot10^{-26}}\cdot\frac{7.52\cdot10^{-24}}{7.97\cdot10^{-41}}\left(\frac{1.43\cdot10^9}{2\cdot6.27\cdot10^{18}}\right)^{5/2}\frac{\pi\cdot0.0123^2}{(3.6\cdot10^6)^2}\right]^2$

$\approx 0.41$ [$erg\,s^{-1}\,cm^{-2}\,\cancel{Hz^{-1}}\,\cancel{Hz}$] =