HOMEWORK 9 (Non Coding)

1. $\frac{2+i}{3-2i}$ in $a+bi$ form: $\frac{2+i}{3-2i} = \frac{2+i}{3-2i} \cdot \frac{3+2i}{3+2i} = \frac{6+4i+3i-2}{9+4}$

$\Rightarrow \frac{2+i}{3-2i} = \frac{4+7i}{13} = \frac{4}{13} + \frac{7}{13}i = a+bi$ for $a = \frac{4}{13}$, $b = \frac{7}{13}$

2. Find $\left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^8$ (No brute force).

$z = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \Rightarrow |z| = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$

$\Rightarrow \theta = \tan^{-1}\left(\frac{1/\sqrt{2}}{1/\sqrt{2}}\right) = \frac{\pi}{4} \Rightarrow$ Polar form: $z = |z|(\cos\theta + i\sin\theta)$
$= 1 \cdot (\cos \pi/4 + i \sin \pi/4)$

$\Rightarrow z^8 = (\cos \pi/4 + i\sin \pi/4)^8 = (\cos 2\pi + i\sin 2\pi) = 1 + 0i$

$\Rightarrow \left(\frac{1}{\sqrt{2}} - \frac{i}{\sqrt{2}}\right)^8 = 1$

3. $T: \mathbb{P}_2 \to \mathbb{P}_2$, $T(p) = p(0) - \frac{1}{2}p(2)t$. Calculate $T^{167}(-16+3t)$

$p(t) = a + bt \Rightarrow p(0) = a$, $p(2) = a + 2b$

$\Rightarrow T(p) = a - \frac{1}{2}(a+2b)t = \begin{bmatrix} 1 & 0 \\ -1/2 & -1 \end{bmatrix}^{167}$

$$\underbrace{\begin{bmatrix} 1 & 0 \\ -1/2 & -1 \end{bmatrix}}_{T(p)} \underbrace{\begin{bmatrix} a \\ b \end{bmatrix}}_{P(t)} = \begin{bmatrix} a \\ -\frac{1}{2}(a+2b) \end{bmatrix}$$

Since $T(P) \cdot T(P) = I \Rightarrow T^{167}(p) = \begin{bmatrix} 1 & 0 \\ -1/2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{83} = \begin{bmatrix} 1 & 0 \\ -1/2 & 1 \end{bmatrix}$

$\Rightarrow T^{167}(-16+3t) = \begin{bmatrix} 1 & 0 \\ -1/2 & 1 \end{bmatrix} \cdot \begin{bmatrix} -16 \\ 3 \end{bmatrix} = \begin{bmatrix} -16 \\ 11 \end{bmatrix}$

$= -16 + 11t$

4. Matlab submitted in a seperate file.