Stability and the Cases of Stellar Mass Transfer

The majority of known star systems are binary, consisting of two stars orbiting a shared center of mass. Some of these stars will orbit sufficiently close to each other at given distance scales for one star to take mass from the partner star, transferring it through the Lagrange 1 point, the shared center of mass. This mass transfer is able to significantly adjust the evolutionary path of one or both of the stars, or even to strip the outer layers of the star losing mass, exposing the core and allowing observations of the core directly. This paper will use the term 'donor' for the star giving mass, and the term 'accretor' for the star receiving the mass, although some other papers will also use the terms secondary and primary respectively.

There are three primary cases of mass transfer between stellar binary systems; A, B, and C; as delineated by the evolutionary period of the donor star at the time of mass transfer.

Case A occurs for binary stars with an orbital period of a few days.

Case B occurs for orbital periods between a few to a hundred days.

Case C occurs for orbital periods greater than a hundred days.

These three cases also require different orbital periods to occur, a few days for Case, between several and a hundred days for Case B, and more than a hundred days for Case C.

A Note on Stability

Because of the importance of the semi-major axis, and thus orbital period, on both the cases and mass transfer itself, it is relevant to touch on the mathematical equations for mass transfer. To simplify things, conservative mass transfer and an already circularized orbit (e=0) are assumed. The Roche Lobe for a star is:

$$R_{L_d} = a * 0.49q^{2/3}/(0.6q^{2/3} + ln(1 + q)^{1/3}) \tag{1}$$

For a as the orbital radius (given that the orbits are circular) and $q = m_d/m_a$, the ratio of the donor mass m_d and accretor mass m_a. The Roche Lobe changes as

$$R'_{L_d}/R_{L_d} = (2q - 5/3)(m'_d/m_d) \tag{2}$$

For R' and m' as first derivatives with respect to time. Similarly:

$$a'/a = 2(q - 1)(m'_d/m_d) \tag{3}$$

And via Kepler's Third Law:

$$P'/P = 3(q - 1)(m'_d/m_d) \tag{4}$$

Equations (2) and (3) give crucial information about the stability of mass transfer. From equation (2) it becomes apparent that for $q > 5/6$ the Roche Lobe will contract as a result of the mass loss. At $q > 4/3$ the Roche Lobe contracts faster than the donor star itself does, resulting in unstable, runaway mass transfer.

From equation (3) it is apparent that for $q > 1$ the orbital radius, and thus the period as well, contracts with mass loss.

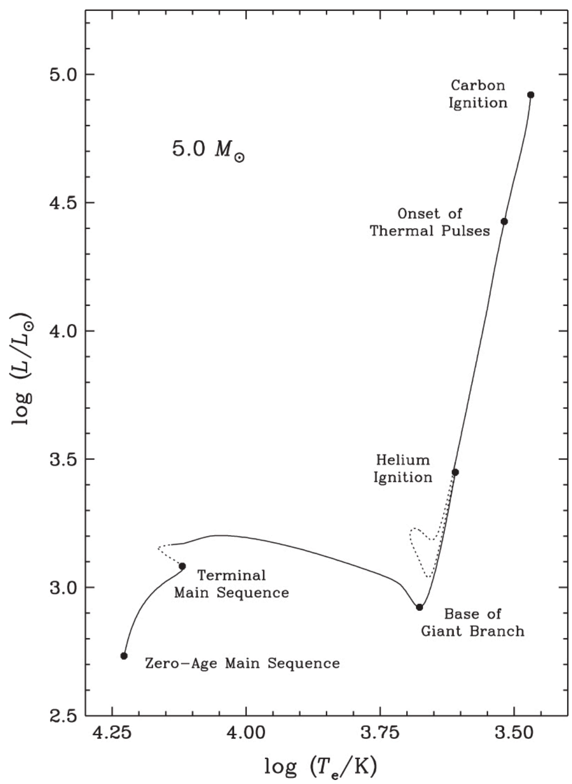

Figure 1. Evolutionary HR diagram track of a 5 solar mass star with some important phases labeled. (Hongwei Ge *et al.* 2015)

Case A

Case A mass transfer occurs during the normal main sequence of a star, when hydrogen burning creates a gradual radius growth over long timescales, as shown in Figure 2 from $t = 0$ to $t = 8 * 10^7$ years. This type of mass transfer necessitates short orbital periods for the stars to be close enough for a main sequence star to fill its Roche Lobe as directly related to the orbital radius.

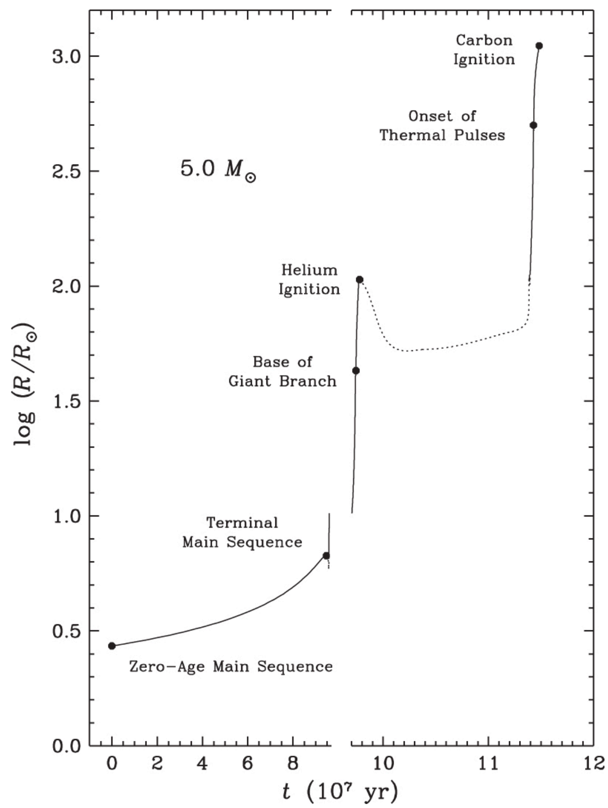

Figure 2. Radius vs time for a 5 stellar mass star with the same phases marked. (Hongwei Ge *et al.* 2015)

Case B

Case B mass transfer occurs during the initial major radius expansion after moving into the giant branch, in the range of $t = 9 * 10^7$ years in Figure 2. As mentioned earlier this case has orbital periods long enough that the gradual growth during the main sequence does not fill the Roche Lobe as with case A, but less than a hundred days period so that the growth into the giant branch does fill the Roche Lobe. When the donor star begins to properly burn helium it can shrink again around $t = 10^8$ years in Figure 2, possibly causing it to stop transferring mass.

Case C

Case C occurs during the growth following helium burning as the donor star moves to heavier and heavier elements in the supergiant phase, allowing orbital periods in excess of a hundred days. This case of mass transfer more than any other allows for significant pollution of the accretor's outer surface, improving the cooling and creating a more extreme temperature gradient, leading to convection currents and extending its current evolutionary phase, assuming that the accretor is the initially smaller mass star and so has not reached the later phases itself.

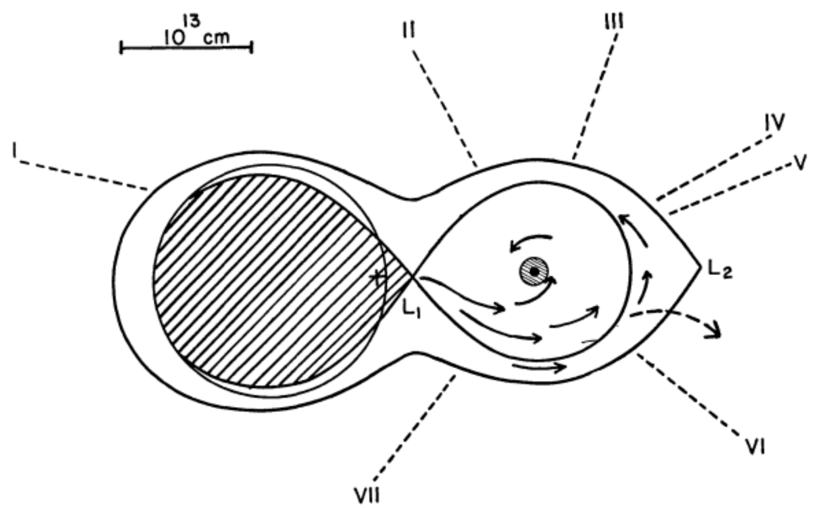

Figure 3. Schematic diagram of the T Coronae Borealis system showing the Roche Lobes of both stars (Kraft 1957).

The mass transfer exhibited in the T Coronae Borealis system (Figure 3) is an example of Case C, with $q = 0.818$ and an orbital period of 227.6 days, indicating a shrinking radius and Roche Lobe for the donor star that is dynamically stable.

References

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Hongwei Ge, Ronald F. Webbink, Xuefei Chen, Zhanwen Han (2015) Adiabatic Mass Loss in Binary Stars. II. From Zero-Age Main Sequence to the Base of the Giant Branch. *The Astrophysical Journal,* 812 40. Doi 10.1088/0004-637X/812/1/40

Robert P. Kraft (1957) The Binary System Nova T Coronae Borealis*.