Degenerate EOS & Chandrasekhar Mass

1. Degen EOS Prob: a) $\frac{dP}{dr} \sim \frac{P_{surf} - P_{center}}{R} \sim \frac{P_{center}}{R}$ ....

Stellar Structure Equation:

$$\frac{dr}{dm} = \frac{1}{4\pi r^2 \rho} \; ; \; \frac{dP}{dm} = \frac{-Gm(r)}{4\pi r^4} \Rightarrow \frac{dP}{dr} = \frac{-Gm(r)\rho}{r^2} \sim \frac{P_{center}}{R} \sim \frac{C_1 \rho^{5/3}}{R}$$

(constant)

b) $\Rightarrow \frac{dP}{dr} \sim C_1 \frac{\left(\frac{M}{4/3\pi R^3}\right)^{5/3}}{R} \sim \frac{C_2 M^{5/3}}{R^6} \sim \frac{G\overbrace{m(r)}\rho}{r^2} \overset{r^2 \approx R^2}{\sim} \frac{G}{4/3\pi} \rightarrow C_3 \frac{M^2}{R^5}$

$$\Rightarrow C_2 M^{5/3} \sim C_3 M^2 R \Rightarrow \boxed{M^{-1/3} \sim C R}$$

↳ constant

Explaination:

a) $dP/dr \sim (P_{surf} - P_{center})/R \sim P_{center}/R$ (1)

b) $P_{center} \sim \rho^{5/3} \cdot Const \sim \frac{M^{5/3}}{R^5}$ const (2)

c) Set $r^2 \sim R^2$ & $m(r) \sim M$ (3)

2. Chandrasekhar Mass Prob:

$P_{center} \approx 1.244 \cdot 10^5 \left(\frac{\rho}{\mu_e}\right)^{4/3}$ dynes $cm^{-2} \approx P$

$M_{chandrasekhar} = 5.80\, M_\odot \cdot \mu_e^{-2} \approx 1.44\, M_\odot$

$\mu_e \approx 1/2$ For ultra-relativistic

$P_{center} \sim \left(\frac{\rho}{\mu_e}\right)^{4/3} \sim P \Rightarrow P_{center} \sim C\rho^{4/3} \sim \frac{M^{4/3}}{R^4} \left(\frac{3C}{4\pi}\right)^{4/3}$ (ignore const)

$\frac{dP}{dr} = -\frac{G m(r) \rho}{r^2} \sim \frac{P_{center}}{R}$; $m(r) \sim M$, $r \sim R$

$\Rightarrow -\frac{GM\rho}{R^2} = \frac{GM^2}{R^5}\left(\rho \sim \frac{M}{R^3}\right) \sim \frac{P_{center}}{R} \Rightarrow P_{center} \sim \frac{GM^2}{R^4}$

$\Rightarrow \left(\frac{\rho}{\mu_e}\right)^{4/3} \sim \frac{GM^2}{R^4} \sim \left(\frac{M}{R^3 \mu_e}\right)^{4/3} \sim \frac{M^{4/3}}{R^4 \mu_e^{4/3}}$

$\Rightarrow M^2 / M^{4/3} \sim (G\mu_e^{4/3})^{-1} \Rightarrow M^{2/3} \sim (G\mu_e^{4/3})^{-1}$

$\Rightarrow M \sim G^{-3/2} \mu_e^{-2} \cdot \text{Constant} \sim K \mu_e^{-2}$ (Const)

$\Rightarrow$ No mass & radius relationship. Only Chandrasekhar mass left: $M_{chandrasekhar} \sim 5.80 M_\odot \mu_e^{-2} \sim \mu_e^{-2}$ ✓