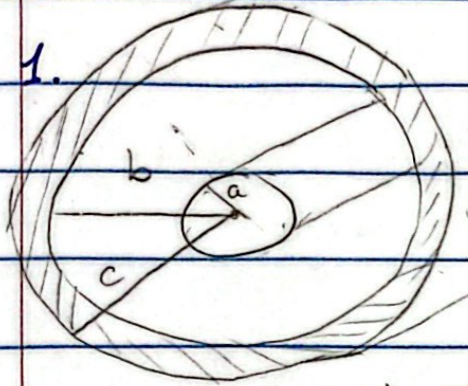
HOMEWORK 3

1.



$$U_e = \frac{1}{2}\int_S \sigma(r)\phi(r)\,da$$

$$C = \frac{2\pi\varepsilon_0 L}{\ln\left(\frac{b}{a}\right)} \quad \text{linear } \lambda \text{ \& } -\lambda = \frac{Q}{l} \Rightarrow \lambda = Ql$$

a) $\Delta\phi = -\int \vec{E}\cdot d\vec{s}$ $\qquad E_{\rho>c} = 0 \qquad E_{\rho<a} = 0$

$$E_{c>\rho>b} = 0 \qquad E_{b>\rho>a} = \frac{\lambda}{2\pi\varepsilon_0\rho}\hat{\rho}$$

$$\Rightarrow \phi_b - \phi_a = \int_a^b \frac{\lambda}{2\pi\varepsilon_0\rho}\hat{\rho}\,d\rho = \frac{\lambda}{2\pi\varepsilon_0}[\ln(b)-\ln(a)]$$

$$\Rightarrow U_e = \frac{1}{2}\int_0^l\int_0^{2\pi} \cancel{\frac{\lambda}{2\pi b}\, 0\, b\,dr\,d\theta}^{\,0} + \frac{1}{2}\int_0^l\int_0^{2\pi} \frac{\lambda}{2\pi a}\frac{\lambda}{2\pi\varepsilon_0}[\ln(b)-\ln(a)]\,a\,dr\,d\theta$$

$$\Rightarrow U_e = \frac{1}{2}\frac{\lambda^2}{4\pi^2\varepsilon_0}\ln\left(\frac{b}{a}\right)\cdot 2\pi\cdot l = \frac{\lambda^2 l}{4\pi\varepsilon_0}\ln\left(\frac{b}{a}\right)$$

$$\Rightarrow C = \frac{\cancel{\lambda^2}\, 4\pi\varepsilon_0 l^{\cancel{2}}}{2\cancel{\lambda^2}\ln\left(\frac{b}{a}\right)\cancel{l}} = \boxed{\frac{2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}} \checkmark$$

Grading space for 1a:

b) $U_e = \int_{\text{allspace}} \frac{\varepsilon_0}{2} E^2 \, d\tau$

$= \int_c^\infty \int_0^{2\pi} \int_0^\ell 0 \, dz\, d\rho\, d\theta + \int_b^c \int_0^{2\pi} \int_0^\ell 0 \, d\rho\, dz\, d\theta + \int_a^b \int_0^{2\pi} \int_0^\ell \frac{\varepsilon_0}{2} \cdot \frac{\lambda^2}{4\pi\varepsilon_0^2 \rho^2} \hat{\rho} z \, d\rho\, dz\, d\theta + \int_a^0 \int_0^{2\pi} \int_0^\ell 0 \, d\rho\, dz\, d\theta$

$= \frac{\lambda^2}{8\pi^2\varepsilon_0} 2\pi \ell \ln\left(\frac{b}{a}\right) = \frac{\lambda^2 \ell}{4\pi\varepsilon_0} \ln\left(\frac{b}{a}\right)$

2. $U_e = \frac{\varepsilon_0}{2} \int_V E^2 \, d\tau + \frac{\varepsilon_0}{2} \oint_S (\phi E) \cdot da$

$E_{r<a} = \frac{\rho}{3\varepsilon_0} r \quad E_{r>a} = \frac{\rho a^3}{3\varepsilon_0} \frac{1}{r^2} \quad \rho = \frac{3Q}{4\pi a^3}$

$\phi_{r>a} = -\int_\infty^r \vec{E} \cdot d\vec{r} = -\frac{\rho a^3}{3\varepsilon_0} \int_\infty^r \frac{1}{r^2} dr = \frac{\rho a^3}{3\varepsilon_0 r}$

$\phi_{r<a} = -\int_\infty^a E_{r>a} \, dr - \int_a^r E_{r<a} \, dr$

$= \frac{\rho a^3}{3\varepsilon_0 a} - \int_a^r \frac{\rho}{3\varepsilon_0} r \, dr = \frac{\rho a^2}{3\varepsilon_0} - \frac{\rho}{6\varepsilon_0}(r^2 - a^2)$

$= \frac{\rho}{6\varepsilon_0}(3a^2 - r^2)$

$\Rightarrow U_e = \frac{\varepsilon_0}{2} \int_V E^2 \, d\tau + \frac{\varepsilon_0}{2} \oint_S (\phi E) \cdot da$

$= \frac{\varepsilon_0}{2} \int_0^a E_{r<a}^2 \, dr + \frac{\varepsilon_0}{2} \int_a^b E_{r>a}^2 \, dr + \frac{\varepsilon_0}{2} \oint_{r=b} \phi_{r>a} E_{r>a} \, da$

$= \frac{\varepsilon_0}{2} \int_0^a \frac{\rho^2}{9\varepsilon_0^2} r^2 \, 4\pi r^2 dr + \frac{\varepsilon_0}{2} \int_a^b \frac{\rho^2 a^6}{9\varepsilon_0^2 r^4} 4\pi r^2 dr + \frac{\varepsilon_0}{2} \left(\frac{\rho^2 a^6}{9\varepsilon_0^2 r^3} 4\pi r^2 \right)_{r=b}$

$= \frac{2\pi\rho^2}{9\varepsilon_0} \int_0^a r^4 dr + \frac{2\pi\rho^2 a^6}{9\varepsilon_0} \int_a^b \frac{1}{r^2} dr + \frac{2\pi\rho^2 a^6}{9\varepsilon_0 b}$

$= \frac{2\pi\rho^2 a^5}{45\varepsilon_0} + \frac{2\pi\rho^2 a^{\cancel{6}\,5}}{\cancel{9}\varepsilon_0(a - \cancel{b})} + \cancel{\frac{2\pi\rho^2 a^6}{9\varepsilon_0 b}}$

$= \frac{2\pi\rho^2 a^5}{9\varepsilon_0} \left(\frac{1}{5} + 1 \right) = \frac{4\pi a^{\cancel{5}}}{\cancel{15}\,5\,\varepsilon_0} \frac{\cancel{9}\,3\, Q^2}{\cancel{16}\pi^2 a^{\cancel{6}}}$

$= \frac{3}{20} \frac{Q^2}{4\pi\varepsilon_0 a}$ $\Rightarrow$ when $b \to \infty$, the answer doesn't change since it's not b dependent. Also because there's no charge outside of $0<r<a$ so that answer makes sense.

Grading space for 1b & 2:

3 (Wdgsness 7-16).

$$U_i = \frac{1}{2} C_0 V_0^2 = \frac{1}{2} \frac{\varepsilon_0 A}{d} V_0^2$$

$$U_f = \frac{1}{2} C_0' V_0^2 = \frac{1}{2} \frac{\varepsilon_0 A}{\alpha d} V_0^2$$

$$\Rightarrow \frac{U_f}{U_i} = \frac{1}{\alpha} \Rightarrow \text{Energy decreased.}$$

The new energy might come from the field generate from the plates.

$$\Rightarrow \Delta U = U_f - U_i = \frac{\varepsilon_0 A V_0^2}{2d}\left(\frac{1}{\alpha} - 1\right)$$

4. $E \cdot 2\pi\rho l = \frac{Q}{\varepsilon_0} \Rightarrow E = \frac{Q}{2\pi\varepsilon_0 \rho l}$

$$\Rightarrow \Delta\phi = -\int_b^a \frac{Q}{2\pi\varepsilon_0 \rho}\, d\rho = \frac{Q}{2\pi\varepsilon_0 l} \ln\left(\frac{b}{a}\right) \Rightarrow Q = \frac{\Delta\phi\, 2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)}$$

$$\Rightarrow F = QE = \frac{\Delta\phi\, 2\pi\varepsilon_0 l}{\ln\left(\frac{b}{a}\right)} \cdot \frac{Q}{2\pi\varepsilon_0 \rho l}, \text{ inner} \Rightarrow \rho = a$$

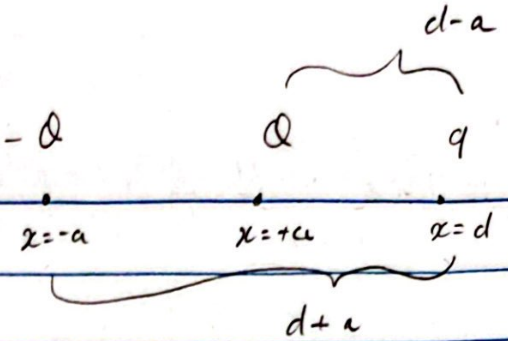
$$= \frac{\Delta\phi\, Q}{\ln\left(\frac{b}{a}\right) a} = \frac{\Delta\phi^2\, 2\pi\varepsilon_0 l}{\left[\ln\left(\frac{b}{a}\right)\right]^2 a}$$

$$\Rightarrow F/A = \frac{\Delta\phi^2\, 2\pi\varepsilon_0 l}{\left[\ln\left(\frac{b}{a}\right)\right]^2 a} \cdot \frac{1}{2\pi a l} = \frac{\Delta\phi^2 \varepsilon_0}{\left[\ln\left(\frac{b}{a}\right)\right]^2 a^2} \text{ (outwards)}$$

$\Rightarrow$ Net force will be 0 / when the outter force cancels out the inner force as the outter force has same magnitude but opposite direction.

Grading space for 3 & 4:

6 a)



$$\vec{F_q} = \vec{F_{qQ}} + \vec{F_{q\text{-}Q}} = \frac{Qq}{4\pi\varepsilon_0 (d-a)^2}\hat{x} + \frac{-Qq}{4\pi\varepsilon_0 (d+a)^2}\hat{x}$$

$$= \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{(d-a)^2} - \frac{1}{(d+a)^2}\right]\hat{x}$$

b)
$$\vec{F_i} = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{24^2} - \frac{1}{26^2}\right)\hat{x}$$
$$\vec{F_f} = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{49^2} - \frac{1}{51^2}\right)\hat{x}$$
$$\Rightarrow \frac{F_i}{F_f} = \frac{\frac{1}{24^2} - \frac{1}{26^2}}{\frac{1}{49^2} - \frac{1}{51^2}} \approx 8.02$$

c)
$$\vec{F_q} = \frac{Qq}{4\pi\varepsilon_0}\left[\frac{1}{(d-a)^2} - \frac{1}{(d+a)^2}\right]\hat{x} = \frac{Qq}{4\pi\varepsilon_0 d^2}\left[\frac{1}{(1-\frac{a}{d})^2} - \frac{1}{(1+\frac{a}{d})^2}\right]\hat{x}$$

$$\Rightarrow F_q = \frac{Qq}{4\pi\varepsilon_0 d^2}\left[\left(1-\frac{a}{d}\right)^{-2} - \left(1+\frac{a}{d}\right)^{-2}\right]$$

For $(1+x)^{-2} \Rightarrow$ Expansion $= 1 - 2x + 3x^2 + 4x^3 \ldots$

$$\Rightarrow \left(1-\frac{a}{d}\right)^{-2} = 1 + \frac{2a}{d} + \frac{3a^2}{d^2} + \frac{4a^3}{d^3}$$
$$\Rightarrow \left(1+\frac{a}{d}\right)^{-2} = 1 - \frac{2a}{d} + \frac{3a^2}{d^2} - \frac{4a^3}{d^3}$$
$$\Rightarrow \left(1-\frac{a}{d}\right)^{-2} - \left(1+\frac{a}{d}\right)^{-2} = 1 - 1 + \frac{2a}{d} + \frac{2a}{d} + \frac{3a^2}{d^2} - \frac{3a^2}{d^2} + \frac{4a^3}{d^3} + \frac{4a^3}{d^3} = \frac{4a}{d} + \frac{8a^3}{d^3}$$

$$\Rightarrow F = \frac{Qq}{4\pi\varepsilon_0 d^2}\cdot\left(\frac{4a}{d} + \frac{8a^3}{d^3}\right) = \frac{Qq \cdot 4a}{4\pi\varepsilon_0 d^3}\left(1 + \frac{2a^2}{d^2}\right)$$
$$= \frac{Qqa}{\pi\varepsilon_0 d^3}\left(1 + \frac{2a^2}{d^2}\right)$$

Grading space for 6:

7. Since only that charge q is there $\Rightarrow Q = q$

$\vec{p} = q \cdot \vec{r} = q(a, b, c)$

$$Q_{JK} = \begin{bmatrix} Q_{xx} & Q_{xy} & Q_{xz} \\ Q_{yx} & Q_{yy} & Q_{yz} \\ Q_{zx} & Q_{zy} & Q_{zz} \end{bmatrix} = q\begin{bmatrix} a^4 & a^3b & a^3c \\ a^3b & b^4 & b^3c \\ a^3c & b^3c & c^4 \end{bmatrix}$$

If added $-q \Rightarrow Q = q - q = 0$

$\vec{p} = q\vec{r} - (-q)(0,0,0) = q\vec{r}$ (unchanged)

Q_{JK} will also remain the same since the charge located at the origin won't affect the components same with $\vec{p}$.

5. The student is wrong since when all the charge were at the surface, the total energy would be at its lowest. This is because it takes a large amount of energy to bring the charges on the sphere's surface from outside. Moving the charges inside would take even more energy, so when the charges are inside, the energy would not be the lowest.

8. (Wangsness 8-8) • $Q = \iint \sigma\, da$, $da = a^2 \sin\theta\, d\theta\, d\phi$

$\Rightarrow Q = \int_0^{\pi}\int_0^{2\pi} a^2 \sigma_0 \cos\theta\, d\theta\, d\phi = 2\pi\sigma_0 a^2 \int_0^{\pi} \frac{\sin^2\theta}{2} d\theta = 0$

• $\vec{p} = \int \vec{r}\,\sigma\, d\vec{a}$, $\vec{r} = a\sin\theta\cos\phi\,\hat{i} + a\sin\theta\sin\phi\,\hat{j} + a\cos\theta\,\hat{k}$

$\Rightarrow \vec{p} = a^3\sigma \int_0^{\pi}\int_0^{2\pi} [(\sin^2\theta\cos\theta\cancel{\cos\phi}^{\,0})\,\hat{i} + (\sin^2\theta\cos\theta\cancel{\sin\phi}^{\,0})\,\hat{j} + (\cos^2\theta\sin\theta)\,\hat{k}]\, d\theta\, d\phi$

$\Rightarrow \vec{p} = a^3\sigma\, 2\pi \int_0^{\pi} \cos^2\theta\sin\theta\, d\theta = \frac{4}{3}\pi a^3 \sigma\, \hat{k}$

• Antisymmetric $\Rightarrow Q_{JK} = 0\ (J \neq K)$ and $Q_{zz} = 0 \Rightarrow Q_{xx} = Q_{yy}$.

$\rho(\vec{r}\,') = 2\pi\sin\theta\,\sigma$ for circle circumference $\Rightarrow Q_{xx} = \int \rho(\vec{r}\,')(3x'^2 - r'^2)\, d\tau'$

$\Rightarrow Q_{xx} = \int_{\theta'} \rho(\theta')(3x'^2 - r'^2)\, d\theta' = 2\pi a^4 \sigma_0 \left(\int_0^{\pi} 3\sin^3\theta' \cancel{\cos\theta'}^{\,0} d\theta' - \int_0^{\pi} \sin\theta' \cancel{\cos\theta'}^{\,0} d\theta'\right)$

$\Rightarrow Q_{xx} = Q_{yy} = 0 \Rightarrow Q_{JK} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Grading space for 7, 5, & 8 (Q, $\vec{p}$, Q_{JK}):

8(cont). $\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\cancel{\frac{Q}{r}}^{\,0} + \frac{\vec{p}\cdot\vec{r}}{r^2} + \frac{1}{2r^3}\cancel{\sum_{JK} \ell_J \ell_K Q_{JK}}^{\,0}\right)$

$\to \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}\cdot\hat{r}$

$= \frac{1}{4\pi\varepsilon_0}\left(\frac{4\pi a^3 \vartheta_0}{3}\boxed{\hat{z}\cdot\hat{r}}\cdot\frac{1}{r^2}\right)$

$= \frac{4\pi a^3 \vartheta_0 \cos\theta}{4\pi\varepsilon_0 r^2\cdot 3} = \frac{a^3 \vartheta_0 \cos\theta}{3\varepsilon_0 r^2}$

9. $\cdot\, Q = \int_{V'} \rho\, dV' = \rho\int_0^a dx' \int_0^b dy' \int_0^c dz' = \rho abc.$

$\cdot\, \vec{p} = \int_{V'} \rho(\vec{r}\,')\vec{r}\,' d\tau' = \rho\int_0^a\int_0^b\int_0^c (x\hat{x} + y\hat{y} + z\hat{z})\, dz'\, dy'\, dx'$

$= \rho\left[\hat{x}\frac{a^2}{2}bc + \hat{y}\,\frac{ab^2c}{2} + \hat{z}\,\frac{abc^2}{2}\right] = \frac{\rho abc}{2}(a\hat{x} + b\hat{y} + c\hat{z})$

$\cdot\, Q_{JK} = \int_{V'} \rho(\vec{r}\,')(3x_J x_K - r'^2 \delta_{JK})\, d\tau'$

$\Rightarrow Q_{xx} = \rho\int_0^a\int_0^b\int_0^c (2x^2 - y^2 - z^2)\, dx\, dy\, dz = \frac{\rho abc}{3}(2a^2 - b^2 - c^2)$

$\Rightarrow Q_{yy} = \frac{Q}{3}(2b^2 - a^2 - c^2), \quad Q_{zz} = \frac{Q}{3}(2c^2 - a^2 - b^2)$

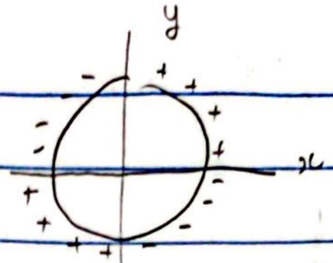
$\Rightarrow Q_{xy} = 3\rho\int_0^a x\,dx \int_0^b y\,dy \int_0^c dz = 3\rho\frac{a^2}{2}\frac{b^2}{2}c = \frac{3}{4}\rho a^2 b^2 c = Q_{yx}$

$\Rightarrow Q_{yz} = \frac{3}{4}\rho a b^2 c^2 = Q_{zy}, \quad Q_{xz} = \frac{3}{4}\rho a^2 b c^2 = Q_{zx}.$

$\Rightarrow Q_{JK} = \begin{bmatrix} \frac{Q}{3}(2a^2 - b^2 - c^2) & \frac{3}{4}\rho a^2 b^2 c & \frac{3}{4}\rho a^2 b c^2 \\ \frac{3}{4}\rho a^2 b^2 c & \frac{Q}{3}(2b^2 - a^2 - c^2) & \frac{3}{4}\rho a b^2 c^2 \\ \frac{3}{4}\rho a^2 b c^2 & \frac{3}{4}\rho a b^2 c^2 & \frac{Q}{3}(2c^2 - a^2 - b^2) \end{bmatrix}$

$= \begin{bmatrix} \frac{Q}{3}(2a^2 - b^2 - c^2) & \frac{3}{4}Qab & \frac{3}{4}Qac \\ \frac{3}{4}Qba & \frac{Q}{3}(2b^2 - a^2 - c^2) & \frac{3}{4}Qbc \\ \frac{3}{4}Qca & \frac{3}{4}Qcb & \frac{Q}{3}(2c^2 - a^2 - b^2) \end{bmatrix}$

Grading space for 8(cont) & 9.

10. 

$\Rightarrow$ quadripole would be the first non-vanishing moment

11. • $\phi(\vec{r}) = \dfrac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3} = \dfrac{p_x x + p_y y + p_z z}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}}$

$$\vec{E} = -\nabla\phi = -\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)\frac{p_x x + p_y y + p_z z}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}}$$

$$\Rightarrow E_x = \frac{p_x}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2}} - \frac{3x(p_x x + p_y y + p_z z)}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{5/2}}$$

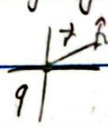
Call $4\pi\varepsilon_0 (x^2+y^2+z^2)^{3/2} = C_1$ & $\dfrac{(p_x x + p_y y + p_z z)}{4\pi\varepsilon_0 (x^2+y^2+z^2)^{5/2}} = C_2$

$\Rightarrow E_y = \dfrac{p_y}{C_1} - 3yC_2$ & $E_z = \dfrac{p_z}{C_1} - 3zC_2$

• $\vec{E} = -\nabla\phi = -\vec{\nabla}\dfrac{\vec{p}\cdot\vec{r}}{4\pi\varepsilon_0 r^3} = -\dfrac{1}{4\pi\varepsilon_0 r^3}\nabla(\vec{p}\cdot\vec{r}) - (\vec{p}\cdot\vec{r})\vec{\nabla}\dfrac{1}{4\pi\varepsilon_0 r^3}$

$$\Rightarrow \vec{E} = -\frac{\vec{p}}{4\pi\varepsilon_0 r^3} + \frac{+3\hat{r}(\vec{p}\cdot\hat{r})}{4\pi\varepsilon_0 r^3} = \boxed{\frac{1}{4\pi\varepsilon_0 r^3}[3(\vec{p}\cdot\hat{r})\hat{r} - p]} \checkmark$$

$\vec{E}$ can't be 0 as the dipole is a point. If $\vec{p}$ @ $\vec{r}' \Rightarrow \vec{E}(\vec{r})$ can be found by changing $\vec{r} \to \vec{r} - \vec{r}'$

12.  $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$ $\Rightarrow \vec{\tau} = \vec{p}\times\vec{E} = \dfrac{pq}{4\pi\varepsilon_0 r^2}\sin\theta$

$\Rightarrow U = -\vec{p}\cdot\vec{E} = \dfrac{-pq}{4\pi\varepsilon_0 r^2}\cos\theta$

$$\vec{F} = \vec{\nabla}(\vec{p}\cdot\vec{E}) = \vec{\nabla}\vec{p}\,\hat{r}\frac{q}{4\pi\varepsilon_0 r^2} = q\frac{-1}{4\pi\varepsilon_0 r^3}[3(\vec{p}\cdot\hat{r})\hat{r} - \vec{p}] = \frac{q}{4\pi\varepsilon_0 r^3}[\vec{p} - 3(\vec{p}\cdot\hat{r})\hat{r}]$$

$\Rightarrow$ Similar to equation 8-84, except for q & -1 factor. This is b/c the charge travel opposite to the field lines.

Grading space for 10, 11, 12:

13. $\cdot\, U = -\vec{p_2}\,\vec{E_1} = -\vec{p_2}\left[\frac{1}{4\pi\varepsilon_0}\left(\frac{3(\vec{p_1}\,\vec{r})\,\vec{r}}{|r|^3} - \frac{\vec{p_1}}{|r|^3}\right)\right]$

$$\Rightarrow U = \frac{-1}{4\pi\varepsilon_0}\left[\frac{3(\vec{p_1}\,\vec{r})\,\vec{p_2}\,\vec{r}}{|r|^5} - \frac{\vec{p_1}\,\vec{p_2}}{|r|^3}\right]$$

$$= \frac{-1}{4\pi\varepsilon_0}\left[\frac{3(\vec{p_1}\,\hat{r})(\vec{p_2}\,\hat{r})\,|r|^2}{|r|^5} - \frac{\vec{p_1}\,\vec{p_2}}{|r|^3}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{\vec{p_1}\,\vec{p_2}}{|r|^3} - \frac{3(\vec{p_1}\,\hat{r})(p_2\,\hat{r})}{|r|^3}\right] \quad \vec{r} = \vec{R} = \vec{r_2} - \vec{r_1}$$

$$\Rightarrow U = \frac{1}{4\pi\varepsilon_0 R^3}\left[\vec{p_1}\cdot\vec{p_2} - 3(\vec{p_1}\,\hat{R})(\vec{p_2}\,\hat{R})\right]$$

U will be the same even from $2\to 1$ b/c: $[\vec{p_1}(\vec{r_1}-\vec{r_2})][\vec{p_2}(\vec{r_1}-\vec{r_2})]$

$= [\vec{p_1}\cdot(\vec{r_2}-\vec{r_1})][\vec{p_2}\cdot(\vec{r_2}-\vec{r_1})]$

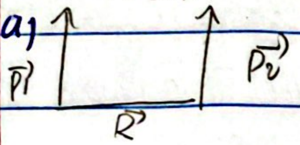
$\cdot\, \vec{F_2} = -\vec{\nabla}U_{DD} = -\vec{\nabla}\frac{1}{4\pi\varepsilon_0 R^3}\left[\vec{p_1}\cdot\vec{p_2} - 3(\vec{p_1}\cdot\vec{R})(\vec{p_2}\cdot\vec{R})\right]$

$$= -\vec{\nabla}\frac{\vec{p_1}\cdot\vec{p_2}}{4\pi\varepsilon_0 R^3} + \vec{\nabla}\frac{3(\vec{p_1}\vec{R})(\vec{p_2}\vec{R})}{4\pi\varepsilon_0 R^3} = \frac{3\vec{p_1}\vec{p_2}\,\hat{R}}{4\pi\varepsilon_0 R^4} + 3(\vec{p_1}\cdot\vec{R})(\vec{p_2}\cdot\vec{R})\,\vec{\nabla}\frac{1}{4\pi\varepsilon_0 R^3} + \frac{1}{4\pi\varepsilon_0 R^5}$$

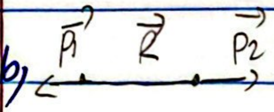
$$\vec{\nabla}3(\vec{p_1}\cdot\vec{R})(\vec{p_2}\cdot\vec{R}) = \frac{3\vec{p_1}\cdot\vec{p_2}\,\hat{R}}{4\pi\varepsilon_0 R^4} - \frac{15(\vec{p_1}\cdot\vec{R})(\vec{p_2}\cdot\vec{R})\hat{R}}{4\pi\varepsilon_0 R^6} + \frac{3}{4\pi\varepsilon_0 R^5}\left[(\vec{p_2}\cdot\vec{R})\vec{\nabla}(\vec{p_1}\cdot\vec{R}) + (\vec{p_1}\cdot\vec{R})\vec{\nabla}(\vec{p_2}\cdot\vec{R})\right]$$

$$\Rightarrow \vec{F_2} = \frac{3\vec{p_1}\cdot\vec{p_2}}{4\pi\varepsilon_0 R^4}\hat{R} - \frac{15(\vec{p_1}\cdot\hat{R})(\vec{p_2}\cdot\hat{R})\hat{R}}{4\pi\varepsilon_0 R^4} + \frac{3}{4\pi\varepsilon_0 R^5}\left[(\vec{p_2}\cdot\vec{R})\vec{p_1} + (\vec{p_1}\cdot\vec{R})\vec{p_2}\right]$$

$$= \frac{3}{4\pi\varepsilon_0 R^4}\left[(\vec{p_1}\cdot\vec{p_2})\hat{R} - 5(\vec{p_1}\cdot\hat{R})(\vec{p_2}\,\hat{R})\hat{R} + (\vec{p_2}\,\hat{R})\vec{p_1} + (\vec{p_1}\cdot\hat{R})\vec{p_2}\right]$$

a) 

$$\vec{F_2} = \frac{3}{4\pi\varepsilon_0 R^4}\left(p_1 p_2 \hat{R} - 0\hat{R} + 0\vec{p_1} + 0\,\vec{p_2}\right) = \frac{-3p_1 p_2}{4\pi\varepsilon_0 R^4}\hat{R}$$

b) 

$$\vec{F_2} = \frac{3}{4\pi\varepsilon_0 R^4}\left(p_1 p_2 \hat{R} - 5p_1 p_2 \hat{R} + \vec{p_2}\vec{p_1} + \vec{p_1}\vec{p_2}\right)$$

$$\Rightarrow \vec{F_2} = \frac{3}{4\pi\varepsilon_0 R^4}\left(-4p_1 p_2 \hat{R} + p_2 p_1 \hat{R} + p_1 p_2 \hat{R}\right) = \frac{-3p_1 p_2}{2\pi\varepsilon_0 R^4}\hat{R}$$