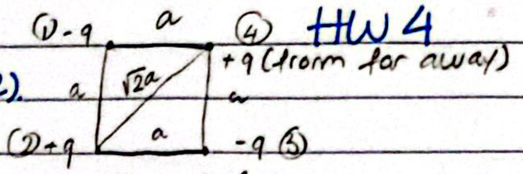
HW 4

1.(2.32).



a) $W = qV$. Electrical potential of a point charge: $V = \frac{Q}{4\pi\varepsilon_0 r}$, r is distance to +q charge from far away

$\Rightarrow W = q\left(\frac{-2q}{4\pi\varepsilon_0 a} + \frac{q}{4\pi\varepsilon_0\sqrt{2}a}\right) = \frac{q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}} - 2\right)$

b) $W_{net} = \underbrace{W_1}_{} + W_2 + W_3 + W_4 = 0 + W_2 + W_3 + W_4$

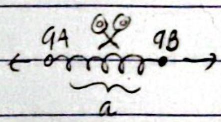
$V = 0$ for 1st charge ; $W_4 = W(\text{part a}) = \frac{q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}} - 2\right)$

V_2: potential @ point ② due to ①: $\frac{q}{4\pi\varepsilon_0 a} \Rightarrow W_2 = \frac{-q^2}{4\pi\varepsilon_0 a}$

V_3: potential @ point ③ due to ① & ②: $\frac{-q}{4\pi\varepsilon_0 \sqrt{2}a} + \frac{q}{4\pi\varepsilon_0 a} = \frac{q}{4\pi\varepsilon_0 a}\left(1 - \frac{1}{\sqrt{2}}\right) \Rightarrow W_3 = \frac{q\cdot(-q)}{4\pi\varepsilon_0 a}\left(1 - \frac{1}{\sqrt{2}}\right)$

$\Rightarrow W_{net} = \frac{-q^2}{4\pi\varepsilon_0 a} + \frac{-q^2}{4\pi\varepsilon_0 a}\left(1 - \frac{1}{\sqrt{2}}\right) + \frac{q^2}{4\pi\varepsilon_0 a}\left(\frac{1}{\sqrt{2}} - 2\right) = \boxed{\frac{q^2}{4\pi\varepsilon_0 a}\left(\frac{2}{\sqrt{2}} - 4\right)} = \frac{-q^2}{4\pi\varepsilon_0 a}\left(4 - \frac{2}{\sqrt{2}}\right)$

2.(2.33).



Conservation of energy $\Rightarrow E_i = E_f \Rightarrow U_i + K_i = U_f + K_f$

$K_i = 0$, $K_f = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$

$U_i = \frac{q_A q_B}{4\pi\varepsilon_0 a}$ (potential NRG of 2 charges), $U_f = 0$ (assuming $a \to \infty$)

$\Rightarrow \frac{q_A q_B}{4\pi\varepsilon_0 a} = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B V_B^2$ Conservation of momentum: $m_A V_A - m_B V_B = 0$

$\Rightarrow V_A^2 = \left(\frac{+m_B V_B}{m_A}\right)^2$, $V_B^2 = \left(\frac{+m_A V_A}{m_B}\right)^2$

$\Rightarrow \frac{q_A q_B}{4\pi\varepsilon_0 a} = \frac{1}{2} m_A V_A^2 + \frac{1}{2} m_B \frac{m_A^2 V_A^2}{m_B^2} = \frac{1}{2} m_A \frac{m_B^2}{m_A^2} V_B^2 + \frac{1}{2} m_B V_B^2$

$\Rightarrow V_A = \left[\frac{q_A q_B}{4\pi\varepsilon_0 a}\left(\frac{1}{2} m_A + \frac{1}{2}\frac{m_A^2}{m_B}\right)^{-1}\right]^{1/2} = \left[\frac{q_A q_B}{2\pi\varepsilon_0 a m_A}\left(1 + \frac{m_A}{m_B}\right)^{-1}\right]^{1/2}$

$V_B = \left[\frac{q_A q_B}{4\pi\varepsilon_0 a}\left(\frac{1}{2}\frac{m_B^2}{m_A} + \frac{1}{2} m_B\right)^{-1}\right]^{1/2} = \left[\frac{q_A q_B}{2\pi\varepsilon_0 a m_B}\left(\frac{m_B}{m_A} + 1\right)^{-1}\right]^{1/2}$

3.(2.36). Bring each charge dq from far away to radius dr

$\Rightarrow$ total charge $- dq_{total}$ $\Rightarrow dW = dq_{total}\, V_{total} = dq_{total} \cdot \frac{q_{total}}{4\pi\varepsilon_0 r}$

$q_{total} = \frac{4}{3}\pi r^3 \rho = 4/3\,\pi r^3 \cdot q/(4/3\,\pi R^3) = q\frac{r^3}{R^3}$

ρ: Charge density of sphere radius R

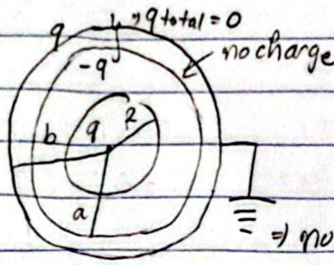
$\Rightarrow dW = dq_{total}\, \frac{q\, r^3/R^3}{4\pi\varepsilon_0 r} = \frac{qr^2}{4\pi\varepsilon_0 R^3}\, dq_{total}$

$dq_{total} = q \cdot 3r^2/R^3\, dr$

$\Rightarrow dW = \frac{q \cdot 3r^2}{R^3} \cdot \frac{qr^2}{4\pi\varepsilon_0 R^3}\, dr = \frac{3r^4 q^2}{4\pi\varepsilon_0 R^6}\, dr$

$\Rightarrow W = \int dW = \int_0^R \frac{3r^4q^2}{4\pi\varepsilon_0 R^6}\, dr = \frac{3q^2}{4\pi\varepsilon_0 R^6}\int_0^R r^4\, dr = \frac{3q^2}{4\pi\varepsilon_0 R^6}\left(\frac{r^5}{5}\right)_0^R$

$\Rightarrow W = \frac{3q^2}{20\pi\varepsilon_0 R}$



4.(2.39). a)

Since the sphere carry charge q on the surface $\Rightarrow$ no charge on surface b.

$\Rightarrow$ It induces a charge $-q$ at the inner surface

$\Rightarrow$ The $-q$ charge will induce a q charge at the outter surface to make the shell carry no net charge.

$\Rightarrow \sigma_R = \frac{q}{4\pi R^2}$, $\sigma_A = \frac{-q}{4\pi a^2}$, $\sigma_b = \frac{+q}{4\pi b^2}$

b) $\Rightarrow E = 0$ at $r<R$ & $a<r<b$ $(q_{net}=0)$; r is the radius of Gaussian Surface

$E\,4\pi r^2 = \frac{q}{\varepsilon_0}$ (Gauss' Law) $\Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$ at $R<r<a$ & $r>b$

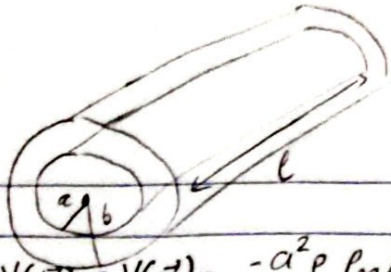
$\Rightarrow V = -\int_\infty^b \frac{q}{4\pi\varepsilon_0 r^2}\, dr - \int_a^R \frac{q}{4\pi\varepsilon_0 r^2}\, dr$ $\left(\Delta\phi = V = -\int \vec{E}\cdot d\vec{\ell}\right)$

$= \frac{q}{4\pi\varepsilon_0}\left(-\int_\infty^b \frac{1}{r^2}\, dr - \int_a^R \frac{1}{r^2}\, dr\right)$

$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{b} + \frac{1}{R} - \frac{1}{a}\right)$

c) Since the outter shell charge got grounded $\Rightarrow q_b = 0 \Rightarrow \sigma_b = 0$ (σ_a & σ_R is same)

$\Rightarrow V = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{a}\right)$ (no need to integrate from $\infty \to b$ since $q_b = 0$)



5. (2.44).

From HW3

problem 4(2.25): $V(\vec{b}) - V(\vec{a}) = \frac{-a^2\rho}{2\varepsilon_0}\ln\left(\frac{b}{a}\right) = \Delta V \approx V = \frac{-a^2 q}{2\varepsilon_0 \pi a^2 l}\ln\left(\frac{b}{a}\right)$

$\Rightarrow C = \frac{q}{V} = 2\varepsilon_0 \pi l\,[\ln(b/a)]^{-1}$

$\Rightarrow C/l = 2\varepsilon_0\pi[\ln(b/a)]^{-1}$ = capacitance/unit length

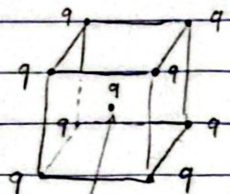
6. (2.60) a) Conductor carries net charge Q, placed in $\vec{E}_e$, experienced $\vec{F}$. If $\vec{E}_e \to -\vec{E}_e$, force would also reverse ($\vec{F} \to -\vec{F}$) b) External field is uniform.

a) When a conductor carries charge Q, it will spread out uniformly on the conductor surface. When we put the conductor into the electric field $\vec{E}_e$, the charge distribution will change, causing the conductor to experience electric force $\vec{F}$. If $\vec{E}_e \to -\vec{E}_e \Rightarrow$ the induced charge on the surface will switch sign but the force

Not now $\to$ ~~b)~~ direction should not change because force depends on the gradient of the electric field, which should remain unchange. $\Rightarrow$ the statement is False.

(continue below)

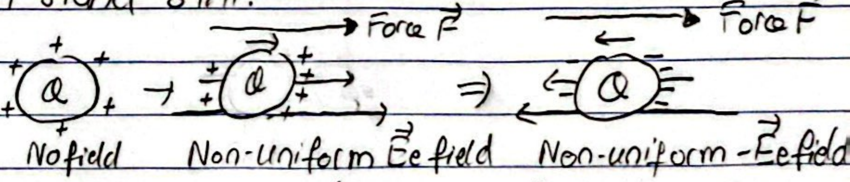
(?) 7. (3.2).



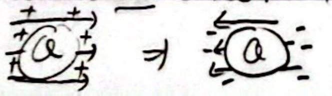
explain why this charge in the middle will not stand still.

To explain why the charge in the center doesn't stay still $\Rightarrow$ explained the potential force in the center is unstable. Since $U = qV \Rightarrow$ we can also explain that U is unstable due to V being unstable. As we've already known: $\nabla^2 V = 0$ inside the volume of this box. Which means that the second derivative of V is equal to $0 \Rightarrow$ Its concavity is an unstable point $\Rightarrow V$ is unstable $\Rightarrow U$ is unstable $\Rightarrow$ the charge in the center won't stand still.

6. (2.60) - Cont. a) Example:



b) Since the field is uniform $\Rightarrow \nabla E = 0 \Rightarrow F = 0 \Rightarrow$ I guess the statement is true since $\vec{F} = -\vec{F} = 0$ (there will be no force)

Ex:  (No force) (I don't know if these crabby illustration is counted as valid Ex so pardon me).