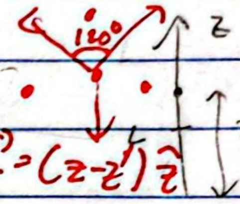
HOMEWORK 1.

1. a, The net force will be 0 because all the charges have equal charge(q) but since they locate symmetrically, they will have opposite direction. Therefore, the forces at Q in the center will all cancelled out ⇒ 0 net force ✓

b, 1 charge remove ⇒ $\vec{E}_{Q \text{ due to } q} = \sum_i \frac{qQ}{4\pi\varepsilon_0 R^2}\hat{R} = \frac{qQ}{4\pi\varepsilon_0 d^2}$ ✓
(only feel force of 1 charge as the other 10 still cancelled each other out). (Doesn't have the direction of the force, towards the gap).

c) Answer'd be different since the charge won't be symmetrically placed anymore if the number is odd. Therefore, they can't cancelled out evenly.
If there're 3 charges then net force on Q = force from each charge $= \sum \frac{qQ\hat{R}}{4\pi\varepsilon_0 R^2}$

I'm wrong about this one. Didn't realize they can be cancelled like this:



I thought it's impossible since it cannot be symmetry for odd #

2. $dq = \lambda ds = Cz\,dz$, $z > L$

$\Rightarrow \vec{E} = \int \frac{Cz}{4\pi\varepsilon_0}\frac{\vec{R}}{R^3}dz'$ $\vec{r} = z\hat{z}$, $\vec{r}\,' = z'\hat{z}$ $\Rightarrow \vec{R} = (z-z')\hat{z}$

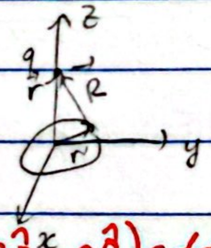
$\Rightarrow \vec{E} = \frac{Cz}{4\pi\varepsilon_0}\int_0^L \cancel{\frac{L^2}{z(z-L)^2}}\hat{z}\,dz$ $\frac{z'dz}{(z-z')^2}\hat{z} = \int_0^L \frac{-dz'}{z-z'} + \int_0^L \frac{z\,dz'}{(z-z')^2}$

somehow I still got the answer right. I didn't plug in the bounds.

$\Rightarrow \vec{E} = \frac{Cz}{4\pi\varepsilon_0}\hat{z}\left[\cancel{\ln}\left(\frac{z-L}{z} + \frac{L}{z-L}\right)\right] = \frac{C\hat{z}}{4\pi\varepsilon_0}\left[\ln\frac{z-L}{z} + \frac{z}{z-L} - 1\right]$

I don't know what is $\vec{r}\,'$ for this problem. I was honestly stuck. But now knowing what $\vec{r}\,'$ is, the integration is easy to get done.

3. a)



$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{\vec{R}}{|\vec{R}|^3}$, $\vec{R} = \vec{r} - \vec{r}\,'$
$\vec{r} = z\hat{z}$ & $\vec{r}\,' = a\hat{y}$
$\Rightarrow \vec{r} - \vec{r}\,' = z\hat{z} - a\hat{y} \Rightarrow |\vec{r} - \vec{r}\,'| = \sqrt{z^2+a^2}$ ✓

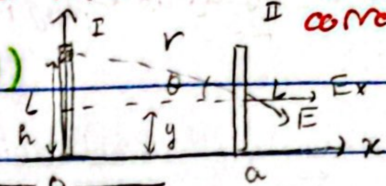
$\vec{E} = \frac{\lambda}{4\pi\varepsilon_0}\int_0^{2\pi}\frac{(z\hat{z} - a\hat{\rho})a\,d\phi'}{(z^2+a^2)^{3/2}}$, $\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{z\hat{z} - a\hat{y}}{(\sqrt{z^2+a^2})^3}$ → 0 (all y components cancelled out)

$\Rightarrow \vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{z\hat{z}}{(z^2+a^2)^{3/2}}$ $\Rightarrow \vec{F} = \vec{E}_q\, q = \frac{q_1 q_2 z\hat{z}}{4\pi\varepsilon_0 (z^2+a^2)^{3/2}}$ ✓

I did not plug in the integral. I should do that, I just copy from the lecture's formula. I also missed $z \gg a \Rightarrow \vec{F} = \frac{qQ\hat{z}}{4\pi\varepsilon_0 z^2}$ & $z = 0 \Rightarrow \vec{F} = 0$

I honestly don't know how to grade this one. My method is the same as the finite linear charge for 263 and not the one where you defined R, r, r' so my integral set up and results are very different. I picked that method since I don't even know how to take r, r' & R correctly.

4.(2-9)



E_y cancelled out due to symmetry

$\Rightarrow \vec{F_y} = 0 \Rightarrow \vec{E} = \vec{E_x}$

$r = \sqrt{a^2 + (h-y)^2}$, $E_x = \frac{\lambda a}{4\pi\varepsilon_0} \int_0^L \frac{dh}{r^3} = \int_0^L \frac{dh}{[a^2 + (h-y)^2]^{3/2}}$

$\Rightarrow E_x = \frac{\lambda}{4\pi\varepsilon_0 a}\left[\frac{L-y}{\sqrt{a^2+(L-y)^2}} + \frac{y}{\sqrt{a^2+y^2}}\right] \Rightarrow dF = dF_x = dq\,E_x = \lambda dy\,E_x$

$\Rightarrow F_x = \int dF_x = \int_0^L \frac{\lambda^2}{4\pi\varepsilon_0 a}\left[\frac{L-y}{\sqrt{a^2+(L-y)^2}} + \frac{y}{\sqrt{a^2+y^2}}\right] dy$

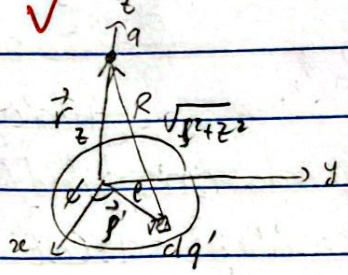
I got this correct for $\vec{F}$. $= \frac{\lambda^2}{4\pi\varepsilon_0 a}\int_0^L \frac{2y}{\sqrt{a^2+y^2}}dy = \frac{\lambda^2}{2\pi\varepsilon_0 a}\left(\sqrt{a^2+L^2} - a\right)$

? forgot to do $a \gg L$ + $\sqrt{1+L^2/a^2} = 1 + 1/2 + L^2/a^2 + \ldots$

$\Rightarrow \vec{F_x} = \frac{\lambda^2}{2\pi\varepsilon_0}\left(\sqrt{a^2+L^2} - a\right)\hat{x}$ ✓

$\Rightarrow \vec{F} = \frac{\lambda^2 L^2}{2\pi\varepsilon_0}\hat{x} = \frac{Q^2\hat{x}}{4\pi\varepsilon_0 a^2}$

5.(2-11)



$\sigma = A\rho^2$, σ is in Cm^{-2}, ρ is in m

$\Rightarrow Cm^{-2} = [A]m^2 \Rightarrow [A] = Cm^{-4}$ (unit)

$da = \rho\,d\rho\,d\phi$. $dQ = \sigma da = A\rho^2(\rho\,d\rho\,d\phi) = A\rho^3 d\rho\,d\phi$

$\Rightarrow Q = \int dQ = \int_0^a\int_0^{2\pi} dQ = \int_0^a\int_0^{2\pi} A\rho^3 d\rho\,d\phi = A\int_0^a \rho^3 d\rho \int_0^{2\pi} d\phi$

$\Rightarrow Q = A\frac{a^4}{4}2\pi = \frac{A\pi a^4}{2}$ (total charge)

$dF = \frac{kq\,dQ}{r^2} = \frac{kqA\rho^3 d\rho\,d\phi}{\rho^2+z^2}$, $dF_z = dF\cos\theta = \frac{kqA\rho^3 d\rho\,d\phi}{\rho^2+z^2}\frac{z}{\sqrt{\rho^2+z^2}} = \frac{\rho^3 kqAz\,d\rho\,d\phi}{(\rho^2+z^2)^{3/2}}$

$\Rightarrow F = \int_0^{2\pi}\int_0^a dF_z = kqAz\int_0^a \frac{\rho^3 d\rho}{(\rho^2+z^2)^{3/2}}\int_0^{2\pi} d\phi$

$\Rightarrow F = 2\pi Azkq\left[\frac{(a^2+2z^2)}{(a^2+z^2)^{1/2}} - 2z\right]\hat{z}$ ✓

I did think I got this one correct.

I don't know how to Coloumb's law this one so I just Gauss' Law it & I didn't even get it right. $|\vec{R}'| = \sqrt{x^2+\rho'^2+z'^2-2x\rho'\cos\phi'}$

$\Rightarrow \vec{E} = \frac{\rho_{ch}}{4\pi\varepsilon_0}\int_0^{2\pi}\int_{-\infty}^{\infty}\int_0^{a} \frac{\vec{R}\rho' d\rho' dz' d\phi'}{R^3}$ ← plug in & solve.

My problem with this problem's that I don't know how to identify $\vec{r}, \vec{r}' \& \vec{R}$.

6(3-13). Don't know how to use Coloumb's law so I will use Gauss' law here (I know I shouldn't but it's 4 and I wanna sleep).

$\oint \vec{E}\cdot d\vec{A} = \frac{Q}{\varepsilon_0}$, ρ_{ch} : volume density.

$r<a$ ($x<a$) $\Rightarrow Q_{in} = \rho\frac{4}{3}\pi r^3 \Rightarrow E_{in}4\pi r^2 = \frac{\rho\frac{4}{3}\pi r^3}{\varepsilon_0}$ $\quad \frac{\rho_{ch} x}{2\varepsilon_0}\hat{x}$

$\Rightarrow \vec{E}_{in} = \frac{\rho r}{3\varepsilon_0}\hat{r} \Rightarrow$ x-axis: $\vec{E}_{in} = \frac{\rho_{ch} x}{3\varepsilon_0}\hat{x}$ ✗

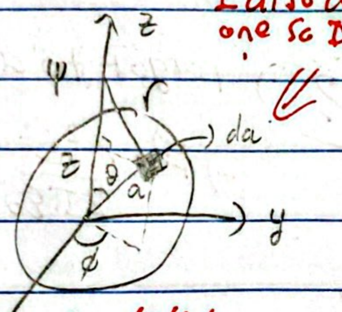
$x>a$

$r>a \Rightarrow E_{out}4\pi r^2 = \rho\frac{4}{3}\pi a^3$

$\Rightarrow \vec{E}_{out} = \frac{\rho a^3}{3\varepsilon_0 r^2}\hat{r}\,\varepsilon_0 \Rightarrow$ x-axis: $\vec{E}_{out} = \frac{\rho_{ch} a^3}{2\varepsilon_0 x}\hat{x}$ ✗ $\frac{\rho_{ch} a^2}{2\varepsilon_0 x}\hat{x}$

I also did not do the $\vec{r}, \vec{r}' \& \vec{R}$ method on this one so I'm not sure how to grade this.

7(2-8).



$\vec{r} = z\hat{z}$
$\vec{r}' = a\hat{r}'$
$\vec{R} = z\hat{z} - a\hat{r}'$

Total charge on sphere:
$Q' = \int \sigma da = \int \sigma a^2 \sin\theta\, d\theta\, d\phi$
$= 4\pi a^2 \sigma$

$\Rightarrow \vec{F} = \frac{q}{4\pi\varepsilon_0}\sigma\int_0^{2\pi}\int_0^{\pi}\frac{(z\hat{z}-a\hat{r}')}{(z^2+a^2-2az\cos\theta')^{3/2}} \times a^2\sin\theta' d\theta' d\phi'$ (solve for $\vec{F}$)

$dq = \sigma da = \sigma a^2 \sin\theta\, d\theta\, d\phi$ $\quad \cos\psi = \frac{z - a\cos\theta}{r}$ $\quad$?

$r^2 = a^2 + z^2 - az\cos\theta$

$F_z = \frac{q}{4\pi\varepsilon_0}\int_0^{\pi}\frac{\sigma a^2 \sin\theta\, d\theta \cdot (z-a\cos\theta)}{(a^2+z^2-2az\cos\theta)^{3/2}}\int_0^{2\pi} d\phi$

I still replace $\cos\theta = \mu$ for this one.

$\mu = \cos\theta \Rightarrow F_z = \frac{q}{4\pi\varepsilon_0}2\pi a^2\sigma\int_{-1}^{1}\frac{(z-a\mu)}{(a^2+z^2-2az\mu)^{3/2}}d\mu$

$\Rightarrow F_z = \frac{qQ'}{8\pi\varepsilon_0}\left[\frac{1}{z^2}\frac{zu-a}{\sqrt{a^2+z^2-2a\mu z}}\right]_{-1}^{1} = \frac{qQ'}{8\pi\varepsilon_0 z^2}\left[\frac{z-a}{|z-a|} - \frac{z-a}{|z+a|}\right]$

(*) $z>a$: $F_z = \frac{qQ'}{4\pi\varepsilon_0 z^2}$ × $\frac{q\sigma a^2}{\varepsilon_0 z^2}\hat{z}$ Sphere acts as point charge when you're outside.

(*) $z<a$: $F_z = 0$ ✓ field inside spherically symmetric distribution $F_z = 0$.

Conclusion: I did pretty bad ly this homework. ~~I just~~ My biggest weakness is learning how to find $\vec{r}, \vec{r}' \& \vec{R}$. I need to work more on understanding how to find it.