Exercises on binary mass determinations:

Ex 7.4: $P = 49.94$ years

$\pi = 0.37921'' \pm 0.00158''$ (trigonometric parallax, not pi!)

$a = 7.61''$; $\frac{a_A}{} = 0.466$

1. Kepler's 3rd Law: $T^2 = \frac{4\pi^2}{GM} a^3 \Rightarrow M = \frac{4\pi^2}{G} \cdot \frac{7.61^3 \cdot \pi^3}{49.94^2} \frac{AU^3}{yr^2} \approx 3.24 M_\odot$

(pi; parallax)

$\frac{M_A}{M_B} = \frac{a_B}{a_A} = \frac{1}{0.466} \approx 2.146 \Rightarrow M_B = \frac{M_A}{2.146}$

$\Rightarrow M = M_A + M_B = M_A + \frac{M_A}{2.15} = 3.24$

$\Rightarrow M_A = 3.24 / (1 + 0.466^{2.15}) \approx 2.21 M_\odot$ (Sirius A)

$\Rightarrow M_B = M - M_A = 3.24 - 2.21 \approx 1.03 M_\odot$ (Sirius B)

2. $M_{bol A} = 1.36$, $M_{bol B} = 8.79$; $M_{bol S} = 4.75$

$M_{bol} = -2.5 \log(L/L_\odot) \Rightarrow \frac{L}{L_\odot} = 10^{-\frac{M_{bol}}{2.5}}$

$\Rightarrow \frac{L_A}{L_\odot} = 10^{\frac{M_{bol S} - M_{bol A}}{2.5}} = 10^{(4.75-1.36)/2.5} \approx 22.7$

$\Rightarrow L_A = 22.7 L_\odot$

$\Rightarrow L_B = 10^{\frac{M_{bol S} - M_{bol B}}{2.5}} L_\odot = 10^{(4.75-8.79)/2.5} \approx 0.0242 L_\odot$

3. $T_B \approx 24790K \pm 100K$

$L_B = 4\pi R_B^2 \sigma T_B^4 \Rightarrow R_B = \sqrt{\frac{L_B}{4\pi\sigma T_B^4}} = \sqrt{\frac{0.0242 \cdot 3.846 \cdot 10^{26}}{4\pi (5.67 \cdot 10^{-8}) \cdot 24790^4}}$

$\approx 588157.47 \approx 5.88 \cdot 10^6$ m $\Rightarrow$ White dwarf, very small & hot, only $\approx 0.92 R_\oplus$ & $\approx 0.0084 R_\odot$

Ex7.6 $P = 6.31$ yr, $v_A = 5.4$ km/s, $v_B = 22.4$ km/s

$(t_b - t_a) = 0.58$ d, $(t_c - t_b) = 0.64$ d

$m_{bol\,max} = 5.40$, $m_{bol\,pmin} = 9.20$, $m_{bol\,smin} = 5.44$

1. $\frac{M_B}{M_A} = \frac{v_A}{v_B} = \frac{5.4}{22.4} \approx 0.24$

2. $\frac{(M_B + M_A)}{M} \frac{\sin^3 i}{\sin^3 90^\circ} = \frac{P}{2\pi G} (v_A + v_B)^3$ (Kepler binary sys)

$\Rightarrow M = \frac{6.31 \cdot 3.154 \cdot 10^7}{2\pi\, 6.67 \cdot 10^{-11}} (5.4 \cdot 10^3 + 22.4 \cdot 10^3)^3$

$\approx 1.020 \cdot 10^{31}$ kg $\approx 5.101 M_\odot$

3. $M_A + M_B = 5.101\, M_\odot$, $M_B = 0.24 M_A \Rightarrow M_A = 5.101/1.24 \approx 4.114 M_\odot$

$\Rightarrow M_B = M - M_A = 5.101 - 4.114 \approx 0.987\, M_\odot$

4. $r_A = \frac{V_A + V_B}{2}(t_b - t_a) = \frac{5.4 + 22.4}{2} \cdot (0.58 \cdot 86400) \cdot 10^3$

$= 69655680000\,m \approx 1 R_\odot$

$r_B = r_A + \frac{V_A + V_B}{2}(t_c - t_b) = 1 + \frac{5.4 + 22.4}{2} \cdot 10^3 (0.64 \cdot 86400)$

$\approx 1 R_\odot + \frac{76861440000}{2} \approx 1 R_\odot + 1.12 R_\odot = 2.12 R_\odot$

(Assuming $r_A < r_B$, could be either way)

5. $\frac{F_A}{F_B} = 10^{.4(m_{bol\,B} - m_{bol\,A})}$

* prime min: $\frac{L_A}{L_B} = 10^{.4(m_{bol\,pmin} - m_{bol\,max})} = 10^{.4(9.2 - 5.4)} \approx 33.11$

$\rightarrow \approx$ Flux $= 4\pi R^2 \sigma T^4$

* Sec min: $\frac{L_A}{L_B} = 10^{.4(m_{bol\,smin} - m_{bol\,max})} = 10^{.4(5.44 - 5.40)} \approx 1.04$

Don't need secondary, need for $\frac{R_A}{R_B}$ tho

$\Rightarrow \frac{T_A}{T_B} \approx \frac{L_A}{L_B} = (33.11)^{1/4} \approx 2.02$