Eddington Luminosity HW

1. $P_{rad} = \frac{aT^4}{3} \gg P_{gas}$; $\frac{dP_{rad}}{dr} = ?$ in L, κ, ρ ; $\frac{dT}{dr} = \frac{-3\kappa\rho L}{16\pi a c r^2 T^3}$

i) $\Rightarrow \frac{dP_{rad}}{dr} = \frac{4aT^3}{3} \frac{dT}{dr} = \frac{-\kappa\rho L}{4\pi c r^2}$

ii) $L = L_{edd}$; $r \sim R$

We have $\frac{dP}{dr} = -\frac{GM\rho}{R^2}$ (hydrostatic equilibrium)

$\Rightarrow \frac{dP}{dr} = \frac{dP_{rad}}{dr} \Rightarrow \frac{-GM\cancel{\rho}}{\cancel{R^2}} = \frac{-\kappa\cancel{\rho} L_{edd}}{4\pi c \cancel{R^2}} \Rightarrow \boxed{L_{edd} = \frac{4\pi c G M}{\kappa}}$

2. i) Previously: $\frac{dT}{dr} = -\frac{K\rho L}{4\pi c r^2} \Rightarrow dT = \frac{-K\rho L}{4\pi c r^2} dr$

$\Rightarrow T = \frac{K\rho L}{4\pi c r} \Rightarrow T \propto \frac{L}{c}$

ii) $r \sim R$; $T_{center} \propto T \propto \frac{L}{c}$, $L(m) \sim L$, $\frac{dT}{dr} \sim \frac{-T_{center}}{R}$

iii) $\frac{dP}{dr} = \frac{-GM\rho}{r^2}$; $P = \frac{k \rho T}{\mu m_H}$ ↳ constant C

$\Rightarrow C\frac{d\rho T}{dr} = \frac{-GM\rho}{R^2} \Rightarrow dT = \frac{-GM}{CR^2} dR \Rightarrow T \propto \frac{M}{R}$

iv) We know $T \propto \frac{L}{C} \propto \frac{M}{R} \propto T_{center} \Rightarrow \frac{L}{R} \propto \frac{M}{R} \Rightarrow \boxed{L \propto M.}$