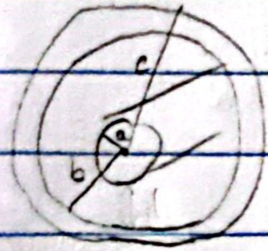
# HOMEWORK6

**1. Wangsness 12-3** $|\vec{v}| = \omega r = \omega a$

$$I = \frac{dq}{dt} = \underbrace{\frac{Q}{2\pi a}}_{\lambda} \cdot \frac{dr}{dt} = \frac{Q}{2\pi}\omega$$

$$\vec{J} = \rho\vec{v} = \frac{Q}{4\pi a^3}\omega\vec{r} = \frac{Q}{4\pi a^3} 3\omega r \sin\theta\,\hat{\phi}$$




**2. Wangsness 12-10.** $E_{r>c} = 0$ (No charge enclose)

$E_{r<a} = E_{c>r>b} = 0$ (conductor)

$$\Rightarrow E_{b>r>a} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}, \quad \Delta\phi = -\int \vec{E}\cdot d\vec{s}$$

$$\Rightarrow \phi_b - \phi_a = \int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}\,dr = \frac{\lambda}{2\pi\varepsilon_0}[\ln(b) - \ln(a)] = \Delta\phi \to V$$

$$\Rightarrow C = \frac{Q}{V} = \frac{\lambda l}{\frac{\lambda \ln(b/a)}{2\pi\varepsilon_0}} = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}.$$

$$\Rightarrow I = C\frac{dV}{dt} = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}\Delta\phi$$

**Grading Space for 1&2:**

3. $\vec{E} = E_0\hat{z}$, $\vec{B} = B_0\hat{x}$, $\omega = \frac{qB}{m}$

Lorentz force: $\vec{F} = q\vec{E} + q(\vec{v}\times\vec{B}) = qE_0\hat{z} + q(\vec{v}\times B_0\hat{x})$

$\vec{v}\times B_0\hat{x} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ B_0 & 0 & 0 \end{vmatrix} = v_z B_0\hat{y} - v_y B_0\hat{z}$

$\Rightarrow \vec{F} = qE_0\hat{z} + qB_0 v_z\hat{y} - qB_0 v_y\hat{z} = (qE_0 - qB_0 v_y)\hat{z} + qB_0 v_z\hat{y} = m\frac{d\vec{v}}{dt}$

$m\frac{dv_x}{dt} = 0$ ①, $m\frac{dv_y}{dt} = qB_0 v_z$ ②, $m\frac{dv_z}{dt} = qE_0 - qB_0 v_y$ ③

Initial Con: $t=0 \Rightarrow v_x = v_y = v_z = 0$.

$\frac{dv_x}{dt} = 0 \Rightarrow v_x = C_1 = 0$ at initial con.

③: $\frac{d^2v_z}{dt^2} = \cancel{\frac{qE_0}{m}}^{0} - \frac{qB_0}{m}\frac{dv_y}{dt} = \frac{-q^2B_0^2}{m^2}v_z$ (from ②)

$= -\omega^2 v_z \Rightarrow v_z = A\sin(\omega t + C_2)$

At $t=0$, $v_z = 0 = A\sin C_2 \Rightarrow C_2 = 0 \Rightarrow v_z = A\sin\omega t$

Substitute $v_z$ to ② $\Rightarrow \frac{m\,dv_y}{dt} = qB_0 A\sin\omega t \Rightarrow \frac{dv_y}{dt} = \omega A\sin\omega t$

$\Rightarrow v_y = -A\cos\omega t + C_3$. Initial Con $\Rightarrow 0 = -A + C_3 \Rightarrow C_3 = A$

$\Rightarrow v_y = A(1-\cos\omega t)$

Substitute $v_z, v_y$ to ③ $\Rightarrow m\frac{d}{dt}A\sin\omega t = qE_0 - qB_0A(1-\cos\omega t)$

$\Leftrightarrow mA\omega\cos\omega t = qE_0 - qB_0A(1-\cos\omega t)$

$t=0 \Rightarrow mA\omega = qE_0 \Rightarrow A = \frac{qE_0}{m\omega} = \frac{qE_0}{m}\cdot\frac{m}{qB_0} = \frac{E_0}{B_0}$

$\Rightarrow v_x = 0$, $v_y = \frac{E_0}{B_0}(1-\cos\omega t)$, $v_z = \frac{E_0}{B_0}\sin\omega t$

Initial Con: $t=0 \Rightarrow x=y=z=0$ (particle @ origin)

$\Rightarrow \frac{dx}{dt} = 0 \Rightarrow x = C_4 = 0$.

$\Rightarrow \frac{dy}{dt} = \frac{E_0}{B_0}(1-\cos\omega t) \Rightarrow y = \frac{E_0}{B_0}\left(t - \frac{\sin\omega t}{\omega}\right) + C_5$

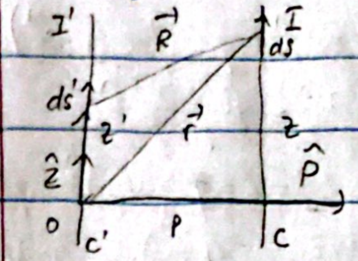
At $t=0 \Rightarrow C_5 = 0 \Rightarrow y = \frac{E_0}{B_0}\left(t - \frac{\sin\omega t}{\omega}\right)$

$\Rightarrow \frac{dz}{dt} = \frac{E_0}{B_0}\sin\omega t \Rightarrow z = -\frac{E_0}{B_0}\frac{\cos\omega t}{\omega} + C_6$. At $t=0 \Rightarrow C_6 = \frac{E_0}{B_0\omega}$

$\Rightarrow z = \frac{E_0}{B_0\omega}(1-\cos\omega t)$

Grading Space for 3:

4. Wangsness 13-1.




According to Biot-Sawarts law, magnetic field due to I is: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s}' \times \hat{R}}{R^2}$

since $\vec{B} = \frac{\mu_0}{4\pi} \oint \frac{I d\vec{s} \times \hat{R}}{R^2}$

$d\vec{F} = \vec{J} \times \vec{B}\, d\tau = I(d\vec{s} \times \vec{B})$ for current I

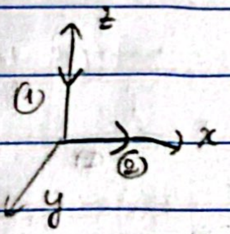
$\Rightarrow$ The force between 2 wires: $\vec{F} = \frac{\mu_0}{4\pi} I I' \oint \oint_{c'} \frac{ds \times ds' \times \vec{R}}{R^3}$

$d\vec{s} = dz\,\hat{z}$ & $d\vec{s}' = dz'\,\hat{z}$ & $R = \sqrt{\rho^2 + (z-z')^2}$

$\vec{R} = \rho\hat{\rho} + (z-z')\hat{z}$

$\Rightarrow \vec{F} = -\frac{\mu_0 I I' \rho \hat{\rho}}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dz\,dz'}{[\rho^2 + (z-z')^2]^{3/2}}$

5. a)




① Magnetic field is along $\hat{y}$

Current is along $-\hat{z}$

$\Rightarrow$ Force is along $-\hat{z} \times \hat{y} = -\hat{x}$

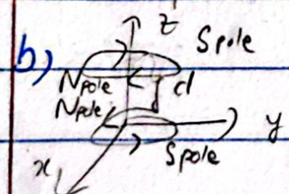
② Magnetic field is along $\hat{y}$

Current is along $\hat{x}$

$\Rightarrow$ Force is along $\hat{x} \times \hat{y} = -\hat{z}$

The answer does seem to contradict the conservation law since the forces are not equal & opposite to each other. However, I don't think it's the case since we did not account for electric force, and the net force between ① & ② would obey the conservation law.

b)




The current of 2 loops are repelling each other for CCW N-pole & CW S pole.

Since the 2 N poles repell $\Rightarrow$ force on top ring will be in $\hat{z}$ (the bottom should be in $-\hat{z}$).

Grading Space for 4 & 5:

7. Wangsness 14-9.

Sheet of current lie in xy plane, current in y-direction:

$$\Rightarrow \vec{B} = \frac{\mu_0 K'}{2} \begin{cases} \hat{x} & \text{for } z>0 \\ -\hat{x} & \text{for } z<0 \end{cases}$$

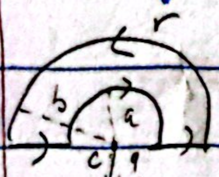
Sheet of current at $z=d \Rightarrow \vec{B}' = -\frac{\mu_0 K'}{2} \begin{cases} \hat{x} & \text{for } z>d \\ -\hat{x} & \text{for } z<d \end{cases} = \frac{\mu_0 K'}{2} \begin{cases} -\hat{x} & (z>d) \\ \hat{x} & (z<d) \end{cases}$

$$\Rightarrow \vec{B}_{net} = \vec{B} + \vec{B}' = \frac{\mu_0 K'}{2}\hat{x} - \frac{\mu_0 K'}{2}\hat{x} = 0 \quad (\text{for } z>d)$$

$$\Rightarrow \vec{B}_{net} = \vec{B} + \vec{B}' = -\frac{\mu_0 K'}{2}\hat{x} + \frac{\mu_0 K'}{2}\hat{x} = 0 \quad (\text{for } z<0)$$

$$\Rightarrow \vec{B}_{net} = \vec{B} + \vec{B}' = \frac{\mu_0 K'}{2}\hat{x} + \frac{\mu_0 K'}{2}\hat{x} = \mu_0 K'\hat{x} \quad (\text{for } 0<z<d)$$

6. Wangsness 14-15 (Correct this time).




Magnetic field at c due to a: $\vec{B} = \frac{\mu_0 I}{4a}(-\hat{z})$

b: $\vec{B}' = \frac{\mu_0 I}{4b}(-\hat{z})$

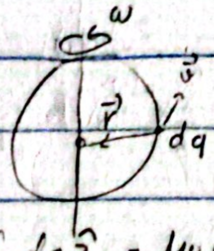
No horizontal field b/c they're along the point c.

$$\Rightarrow \vec{B}_{net} = \vec{B} - \vec{B}' = \left(\frac{\mu_0 I}{4a} - \frac{\mu_0 I}{4b}\right)\hat{z} = \frac{\mu_0 I}{4ab}(b-a)(-\hat{z})$$

$$\Rightarrow \vec{F} = q\vec{v} \times \vec{B}_{net} = q\, v(-\hat{y}) \times \frac{\mu_0 I}{4ab}(b-a)\cdot(-\hat{z}) = \frac{qv\mu_0 I}{4ab}(b-a)\hat{x}$$

Grading Space for 6&7:

8. Wangsness 14-12.



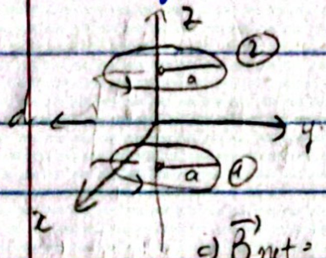

$$d\vec{B} = \frac{\mu_0}{4\pi} dq \frac{\vec{v}\times\vec{r}}{a^3}$$

$$\vec{v}\perp\vec{r} \Rightarrow dB = \frac{\mu_0}{4\pi} dq \frac{va\sin 90^\circ}{a^3}$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi} dq \frac{v}{a^2} = \frac{\mu_0}{4\pi} dq \frac{\omega}{a}$$

$$\Rightarrow \vec{B} = \int d\vec{B} = \frac{\mu_0 \omega}{4\pi a}\int dq\,\hat{z} = \frac{\mu_0\omega Q}{4\pi a}\hat{z}$$

9. Wangsness 14-6




$$\vec{B_1} = \frac{\mu_0}{4\pi}\frac{2\pi a^2 I}{[a^2+(z-d/2)^2]^{3/2}}\hat{z} \text{ (loop 2)}$$

$$\vec{B_2} = \frac{\mu_0}{4\pi}\frac{2\pi a^2 I}{[a^2+(z+d/2)^2]^{3/2}}\hat{z} \text{ (loop 2)}$$

$$\Rightarrow \vec{B}_{net} = \frac{\mu_0}{2} a^2 I \left\{ \frac{1}{[a^2+(z-d/2)^2]^{3/2}} + \frac{1}{[a^2+(z+d/2)^2]^{3/2}} \right\}\hat{z}$$

$z=0$ at origin $\Rightarrow \vec{B}_{net} = \frac{\mu_0 a^2 I}{2}\left\{\frac{1}{[a^2+d^2/4]^{3/2}} + \frac{1}{[a^2+d^2/4]^{3/2}}\right\}\hat{z}$

$$\Rightarrow \vec{B}_{net} = \frac{\mu_0 a^2 I}{(a^2+d^2/4)^{3/2}}$$

Generally: $\vec{B}_{net} = \frac{\mu_0 a^2 I}{2}\left\{\frac{1}{[a^2+(z+l)^2]^{3/2}} + \frac{1}{[a^2+(z-l)^2]^{3/2}}\right\}$

where $l$ is the distance from $z$.

$$\Rightarrow \frac{dB_z}{dz} = \frac{\mu_0 a^2 I}{2}\left\{-\frac{3}{2}[a^2+(z+l)^2]^{-5/2}\, 2(z+l) - \frac{3}{2}[a^2+(z-l)^2]^{-5/2}\cdot 2(z-l)\right\}$$

$$= -\frac{3\mu_0 a^2 I}{2}\left\{\frac{z+l}{[a^2+(z+l)^2]^{5/2}} + \frac{z-l}{[a^2+(z+l)^2]^{5/2}}\right\}$$

$z=0$ at origin

$$\Rightarrow \frac{dB_z}{dz} = -\frac{3\mu_0 a^2 I}{2}\cdot 0 = 0$$

$$\Rightarrow \frac{d^2B_z}{dz^2} = \frac{d}{dz}\frac{dB_z}{dz} = -\frac{3\mu_0 a^2 I}{2}\left\{[a^2+(z+l)^2]^{-5/2} - 5(z+l)^2[a^2+(z+l)^2]^{-7/2} + [a^2+(z-l)^2]^{-5/2} - 5(z-l)^2[a^2+(z-l)^2]^{-7/2}\right\}$$

$l = d/2,\ z=0 \Rightarrow \frac{d^2B_z}{dz^2} = -3\mu_0 a^2 I\left(a^2+\frac{d^2}{4}\right)^{-5/2}\left(\frac{a^2-d^2}{a^2+\frac{d^2}{4}}\right)$

$$\Rightarrow \frac{d^2B_z}{dz^2} = 0 \text{ when } a=d$$