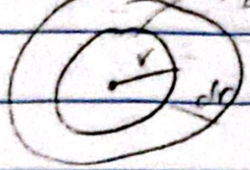
HOMEWORK 2

1. Since the charge lies offcenter inside the cube, we cannot use Gauss' Law to calculate the flux as this situation's not symmetrical.

→ The side of the cube closest to the charge will contain more flux. The total flux is unknown as Gauss' Law cannot be used but it will be independent from the charge's position (total flux will stay the same as long as charge still in box). ✗

2. $\rho_{charge} = \rho_0 r^n / R^n$, $n > -3$. $q = \rho V = \rho 4\pi r^2$ → $dq = 4\pi r^2 dr$



$dq = \rho 4\pi r^2 dr$ $\quad \oint \vec{E} \cdot d\vec{s} = \frac{Q_{encl}}{\varepsilon_0}$ → $\vec{E} \cdot d\vec{s} = \int \frac{Q_{encl}}{\varepsilon_0}$

$dS = 4\pi r^2$

$$\Rightarrow E 4\pi r^2 = \frac{4\pi}{\varepsilon_0} \int_0^r \rho_{charge} r^2 dr = \frac{4\pi \rho_0}{\varepsilon_0 R^n} \int_0^r r^n r^2 dr$$

$$\Rightarrow E = \frac{\rho_0}{\varepsilon_0 R^n r^2} \int_0^r r^{n+2} dr = \frac{\rho_0}{\varepsilon_0 R^n r^2} \frac{r^{n+3}}{n+3},$$

$$\Rightarrow \vec{E} = \frac{\rho_0 r^{n+1}}{\varepsilon_0 R^n (n+3)} \hat{r} \text{ (radially outward)}. \checkmark$$

$\frac{dE}{dr} = 0$ for constant magnitude of electric field. ✓

$$\Rightarrow \frac{dE}{dr} = (n+1) \frac{\rho_0 r^n}{\varepsilon_0 R^n (n+3)} = 0 \Leftrightarrow (n+1) r^n = 0$$

$$\Rightarrow n r^n + r^n = 0$$

$$\Rightarrow n r^n = -r^n \Rightarrow n = -1 \checkmark$$

Grading Space For 1 & 2: 1. I was wrong for 1 since I thought Gauss' Law cannot be used due to not symmetry.

Answer: $\frac{q}{6\varepsilon_0} < \text{flux} < \frac{q}{2\varepsilon_0}$

2. I think I'm correct for 2.

✓

3. $\oint \vec{E} \cdot ds = \frac{Q_{encl}}{\varepsilon_0} = \boxed{E \cdot 2\pi r h} \Rightarrow E = \frac{Q_{encl}}{2\pi r h \varepsilon_0}$

$Q_{encl} = \rho_{ch} \oint_V r\,dr\,d\phi\,dz = \rho_{ch} \int_0^{a} \int_0^{2\pi} \int_0^{h} r\,dr\,d\phi\,dz = \boxed{\frac{\cancel{2}\pi h \rho_{ch} a^2}{\cancel{2}}}$ ✓

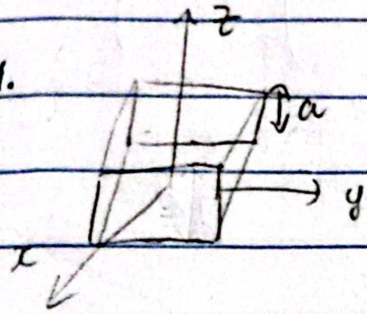
$\Rightarrow E = \frac{\pi h \rho_{ch} a^2}{2\pi r h \varepsilon_0} = \frac{\rho_{ch} a^2}{2 r \varepsilon_0}$. Inside $\Rightarrow r < a \Rightarrow \boxed{\frac{\rho_{ch} r^2}{2 r \varepsilon_0} = \frac{\rho_{ch} r}{2\varepsilon_0}}$ ✗ ✓

Outside $\Rightarrow r > a \Rightarrow \frac{\pi h \rho_{ch} a^2}{2\pi r h \varepsilon_0} = \boxed{\frac{\rho_{ch} a^2}{2\varepsilon_0 r}}$ ✓

$\Rightarrow \vec{E}_{in} = \frac{\rho_{ch} x}{2\varepsilon_0} \hat{x}$ $\Rightarrow \vec{E}_{out} = \frac{\rho_{ch} a^2}{2\varepsilon_0 x} \hat{x}$.

$\rho \geq a$: charge between these aren't enclosed $\Rightarrow$ same solution as $\rho = a$ $(r = a)$

4.



$Q_{encl\,(in)} = \rho_{ch} \oint_V dx\,dy\,dz = \rho_{ch} \int_{-a/2}^{a/2} \int_{-y}^{y} \int_{-x}^{x} dx\,dy\,dz$
$= \rho_{ch}\, 2x\, 2y\, a$

$\oint \vec{E} \cdot dV = E \cdot \frac{x}{2} \frac{y}{2} a = \frac{\rho_{ch}\, 4xy\, a}{\varepsilon_0}$

$\Rightarrow \vec{E}(z \leq a/2) = \boxed{\frac{\rho_{ch}}{\varepsilon_0} z\hat{z}}$ ✓

No integration?

is my Q_{encl} wrong?

why $2z$?

$Q_{encl\,(out)} = \rho_{ch} \int_a^z \int_{-y}^{y} \int_{-x}^{x} dx\,dy\,dz = \rho_{ch}\, 4xy\,(z - a)$ ✓

$z < a/2 \Rightarrow 2EA = \rho_{ch}(2z)A$

$\Rightarrow \vec{E}(z \geq a/2) = \boxed{\frac{\rho_{ch} \cdot (z - a)}{\varepsilon_0} z\hat{z}}$ ✗ $\Rightarrow \vec{E} = \frac{\rho_{ch}|z|}{\varepsilon_0} \frac{z}{|z|} \hat{z}$ ε_0 $= \frac{\rho_{ch} z}{\varepsilon_0} \hat{z}$

Grading Space For 3 & 4 3. I think I did correctly.

4. I got $E(z > a/2)$ wrong because I honestly didn't know how to calculate charge enclosed for that. The solution doesn't even calculate charge enclosed as well so I'm very confused how to get the right answer.

why there's no $2z$?

$z > a/2 \Rightarrow 2EA = \rho_{ch}(\cancel{2z})A/\varepsilon_0 \Rightarrow \vec{E} = \frac{\rho_{ch} a}{2\varepsilon_0} \frac{z}{|z|} \hat{z}$

(*) will see this field again. Field due to a "point" electric dipole @ the origin, empty space 4 the rest.

5. $E_r = \frac{2A\cos\theta}{r^3}$, $E_\theta = \frac{A\sin\theta}{r^3}$, $E_\phi = 0$, $A = \text{const for } r > a$

$$\oint \vec{E}\cdot d\vec{l} = \int_a^r \frac{2A\cos\theta}{r^3} dr + \int_0^{2\pi} \frac{A\sin\theta}{r^3} r\,d\theta + 0$$

$$= -\frac{2A\cos\theta}{2r^2}\Big|_a^r + \cancel{\frac{A}{r^2}(-\cos\theta)\Big|_0^{2\pi}}^{\,0}$$

I should do $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$ instead. Then $\rho = \varepsilon_0 . \vec{\nabla}\cdot\vec{E}$

I think I did the whole thing wrong

$$= A\cos\theta\left(\frac{1}{r^2} - \frac{1}{a^2}\right)$$

$$\oint \vec{E}\cdot d\vec{l} = W = \int E\cdot dr = \int_a^r \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2} dr = -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{a}\right)$$

$$\Rightarrow A\cos\theta\left(\frac{1}{r^2} - \frac{1}{a^2}\right) = -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{a}\right)$$

$$\Leftrightarrow A\cos\theta\left(\frac{r^2-a^2}{r^2a^2}\right) = \boxed{\frac{Q}{4/3\,\pi a^3}} \cdot \frac{a^3}{3\varepsilon_0}\;\frac{a-r}{ar}$$

$$\Leftrightarrow A\cos\theta\frac{\cancel{(r-a)}(r+a)}{r^2a^2} = \frac{\sigma a^3}{3\varepsilon_0}\cdot\frac{\cancel{r-a}}{ar}$$

$$\Rightarrow \boxed{\sigma = \frac{3A\cos\theta\,(r+a)\,\varepsilon_0\,a}{r}} \times \; = 0.$$

6. $\vec{E} = (yz - 2x)\hat{x} + xz\hat{y} + xy\hat{z}$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz-2x & xz & xy \end{vmatrix} = (x-x)\hat{x} - (y-y)\hat{y} + (z-z)\hat{z} = 0$$

$\Rightarrow$ It is possible for the electrostatic field. ✓

$$d\phi = -\vec{E}\cdot d\vec{l} = -(yz-2x\,dx + xz\,dy + xy\,dz)$$

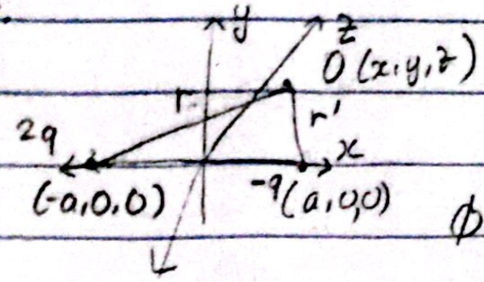
$$\Rightarrow \phi = -\int(yz-2x)dx - \int xz\,dy - \int xy\,dz$$

$$= -yzx + x^2 - xzy - xyz$$

My mistake starts here. Just mathematics. I should do them separately.

× $= x^2 - \cancel{3}xyz + C$ (No the 3 xyz b/c they are just going to repeated them selves.)

Grading Space for 5&6: I completely butchered #5.

7. 

O: A random point on the surface.

$r = \sqrt{(x+a)^2 + y^2 + z^2}$

$r' = \sqrt{(x-a)^2 + y^2 + z^2}$

$\phi = \dfrac{Q}{4\pi\varepsilon_0 R}$

$\rightarrow \phi_P = \left(\underbrace{\dfrac{2q}{\sqrt{(x+a)^2+y^2+z^2}}}_{r} - \underbrace{\dfrac{q}{\sqrt{(x-a)^2+y^2+z^2}}}_{r'} \right) \dfrac{1}{4\pi\varepsilon_0}$ ✓

$\phi = 0 \rightarrow \left(\dfrac{2q}{\sqrt{(x+a)^2+y^2+z^2}} - \dfrac{q}{\sqrt{(x-a)^2+y^2+z^2}} \right) \dfrac{1}{4\pi\varepsilon_0} = 0$

$\Rightarrow 2\sqrt{(x-a)^2+y^2+z^2} = \sqrt{(x+a)^2+y^2+z^2}$

$\Leftrightarrow 4[(x-a)^2+y^2+z^2] = (x+a)^2+y^2+z^2$

$\Leftrightarrow 4x^2 - 8xa + 4a^2 + 4y^2 + 4z^2 = x^2 + 2xa + a^2 + y^2 + z^2$

$\Leftrightarrow 3x^2 - 10xa + 3a^2 + 3y^2 + 3z^2 = 0$

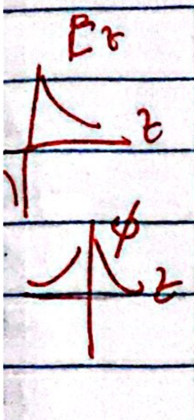
$\Leftrightarrow x^2 + y^2 + z^2 + a^2 - \frac{10}{3}xa = 0 \Rightarrow$ This surface is a sphere. ✓

8. Circular disk with radius r & thickness dr & $d\phi$ $2\pi\sigma\, r dr$

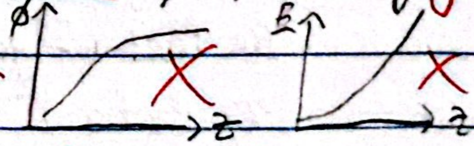
$\Rightarrow \phi = \int d\phi = \int_0^a \dfrac{2\pi\sigma}{4\pi\varepsilon_0\sqrt{r^2+z^2}}\, r dr = \boxed{\dfrac{\sigma}{2\varepsilon_0}\left(\sqrt{a^2+z^2} - |z|\right)}$ ✓

For $a \gg z$:

$\sqrt{a^2+z^2} = a \Rightarrow \phi = \dfrac{\sigma a}{2\varepsilon_0}$. This just means the potential in the center of the disk as a point charge. I don't really know if this can use to calculate x & y, I assume we can't since by symmetry the potential should cancel out for x & y. ?

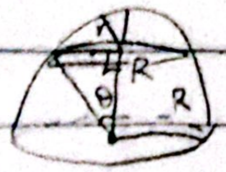


Grading space for 7 & 8: $(E_x = E_y = 0)$

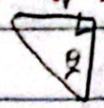


7. I got everything correct, but I did not say the size & location of the sphere (radius 4/3, center at (5/3 a, 0, 0)). I wonder if it's ok?

8. I got the graph wrong and the explaination of why we can't calculate for x & y potential wrong somewhat.

9.  $\Rightarrow \phi_{center} = \frac{\sigma}{4\pi\varepsilon_0} \int \frac{1}{R}\, da$ ↳ surface area $= 2\pi R^2$

$\Rightarrow \phi_{center} = \frac{\sigma}{4\pi\varepsilon_0} \frac{2\pi R^2}{R} = \boxed{\frac{\sigma R}{2\varepsilon_0}}$

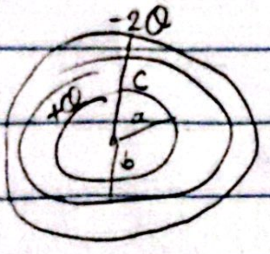
$\Rightarrow \phi_{NP} = \frac{\sigma}{4\pi\varepsilon_0} \int \frac{1}{r}\, da$ ↳ $2\pi R^2 \sin\theta\, d\theta$ 

$\Rightarrow r^2 = R^2 + R^2 - 2R^2\cos\theta = 2R^2(1-\cos\theta)$

$= \frac{\sigma}{4\pi\varepsilon_0} \frac{2\pi R^2}{R\sqrt{2}} \int_0^{\pi/2} \frac{\sin\theta}{\sqrt{1-\cos\theta}}\, d\theta = \frac{\sigma R}{2\sqrt{2}\varepsilon_0}\left(2\sqrt{1-\cos\theta}\right)\Big|_0^{\pi/2}$

$= \boxed{\frac{\sigma R}{\sqrt{2}\varepsilon_0}}$

$\Rightarrow \phi_{NP} - \phi_{center} = \frac{\sigma R}{\sqrt{2}\varepsilon_0} - \frac{\sigma R}{2\varepsilon_0} = \boxed{\frac{\sigma R}{\varepsilon_0}\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)}$

10. 

a) Since a net charge is placed on the sphere
⇒ +Q charge must be on the surface of a
⇒ −Q charge must be on the surface of b to cancel out the one on a.

⇒ $-2Q - (-Q) = -Q$ must be on the surface of c to make up the net charge $-2Q$.

b) $\oint \vec{E}\cdot d\vec{a} = \frac{Q_{encl}}{\varepsilon_0} = E4\pi r^2 \Rightarrow E = \frac{Q_{encl}}{\varepsilon_0 4\pi r^2}$

- $r<a$ & $b<r<c$: $E=0$ (inside conductor)
- $a<r<b$: $Q_{encl} = Q4\pi r^3 \Rightarrow \vec{E} = \frac{Qr}{\pi\varepsilon_0}\hat{r}$ ✗ $\vec{E} = \frac{Q\hat{r}}{4\pi\varepsilon_0 r^2}$
- $r>c \Rightarrow Q_{encl} = Q - Q + Q - 2Q = -Q \Rightarrow \vec{E} = \frac{-Q}{4\pi r^2\varepsilon_0}\hat{r}$

Grading Space for 9&10ab: 9. I got 9 all correct I think.
10. Part a I got correct but questionable explaination.
10. Part b I got $a<r<b$ incorrect since my Q_{encl} is calculated incorrectly. The charge should only = +Q since that's the Q_{encl} out of the surface of a. I think the −Q charge at b kinda threw me off.

$E=0 \Rightarrow \phi$ should be the same!

9. $a<r \Rightarrow \phi(r) = -\int_{\infty}^{r} \vec{E}\cdot d\vec{r} = -\int_{\infty}^{c} \vec{E}\cdot d\vec{r} - \int_{c}^{b} \vec{E}\cdot d\vec{r} - \int_{b}^{a} \vec{E}\cdot d\vec{r} - \int_{a}^{r} \vec{E}\cdot d\vec{r}$

$= \frac{-Q}{4\pi\varepsilon_0 c} - \frac{Q}{2\pi\varepsilon_0}(a^2-b^2)$ ✗

• $a<r<b \Rightarrow \phi(r) = -\int_{\infty}^{c} \vec{E}\cdot d\vec{r} - \int_{b}^{r} \vec{E}\cdot d\vec{r} = -\frac{Q}{4\pi\varepsilon_0 c} - \frac{Q}{\pi\varepsilon_0}\int_{b}^{r} r\,dr$

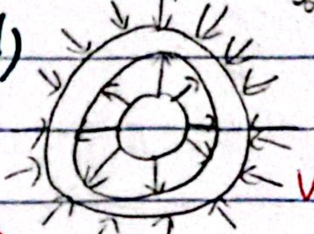
$= \frac{-Q}{4\pi\varepsilon_0 c} - \frac{Q}{\pi\varepsilon_0}\left(\frac{r^2}{2}\right)_b^r = \frac{-Q}{4\pi\varepsilon_0} - \frac{Q}{2\pi\varepsilon_0}(r^2-b^2)$ ✗

• $b<r<c \Rightarrow \phi(r) = -\int_{\infty}^{c} \vec{E}\cdot d\vec{r} = \frac{-Q}{4\pi\varepsilon_0 c}$ ✓

• $r>c \Rightarrow \phi(r) = \int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{-1}{r}\right)_{\infty}^{r} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{-1}{r} - \frac{1}{\infty}\right)$ ✓

Maybe right $\approx \frac{-Q}{4\pi\varepsilon_0 r}$

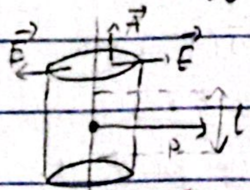
d)



✓

Is # of arrows matter?

11.



radius a. Charge per unit length q_e

perpendicular distant from axis p.

~~$\oint \vec{E}\cdot ds = \frac{Q_{encl}}{\varepsilon_0} = E\,2\pi p l = \frac{q_e l}{\varepsilon_0}$~~

~~$\Rightarrow E = \frac{q_e}{2\pi\varepsilon_0 p}$~~

~~$E = \frac{-d\phi}{dr} \Rightarrow \int d\phi - \int E\,dr = -\int_{p_0}^{p} \frac{q_e}{2\pi\varepsilon_0 p}\,dp = \frac{-q_e}{2\pi\varepsilon_0} \ln p \Big|_{p_0}^{p}$~~

~~$\Rightarrow \phi(p) = \frac{q_e}{2\pi\varepsilon_0} \ln\left(\frac{p_0}{p}\right)$~~

~~Since electric field inside cylinder $=0 \Rightarrow \phi = $ const.~~

~~$\Rightarrow$ Potential inside = Potential at surface of cylinder $\Rightarrow p=a$~~

$\Rightarrow \phi(a) = \frac{q_e}{2\pi\varepsilon_0} \ln\left(\frac{p_0}{a}\right)$

12. $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$. If a sheet thickness t is inserted $\Rightarrow C' = \frac{Q}{V'}$ where $V' = E(d-t)$

$= \frac{Q}{\varepsilon_0 A}(d-t)$

$\Rightarrow C' = \frac{\varepsilon_0 A}{d-t}$

$\Rightarrow \Delta C = C' - C = \frac{\varepsilon_0 A}{d-t} - \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A(d-d+t)}{d(d-t)}$

$\boxed{\Delta C = \frac{\varepsilon_0 A t}{d(d-t)}}$ ✓

I don't know if I can do this by combining 2 capacitors in someway but I think we might can? I don't know what way though. I really was clueless about this part to be honest.

$\frac{1}{C_{new}} = \frac{1}{C_1} + \frac{1}{C_2}$, $C_1 = \frac{\varepsilon_0 A}{x}$, $C_2 = \frac{\varepsilon_0}{d-x-t}$

Grading space for 10cd, 11, 12.

10c: I got $a<r$ & $a<r<b$ scenario wrong. They should both $= \frac{Q}{4\pi\varepsilon_0 abc}(bc-ac-ab)$

I think I took the integral incorrectly.

11. I think working on HW at 4 in the morning got me because I didn't even work on the correct problem. What did I do is still a mystery.

13. a) charge density: $\sigma_b = \frac{q_b}{4\pi b^2}$ & $\sigma_c = \frac{q_c}{4\pi c^2}$

$\rightarrow$ Area $\leftarrow$

+Charge density of the outer surface = $\frac{q_b + q_c}{4\pi a^2}$

b) $\vec{E}_a = \frac{q_b + q_c}{4\pi\varepsilon_0 a^2}\hat{r}$ for $r > a$

r^2?

c) $\vec{E}_b = \frac{q_b}{4\pi\varepsilon_0 b^2}\hat{r}$ $0 < r \leq b$ & $\vec{E}_c = \frac{q_c}{4\pi\varepsilon_0 c^2}\hat{r}$ $0 < r \leq c$

r^2 r^2

?

d) The charges will not move as field caused by each charge doesn't effect other field → force = 0.

e) It would probably would. +charge will become $\frac{Q + q_b + q_c}{4\pi a^2}$

$\Rightarrow E = \frac{Q + q_b + q_c}{4\pi\varepsilon_0 a^2} > E_a \Rightarrow$ it would move?

?

Grading space for 13. For part b & c, I wonder if it's ok to use b & c & a distant instead of r. But then the answer is that they suppose to cancel each other so I might be wrong there. I'm so confused about part e, if σ_a is not constant anymore, never mid, I think I got it now. It's only affected the field outside only, I got it.

Overall performance: I really don't understand the material that well, especially conceptual stuffs. I just looked up similar problems on YouTube to understand the math and taking integrals but I really stuck with conceptual question. I really need to catch up with the materials in office hours soon otherwise I will fail my first midterm. Nevertheless, I did quite well this homework despite not understanding a lot.