HOMEWORK 2

1.3 #18. $v_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}$, $y = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}$

$$\Rightarrow [v_1\ v_2\ y] = \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ -2 & 8 & -3 \end{bmatrix} \underset{R_3 \to 2R_1 + R_3}{\sim} \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 2 & 2h-3 \end{bmatrix}$$

$$\underset{R_3 \to 2R_2 - R_3}{\sim} \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 0 & -10-2h+3 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & h \\ 0 & 1 & -5 \\ 0 & 0 & -2h-7 \end{bmatrix} \Rightarrow -2h-7=0 \Rightarrow h = \frac{-7}{2}$$

1.4 #10.

$$8x_1 - x_2 = 4$$
$$5x_1 + 4x_2 = 1$$
$$x_1 - 3x_2 = 2$$

$Ax = B \Rightarrow A = \begin{bmatrix} 8 & -1 \\ 5 & 4 \\ 1 & -3 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $b = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$\Rightarrow$ vector equation: $x_1 \begin{bmatrix} 8 \\ 5 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$\Rightarrow$ matrix equation: $\begin{bmatrix} 8 & -1 \\ 5 & 4 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

1.4 #18.

$$B = \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 1 & 2 & -3 & 7 \\ -2 & -8 & 2 & -1 \end{bmatrix} \underset{\substack{R_3 \to R_1 - R_3 \\ R_4 \to 2R_1 + R_4}}{\sim} \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 1 & 1 & -5 \\ 0 & -2 & -2 & 3 \end{bmatrix} \underset{\substack{R_3 \to R_2 - R_3 \\ R_4 \to R_4 + R_1}}{\sim}$$

$$\begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -2 \end{bmatrix} \underset{R_4 \to 2R_2 + R_4}{\sim} \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -12 \end{bmatrix} \underset{R_4 \to R_4 / -12}{\sim} \begin{bmatrix} 1 & 3 & -2 & 2 \\ 0 & 1 & 1 & -5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\overset{R_1\to 3R_2-R_1}{\sim}\begin{bmatrix}1&0&5&-17\\0&1&1&-5\\0&0&0&1\\0&0&0&0\end{bmatrix}\overset{\substack{R_1\to 17R_3+R_1\\R_2\to 5R_3+R_2}}{\sim}\begin{bmatrix}1&0&5&0\\0&1&1&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$$ Reduced echelon form

(I forgot to reduce all the way, sorry).

$$\overset{R_4\leftrightarrow R_3}{\sim}\begin{bmatrix}1&3&-2&2\\0&1&1&-5\\0&0&0&1\\0&0&0&0\end{bmatrix}$$

B has pivot points in columns 1,2,4.

$\Rightarrow$ Does Not have pivot points in every columns

$\Rightarrow$ Does not have a solution for each y in $\mathbb{R}^4$ for equation $Bx=y$

$\Rightarrow$ Do not span $\mathbb{R}^4$ for the columns of B.

1.5 #6.
$$\begin{aligned}x_1+3x_2-5x_3&=0\\x_1+4x_2-8x_3&=0\\-3x_1-7x_2+9x_3&=0\end{aligned}\Rightarrow\begin{bmatrix}1&3&-5&0\\1&4&-8&0\\-3&-7&9&0\end{bmatrix}\text{(augmented)}$$

$$\overset{\substack{R_2\to R_1-R_2\\R_3\to 3R_1+R_3}}{\sim}\begin{bmatrix}1&3&-5&0\\0&-1&3&0\\0&2&-6&0\end{bmatrix}\overset{\substack{R_1\to 3R_2+R_1\\R_2\to R_2/-1\\R_3\to 2R_2+R_3}}{\sim}\begin{bmatrix}1&0&4&0\\0&1&-3&0\\0&0&0&0\end{bmatrix}$$

$$\Rightarrow\begin{aligned}x_1+4x_3&=0\\x_2-3x_3&=0\end{aligned}\Rightarrow\begin{cases}x_1=-4x_3\\x_2=3x_3\\x_3\text{ is free}\end{cases}$$

$\Rightarrow$ Solutions: $\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}-4x_3\\3x_3\\x_3\end{bmatrix}=x_3\begin{bmatrix}-4\\3\\1\end{bmatrix}$

1.5 #20.
$$\begin{aligned}x_1+3x_2-5x_3&=4\\x_1+4x_2-8x_3&=7\\-3x_1-7x_2+9x_3&=-6\end{aligned}\Rightarrow\begin{bmatrix}1&3&-5&4\\1&4&-8&7\\-3&-7&9&-6\end{bmatrix}\text{(augmented)}$$

$$\overset{\substack{R_2\to R_1-R_2\\R_3\to 3R_1+R_3}}{\sim}\begin{bmatrix}1&3&-5&4\\0&-1&3&-3\\0&2&-6&6\end{bmatrix}\overset{\substack{R_2\to R_2/-1\\R_3\to R_3/2}}{\sim}\begin{bmatrix}1&3&-5&4\\0&1&-3&3\\0&1&-3&3\end{bmatrix}$$

$$\overset{\substack{R_1\to R_1-3R_2\\R_3\to R_2-R_3}}{\sim}\begin{bmatrix}1&0&4&-5\\0&1&-3&3\\0&0&0&0\end{bmatrix}\Rightarrow\begin{aligned}x_1+4x_3&=-5\\x_2-3x_3&=3\end{aligned}$$

$\Rightarrow x_1=-5-4x_3,\ x_2=3+3x_3$

x_3 is free

$\Rightarrow$ Solutions: $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5-4x_3 \\ 3+3x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$

$= a + x_3 b$

According to problem 1.5 #6, the homogenous equation's solution is:

$x = x_3 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} = x_3 c$ for $c = \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix} = b$

$\Rightarrow x = a + x_3 b \,//\, x = x_3 c$