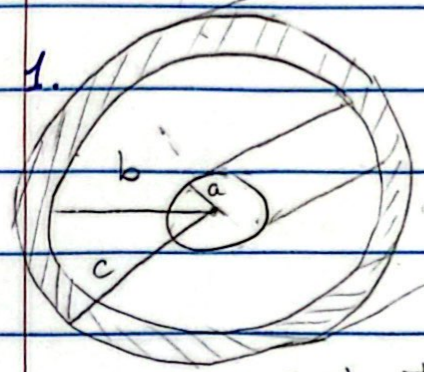
HOMEWORK 3

1.



$U_e = \frac{1}{2}\int_S \sigma(r)\phi(r)\,da$

$C = \frac{2\pi\varepsilon_0 L}{\ln(\frac{b}{a})}$ linear λ & $-\lambda$ $= \frac{Q}{l} \Rightarrow \lambda = Ql$

No charge enclosed

a) $\Delta\phi = -\int \vec{E}\cdot d\vec{s}$ $\qquad E_{\rho>c} = 0$ ✓ $\qquad E_{\rho<a} = 0$ conductor.

$E_{c>\rho>b} = 0$ $\qquad E_{b>\rho>a} = \frac{\lambda}{2\pi\varepsilon_0\rho}\hat{\rho}$ ✓

conductor

$\Rightarrow \phi_b - \phi_a = \int_a^b \frac{\lambda}{2\pi\varepsilon_0\rho}\hat{\rho}\,d\rho = \frac{\lambda}{2\pi\varepsilon_0}[\ln(b)-\ln(a)]$

$\Rightarrow U_e = \cancel{\frac{1}{2}\int_0^l\int_0^{2\pi}\frac{\lambda}{2\pi b}\,0\,b\,dr\,d\theta} + \frac{1}{2}\int_0^l\int_0^{2\pi}\frac{\lambda}{2\pi a}\cdot\frac{\lambda}{2\pi\varepsilon_0}[\ln(b)-\ln(a)]\,a\,dr\,d\theta$

$\Rightarrow U_e = \frac{1}{2}\frac{\lambda^2}{4\pi^2\varepsilon_0}\ln\left(\frac{b}{a}\right)\cdot 2\pi\cdot l = \boxed{\frac{\lambda^2 l}{4\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)}$ ✓

$\Rightarrow C = \frac{\lambda^2 4\pi\varepsilon_0 l^2}{2\lambda^2\ln(\frac{b}{a})l} = \boxed{\frac{2\pi\varepsilon_0 l}{\ln(\frac{b}{a})}}$ ✓ ✓

Grading space for 1a: I think I have done all correctly.

I didn't do c since I didn't know how to ~~integrate~~ More NRG integrate $\hat{\rho}$ from 0 → a

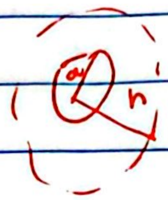
$E\ \rho<a \Rightarrow \rho_{ch}^{new} = \frac{\lambda}{\pi a^2}$ $\oint \vec{E}\cdot d\vec{a} = \frac{Q_{en}}{\varepsilon_0}$, $\vec{E} = \frac{\lambda\rho}{2\pi\varepsilon_0 a^2}\hat{\rho}$

b) $U_e = \int_{allspace} \frac{\varepsilon_0}{2} E^2 d\tau$ $\Rightarrow U_c^{new} = \frac{\lambda^2 \ell}{4\pi\varepsilon_0}\ln\left(\frac{b}{a}\right) + \frac{1}{2}\varepsilon_0 \int_0^{2\pi}\int_0^{a}\int_0^{\ell} E\rho\, dz\, d\rho\, d\phi$

$= \frac{\lambda^2\ell}{4\pi\varepsilon_0}\ln(b/a) + \frac{\lambda^2\ell}{16\pi\varepsilon_0}$.

$$= \int_c^{\infty}\int_0^{2\pi}\int_0^{\ell} 0\, d\rho\, d\rho\, d\theta + \int_b^{c}\int_0^{2\pi}\int_0^{\ell} 0\, d\rho\, d\rho\, d\theta + \int_a^{b}\int_0^{2\pi}\int_0^{\ell} \frac{\varepsilon_0}{2}\cdot\frac{\lambda^2}{4\pi\varepsilon_0^2\rho^2}\,\hat{\rho}\, \rho\, d\rho\, d\rho\, d\theta + \int_0^{a}\int_0^{2\pi}\int_0^{\ell} 0\, d\rho\, d\rho\, d\theta$$

$$= \frac{\lambda^2}{8\pi^2\varepsilon_0}\, 2\pi\ell\ln\left(\frac{b}{a}\right) = \boxed{\frac{\lambda^2\ell}{4\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)}$$ ✓



2. $U_e = \frac{\varepsilon_0}{2}\int_V E^2 d\tau + \frac{\varepsilon_0}{2}\oint_S (\phi E)\cdot da$

$E_{r<a} = \frac{\rho}{3\varepsilon_0} r$ $= \frac{Q}{4\pi a^3} r$ ✓ $\quad E_{r>a} = \frac{\rho a^3}{3\varepsilon_0}\frac{1}{r^2}$ $= \frac{Q}{4\pi\varepsilon_0 r^2}$ ✓ $\quad \rho = \frac{3Q}{4\pi a^3}$

$$\phi_{r>a} = -\int_{\infty}^{r} \vec{E}\cdot d\vec{r} = -\frac{\rho a^3}{3\varepsilon_0}\int_{\infty}^{r}\frac{1}{r^2}dr = \frac{\rho a^3}{3\varepsilon_0 r} = \frac{Q}{4\pi\varepsilon_0 r}$$ ✓

$$\phi_{r<a} = -\int_{\infty}^{a} E_{r>a}\, dr - \int_a^{r} E_{r<a}\, dr$$
$$= \frac{\rho a^3}{3\varepsilon_0 a} - \int_a^{r}\frac{\rho}{3\varepsilon_0} r\, dr = \frac{\rho a^2}{3\varepsilon_0} - \frac{\rho}{6\varepsilon_0}(r^2 - a^2)$$
$$= \frac{\rho}{6\varepsilon_0}(3a^2 - r^2)$$

$$\Rightarrow U_e = \frac{\varepsilon_0}{2}\int_V E^2 d\tau + \frac{\varepsilon_0}{2}\oint_S (\phi E)\cdot da$$
$$= \frac{\varepsilon_0}{2}\int_0^{a} E_{r<a}^2\, dr + \frac{\varepsilon_0}{2}\int_a^{b} E_{r>a}^2\, dr + \frac{\varepsilon_0}{2}\oint_{r=b} \phi_{r>a} E_{r>a}\, da$$ ✓

integrating over angles

$$= \frac{\varepsilon_0}{2}\int_0^{a}\frac{\rho^2}{9\varepsilon_0^2} r^2\, 4\pi r^2 dr + \frac{\varepsilon_0}{2}\int_a^{b}\frac{\rho^2 a^6}{9\varepsilon_0^2 r^4}4\pi r^2 dr + \frac{\varepsilon_0}{2}\left(\frac{\rho^2 a^6}{9\varepsilon_0^2 r^3}4\pi r^2\right)_{r=b}$$

$$= \frac{2\pi\rho^2}{9\varepsilon_0}\int_0^{a} r^4 dr + \frac{2\pi\rho^2 a^6}{9\varepsilon_0}\int_a^{b}\frac{1}{r^2}dr + \frac{2\pi\rho^2 a^6}{9\varepsilon_0 b}$$

$$= \frac{2\pi\rho^2 a^5}{45\varepsilon_0} + \frac{2\pi\rho^2 \cancel{a^6}\, a^5}{9\varepsilon_0 \cancel{(a-b)}} + \frac{2\pi\rho^2 a^6}{\cancel{9}\varepsilon_0 b}$$

$$= \frac{2\pi\rho^2 a^5}{9\varepsilon_0}\left(\frac{1}{5} + 1\right) = \frac{4\pi a^6}{15\varepsilon_0}\,\frac{\cancel{9}^{3} Q^2}{16\pi^2 a^{6}}$$

~kinda?

surface integral → 0

I forgot. I multiplied 4 already mercy please!

$\frac{3Q^2}{20\cancel{4}\pi\varepsilon_0 a}$ ✓ $\Rightarrow$ when $b\to\infty$, the answer doesn't change since it's ~~1~~ none in the answer?

not b dependent. Also because there's no charge outside of $0<r<a$ so that answer makes sense.

Grading space for 1b & 2: I got 1b correct and 2 mostly correct. I'm still confused about the final part of 2, I got that wrong caused I thought the answer is b independence.

3 (Wdgsness 7-16).

$$U_i = \frac{1}{2} C_0 V_0^2 = \frac{1}{2} \frac{\varepsilon_0 A}{d} V_0^2$$

$$U_f = \frac{1}{2} C' V_0^2 = \frac{1}{2} \frac{\varepsilon_0 A}{\alpha d} V_0^2$$

$$\Rightarrow \frac{U_f}{U_i} = \frac{1}{\alpha} \Rightarrow \text{Energy decreased.}$$

? $U_f/U_i = \alpha$

$U_f/U_i = \alpha$ in answer ~~why~~?

I got stupid for a bit there. It is $=\alpha$. I was wrong.

The new energy might come from the field generate from the plates.

$$\Rightarrow \Delta U = U_f - U_i = \frac{\varepsilon_0 A V_0^2}{2d}\left(\frac{1}{\alpha} - 1\right)$$

? I am confused why in the answer we have $U_f/U_i = \alpha$ and then energy decreased? Doesn't make sense. I still think I'm right tho.

4. $E \, 2\pi \rho l = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{2\pi\varepsilon_0 \rho l} = \frac{\lambda}{2\pi\varepsilon_0 \rho}$

$$\Rightarrow \Delta\phi = -\int_b^a \frac{Q}{2\pi\varepsilon_0 l \rho} d\rho = \frac{Q}{2\pi\varepsilon_0 l} \ln\left(\frac{b}{a}\right) \Rightarrow Q = \frac{\Delta\phi \, 2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)} \; ?$$

$$\Rightarrow F = QE = \frac{\Delta\phi \, 2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)} \cdot \frac{Q}{2\pi\varepsilon_0 \rho l}, \text{ inner} \Rightarrow \rho = a$$

don't think so?

$$= \frac{\Delta\phi \, Q}{\ln\left(\frac{b}{a}\right) a} = \frac{\Delta\phi^2 \, 2\pi\varepsilon_0 l}{\left[\ln\left(\frac{b}{a}\right)\right]^2 a} \stackrel{?}{=} \frac{\lambda^2}{8\pi^2\varepsilon_0 a^2} \; ?$$

$$\Rightarrow F/A = \frac{\Delta\phi^2 \, 2\pi\varepsilon_0 l}{\left[\ln\left(\frac{b}{a}\right)\right]^2 a} \cdot \frac{1}{2\pi a l} = \frac{\Delta\phi^2 \varepsilon_0}{\left[\ln\left(\frac{b}{a}\right)\right]^2 a^2} \text{ (outwards)} \stackrel{?}{=} \frac{1}{2}\varepsilon_0 E^2$$

highly not equal tho...

$\Rightarrow$ Net force will be 0, when the outter force cancels out the inner force as the outter force has same magnitude but opposite direction.

Grading space for 3 & 4: I messed up the math a bit for part 3. I think I did it wrong but I went with calculating the potential difference so I don't know how to grade ... [illegible]

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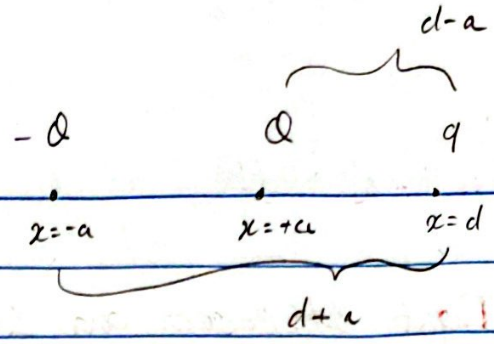
6 a)



$$\vec{F}_q = \vec{F}_{qQ} + \vec{F}_{q-Q} = \frac{Qq}{4\pi\varepsilon_0 (d-a)^2}\hat{x} + \frac{-Qq}{4\pi\varepsilon_0 (d+a)^2}\hat{x}$$

$$= \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{(d-a)^2} - \frac{1}{(d+a)^2}\right]\hat{x} \checkmark$$

b) $$\vec{F}_i = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{24^2} - \frac{1}{26^2}\right)\hat{x}$$

$$\vec{F}_f = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{49^2} - \frac{1}{51^2}\right)\hat{x}$$

$$\Rightarrow \frac{F_i}{F_f} = \frac{\frac{1}{24^2} - \frac{1}{26^2}}{\frac{1}{49^2} - \frac{1}{51^2}} \approx 8.02 \checkmark$$

c) $$\vec{F}_q = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{(d-a)^2} - \frac{1}{(d+a)^2}\right]\hat{x} = \frac{Qq}{4\pi\varepsilon_0 d^2}\left[\frac{1}{(1-\frac{a}{d})^2} - \frac{1}{(1+\frac{a}{d})^2}\right]\hat{x}$$

$$\Rightarrow F_q = \frac{Qq}{4\pi\varepsilon_0 d^2}\left[\left(1-\frac{a}{d}\right)^{-2} - \left(1+\frac{a}{d}\right)^{-2}\right] \approx \frac{Qqa}{\pi\varepsilon_0 d^3}\left(1+2\frac{a^2}{d^2}\right) \checkmark$$

For $(1+x)^{-2} \Rightarrow$ Expansion $= 1-2x+3x^2+4x^3\ldots$

$$\Rightarrow \left(1-\frac{a}{d}\right)^{-2} = 1 + \frac{2a}{d} + \frac{3a^2}{d^2} + \frac{4a^3}{d^3}$$

$$\Rightarrow \left(1+\frac{a}{d}\right)^{-2} = 1 - \frac{2a}{d} + \frac{3a^2}{d^2} - \frac{4a^3}{d^3}$$

$$\Rightarrow \left(1-\frac{a}{d}\right)^{-2} - \left(1+\frac{a}{d}\right)^{-2} = 1-1+\frac{2a}{d}+\frac{2a}{d}+\frac{3a^2}{d^2}-\frac{3a^2}{d^2}+\frac{4a^3}{d^3}+\frac{4a^3}{d^3} = \frac{4a}{d}+\frac{8a^3}{d^3}$$

$$\Rightarrow F = \frac{Qq}{4\pi\varepsilon_0 d^2}\left(\frac{4a}{d} + \frac{8a^3}{d^3}\right) = \frac{Qq \cdot 4a}{4\pi\varepsilon_0 d^3}\left(1 + \frac{2a^2}{d^2}\right)$$

$$= \frac{Qqa}{\pi\varepsilon_0 d^3}\left(1 + \underbrace{\frac{2a^2}{d^2}}_{\ll 1}\right) \approx \frac{Qqa}{\pi\varepsilon_0 d^3} \quad (a/d \ll 1) \checkmark$$

Grading space for 6: I think I got this question all correct.

$$Q_{JK} = q \begin{bmatrix} 2a^2-b^2-c^2 & 3qab & 3qac \\ 3qab & 2b^2-a^2-c^2 & 3qbc \\ 3qac & 3qbc & 2c^2-a^2-b^2 \end{bmatrix}$$

7. Since only that charge q is there $\Rightarrow Q = q$ ✓

$\vec{p} = q \cdot \vec{r} = q(a,b,c)$ ✗ (should be vector) $= q(a\hat{x} + b\hat{y} + c\hat{z})$

$$Q_{JK} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xz} \\ Q_{yx} & Q_{yy} & Q_{yz} \\ Q_{zx} & Q_{zy} & Q_{zz} \end{bmatrix} = q \begin{bmatrix} a^4 & a^3b & a^3c \\ a^3b & b^4 & b^3c \\ a^3c & b^3c & c^4 \end{bmatrix}$$ ✗

If added $-q \Rightarrow Q = q - q = 0$

$\vec{p} = q\vec{r} - (-q)(0,0,0) = q\vec{r}$ (unchanged)

Q_{JK} will also remain the same since the charge ✓
located at the origin won't affect the components same with $\vec{p}$.

$a \to \infty$
inside $\Rightarrow Q/4$
outside $\Rightarrow Q$.
$a \to a/2$

5. The student is wrong since when all the charge were at the surface, the total energy would be at its lowest. This is because it takes a large amount of energy to bring the charges on the sphere's surface from outside. Moving the charges inside would take even more energy, so when the charges are inside, the energy would not be the lowest. ?

8 (Wangsness 8-8) $\cdot Q = \iint \sigma\, da$, $da = a^2 \sin\theta\, d\theta\, d\phi$

$$\Rightarrow Q = \int_0^{\pi}\int_0^{2\pi} a^2 \sigma_0 \cos\theta\, d\theta\, d\phi = 2\pi\sigma_0 a^2 \int_0^{\pi} \frac{\sin^2\theta}{2} d\theta = 0$$ ✓

$\cdot \vec{p} = \int \vec{r}\, \sigma\, d\vec{a}$, $\vec{r} = a\sin\theta\cos\phi\,\hat{i} + a\sin\theta\sin\phi\,\hat{j} + a\cos\theta\,\hat{k}$

$$\Rightarrow \vec{p} = a^3\sigma \int_0^{\pi}\int_0^{2\pi} [(\sin^2\theta\cos\theta\cos\phi)\hat{i} + (\sin^2\theta\cos\theta\sin\phi)\hat{j} + (\cos^2\theta\sin\theta)\hat{k}]\, d\theta\, d\phi$$

$$\Rightarrow \vec{p} = a^3\sigma\, 2\pi \int_0^{\pi} \cos^2\theta \sin\theta\, d\theta = \boxed{\frac{4}{3}\pi a^3 \sigma\, \hat{k}}$$ ✓

$\cdot$ Antisymmetric $\Rightarrow Q_{JK} = 0\ (J \neq K)$ and $Q_{zz} = 0 \Rightarrow Q_{xx} = Q_{yy}$.

$\rho(\vec{r}') = 2\pi \sin\theta\, \sigma$ for circle circumference $\Rightarrow Q_{xx} = \int \rho(\vec{r}')(3x'^2 - r'^2)\, d\tau'$

$$\Rightarrow Q_{xx} = \int_{\theta'} \rho(\theta')(3x'^2 - r'^2)\, d\theta' = 2\pi a^4 \sigma_0 \left(\int_0^{\pi} 3\sin^3\theta' \cos\theta'\, d\theta' - \int_0^{\pi} \sin\theta' \cos\theta'\, d\theta' \right)$$

$$\Rightarrow Q_{xx} = Q_{yy} = 0 \Rightarrow Q_{JK} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ ✓

Grading space for 7, 5, & 8 (Q, $\vec{p}$, Q_{JK}): $\cdot$5: Fundamentally my answer is correct but I don't know if the logic is right so I will let you decide. I need mathematical proof like the answer tho so I dk.

$\cdot$ I messed up the Q_{JK} for 7, I didn't remember what went thru my head @ that time. I probably watched a Youtube video and applied the wrong situation since that's my first tensor problem.

8(cont). $\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r}^{\,\to 0} + \frac{\vec{p}\cdot\vec{r}}{r^2} + \frac{1}{2r^3}\sum_{JK} l_J l_K Q_{JK}^{\,\to 0}\right)$

$\to \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}\cdot\hat{r}$

$= \frac{1}{4\pi\varepsilon_0}\left(\frac{4\pi a^3 \vartheta_0}{3}\boxed{\hat{z}\cdot\hat{r}}\cdot\frac{1}{r^2}\right)$

$= \frac{4\pi a^3 \vartheta_0 \cos\theta}{4\pi\varepsilon_0 r^2 \cdot 3} = \boxed{\frac{a^3\vartheta_0\cos\theta}{3\varepsilon_0 r^2}}$ ✓

9. $Q = \int_{V'} \rho\, dV' = \rho\int_0^a dx'\int_0^b dy'\int_0^c dz' = \rho abc.$ ✓

- $\vec{p} = \int_{V'} \rho(\vec{r}\,')\vec{r}\,' d\tau' = \rho\int_0^a\int_0^b\int_0^c (x\hat{x} + y\hat{y} + z\hat{z})\, dz'\, dy'\, dx'$ ✓

$= \rho\left[\hat{x}\frac{a^2}{2}bc + \hat{y}\frac{ab^2c}{2} + \hat{z}\frac{abc^2}{2}\right] = \frac{\rho abc}{2}(a\hat{x} + b\hat{y} + c\hat{z})$ ✓

- $Q_{JK} = \int_{V'} \rho(\vec{r}\,')(3x_J x_K - r'^2\delta_{JK})\, d\tau'$ $\to Q$

$\Rightarrow Q_{xx} = \rho\int_0^a\int_0^b\int_0^c (2x^2 - y^2 - z^2)\, dx\, dy\, dz = \frac{\rho abc}{3}(2a^2 - b^2 - c^2)$

$\Rightarrow Q_{yy} = \frac{Q}{3}(2b^2 - a^2 - c^2),\quad Q_{zz} = \frac{Q}{3}(2c^2 - a^2 - b^2)$

$\Rightarrow Q_{xy} = 3\rho\int_0^a x\,dx\int_0^b y\,dy\int_0^c dz = 3\rho\frac{a^2}{2}\frac{b^2}{2}c = \frac{3}{4}\rho a^2b^2c = Q_{yx}$

$\Rightarrow Q_{yz} = \frac{3}{4}\rho ab^2c^2 = Q_{zy},\quad Q_{xz} = \frac{3}{4}\rho a^2bc^2 = Q_{zx}.$

$$\Rightarrow Q_{JK} = \begin{bmatrix} \frac{Q}{3}(2a^2-b^2-c^2) & \frac{3}{4}\rho a^2b^2c & \frac{3}{4}\rho a^2bc^2 \\ \frac{3}{4}\rho a^2b^2c & \frac{Q}{3}(2b^2-a^2-c^2) & \frac{3}{4}\rho ab^2c^2 \\ \frac{3}{4}\rho a^2bc^2 & \frac{3}{4}\rho ab^2c^2 & \frac{Q}{3}(2c^2-a^2-b^2) \end{bmatrix}$$

I forgot to do ϕ...

$$= \begin{bmatrix} \frac{Q}{3}(2a^2-b^2-c^2)\ ✓ & \frac{3}{4}Qab\ ✓ & \frac{3}{4}Qac\ ✓ \\ \frac{3}{4}Qba\ ✓ & \frac{Q}{3}(2b^2-a^2-c^2)\ ✓ & \frac{3}{4}Qbc\ ✓ \\ \frac{3}{4}Qca\ ✓ & \frac{3}{4}Qcb\ ✓ & \frac{Q}{3}(2c^2-a^2-b^2)\ ✓ \end{bmatrix}$$

Grading space for 8(cont) & 9 : All correct for 8.

9. $\phi(\vec{r}) = \frac{3Qa^2}{16\pi\varepsilon_0 r^3}\sin\theta\,[\sin\theta\sin\phi\cos\phi + \cos\theta\cos\phi + \cos\theta\sin\phi]$

for $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$

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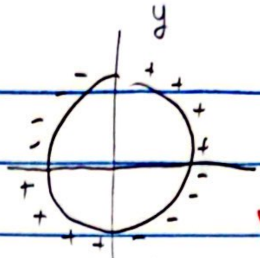
10.  $\Rightarrow$ quadripole would be the first non-vanishing moment

11. • $\phi(\vec{r}) = \dfrac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3} = \dfrac{p_x x + p_y y + p_z z}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}}$

$$\vec{E} = -\nabla\phi = -\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\frac{p_x x + p_y y + p_z z}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}}$$

$$\Rightarrow E_x = \frac{p_x}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}} - \frac{3x(p_x x + p_y y + p_z z)}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{5/2}}$$

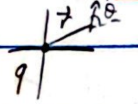
Call $4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2} = C_1$ & $\dfrac{(p_x x + p_y y + p_z z)}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{5/2}} = C_2$

$\Rightarrow E_y = \dfrac{p_y}{C_1} - 3yC_2$ & $E_z = \dfrac{p_z}{C_1} - 3zC_2$

• $\vec{E} = -\nabla\phi = -\vec{\nabla}\dfrac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3} = -\dfrac{1}{4\pi\varepsilon_0 r^3}\nabla(\vec{p}\cdot\vec{r}) - (\vec{p}\cdot\vec{r})\vec{\nabla}\dfrac{1}{4\pi\varepsilon_0 r^3}$

$$\Rightarrow \vec{E} = -\frac{\vec{p}}{4\pi\varepsilon_0 r^3} + \frac{+3\hat{r}(\vec{p}\cdot\vec{r})}{4\pi\varepsilon_0 r^3} = \boxed{\frac{1}{4\pi\varepsilon_0 r^3}[3(\vec{p}\cdot\hat{r})\hat{r} - p]}$$

$\vec{E}$ can't be 0 $\rightarrow$ only when $r \to \infty$ as the dipole is a point. If $\vec{p}$ @ $\vec{r}' \Rightarrow \vec{E}(\vec{r})$ can be found by changing $\vec{r} \to \vec{r}' - \vec{r}$ (should this be correct according to the answer).

12.  $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$ $\Rightarrow \vec{\tau} = \vec{p}\times\vec{E} = \dfrac{pq}{4\pi\varepsilon_0 r^2}\sin\theta$ (can I do this since $\vec{p}\times\vec{E} = pE\sin\theta$?

$\Rightarrow U = -\vec{p}\cdot\vec{E} = \dfrac{-pq}{4\pi\varepsilon_0 r^2}\cos\theta$ ($\vec{p}\cdot\vec{E} = pE\cos\theta$.)

$$\vec{F} = \vec{\nabla}(\vec{p}\cdot\vec{E}) = \vec{\nabla}\vec{p}\,\hat{r}\frac{q}{4\pi\varepsilon_0 r^2} = q\frac{-1}{4\pi\varepsilon_0 r^3}[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}] = \boxed{\frac{q}{4\pi\varepsilon_0 r^3}[\vec{p} - 3(\vec{p}\cdot\hat{r})\hat{r}}$$

$\Rightarrow$ Similar to equation 8-84, except for q & -1 factor. This is b/c the charge travel opposite to the field lines.

Grading space for 10, 11, 12: • All correct for 10. I think it's also all correct for 11 as well (maybe bad explaination on my end. I also think I did correct for 12 but please check it for me.

13. $\bullet\ U = -\vec{p_2}\vec{E_1} = -\vec{p_2}\left[\frac{1}{4\pi\varepsilon_0}\left(\frac{3(\vec{p_1}\vec{r})\vec{r}}{|r|^3} - \frac{\vec{p_1}}{|r|^3}\right)\right]$

$\Rightarrow U = \frac{-1}{4\pi\varepsilon_0}\left[\frac{3(\vec{p_1}\vec{r})\vec{p_2}\vec{r}}{|r|^5} - \frac{\vec{p_1}\vec{p_2}}{|r|^3}\right]$

$= \frac{-1}{4\pi\varepsilon_0}\left[\frac{3(\vec{p_1}\hat{r})(\vec{p_2}\hat{r})|r|^2}{|r|^5} - \frac{\vec{p_1}\vec{p_2}}{|r|^3}\right]$

$= \frac{1}{4\pi\varepsilon_0 R^3}\left[\frac{\vec{p_1}\vec{p_2}}{\underbrace{|r|^3}_{=R^3}} - \frac{3(\vec{p_1}\hat{r})(p_2\hat{r})}{\underbrace{|r|^3}_{=R^3}}\right] \quad \vec{r} = \vec{R} = \vec{r_2} - \vec{r_1}$ ✓

$\Rightarrow U = \frac{1}{4\pi\varepsilon_0 R^3}\left[\vec{p_1}\cdot\vec{p_2} - 3(\vec{p_1}\hat{R})(\vec{p_2}\hat{R})\right]$

U will be the same even from $2 \to 1$ b/c: $[\vec{p_1}(\vec{r_1}-\vec{r_2})][\vec{p_2}(\vec{r_1}-\vec{r_2})]$ ✓

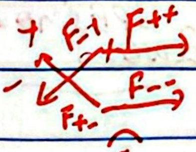
$= [\vec{p_1}\cdot(\vec{r_2}-\vec{r_1})][\vec{p_2}\cdot(\vec{r_2}-\vec{r_1})]$

$\bullet\ \vec{F_2} = -\vec{\nabla}U_{DD} = -\vec{\nabla}\frac{1}{4\pi\varepsilon_0 R^3}\left[\vec{p_1}\cdot\vec{p_2} - 3(\vec{p_1}\cdot\vec{R})(\vec{p_2}\cdot\vec{R})\right]$

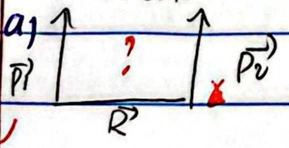
$= -\vec{\nabla}\frac{\vec{p_1}\cdot\vec{p_2}}{4\pi\varepsilon_0 R^3} + \vec{\nabla}\frac{3(\vec{p_1}\vec{R})(\vec{p_2}\vec{R})}{4\pi\varepsilon_0 R^3} = \frac{3\vec{p_1}\vec{p_2}\hat{R}}{4\pi\varepsilon_0 R^4} + 3(\vec{p_1}\cdot\vec{R})(\vec{p_2}\cdot\vec{R})\vec{\nabla}\frac{1}{4\pi\varepsilon_0 R^5} + \frac{1}{4\pi\varepsilon_0 R^5}$

$\vec{\nabla}3(\vec{p_1}\cdot\vec{R})(\vec{p_2}\cdot\vec{R}) = \frac{3\vec{p_1}\cdot\vec{p_2}\hat{R}}{4\pi\varepsilon_0 R^4} - \frac{15(\vec{p_1}\cdot\vec{R})(\vec{p_2}\cdot\vec{R})\hat{R}}{4\pi\varepsilon_0 R^6} + \frac{3}{4\pi\varepsilon_0 R^5}[(\vec{p_2}\cdot\vec{R})\vec{\nabla}(\vec{p_1}\cdot\vec{R})$
$+(\vec{p_1}\cdot\vec{R})\vec{\nabla}(\vec{p_2}\cdot\vec{R})]$

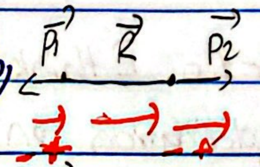
$\Rightarrow \vec{F_2} = \frac{3\vec{p_1}\cdot\vec{p_2}}{4\pi\varepsilon_0 R^4}\hat{R} - \frac{15(\vec{p_1}\cdot\hat{R})(\vec{p_2}\cdot\hat{R})\hat{R}}{4\pi\varepsilon_0 R^4} + \frac{3}{4\pi\varepsilon_0 R^5}[(\vec{p_2}\cdot\vec{R})\vec{p_1} + (\vec{p_1}\cdot\vec{R})\vec{p_2}]$



$= \frac{3}{4\pi\varepsilon_0 R^4}[(\vec{p_1}\cdot\vec{p_2})\hat{R} - 5(\vec{p_1}\cdot\hat{R})(\vec{p_2}\hat{R})\hat{R} + (\vec{p_2}\hat{R})\vec{p_1} + (\vec{p_1}\cdot\hat{R})\vec{p_2}]$ ✓

a) 

$\vec{F_2} = \frac{3}{4\pi\varepsilon_0 R^4}(p_1p_2\hat{R} - 0\hat{R} + 0\vec{p_1} + 0\vec{p_2}) = \frac{3p_1p_2}{4\pi\varepsilon_0 R^4}\hat{R}$ ✓

b) 

$\vec{F_2} = \frac{3}{4\pi\varepsilon_0 R^4}(p_1p_2\hat{R} - 5p_1p_2\hat{R} + \vec{p_2}\vec{p_1} + \vec{p_1}\vec{p_2})$

$\Rightarrow \vec{F_2} = \frac{3}{4\pi\varepsilon_0 R^4}(-4p_1p_2\hat{R} + p_2p_1\hat{R} + p_1p_2\hat{R}) = \frac{-3p_1p_2}{2\pi\varepsilon_0 R^4}\hat{R}$ ✓

Scanned with CamScanner

Overall Performance: I think this is by far my best homework. There are still many things I need to improve like how to find Q_{JK} (problem 7 is just so embarassing) and how to find $\vec{E}$, $\vec{U}$ (or $\vec{\phi}$) thru symmetrical surface as I think that will be a stable throughout the course. I also need to be more careful with my math (problem 3 is also quite embrassing).