HOMEWORK 4

2.2 #41.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right] \underset{\substack{R_2 \to 3R_1+R_2 \\ R_3 \to 2R_1-R_3}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 3 & -8 & 2 & 0 & -1 \end{array}\right]$$

$$\underset{R_3 \to 3R_2-R_3}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right] \underset{\substack{R_2 \to R_3+R_2 \\ R_1 \to R_3+R_1}}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right]$$

$$\underset{R_3 \to R_3/2}{\sim} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & 7/2 & 3/2 & 1/2 \end{array}\right] \Rightarrow \text{The inverse matrix is: } \begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{bmatrix}$$

2.3 #21. Matrix like exercise 8:

A row echelon?

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ 0 & a_{22} & a_{23} & a_{24} \\ 0 & 0 & a_{33} & a_{34} \\ 0 & 0 & 0 & a_{44} \end{bmatrix}$$

- The first condition for this matrix to be invertible is that the # of rows = # of columns → A square ($n \times n$) matrix. (this one satisfy

- The second condition is that this matrix has to have n # of pivot position (# pivot position = # of rows/columns) according to the Invertible Matrix Theorem. The matrix above has 4 rows/columns & 4 pivot positions along the diagonal, therefore it is an invertible matrix. To justify my answer, let's find the invertible matrix from problem 8:

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$$\left[\begin{array}{cccc|cccc} 1&3&7&4&1&0&0&0\\ 0&5&9&6&0&1&0&0\\ 0&0&2&8&0&0&1&0\\ 0&0&0&10&0&0&0&1 \end{array}\right] \underset{R_2\to R_2/5}{\sim} \left[\begin{array}{cccc|cccc} 1&3&7&4&1&0&0&0\\ 0&1&9/5&6/5&0&1/5&0&0\\ 0&0&2&8&0&0&1&0\\ 0&0&0&10&0&0&0&1 \end{array}\right] \underset{R_1\to -3R_2+R_1}{\sim}$$

$$\left[\begin{array}{cccc|cccc} 1&0&8/5&2/5&1&-3/5&0&0\\ 0&1&9/5&6/5&0&1/5&0&0\\ 0&0&2&8&0&0&1&0\\ 0&0&0&10&0&0&0&1 \end{array}\right] \underset{R_3\to R_3/2}{\sim} \left[\begin{array}{cccc|cccc} 1&0&8/5&2/5&1&-3/5&0&0\\ 0&1&9/5&6/5&0&1/5&0&0\\ 0&0&1&4&0&0&1/2&0\\ 0&0&0&10&0&0&0&1 \end{array}\right]$$

$R_1\to R_1-8/5R_3$

$R_2\to R_2-9/5R_3$

$$\sim \left[\begin{array}{cccc|cccc} 1&0&0&-6&1&-3/5&-4/5&0\\ 0&1&0&-6&0&1/5&-9/10&0\\ 0&0&1&4&0&0&1/2&0\\ 0&0&0&10&0&0&0&1 \end{array}\right] \underset{R_4\to R_4/10}{\sim} \left[\begin{array}{cccc|cccc} 1&0&0&-6&1&-3/5&-4/5&0\\ 0&1&0&-6&0&1/5&-9/10&0\\ 0&0&1&4&0&0&1/2&0\\ 0&0&0&1&0&0&0&1/10 \end{array}\right]$$

$R_1\to R_1+6R_4$

$R_2\to R_2+6R_4$

$R_3\to R_3-4R_4$

$$\sim \left[\begin{array}{cccc|cccc} 1&0&0&0&1&-3/5&-4/5&3/5\\ 0&1&0&0&0&1/5&-9/10&3/5\\ 0&0&1&0&0&0&1/2&-2/5\\ 0&0&0&1&0&0&0&1/10 \end{array}\right]$$

So the matrix $\begin{bmatrix} 1&3&7&4\\ 0&5&9&6\\ 0&0&2&8\\ 0&0&0&10 \end{bmatrix}$ has an inverse of $\begin{bmatrix} 1&-3/5&-4/5&3/5\\ 0&1/5&-9/10&3/5\\ 0&0&1/2&-2/5\\ 0&0&0&1/10 \end{bmatrix}$

which justified my answer.

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2.3. #18+10=28 E is invertible since according to the Invertible Matrix Theorem, if E is a square $n \times n$ matrix then E is invertible iff there exists another $n \times n$ matrix that its product with $E = I$. $EF = I \Rightarrow E$ is invertible.

$\Rightarrow EF = EE^{-1} = E^{-1}E = I$ (E is invertible).

$\Rightarrow E^{-1}(EF) = E^{-1}I$ (times E^{-1} both sides)

$\Rightarrow IF = E^{-1} \Rightarrow F$ is also an inverse of E. ($F = E^{-1}$)

Since F is an inverse of $E \Rightarrow EF = FE$ (inverse property) $\Rightarrow E$ & F commute.

4.1. #3. $H = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x^2 + y^2 \le 1 \right\}$

I will use vector $\vec{v} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$ which $\in H$ since $0.1^2 + 0.1^2 = 0.02 \le 1$.

Pick a random scalar: $c = 10 \Rightarrow c\vec{v} = 10 \cdot \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$c\vec{v}$ doesn't belong to H since $1^2 + 1^2 = 2 > 1$.

$\Rightarrow H$ is not closed under scalar multiplication $\Rightarrow H$ is not an $\mathbb{R}^2$'s subspac

4.1. #41. $H + K = \{\vec{w} : \vec{w} = \vec{u} + \vec{v}$ for some $\vec{u}$ in H and some $\vec{v}$ in $K\}$

a) • Call $\vec{u_1}$ & $\vec{u_2}$ vectors $\in H$, $\vec{v_1}$ & $\vec{v_2}$ vectors $\in K$, and $\vec{w_1}$ & $\vec{w_2}$ vectors $\in V$ such that $\vec{w_1} = \vec{u_1} + \vec{v_1}$ & $\vec{w_2} = \vec{u_2} + \vec{v_2}$

$\Rightarrow \vec{w_1} + \vec{w_2} = (\vec{u_1} + \vec{v_1}) + (\vec{u_2} + \vec{v_2}) = \vec{u_1} + \vec{v_1} + \vec{u_2} + \vec{v_2} = \underbrace{(\vec{u_1} + \vec{u_2})}_{\in H} + \underbrace{(\vec{v_1} + \vec{v_2})}_{\in K}$

$\Rightarrow \vec{w_1} + \vec{w_2} \in H + K \Rightarrow H + K$ is closed under vector addition.

• Call $c \in \mathbb{R}$ a real scalar such that $c\vec{u_1} \in H$ & $c\vec{v_1} \in K$

$\Rightarrow \underbrace{c\vec{u_1}}_{\in H} + \underbrace{c\vec{v_1}}_{\in K} = c(\vec{u_1} + \vec{v_1}) = \underbrace{c\vec{w_1}}_{\in V} \Rightarrow H + K$ is closed under vector multiplication.

• $H \in V$ & $K \in V$. Also within subspace H & K, there are $\vec{0}$ belongs to V such that $\vec{0} \in H$, $\vec{0} \in K \Rightarrow \vec{0} \in H + K$

$\Rightarrow \vec{0}$ of V are in $H + K$

$\Rightarrow H + K$ is a subspace of V.

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b/ • $\vec{u} \in H$, $\vec{0} \in K$, $\vec{0} \in H$, $\vec{0} \in H+K$

$\Rightarrow \vec{u} + \vec{0} = \vec{u} \in H+K \Rightarrow H \in H+K \Rightarrow H$ is a subset of $H+K$

for $\vec{u_1}, \vec{u_2} \in H \Rightarrow \vec{u_1} + \vec{u_2} \in H \Rightarrow H$ is closed under vector addition

for $c \in \mathbb{R}$ and $c \in H \Rightarrow c\vec{u_1} \in H \Rightarrow H$ is closed under scalar multiplication

$\Rightarrow H$ is a subspace of $H+K$

• $\vec{v} \in K$, $\vec{0} \in K$, $\vec{0} \in H$, $\vec{0} \in H+K$

$\Rightarrow \vec{v} + \vec{k} = \vec{v} \in H+K \Rightarrow K \in H+K \Rightarrow K$ is a subset of $H+K$

for $\vec{v_1}, \vec{v_2} \in K \Rightarrow \vec{v_1} + \vec{v_2} \in K \Rightarrow K$ is closed under vector addition

for $c \in \mathbb{R}$ and $c \in K \Rightarrow c\vec{v_1} \in K \Rightarrow K$ is closed under scalar multiplica-tion

$\Rightarrow K$ is a subspace of $H+K$.

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1. I have reviewed and understood the comments I received on Exam 1. I have also figured out the correct way todo each problem on that exam.

Ben Phan

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