HOMEWORK 7

1. Wangsness 15-7.

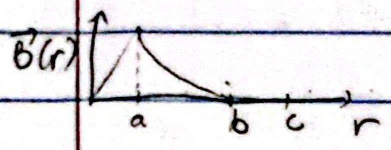
Using Ampere's circular law: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

$\vec{B}(r) = \frac{\mu_0 I r}{2\pi a^2} \hat{z}$ (for $r<a$)

$\vec{B}(r) = \frac{\mu_0 I}{2\pi r} \hat{z}$ (for $a<r<b$)

$\vec{B}(r) = 0$ (for $r>b$) $= \frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{2\pi r}$

contribution from 2 cylinder.



2. Wangsness 15-8.

$\vec{B} = 0$ for $0<\rho<a$, $\vec{B} = \frac{\mu_0 I}{2\pi\rho} \frac{\rho^2 - a^2}{b^2 - a^2} \hat{\phi}$ for $a<\rho<b$, and $\vec{B} = \frac{\mu_0 I}{2\pi\rho} \hat{\phi}$ for $b<\rho$.

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$. In cylindrical coord: $\frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho \vec{B}) \hat{z} = \mu_0 \vec{J}$

For $a<\rho<b$: $\frac{1}{\rho} \left(\vec{B} + \rho \frac{\partial \vec{B}}{\partial \rho} \right) \hat{z} = \mu_0 \vec{J} = \frac{\mu_0 I}{\pi (b^2 - a^2)} \Rightarrow \vec{J} = \frac{I}{\pi (b^2 - a^2)} \hat{z}$

$\vec{B} = \frac{\mu_0 I}{2\pi\rho} \frac{\rho^2 - a^2}{b^2 - a^2} \hat{\phi}$ for $a<\rho<b$

$\Rightarrow \vec{J} = \frac{1}{\mu_0} (\vec{\nabla} \times \vec{B}) = \hat{z} \frac{I}{2\pi\rho} \frac{\partial}{\partial \rho} \left[\frac{\rho^2 - a^2}{b^2 - a^2} \right] = \hat{z} \frac{I}{\pi (b^2 - a^2)}$

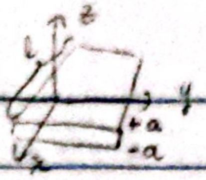
For $b<\rho$: $\vec{J} = 0$ because $\vec{B} = \frac{\mu_0 I}{2\pi\rho}$

$\Rightarrow \frac{\partial}{\partial \rho} (\rho \vec{B}) = \frac{\partial}{\partial \rho} \left(\frac{\mu_0 I}{2\pi} \right) = 0$

You could produce this kind of field by having coaxial cables carrying current in the opposite direction.

Grading space for 1 & 2:

Scanned with CamScanner

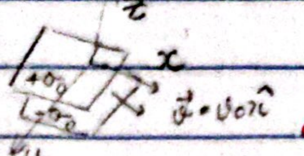
3.  $\vec{J} = J_0\hat{x}$ $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$

Using the right hand rule → $\vec{B}$ in $-\hat{y}$ for $z > 0$

$I_{encl} = \int \vec{J}\cdot d\vec{a} = J_0 z l$ $\vec{B}$ in $\hat{y}$ for $z < 0$

→ $\oint \vec{B}\cdot d\vec{l} = B\cdot l = \mu_0 J_0 z l$ → $B = \mu_0 z J_0$ → $\vec{B} = \begin{cases} -\mu_0 J_0 a\hat{y} & (z>a) \\ \mu_0 J_0 a\hat{y} & (z<-a) \end{cases}$

For $-a<z<a$: $\vec{B} = -\mu_0 z J_0 \hat{y}$

4.  $\vec{K} = \sigma\vec{v}$

a) → $\oint \vec{B}\cdot d\vec{l} = 2Bl = \mu_0 I_{encl} = \mu_0 K l$ → $B = \frac{\mu_0 K}{2}$

• Above & below the capacitors: B-field = 0 since K points to opposite directions from the 2 plates.

into the page

• In between: $\vec{B} = \frac{\mu_0}{2}\sigma_0 v_0(-\hat{y}) - \frac{\mu_0}{2}\cdot(-\sigma_0)v_0(-\hat{y}) = \mu_0 \sigma_0 v_0 (-\hat{y})$

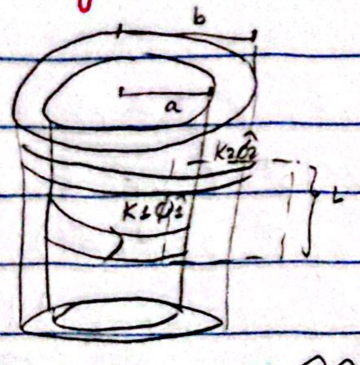
b) $\vec{f} = \vec{K}\times\vec{B} = \sigma v_0 \cdot \frac{\mu_0 \sigma_0 v_0}{2} = \frac{\mu_0 \sigma_0^2 v_0^2}{2}\hat{z}$ (using right hand rule)

c) Electric field @ bottom plate: $E = \frac{\sigma_0}{2\epsilon_0}$; Force on the upper plate: $F = \frac{\sigma_0^2 A}{2\epsilon_0}$

$f = \frac{\sigma_0^2}{2\epsilon_0}$ (force/unit area)

$= \frac{\mu_0 \sigma_0^2 v_0^2}{2}$ ⇒ $v_0 = \frac{1}{\sqrt{\mu_0\epsilon_0}} = c$ → forces would never balance.

5. Wangsness 15-10.



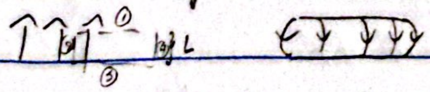
$\oint B\cdot dl = \mu_0 I_{inc} = \mu_0(K_1+K_2)L$

⇒ $\vec{B} = \mu_0(K_1+K_2)\hat{z}$ (right hand rule)

→ Magnetic field between 2 cylinders: $\vec{B} = \mu_0 K_2 \hat{z}$

⇒ Magnetic field outside = 0.

Magnetic field from loop:

6. Wangsness 15-11. 

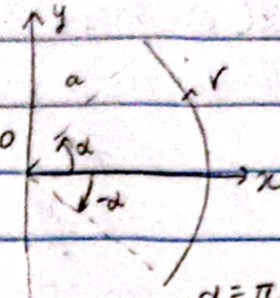
$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$ ⇒ $B = \frac{\mu_0 I_{encl}}{l}$. At ② & ④ → $B = 0$

⇒ $B_{loop} = B_1 + B_3 = \frac{2\mu_0 I_{encl}}{l}$ ⇒ induction cannot drop to 0.

Change in B is also consistent with capacitor image as current move parallel in the same direction → Additive B → not drop to 0.

Grading space for 3→6:

7. Wangsness 16-10.



$r = z\hat{z}$, $r' = a\hat{\phi} \Rightarrow R = z\hat{z} - a\hat{\phi} = \sqrt{z^2+a^2}$

$$\vec{A} = \frac{\mu_0 I}{4\pi}\int \frac{ds}{R} = \frac{\mu_0 I}{4\pi}\int \frac{a\,d\phi\,d\theta}{\sqrt{z^2+a^2}}$$

$$= \frac{\mu_0 I a}{4\pi\sqrt{z^2+a^2}}\,2\pi \cdot \sin\alpha\,\hat{y} = \frac{\mu_0 I a \sin\alpha}{2\pi\sqrt{z^2+a^2}}\,\hat{y}$$

$\alpha = \pi \Rightarrow \vec{A} = 0$ That doesn't make sense, I just don't know why.

8. Wangsness 16-14.

Uniform ∞ sheet of current in xy plane, carrying current in y-direction $\Rightarrow$

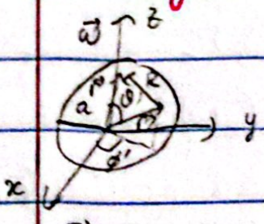
$$B = \begin{cases} -\frac{\mu_0}{2} k\hat{x} & (z>0) \\ \frac{\mu_0}{2} k\hat{x} & (z<0) \end{cases}$$

$\vec{\nabla}\times\vec{A} = \vec{B}$, using Stock's Theorem: $\oint A\cdot dl = \int_A B\cdot dA$

$$\Rightarrow A = \frac{\vec{B}\times\vec{r}}{2} = -\frac{\mu_0 k}{4}(\hat{x}\times\vec{r})$$

$$\Rightarrow \begin{cases} A = -\frac{\mu_0 k}{4}(z\hat{y} - \hat{k}y) & \text{for } z>0 \\ A = \frac{\mu_0 k}{4}(z\hat{y} - \hat{k}y) & \text{for } z<0 \end{cases}$$

9. Wangsness 16-15.



Assuming $\vec{\omega} = \omega\hat{z}$

$$\vec{r}' = r\sin\theta'\cos\phi'\,\hat{x} + r\sin\theta'\sin\phi'\,\hat{y} + r\cos\theta'\,\hat{z}$$

$$\vec{v} = \vec{\omega}\times\vec{r} = (-\omega r\sin\theta'\sin\phi')\,\hat{x} - (-\omega r\sin\theta'\cos\phi')\,\hat{y}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')}{R}\,d\tau, \text{ where } J(\vec{r}') = \rho\vec{v} = \rho(\vec{\omega}\times\vec{r}')$$

$$= \frac{\mu_0}{4\pi}\int_0^R\int_0^\pi\int_0^{2\pi} (-\omega r'\sin\theta'\sin\phi'\,\hat{x} + \omega r\sin\theta'\cos\theta'\,\hat{y}) \ldots r^2\sin\theta'\,dr\,d\theta\,d\phi$$

But since $\int_0^{2\pi}\sin\phi'\,d\phi = 0 = \int_0^{2\pi}\cos\phi'\,d\phi'$

$\Rightarrow \vec{A} = 0$

Grading space for 7→9.

10. $L = \int \frac{dL}{dt}\,dt = \int \tau\,dt = \int (r \times F)\,dt$ (Angular momentum of particle).
Initial velocity: $\vec{v_0} = v_0 \hat{\rho}$. Since the particle is moving in B-field
$\Rightarrow \vec{F} = q(\vec{v_0} \times \vec{B}) \Rightarrow L = \int r \times q(v \times B)\,dt = \int r \times q(d\ell \times B) = q\left[\int \underbrace{(r \cdot B)}_{0 \text{ since } r \perp B} d\ell - \int B(r \cdot d\ell)\right]$
$r \cdot d\ell = r \cdot dr = \frac{1}{2} d(r^2) = r\,dr = \frac{1}{2\pi}(2\pi r\,dr)$
$\Rightarrow L = \frac{-q}{2\pi} \int_0^R B\,2\pi r\,dr = \frac{-q}{2\pi} \int B\,da = \frac{-q}{2\pi} \cdot \phi \Big] \rightarrow \text{flux} = \int B\,da.$
$\Rightarrow$ If $\phi = 0 \Rightarrow L = 0 \Rightarrow \iint B \cdot da = 0$. Therefore, we can conclude that the charge leave the region pretty radially.

11. Wangsness 16-5.

$\vec{B} = B\hat{z} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \Rightarrow \begin{cases} \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = 0 & (1) \\ \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = 0 & (2) \\ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = B & (3) \end{cases}$

From Section 16-3:

(a) $A_x = 0,\ A_y = Bx,\ A_z = \text{const}$. From (1) $\Rightarrow \frac{\partial A_z}{\partial y} = 0,\ \frac{\partial A_y}{\partial z} = 0$
$\frac{\partial A_x}{\partial z} = 0,\ \frac{\partial A_z}{\partial x} = 0\ ;\ \frac{\partial A_y}{\partial x} = B\frac{\partial x}{\partial x} = B\ ;\ \frac{\partial A_x}{\partial y} = 0$
$\Rightarrow \frac{\partial A_y}{\partial x} - \frac{\partial A_y}{\partial y} = B - 0 = B \Rightarrow \vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 0 + 0 + 0 = 0$ $\begin{cases} A_x = 0 \\ A_y = Bx \\ A_z = \text{const} \end{cases}$

(b) $A_x = -By,\ A_y = 0,\ A_z = \text{const}. \Rightarrow \frac{\partial A_x}{\partial y} = -B,\ \frac{\partial A_x}{\partial z} = 0,\ \frac{\partial A_y}{\partial z} = 0 = \frac{\partial A_y}{\partial x}$
$\frac{\partial A_z}{\partial y} = 0,\ \frac{\partial A_z}{\partial x} = 0 \Rightarrow$ (1) & (2) satisfied $\Rightarrow$ (3) $: \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \Rightarrow 0 - (-B) = B$
$\Rightarrow \vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = -B\frac{\partial y}{\partial x} + 0 + \frac{\partial\,\text{const}}{\partial z} = 0$

(c) $A_x = -\frac{1}{2}By,\ A_y = \frac{1}{2}Bx,\ A_z = \text{Const}$
$\Rightarrow \vec{\nabla} \cdot \vec{A} = -\frac{1}{2}B\frac{\partial y}{\partial x} + \frac{1}{2}B\frac{\partial x}{\partial y} + \frac{\partial}{\partial z}\text{const} = 0$
$\Rightarrow \frac{\partial A_x}{\partial A_y} = -\frac{1}{2}B,\ \frac{\partial A_y}{\partial x} = \frac{1}{2}B$, else $= 0$. Satisfied (1) & (2) & (3)

From Gauge transformation $\Rightarrow \vec{A_1} = (0, Bx, \text{const})$
$\vec{A_2} = (-By, 0, \text{const}) = \vec{A_1} + \vec{\nabla}\chi_{12}$

$\Rightarrow (-By, 0, \overset{=c}{\text{const}}) = (0, Bx, c) + \left(\frac{\partial \chi_{12}}{\partial x}, \frac{\partial \chi_{12}}{\partial y}, \frac{\partial \chi_{12}}{\partial z}\right) = \left(\frac{\partial \chi_{12}}{\partial x}, Bx + \frac{\partial \chi_{12}}{\partial y}, c + \frac{\partial \chi_{12}}{\partial z}\right)$

$\Rightarrow \begin{cases} \frac{\partial \chi_{12}}{\partial x} = -By \Rightarrow \chi_{12} = -Bxy + c' \\ \frac{\partial \chi_{12}}{\partial y} = -Bx \\ \frac{\partial \chi_{12}}{\partial z} = 0 \end{cases}$

- $\vec{A_2} \to \vec{A_3} \Rightarrow \vec{A_3} = \vec{A_2} + \vec{\nabla}\chi_{23}$

$\Rightarrow \left(-\frac{1}{2}By, \frac{1}{2}Bx, c\right) = (-By, 0, c) + \left(\frac{\partial \chi_{23}}{\partial x}, \frac{\partial \chi_{23}}{\partial y}, \frac{\partial \chi_{23}}{\partial z}\right)$

$\Rightarrow \left(-\frac{1}{2}By, \frac{1}{2}Bx, c\right) = \left(-By + \frac{\partial \chi_{23}}{\partial x}, \frac{\partial \chi_{23}}{\partial y}, c + \frac{\partial \chi_{23}}{\partial z}\right)$

$\Rightarrow -\frac{1}{2}By = -By + \frac{\partial \chi_{23}}{\partial x} \Rightarrow \frac{\partial \chi_{23}}{\partial x} = \frac{1}{2}By \to \chi_{23} = \frac{1}{2}Bxy + c''$

- $\vec{A_3} \to \vec{A_1} \Rightarrow \vec{A_1} = \vec{A_3} \cdot \vec{\nabla}\chi_{13}$

$\Rightarrow (0, Bx, c) = \left(-\frac{1}{2}By \cdot \frac{\partial \chi_{13}}{\partial x}, \frac{1}{2}Bx \cdot \frac{\partial \chi_{13}}{\partial y}, c \cdot \frac{\partial \chi_{13}}{\partial z}\right)$

$\Rightarrow -\frac{1}{2}By \cdot \frac{\partial \chi_{13}}{\partial x} = 0 \Rightarrow \frac{\partial \chi_{13}}{\partial x} = -\frac{1}{2}By \Rightarrow \chi_{13} = -\frac{1}{2}Bxy + c''$

Grading space for 10+11:

Conclusion: