3.3 #24. Vertices of parallelepiped: $(1,5,0), (-3,0,3), (-1,4,-1) \Rightarrow A = \begin{bmatrix} 1 & -3 & -1 \\ 5 & 0 & 4 \\ 0 & 3 & -1 \end{bmatrix}$

$\Rightarrow$ Volume $V = |\det A| = \begin{vmatrix} 1 & -3 & -1 \\ 5 & 0 & 4 \\ 0 & 3 & -1 \end{vmatrix} = \det \begin{vmatrix} 1 & -3 & -1 \\ 0 & 15 & 9 \\ 0 & 3 & -1 \end{vmatrix} \Rightarrow$ cofactor expansion of the first column: $+\begin{vmatrix} 15 & 9 \\ 3 & -1 \end{vmatrix} = -15 - 27 = -42 \Rightarrow V = |-42| = 42$

$\Rightarrow$ Volume of the parallelepiped is 42.

Tutorial 0: I have read and understand what the code is doing (the for loop by the end confused me a little bit but I understand it now).

HOMEWORK 7 (For Linear Algebra)

5.1. #6 $A=\begin{bmatrix}3&6&7\\3&3&7\\5&6&5\end{bmatrix}$, $\vec{x}=\begin{bmatrix}1\\-2\\1\end{bmatrix}$ $\Rightarrow A\vec{x}=\begin{bmatrix}-2\\4\\-2\end{bmatrix}=-2\begin{bmatrix}1\\-2\\1\end{bmatrix}=\lambda\vec{x}$

$\Rightarrow$ $\vec{x}$ is an eigen vector of A with the eigen value $\lambda=-2$.

5.1. #16 $A=\begin{bmatrix}3&0&2&0\\1&3&1&0\\0&1&1&0\\0&0&0&4\end{bmatrix}$, $\lambda=4$

$$\Rightarrow A-\lambda I=\begin{bmatrix}3&0&2&0\\1&3&1&0\\0&1&1&0\\0&0&0&4\end{bmatrix}-\begin{bmatrix}4&0&0&0\\0&4&0&0\\0&0&4&0\\0&0&0&4\end{bmatrix}=\begin{bmatrix}-1&0&2&0\\1&-1&1&0\\0&1&-3&0\\0&0&0&0\end{bmatrix}$$

$\Rightarrow$ Augmented matrix for $(A-4I)\vec{x}=\vec{0}$:

$$\begin{bmatrix}-1&0&2&0&0\\1&-1&1&0&0\\0&1&-3&0&0\\0&0&0&0&0\end{bmatrix}\sim\begin{bmatrix}1&0&-2&0&0\\0&-1&3&0&0\\0&1&-3&0&0\\0&0&0&0&0\end{bmatrix}$$

$$\sim\begin{bmatrix}1&0&-2&0&0\\0&1&-3&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}\Rightarrow\begin{cases}x_1-2x_3=0\\x_2-3x_3=0\\x_3=x_3\\x_4=x_4\end{cases}\Rightarrow\begin{cases}x_1=2x_3\\x_2=3x_3\\x_3=x_3\\x_4=x_4\end{cases}\Rightarrow\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=x_3\begin{bmatrix}2\\3\\1\\0\end{bmatrix}+x_4\begin{bmatrix}0\\0\\0\\1\end{bmatrix}$$

$\Rightarrow$ The basis for the eigenspace is $\left\{\begin{bmatrix}2\\3\\1\\0\end{bmatrix},\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right\}$

Co-factor Expansion of:

3.3 #6 $\begin{matrix}x_1+3x_2+x_3=8\\-x_1+2x_3=4\\3x_1+x_2=4\end{matrix}$ $\quad A\vec{x}=\vec{b}\Rightarrow A=\begin{bmatrix}1&3&1\\-1&0&2\\3&1&0\end{bmatrix}$, $\vec{b}=\begin{bmatrix}8\\4\\4\end{bmatrix}$

$$\Rightarrow A_1(\vec{b})=\begin{bmatrix}8&3&1\\4&0&2\\4&1&0\end{bmatrix},\quad A_2(\vec{b})=\begin{bmatrix}1&8&1\\-1&4&2\\3&4&0\end{bmatrix},\quad A_3(\vec{b})=\begin{bmatrix}1&3&8\\-1&0&4\\3&1&4\end{bmatrix}$$

→ 3rd column: $\det A=1\cdot(-1)^{1+3}\begin{vmatrix}-1&0\\3&1\end{vmatrix}+2(-1)^{2+3}\begin{vmatrix}1&3\\3&1\end{vmatrix}=(-1-0)-2(1-9)=15$

→ 3rd column: $\det A_1(\vec{b})=1(-1)^{1+3}\begin{vmatrix}4&0\\4&1\end{vmatrix}+2(-1)^{2+3}\begin{vmatrix}8&3\\4&1\end{vmatrix}=4-2(8-12)=4+8=12$

→ 3rd column: $\det A_2(\vec{b})=1(-1)^{1+3}\begin{vmatrix}-1&4\\3&4\end{vmatrix}+2(-1)^{2+3}\begin{vmatrix}1&8\\3&4\end{vmatrix}=-16-2(-20)=-16+40=24$

→ 2nd column: $\det A_3(\vec{b})=3(-1)^{1+2}\begin{vmatrix}-1&4\\3&4\end{vmatrix}+1(-1)^{3+2}\begin{vmatrix}1&8\\-1&4\end{vmatrix}=-3(-16)-1(12)=48-12=36$

$$\Rightarrow x_1=\frac{\det A_1(\vec{b})}{\det A}=\frac{12}{15}=\frac{4}{5},\quad x_2=\frac{\det A_2(\vec{b})}{\det A}=\frac{24}{15}=\frac{8}{5},\quad x_3=\frac{\det A_3(\vec{b})}{\det A}=\frac{36}{15}=\frac{12}{5}$$

3.3 #14 $\begin{bmatrix}1&-1&2\\0&2&1\\3&0&6\end{bmatrix}=A\Rightarrow$ Co-factor expansion of column 1:

$\det A=(2\cdot6)+3(-1\cdot1-2\cdot2)=12-15=-3$

- $C_{11}=+\begin{vmatrix}2&1\\0&6\end{vmatrix}=12$ • $C_{12}=-\begin{vmatrix}0&1\\3&6\end{vmatrix}=3$ • $C_{13}=+\begin{vmatrix}0&2\\3&0\end{vmatrix}=-6$
- $C_{21}=-\begin{vmatrix}-1&2\\0&6\end{vmatrix}=6$ • $C_{22}=+\begin{vmatrix}1&2\\3&6\end{vmatrix}=0$ • $C_{23}=-\begin{vmatrix}1&-1\\3&0\end{vmatrix}=-3$
- $C_{31}=+\begin{vmatrix}-1&2\\2&1\end{vmatrix}=-5$ • $C_{32}=-\begin{vmatrix}1&2\\0&1\end{vmatrix}=-1$ • $C_{33}=+\begin{vmatrix}1&-1\\0&2\end{vmatrix}=2$

$$\Rightarrow \operatorname{adj}A=\begin{bmatrix}12&6&-5\\3&0&-1\\-6&-3&2\end{bmatrix}\Rightarrow A^{-1}=\frac{1}{\det A}\operatorname{adj}A=\frac{1}{-3}\begin{bmatrix}12&6&-5\\3&0&-1\\-6&-3&2\end{bmatrix}=\begin{bmatrix}-4&-2&5/3\\-1&0&1/3\\2&1&-2/3\end{bmatrix}$$