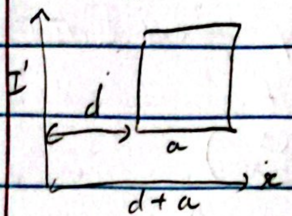
HOMEWORK 8

1. Wangsness 17-3.



$I' = I_0 e^{-\lambda t}$

$B = \frac{\mu_0 I'}{2\pi x} \Rightarrow \phi = \int d\phi = \int_d^{d+a} B\,dA = \int_d^{d+a} \frac{\mu_0 I'}{2\pi x} b\,dx$

$\Rightarrow \phi = \frac{\mu_0 I'}{2\pi} b[\ln(a+d) - \ln d] = \frac{\mu_0 I'}{2\pi} b \ln\left(\frac{a+d}{d}\right)$

$\Rightarrow \mathcal{E} = \frac{d\phi}{dt} = \frac{d\left(\frac{\mu_0 I'}{2\pi}\right) b \ln\left(\frac{a+d}{d}\right)}{dt} = \frac{\mu_0}{2\pi} b \ln\left(\frac{a+d}{d}\right) \frac{dI'}{dt}$

$\Rightarrow \mathcal{E} = -\frac{\mu_0}{2\pi} b \ln\left(\frac{a+d}{d}\right) I_0 e^{-\lambda t} \cdot \lambda$ ✓ Induced current direction: CW ✓

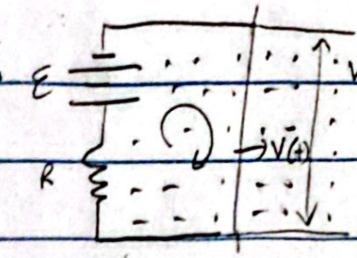
2. a) Current develop in loop 2 only in $-\hat{y}$ direction. Since $\vec{B} \perp d\vec{a} \Rightarrow$ if $k\uparrow$, there will be an induced current. ✓

Flux = 0 in loop 1 & 3 when $k\uparrow \Rightarrow$ no induced current. ✓

b) $P = \frac{\mathcal{E}^2}{R}$, $|\mathcal{E}| = \left|\frac{d\phi}{dt}\right|$, $\Phi = \frac{\mu_0 \alpha t}{2} LH \Rightarrow \mathcal{E} = \frac{\alpha LH}{2} \Rightarrow P = \frac{\mu_0^2 \alpha^2 L^2 H^2}{4R}$

$\Rightarrow NRG_T = \int P\,dt = \frac{\mu_0^2 \alpha^2 L^2 H^2 T}{4R}$

Grading Space for 1 & 2: All correct for 1. I did not calculate the potential, emf, power, and NRG for 2b but my 2a is correct.

3.  Induced emf: $\mathcal{E} = \frac{d\phi}{dt} = \frac{d}{dt}(Blx) = BlV$

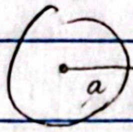
Effective current: $\frac{V}{R} = I = \frac{\mathcal{E} - BlV}{R}$

Force on the wire from B: $F = BIl = m\frac{dV}{dt} = \left(\frac{\mathcal{E} - BlV}{R}\right)Bl$

$$\Rightarrow \int_0^V \frac{dV}{\frac{\mathcal{E}Bl}{R} - \frac{B^2l^2V}{R}} = \int_0^t \frac{dt}{m} \Rightarrow -\frac{R}{B^2l^2} \ln\left|\frac{\frac{\mathcal{E}Bl}{R} - \frac{B^2l^2V}{R}}{\mathcal{E}Bl/R}\right| = \frac{t}{m}$$

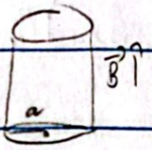
$$\Rightarrow 1 - \frac{BlV}{\mathcal{E}} = e^{-\frac{B^2l^2t}{Rm}} \Rightarrow V = \frac{\mathcal{E}}{Bl}\left(1 - e^{\frac{-B^2l^2t}{mR}}\right) ✓$$

4. $Q = \int I\,dt$, $I = \frac{\mathcal{E}}{R} \Rightarrow Q = \int \frac{1}{R}\frac{d\phi}{dt}\,dt = \frac{\phi_f - \phi_i}{R}$ (Wangsness 17-6). ✓

5.  $\vec{B} = B_0\hat{z}$

$$\vec{F_B} = q\vec{v}\times\vec{B} = q\rho\omega\hat{\phi}\,B_0\hat{z} = qa\omega\sin\theta B_0\hat{\rho} \Rightarrow \vec{F_E} = -\vec{F_B} = -a\omega B_0\sin\theta\hat{\rho}$$

$$\Rightarrow \phi_{equator} - \phi_{North} = -\int_0^{\pi/2} (-a\omega B_0\sin\theta\,\hat{\rho})\,a\,d\theta\,\hat{\theta} = \frac{1}{2}\omega B_0 a^2 ✓$$

6. 

$\vec{F_B} = q\vec{v}\times\vec{B} = q\rho\omega B\hat{\rho}$ $\quad\sigma_b = (K_e - 1)\varepsilon_0 a\omega B$ ✓

$\vec{P} = \chi_e\varepsilon_0\vec{E} = (K_e - 1)\varepsilon_0\rho\omega B\hat{\rho}$ ✓ $\quad\rho_b = -\vec{\nabla}\cdot\vec{P} = -(K_e - 1)\varepsilon_0\omega B(2)$ ✓

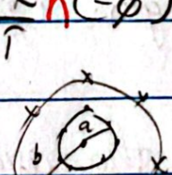
$Q = \sigma_0(2\pi a l) + \rho_b(\pi a^2 l) = 0$ ✓ (Wangsness 17-7).

7. a) $\oint \vec{E}\cdot d\vec{s} = -\frac{d\phi_E}{dt} \Rightarrow E\,2\pi\rho = \frac{-d}{dt}\left(\mu_0 n I_0 e^{-t/\tau}\pi\rho^2\right)$

$\Rightarrow \vec{E} = \frac{\mu_0 n I_0 \rho e^{-t/\tau}}{2\tau}\hat{\phi}$ ✓

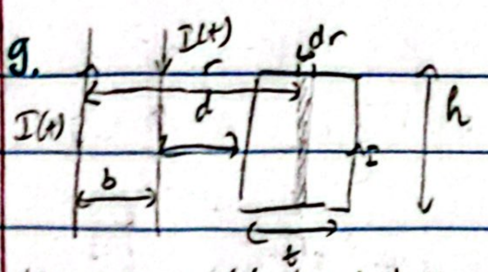
b) $E(2\pi\rho) = -\frac{d}{dt}\left(\mu_0 I_0 e^{-t/\tau} n\pi R^2\right) \Rightarrow \vec{E} = \frac{\mu_0 n I_0 R^2 e^{-t/\tau}}{2\rho\tau}\hat{\phi}$

$\Rightarrow \vec{a_e} = \frac{e\vec{E}}{m} = \frac{e\mu_0 I_0 R^2}{2m\rho\tau}n(-\hat{\phi})$ ✓

8. (Wangsness 17-24).  $\oint \vec{B}\cdot d\vec{s} = \mu_0 I_{encl} \Rightarrow \vec{B} = \begin{cases} 0 \text{ for } \rho < a ✓ \\ \frac{\mu_0 I}{2\pi\rho}\hat{\phi} \text{ for } a < \rho < b ✓ \\ 0 \text{ for } \rho > b ✓ \end{cases}$

Grading Space for 3-8: All correct. Just some minor mistakes for 7b (I forgot to write n at the end) and the $\hat{\phi}$ for $\vec{B}$ in 8.

9.



Let current goes up in left & down in right wire. $dB = \frac{\mu_0 I}{2\pi r}$

Magnetic field due to left wire: $B_L = \frac{\mu_0 I}{2\pi r}$

" right wire: $B_R = \frac{\mu_0 I}{2\pi(r-b)}$

$\Rightarrow B_{net} = B_R - B_L$ (opposite direction) $= \frac{\mu_0 I}{2\pi}\left(\frac{1}{r-b} - \frac{1}{r}\right)$

Magnetic flux through the strip: $d\phi = B_{net} \times dA = B_{net}\, h\, dr$

$$\Rightarrow \phi = \int d\phi = \frac{\mu_0 I h}{2\pi}\int_{b+d}^{b+d+t}\left(\frac{1}{r-b} - \frac{1}{r}\right) dr = \frac{\mu_0 I h}{2\pi}\left[\ln\left(\frac{d+t}{b+d+t}\right) - \ln\left(\frac{d}{b+d}\right)\right]$$

$$= \frac{h\mu_0 I_0 \sin\omega t}{2\pi} \ln\frac{(d+t)(b+d)}{d(b+d+t)} \quad (I = I_0 \sin\omega t)$$

$$\Rightarrow \mathcal{E} = \frac{d\phi}{dt} = \frac{d}{dt}\left[\frac{h\mu_0 I_0 \sin\omega t}{2\pi} \ln\frac{(d+t)(b+d)}{d(b+d+t)}\right], \quad \mathcal{E}_{max} \text{ when } \omega t = \pi/2$$

$$\Rightarrow \mathcal{E}_{max} = \frac{\mu_0 I_0 h \omega}{2\pi} \ln\frac{(d+t)(d+b)}{d(b+d+t)}$$ ✓

Grading Space for 9.: All correct. The way I did it is a bit different than the answer though since I went from $\vec{B} \to \phi \to \mathcal{E}$, the answer calculate through M instead.

Overale: I think this is the easiest homework up to date and I got most of the homework correct but some small silly mistakes.