HOMEWORK 1.

1.1 #13

$$\begin{cases} x_1 - 3x_3 = 8 \\ 2x_1 + 2x_2 + 9x_3 = 7 \\ x_2 + 5x_3 = -2 \end{cases} \Leftrightarrow \begin{cases} x_1 - 3x_3 = 8 \\ 2x_2 + 15x_3 = -9 \\ x_2 + 5x_3 = -2 \end{cases} \Leftrightarrow \begin{cases} x_1 - 3x_3 = 8 \\ -5x_3 = -5 \\ x_2 + 5x_3 = -2 \end{cases}$$

$$\Leftrightarrow \begin{cases} x_1 - 3(-1) = 8 \\ x_3 = -1 \\ x_2 + 5(-1) = -2 \end{cases} \Leftrightarrow \begin{cases} x_1 = 5 \\ x_2 = 3 \\ x_3 = -1 \end{cases}$$

1.1 #26

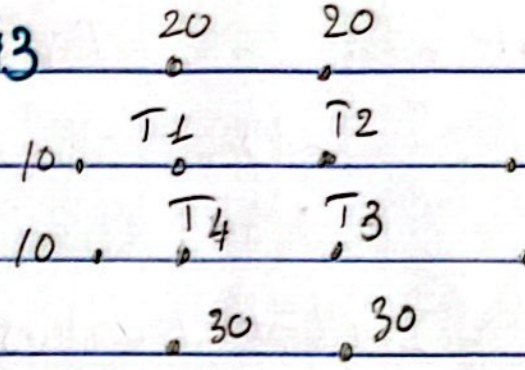
$$\begin{bmatrix} 2 & -3 & h \\ -6 & 9 & 5 \end{bmatrix} \xrightarrow{R_2 + 3R_1} \begin{bmatrix} 2 & -3 & h \\ 0 & 0 & 3h+5 \end{bmatrix} \Rightarrow 0 = 3h + 5 \Rightarrow h = -5/3$$

1.1 #13 (Matrix Style)

$$\begin{bmatrix} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 2 & 5 & -2 \end{bmatrix} \xrightarrow{2R_1 - R_2} \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & -2 & -15 & 9 \\ 0 & 2 & 5 & -2 \end{bmatrix} \xrightarrow{R_2 / -2} \xrightarrow{R_2 + R_3} \begin{bmatrix} 1 & 0 & -3 & 8 \\ 0 & 1 & 15/2 & -9/2 \\ 0 & 0 & -7 & 7 \end{bmatrix}$$

$$\xrightarrow{3/7 R_3 + R_1} \xrightarrow{R_3 / -7} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 15/2 & -9/2 \\ 0 & 0 & 1 & -1 \end{bmatrix} \xrightarrow{-15/2 R_3 + R_2} \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix} \Rightarrow \begin{matrix} x_1 = 5 \\ x_2 = 3 \\ x_3 = -1 \end{matrix}$$

1.1#43



$\Rightarrow T_1 = \frac{10+20+T_2+T_4}{4}$

$\Rightarrow T_2 = \frac{T_1+20+40+T_3}{4}$

$\Rightarrow T_3 = \frac{T_4+T_2+40+30}{4}$

$\Rightarrow T_4 = \frac{10+T_1+T_3+30}{4}$

$\Rightarrow 4T_1 - T_2 - T_4 = 30$

$4T_2 - T_1 - T_3 = 60 \Rightarrow$

$4T_3 - T_4 - T_2 = 70$

$4T_4 - T_1 - T_3 = 40$

$$\begin{cases} 4T_1 - 2T_2 - T_4 = 30 \\ -T_1 + 4T_2 - T_3 = 60 \\ -T_2 + 4T_3 - T_4 = 70 \\ -T_1 - T_3 + 4T_4 = 40 \end{cases}$$

1.2 #4

$$\begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix} \xrightarrow{\substack{3R_1-R_2 \\ 5R_1-R_3}} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 4 & 8 & 12 \\ 0 & 8 & 16 & 34 \end{bmatrix} \xrightarrow[R_3/2]{R_2/4} \begin{bmatrix} 1 & 3 & 5 & 7 \\ 0 & 1 & 2 & 3 \\ 0 & 4 & 8 & 17 \end{bmatrix} \xrightarrow{\substack{3R_2-R_1 \\ 4R_2-R_3}}$$

$$\begin{bmatrix} -1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix} \xrightarrow{R_3/5} \begin{bmatrix} -1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \xrightarrow[-3R_3+R_2]{2R_3-R_1} \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⇒ 1, 2, 4 are pivot columns

1.2 #14

$$\begin{bmatrix} 1 & 2 & -5 & -4 & 0 & -5 \\ 0 & 1 & -6 & -4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_1-2R_2} \begin{bmatrix} 1 & 0 & 7 & 4 & 0 & -9 \\ 0 & 1 & -6 & -4 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow x_5 = 0$$

5 unknowns, only 3 equations ⇒ cannot get specific solutions.

⇒ $\begin{cases} x_1 + 7x_3 + 4x_4 = -9 \\ x_2 - 6x_3 - 4x_4 = 2 \end{cases}$ ⇒ General Solution: $\begin{cases} x_1 = -9 - 7x_3 - 4x_4 \\ x_2 = +6x_3 + 4x_4 + 2 \\ x_3 = c_1 \\ x_4 = c_2 \\ x_5 = 0 \end{cases}$

c_1, c_2 = some const.