VIRIAL THEOREM & THERMAL TIMESCALE HW

1. (Avg Temp of Sun)

Virial theorem: $2K + U = 0 \Rightarrow 2\,\frac{3}{2}\,N k_B T_{avg} - \frac{GM^2}{R} = 0$

molecular weight

$\Rightarrow T_{avg} = \frac{GM^2}{3RNk_B}$. Assuming most of the sun is Hydrogen & $N = \frac{M}{\mu m}$

$= \frac{GM}{3Rk_B}\mu m = \frac{6.67\cdot10^{-11}\cdot1.99\cdot10^{30}}{3\cdot6.96\cdot10^{8}\cdot1.38\cdot10^{-23}}\cdot 0.61\cdot1.67\cdot10^{-27}$ Hydrogen atom mass

$\approx 4.69\cdot10^{6}\,K \gg T_{surface} = 5.78\cdot10^{3}\,K$

I guess the average temp is higher because the core temperature inside the sun is very high ($1.5\cdot10^{7}K$)

For thermal equilibrium, the energy produces by the sun core $\approx$ the luminosity of the sun. Since we assume $C\sim1$ & the sun is in thermal equilibrium as the gravitational potential (U) energy is roughly equal to the thermal energy (K) of the sun through virial theorem. This means that the gravitational forces are balanced by the fusion force (pressure) inside the star, maintaining thermal equilibrium.

2. (Avg Sound Speed vs Escape Velocity)

$C_s = \sqrt{\dfrac{K_B T}{\mu m_H}}$; $V_{esc} = \sqrt{\dfrac{2GM}{R}}$

Assuming the star is mostly Hydrogen

Earlier, in the previous problem, I have: $T_{avg} = \dfrac{GM}{3R K_B} \mu m_H$

$\Rightarrow C_s = \sqrt{\dfrac{K_B GM \cancel{\mu m_H}}{3 \cancel{\mu m_H} K_B R}} = \sqrt{\dfrac{GM}{3R}}$

$\rightarrow C_s / V_{esc} = \sqrt{\dfrac{\cancel{GM}}{3\cancel{R}} \cdot \dfrac{\cancel{R}}{2\cancel{GM}}} = \sqrt{\dfrac{1}{6}}$

$\rightarrow C_s \sim \dfrac{1}{\sqrt{6}} V_{esc}$