HOMEWORK 3

1.7 #41. If $\vec{v}_1$ & $\vec{v}_2$ are in $\mathbb{R}^4$ & $\vec{v}_2$ is not scalar multiple of $\vec{v}_1 \Rightarrow \{\vec{v}_1, \vec{v}_2\}$ is linearly independent.

- If $\vec{v}_1$ & $\vec{v}_2 \neq 0 \Rightarrow$ this statement is true since $\vec{v}_1$ & $\vec{v}_2 \neq 0$ and $v_2 \neq k_1 v_1$ and $k_1 v_1 + k_2 v_2 = 0$ iff k_1 & $k_2 = 0$.
- If $\vec{v}_1$ or $\vec{v}_2 = 0 \Rightarrow$ this statement is false since $\vec{v}_1$ or $\vec{v}_2 = 0$ then $\vec{v}_1 = k_1 \vec{v}_2$ if $\vec{v}_1 = 0$ and $k_1 v_1 + k_2 v_2 = 0$ for any value of k_1 or k_2 (depending on which $\vec{v}$ is 0).

$\Rightarrow$ Answer: depending on if $\vec{v}_1$ or $\vec{v}_2 = 0$ or not.

1.8. #19.

$\vec{e_1} = \begin{bmatrix}1\\0\end{bmatrix}$, $\vec{e_2} = \begin{bmatrix}0\\1\end{bmatrix}$, $\vec{y_1} = \begin{bmatrix}2\\5\end{bmatrix}$, $\vec{y_2} = \begin{bmatrix}-1\\6\end{bmatrix}$, $T: \mathbb{R}^2 \to \mathbb{R}^2$

$T(\vec{e_1}) = \vec{y_1}$ & $T(\vec{e_2}) = \vec{y_2}$.

$$\underbrace{\begin{bmatrix}5\\-3\end{bmatrix}} = \underbrace{5\begin{bmatrix}1\\0\end{bmatrix}} - \underbrace{3\begin{bmatrix}0\\1\end{bmatrix}} = 5\vec{e_1} - 3\vec{e_2}$$

$T(c\vec{u} + d\vec{v}) = cT(\vec{u}) + dT(\vec{v})$ (This is a linear transformation)

$\Rightarrow$ Image of $\begin{bmatrix}5\\-3\end{bmatrix}$: $T\left(\begin{bmatrix}5\\-3\end{bmatrix}\right) = 5T(\vec{e_1}) - 3T(\vec{e_2})$

$= 5\vec{y_1} - 3\vec{y_2}$

$= 5\begin{bmatrix}2\\5\end{bmatrix} - 3\begin{bmatrix}-1\\6\end{bmatrix} = \begin{bmatrix}10\\25\end{bmatrix} - \begin{bmatrix}-3\\18\end{bmatrix}$

$= \begin{bmatrix}13\\7\end{bmatrix}$

$$\begin{bmatrix}x_1\\x_2\end{bmatrix} = x_1\begin{bmatrix}1\\0\end{bmatrix} + x_2\begin{bmatrix}0\\1\end{bmatrix} = x_1\vec{e_1} + x_2\vec{e_2}$$

$\Rightarrow$ Image of $\begin{bmatrix}x_1\\x_2\end{bmatrix}$: $T\left(\begin{bmatrix}x_1\\x_2\end{bmatrix}\right) = x_1\underbrace{T(\vec{e_1})}_{\vec{y_1}} + x_2\underbrace{T(\vec{e_2})}_{\vec{y_2}}$

$= x_1\begin{bmatrix}2\\5\end{bmatrix} + x_2\begin{bmatrix}-1\\6\end{bmatrix}$

$= \begin{bmatrix}2x_1 - x_2\\5x_1 + 6x_2\end{bmatrix}$

1.8 #41. $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$

Set $x_1 = 1$, $x_2 = 2$

$\Rightarrow T(x_1 + x_2) = (2.1 - 3.2, 1+4, 5.2) = (-4, 5, 10)$

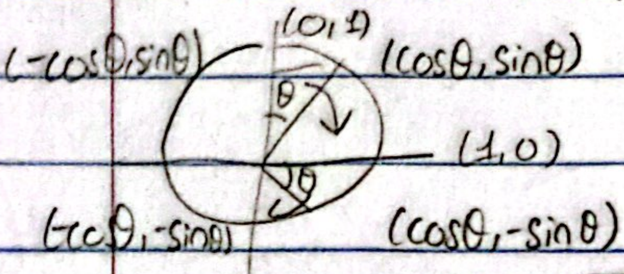
$T(x_1) + T(x_2) = (2.1 - 3.0, 1+4, 5.0) + (2.0 - 3.2, 0+4, 5.2)$

$= (2, 5, 0) + (-6, 4, 10) = (-4, 9, 10)$

Since $T(x_1 + x_2) \neq T(x_1) + T(x_2) \Rightarrow T$ is not linear.

1.9 #4. $T: \mathbb{R}^2 \to \mathbb{R}^2$ rotates point abt the origin thru $\underbrace{-\pi/4}_{\theta}$ rad (cw).

$T(\vec{e_1}) = (1/\sqrt{2}, -1/\sqrt{2})$, $\vec{e_1} = (1,0)$



$\Rightarrow T(\vec{e_1}) = (\cos\theta, -\sin\theta) = (1/\sqrt{2}, -1/\sqrt{2})$

$\Rightarrow T(\vec{e_2}) = (\cos\theta, \sin\theta) = (1/\sqrt{2}, 1/\sqrt{2})$, $\vec{e_2} = (0,1)$

$\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, $\sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, $-\sin\frac{\pi}{4} = \frac{-1}{\sqrt{2}}$

$$\Rightarrow T = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$\begin{bmatrix} \cos\pi/4 \\ -\sin\pi/4 \end{bmatrix} = T(\vec{e_1})$ $\qquad T(\vec{e_2}) = \begin{bmatrix} \cos\pi/4 \\ \sin\pi/4 \end{bmatrix}$

2.1 #29. For b_n is the last column of b

$\Rightarrow b_n \neq 0$ & $Ab_n = 0$. Since $Ab_n = 0$ and $b_n \neq 0 \Rightarrow A$ is linear dep.
A is only linear independent iff $Ab_n = 0$ when $b_n = 0$. But since we know $b_n \neq 0 \Rightarrow$ A can only be linear independent.

$\Rightarrow$ Columns of A must also be linear independent.