HOMEWORK 1.

1. a) The net force will be 0 because all the charges have equal charge(q) but since they locate symmetrically, they will have opposite direction. Therefore, the forces at Q in the center will all cancelled out ⇒ 0 net force.

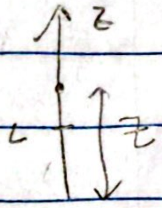
b) 1 charge remove ⇒ $\vec{F}_{Q \text{ due to } q} = \sum_i \frac{qQ\hat{R}}{4\pi\varepsilon_0 R^2} = \frac{qQ}{4\pi\varepsilon_0 d^2}$

(only feel force of 1 charge as the other 10 still cancelled each other out).

c) Answer'd be different since the charge won't be symmetrically placed anymore if the number is odd. Therefore, they can't cancelled out evenly.

If there're 3 charges then net force on Q = force from each charge $= \sum_i \frac{qQ\hat{R}}{4\pi\varepsilon_0 R^2}$

2. $dq = \lambda ds = Cz\,dz, \quad z > L$

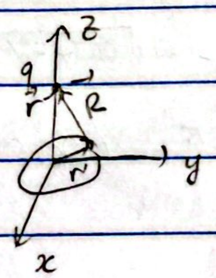


$\Rightarrow \vec{E} = \int \frac{Cz}{4\pi\varepsilon_0} \frac{\vec{R}}{R^3} dz' \qquad \vec{r} = z\hat{z}$

$\Rightarrow \vec{E} = \frac{Cz}{4\pi\varepsilon_0} \int \frac{-L^2}{z(z-L)^2} \hat{z}\, dz$

$\Rightarrow \vec{E} = \frac{Cz}{4\pi\varepsilon_0} \hat{z} \left[\ln\left(\frac{z-L}{z} + \frac{L}{z-L} \right) \right]$

3. a)



$\vec{E} = \frac{q}{4\pi\varepsilon_0} \frac{\vec{R}}{|\vec{R}|^3}, \quad \vec{R} = \vec{r} - \vec{r}'$

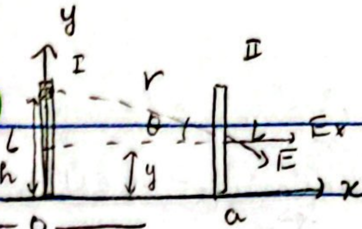
$\vec{r} = z\hat{z}$ & $\vec{r}' = a\hat{y}$

$\Rightarrow \vec{r} - \vec{r}' = z\hat{z} - a\hat{y} \Rightarrow |\vec{r} - \vec{r}'| = \sqrt{z^2 + a^2}$

$\Rightarrow \vec{E} = \frac{q}{4\pi\varepsilon_0} \frac{z\hat{z} - a\hat{y}}{(\sqrt{z^2+a^2})^3}$ → 0 (all y components cancelled out)

$\Rightarrow \vec{E} = \frac{q}{4\pi\varepsilon_0} \frac{z\hat{z}}{(z^2+a^2)^{3/2}} \Rightarrow \vec{F} = \vec{E}_q\, q = \frac{q_1 q_2 z\hat{z}}{4\pi\varepsilon_0 (z^2+a^2)^{3/2}}$

4.(2-9)



Ey cancelled out due to symmetry

$\Rightarrow \vec{F_y}=0 \Rightarrow \vec{E}=\vec{E_x}$

$r=\sqrt{a^2+(h-y)^2}$, $E_x=\dfrac{\lambda a}{4\pi\varepsilon_0}\displaystyle\int_0^L \frac{dh}{r^3} = \int_0^L \frac{dh}{[a^2+(h-y)^2]^{3/2}}$

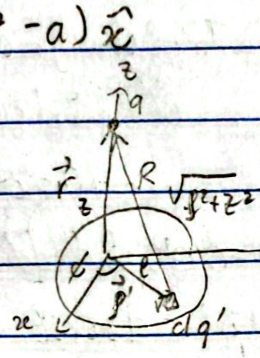
$\Rightarrow E_x = \dfrac{\lambda}{4\pi\varepsilon_0 a}\left[\dfrac{L-y}{\sqrt{a^2+(L-y)^2}}+\dfrac{y}{\sqrt{a^2+y^2}}\right] \Rightarrow dF = dF_x = dq\,E_x = \lambda dy\,E_x$

$\Rightarrow F_x = \int dF_x = \displaystyle\int_0^L \frac{\lambda^2}{4\pi\varepsilon_0 a}\left[\frac{L-y}{\sqrt{a^2+(L-y)^2}}+\frac{y}{\sqrt{a^2+y^2}}\right]dy$

$= \dfrac{\lambda^2}{4\pi\varepsilon_0 a}\displaystyle\int_0^L \frac{2y}{\sqrt{a^2+y^2}}dy = \frac{\lambda^2}{2\pi\varepsilon_0 a}\left(\sqrt{a^2+L^2}-a\right)$

$\Rightarrow \vec{F_x} = \dfrac{\lambda^2}{2\pi\varepsilon_0}\left(\sqrt{a^2+L^2}-a\right)\hat{x}$

5.(2-11).



$\sigma = A\rho^2$, σ is in Cm^{-2}, ρ is in m

$\Rightarrow Cm^{-2} = [A]\,m^2 \Rightarrow [A] = Cm^{-4}$ (unit)

$da = \rho d\rho d\phi$. $dQ = \sigma da = A\rho^2(\rho d\rho d\phi) = A\rho^3 d\rho d\phi$

$\Rightarrow Q = \int dQ = \displaystyle\int_0^a\int_0^{2\pi} dQ = \int_0^a\int_0^{2\pi} A\rho^3 d\rho d\phi = A\int_0^a \rho^3 d\rho \int_0^{2\pi} d\phi$

$\Rightarrow Q = A\dfrac{a^4}{4}2\pi = \dfrac{A\pi a^4}{2}$ (total charge)

$dF = kq\dfrac{dQ}{r^2} = \dfrac{kqA\rho^3 d\rho d\phi}{\rho^2+z^2}$, $dF_z = dF\cos\theta = \dfrac{kqA\rho^3 d\rho d\phi}{\rho^2+z^2}\dfrac{z}{\sqrt{\rho^2+z^2}} = \dfrac{\rho^3 kqAz\,d}{(\rho^2+z^2)^3}$

$\Rightarrow F = \displaystyle\int_0^{2\pi}\int_0^a dF_z = kqAz\int_0^a \frac{\rho^3 d\rho}{(\rho^2+z^2)^{3/2}}\int_0^{2\pi} d\phi$

$\Rightarrow F = 2\pi Azkq\left[\dfrac{(a^2+2z^2)}{(a^2+z^2)^{1/2}} - 2z\right]$

6(3-13). Don't Know how to use Coulomb's law so I will use Gauss' law here (I Know I shouldn't but it's 4 and I wanna sleep).

$\oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$, ρ_{ch} : volume density.

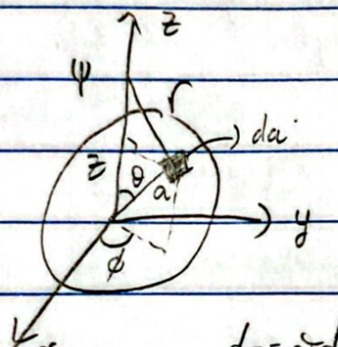
$r<a \Rightarrow Q_{in} = \rho \frac{4}{3}\pi r^3 \Rightarrow E_{in} 4\pi r^2 = \frac{\rho \frac{4}{3}\pi r^3}{\varepsilon_0}$

$\Rightarrow \vec{E}_{in} = \frac{\rho r}{3\varepsilon_0}\hat{r} \Rightarrow$ x-axis: $\vec{E}_{in} = \frac{\rho_{ch} x}{3\varepsilon_0}\hat{x}$

$r>a \Rightarrow E_{out} 4\pi r^2 = \frac{\rho \frac{4}{3}\pi a^3}{\varepsilon_0}$

$\Rightarrow \vec{E}_{out} = \frac{\rho a^3}{3\varepsilon_0 r^2}\hat{r} \Rightarrow$ x-axis: $\vec{E}_{out} = \frac{\rho_{ch} a^3}{2\varepsilon_0 x}\hat{x}$

7(2-8).



Total charge on sphere:

$Q' = \int \sigma da = \int \sigma a^2 \sin\theta \, d\theta \, d\phi$

$= 4\pi a^2 \sigma$

$dq = \sigma da = \sigma a^2 \sin\theta \, d\theta \, d\phi \quad \cos\phi = \frac{z - a\cos\theta}{r}$

$r^2 = a^2 + z^2 - az\cos\theta$

$$F_z = \frac{q}{4\pi\varepsilon_0} \int_0^{\pi} \frac{\sigma a^2 \sin\theta \, d\theta \, (z - a\cos\theta)}{(a^2+z^2-2az\cos\theta)^{3/2}} \int_0^{2\pi} d\phi$$

$$\mu = \cos\theta \Rightarrow F_z = \frac{q}{4\pi\varepsilon_0} 2\pi a^2 \sigma \int_{-1}^{1} \frac{(z - a\mu)}{(a^2+z^2-2az\mu)^{3/2}} d\mu$$

$$\Rightarrow F_z = \frac{qQ'}{8\pi\varepsilon_0}\left[\frac{1}{z^2}\frac{z\mu - a}{\sqrt{a^2+z^2-2a\mu z}}\right]_{-1}^{1} = \frac{qQ'}{8\pi\varepsilon_0 z^2}\left[\frac{z-a}{|z-a|} - \frac{z-a}{|z+a|}\right]$$

(*) $z>a$: $F_z = \frac{qQ'}{4\pi\varepsilon_0 z^2}$

(*) $z<a$: $F_z = 0$