305 HW1 Q5: (Taylor expand)

a) $x \approx 0 \Rightarrow \dfrac{(1-e^{-x})}{x(1+x^2)} \approx \dfrac{1-(1-x)}{x(1+x^2)} \approx \dfrac{1}{1+x^2} \approx 1$

$x \to \infty \Rightarrow \dfrac{(1-e^{-x})}{x(1+x^2)} \approx \dfrac{1-0}{x^3} \approx 0$

b) Yes, we can. We can approx the integral as $x \to \infty$ aka the integral $\dfrac{1}{x^3}$ to get a close enough estimation for λ.

c) Error $< 0.001 \Rightarrow \int_{\lambda}^{\infty} x^{-3}dx = -\dfrac{x^{-2}}{2}\Big|_{\lambda}^{\infty} = 0 + \dfrac{1}{2\lambda^2} < 0.001$

$\Rightarrow 2\lambda^2 > \dfrac{1}{0.001} \Rightarrow \lambda > \left(\dfrac{1}{0.002}\right)^{1/2} \Rightarrow \lambda > 22.361$

d) So I think we should pick Δx as small as possible for an accurate estimation & since Δx constrains the error by the similar amount: $\Delta x < 0.001$. I don't know what's the most important thing is here but I guess I will try to minimize Δx beneath the error limit.

e) I pick $\lambda = 1 \cdot 10^8$, $\Delta x = \dfrac{1000 - 1 \cdot 10^8}{1 \cdot 10^8} \approx 0.0001$

Estimation ≈ 0.9205 for the given integral