HOMEWORK 5

4.2 #4. $A = \begin{bmatrix} 1 & -6 & 4 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix}$ $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$

$\Rightarrow Ax = 0 \Leftrightarrow \begin{bmatrix} 1 & -6 & 4 & 0 \\ 0 & 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow x_1 - 6x_2 + 4x_3 = 0 \Rightarrow x_1 = 6x_2 \Rightarrow \begin{cases} x_1 = 6x_2 \\ x_3 = 0 \\ x_4 \text{ is free } = f \end{cases}$

$2x_3 = 0 \Rightarrow x_3 = 0$

$\Rightarrow \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 6x_2 \\ x_2 \\ 0 \\ f \end{bmatrix} = \underbrace{\begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \end{bmatrix}}_{u} x_2 + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}_{v} f$

$\Rightarrow$ Every linear combination of u & v is an element of Nul A

$\Rightarrow \begin{bmatrix} 6 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ & $\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$ are vectors that span Nul A.

4.2 #8. $\left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} : 5r - 1 = s + 2t \right\}$

If W is a vector space $\Rightarrow$ W contains $\vec{0}$ $\Rightarrow \begin{bmatrix} r \\ s \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow 5 \cdot 0 - 1 = 0 + 2 \cdot 0 \Rightarrow -1 = 0$.

Since the $\vec{0}$ doesn't satisfy the condition of $W \Rightarrow \vec{0} \notin W$

$\Rightarrow$ W is <u>not</u> a vector space

4.3 #3. $\begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ -4 \end{bmatrix} \begin{bmatrix} -3 \\ -5 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 3 & -3 \\ 0 & 2 & -5 \\ -2 & -4 & 1 \end{bmatrix} \overset{R_3 \to 2R_1 + R_3}{\sim} \begin{bmatrix} 1 & 3 & -3 \\ 0 & 2 & -5 \\ 0 & 2 & -5 \end{bmatrix}$

$\overset{R_3 \to R_2 - R_3}{\sim} \begin{bmatrix} 1 & 3 & -3 \\ 0 & 2 & -5 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \left. \begin{matrix} x_1 + 3x_2 - 3x_3 = 0 \\ 2x_2 - 5x_3 = 0 \end{matrix} \right\}$ $\Rightarrow$ System will have a free variable

$\Rightarrow$ These vectors are linearly dependent

$\Rightarrow$ They are not bases of $\mathbb{R}^3$.

4.3. #14. $A = \begin{bmatrix} 1 & 2 & -5 & 11 & -3 \\ 2 & 4 & -5 & 15 & 2 \\ 1 & 2 & 0 & 4 & 5 \\ 3 & 6 & -5 & 19 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Nul A: $Bx = 0 \to Bx = \begin{bmatrix} 1 & 2 & 0 & 4 & 5 \\ 0 & 0 & 5 & -7 & 8 \\ 0 & 0 & 0 & 0 & -9 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$\to x_1 + 2x_2 + 4x_4 + 5x_5 = 0 \to x_1 = -2x_2 - 4x_4$

$5x_3 - 7x_4 + 8x_5 = 0 \to x_3 = 7/5 x_4$

$-9x_5 = 0 \to x_5 = 0$

$\to \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -2x_2 - 4x_4 \\ x_2 \\ 7/5 x_4 \\ x_4 \\ 0 \end{bmatrix} = \begin{bmatrix} -2x_2 \\ x_2 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -4x_4 \\ 0 \\ 7/5 x_4 \\ x_4 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} -4 \\ 0 \\ 7/5 \\ 1 \\ 0 \end{bmatrix} x_4.$

$\to \text{Nul} A = \left\{ \begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} -4 \\ 0 \\ 7/5 \\ 1 \\ 0 \end{bmatrix} \right\}$

Col A: Since B is row echelon form of A $\to$ pivot points on 1st, 3rd, 5th columns. $\to$ Col A = Pivot Columns = $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -5 \\ -5 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 5 \\ 2 \end{bmatrix} \right\}$

Row A: pivot points on 1st, 2nd, 3rd row $\to$ Row A = {[1 2 0 4 5], [0 0 5 -7 8], [0 0 0 0 -9]}

4.3 #16. $\underset{w_1}{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}}, \underset{w_2}{\begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}}, \underset{w_3}{\begin{bmatrix} 6 \\ -1 \\ 2 \\ -1 \end{bmatrix}}, \underset{w_4}{\begin{bmatrix} 5 \\ 3 \\ 3 \\ -4 \end{bmatrix}}, \underset{w_5}{\begin{bmatrix} 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}}$

Space spanned by $\{w_1, \dots, w_5\}$ = Col A where $A = \{w_1, \dots, w_5\}$

$\to A = \begin{bmatrix} 1 & -2 & 6 & 5 & 0 \\ 0 & 1 & -1 & 3 & 3 \\ 0 & -1 & 2 & 3 & -1 \\ 1 & 1 & -1 & -4 & 1 \end{bmatrix} \underset{R_4 \to R_4 - R_1}{\sim} \begin{bmatrix} 1 & -2 & 6 & 5 & 0 \\ 0 & 1 & -1 & 3 & 3 \\ 0 & -1 & 2 & 3 & -1 \\ 0 & 3 & -7 & -9 & 1 \end{bmatrix} \underset{R_3 \to R_3 + R_2}{\sim} \begin{bmatrix} 1 & -2 & 6 & 5 & 0 \\ 0 & 1 & -1 & -3 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 3 & -7 & -9 & 1 \end{bmatrix} \underset{R_4 \to R_4 -}{\sim}$

$\begin{bmatrix} 1 & -2 & 6 & 5 & 0 \\ 0 & 1 & -1 & -3 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & -4 & 0 & 8 \end{bmatrix} \underset{R_4 \to R_4 + 4R_3}{\sim} \begin{bmatrix} 1 & -2 & 6 & 5 & 0 \\ 0 & 1 & -1 & -3 & 3 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$\to \text{Col} A = \left\{ w_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, w_2 = \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, w_3 = \begin{bmatrix} -6 \\ -1 \\ 2 \\ -1 \end{bmatrix} \right\}$

$\to$ Space spanned by $\{w_1, \dots, w_5\}$ is $\{w_1, w_2, w_3\}$.