Hydrostatic Equilibrium & Dynamic Timescales HW

1. $t_{ff} = \sqrt{\frac{1}{G\rho}} = \sqrt{\frac{1}{GM/R^3}} = \sqrt{\frac{R^3}{GM}}$

$t_{Kep}^2 = \frac{4\pi^2}{GM} R^3 \Rightarrow t_{Kep} = 2\pi \sqrt{\frac{R^3}{GM}}$

$\Rightarrow t_{Kep} = 2\pi t_{ff}$

2. Sun : $M_\odot = 1.989 \cdot 10^{33}$ g, $R_\odot = 6.955 \cdot 10^{10}$ cm $\Rightarrow \rho_\odot = \frac{3}{4} \frac{M_\odot}{\pi R_\odot^3} \approx 1.411$ g/cm^3, $\Rightarrow t_{ff} = \sqrt{\frac{R^3}{GM}} \approx 1.59 \cdot 10^3$ s $\approx$ 26 mins

Supergiant : $M_{RS} = 10 M_\odot$, $R_{RS} = 1000 R_\odot$ $\Rightarrow \rho_{RS} \approx 1.411 \cdot 10^{-8}$ g/cm^3, $\Rightarrow t_{ff} \approx 1.59 \cdot 10^7$ s $\approx$ 184 days

White dwarf : $M_{WD} = 1 M_\odot$, $R_{WD} = 1000$ km $\Rightarrow \rho_{WD} \approx 4.748 \cdot 10^8$ g/cm^3, $\Rightarrow t_{ff} \approx 0.087$ s

Neutron : $M_N = 1 M_\odot$, $R_N = 10$ km $\Rightarrow \rho_N \approx 4.748 \cdot 10^{14}$ g/cm^3, $\Rightarrow t_{ff} \approx 8.68 \cdot 10^{-5}$ s

$\Rightarrow \rho_N > \rho_{WD} > \rho_\odot > \rho_{RS}$ $\Rightarrow$ The higher the density, the faster the collapse