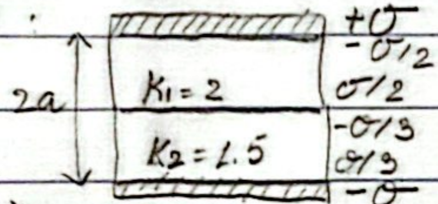
Homework 8

1.(4.18). Head on pov:



a) $\oint D \cdot da = Q_{enc} \Rightarrow DA = \sigma A$

$\Rightarrow D = \sigma$

$\Rightarrow \vec{D_1} = \sigma(-\hat{z}),\ \vec{D_2} = -\sigma\hat{z} = \sigma(-\hat{z})$

b) $\vec{E_1} = \dfrac{\vec{D_1}}{K_1\varepsilon_0} = \dfrac{\sigma}{2\varepsilon_0}(-\hat{z})$ $\boxed{\varepsilon = K\varepsilon_0}$

$\vec{E_2} = \dfrac{\vec{D_2}}{K_2\varepsilon_0} = \dfrac{\sigma}{1.5\varepsilon_0} = \dfrac{2\sigma}{3\varepsilon_0}(-\hat{z})$

c) $\vec{P_1} = \varepsilon_0(K_1 - 1)\vec{E_1} = \varepsilon_0(2-1)\dfrac{\sigma}{2\varepsilon_0}\hat{z} = \dfrac{\sigma}{2}\hat{z}$

$\vec{P_2} = \varepsilon_0(K_2 - 1)\vec{E_2} = \varepsilon_0(1.5-1)\dfrac{2\sigma}{3\varepsilon_0}(-\hat{z}) = \dfrac{\sigma}{3}(-\hat{z})$

d) $\Delta V = E_1 a + E_2 a = \dfrac{\sigma}{2\varepsilon_0}a + \dfrac{2\sigma}{3\varepsilon_0}a = \dfrac{7\sigma a}{6\varepsilon_0}$

e) $\sigma_{b1} = \begin{cases} P_1 \cdot \hat{n} = -\dfrac{\sigma}{2} \text{ (top)} \\ P_1 \cdot \hat{n} = \dfrac{\sigma}{2} \text{ (bottom)} \end{cases}$

$\sigma_{b2} = \begin{cases} P_2 \cdot \hat{n} = -\dfrac{\sigma}{3} \text{ (top)} \\ P_2 \cdot \hat{n} = \dfrac{\sigma}{3} \text{ (bottom)} \end{cases}$

(neg bound charge attract to pos free charge & vice versa).

f) $\vec{E_1} = \dfrac{\sigma}{\varepsilon_0} - \dfrac{\sigma}{2\varepsilon_0} = \dfrac{\sigma}{2\varepsilon_0}$

$\vec{E_2} = \dfrac{\sigma}{\varepsilon_0} - \dfrac{\sigma}{3\varepsilon_0} = \dfrac{2\sigma}{3\varepsilon_0}$

} similar to b

2. (4.20). To get ϕ, we have to get E, to get E, we need D

• $r<R$ (inside)

$$\oint \vec{D}\cdot d\vec{a} = Q_{enc} = \frac{4}{3}\pi r^3\rho = D\cdot 4\pi r^2 \Rightarrow \vec{D}_{in} = \frac{r}{3}\rho\,\hat{r}$$

$$\Rightarrow \vec{E}_{in} = \frac{\vec{D}_{in}}{\epsilon_r\epsilon_0} = \frac{\rho r}{3\epsilon_r\epsilon_0}\hat{r}$$

• $r>R$ (outside)

$$\oint \vec{D}\cdot d\vec{a} = Q_{enc} = \frac{4}{3}\pi R^3\rho \Rightarrow \vec{D}_{out} = \frac{\rho}{3}\frac{R^3}{r^2}\hat{r}$$

$$\Rightarrow \vec{E}_{out} = \frac{\vec{D}_{out}}{\epsilon_0} = \frac{\rho R^3}{3r^2\epsilon_0}$$

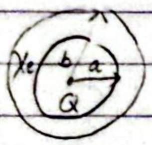
$$\Rightarrow \phi = -\int_\infty^R \frac{\rho R^3}{3r^2\epsilon_0}dr - \int_R^0 \frac{\rho r}{3\epsilon_r\epsilon_0}dr$$

$$= \frac{-\rho}{3\epsilon_0}\left(R^3\left[\frac{-1}{r}\right]_\infty^R + \frac{1}{2\epsilon_r}[r^2]_R^0\right)$$

$$= \frac{-\rho}{3\epsilon_0}\left(-R^2 - \frac{1}{2\epsilon_r}R^2\right) = +\frac{\rho R^2}{3\epsilon_0}\left(1+\frac{1}{2\epsilon_r}\right)$$

3. (4.26).

$$\vec{E} = \frac{\vec{D}}{\epsilon}$$



• $r<a \Rightarrow \vec{E}=0,\ \vec{D}=0$

• $a<r<b \Rightarrow E\cdot 4\pi r^2 = \frac{Q_{encl}}{\epsilon} \Rightarrow \vec{E} = \frac{Q}{4\pi r^2\epsilon}\hat{r} \Rightarrow \vec{D} = \frac{Q}{4\pi r^2}\hat{r}$

• $r>b \Rightarrow E = \frac{Q}{4\pi r^2\epsilon_0}\hat{r} \Rightarrow \vec{D} = \frac{Q}{4\pi r^2}\hat{r}$

Eqn 4.58: $W = \frac{1}{2}\int \vec{D}\cdot\vec{E}\,d\tau = \frac{1}{2}\int_0^{2\pi}\int_0^{\pi}\int_0^\infty |\vec{D}||\vec{E}|\sin\theta\, r^2 dr\, d\theta\, d\phi = 2\pi\int_0^\infty DE r^2 dr$

$$\Rightarrow W = 2\pi\left[\int_0^a 0\, r^2 dr + \int_a^b \frac{Q}{4\pi r^2\epsilon}\frac{Q}{4\pi r^2}r^2 dr + \int_b^\infty \frac{Q}{4\pi r^2\epsilon_0}\frac{Q}{4\pi r^2}r^2 dr\right]$$

$$= \frac{2\pi}{16\pi^2}\left[\int_a^b \frac{Q^2}{r^2\epsilon}dr + \int_b^\infty \frac{Q^2}{r^2\epsilon_0}dr\right]$$

$$= \frac{Q^2}{8\pi}\left[\frac{1}{\epsilon}\int_a^b \frac{1}{r^2}dr + \frac{1}{\epsilon_0}\int_b^\infty \frac{1}{r^2}dr\right] = \frac{Q^2}{8\pi}\left[\frac{1}{\epsilon}\left(\frac{1}{a}-\frac{1}{b}\right) + \frac{1}{\epsilon_0}\left(\frac{1}{b}\right)\right]$$

$$\epsilon = \epsilon_0(1+\chi_e) \Rightarrow W = \frac{Q^2}{8\pi\epsilon_0}\left[\frac{1}{1+\chi_e}\left(\frac{1}{a}-\frac{1}{b}\right) + \frac{1}{b}\right]$$

4.(4.36). $\vec{E} = ?, \vec{P} = ?, \rho_b = ?, \sigma_b = ?, Q_{total\ surf} = ?$

$\oint \vec{D} \cdot d\vec{a} = Q_{fenc}$, $q = D \cdot 4\pi r^2 \Rightarrow \boxed{\vec{D} = \frac{q}{4\pi r^2}\hat{r}} \Rightarrow \vec{E} = \frac{D}{\varepsilon} = \frac{q}{4\pi r^2 \varepsilon}\hat{r} = \boxed{\frac{q}{4\pi r^2 \varepsilon_0 (1+\chi_e)}\hat{r}}$

$\Rightarrow \vec{P} = \varepsilon_0 \chi_e \vec{E} = \boxed{\frac{q\chi_e}{4\pi r^2 (1+\chi_e)}\hat{r}} \Rightarrow \rho_b = -\vec{\nabla}\cdot\vec{P} = -\frac{q\chi_e}{4\pi(1+\chi_e)}\left(\nabla \cdot \frac{\hat{r}}{r^2}\right) = \frac{-q\chi_e}{4\pi(1+\chi_e)} \frac{1}{r^2}\frac{\partial}{\partial r}\frac{r^2}{r^2}$
$= \boxed{0}$

$\Rightarrow \sigma_b = \vec{P}\cdot\hat{n} = \vec{P}\cdot\hat{r} = \boxed{\frac{q\chi_e}{4\pi R^2 (1+\chi_e)}}$ ($r = R$ on the surface)

$\Rightarrow Q_{b\ total\ surf} = \sigma_b \cdot 4\pi R^2 = \frac{q\chi_e}{4\pi R^2(1+\chi_e)} 4\pi R^2 = \boxed{\frac{q\chi_e}{1+\chi_e}}$ (Sorry for the disorganization)

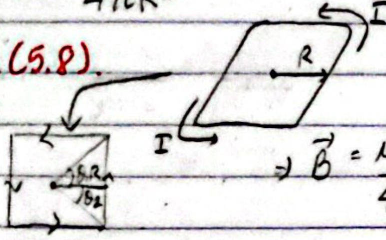
The compensating negative bound charge is located on the surface of the sphere. Since ρ_b is 0, the volume bound charge density is 0 inside the volume of the sphere. The non-zero bound charge is found at the surface instead.

5.(5.6). a) $\vec{K} = \sigma\vec{v} = \sigma\omega r$ (surface current density K, surface charge density σ)

b) $\rho = \frac{Q}{V} = \frac{3}{4}\frac{Q}{\pi R^3}$; $\vec{J} = \rho\vec{v} = \rho\vec{\omega}\times\vec{r} = \rho\,\omega r\sin\theta\,\hat{\phi}$ (current density $\vec{J}$)

$\Rightarrow \vec{J} = \frac{3Q}{4\pi R^3}\omega r\sin\theta\,\hat{\phi}$ at any point (r,θ,ϕ) within the sphere.

6.(5.8).



Magnetic field from a long straight wire carrying current I: $\vec{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\vec{l}'\times\hat{r}'}{r'^2}$; $dl'\sin\alpha = dl'\cos\theta$
lengths

$\Rightarrow \vec{B} = \frac{\mu_0 I}{4\pi}\int_{\theta_1}^{\theta_2}\left(\frac{\cos^2\theta}{s^2}\right)\left(\frac{s}{\cos^2\theta}\right)\cos\theta\, d\theta$; $l' = s\tan\theta \Rightarrow dl' = \frac{s}{\cos^2\theta}d\theta$

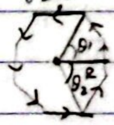
$s = r\cos\theta \Rightarrow \frac{1}{r^2} = \frac{\cos^2\theta}{s^2}$

$\Rightarrow \theta_1 = -45°;\ \theta_2 = 45°$ $= \frac{\mu_0 I}{4\pi s}(\sin\theta_2 - \sin\theta_1)$ Equation 5.37)

a) (initial & final angle) $s = R$

$\Rightarrow \vec{B}_{net} = 4\vec{B}$ (4 wire) $= \frac{\mu_0 I}{\pi R}[\sin 45° - \sin(-45°)] = \frac{\mu_0 I}{\pi R}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}\mu_0 I}{\pi R}$

b) Polygon with n wires $\Rightarrow \vec{B}_{net} = n\vec{B}$; $s = R$: $\theta_1 = -\frac{\pi}{n}$ & $\theta_2 = \frac{\pi}{n}$

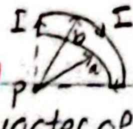


$\Rightarrow \vec{B}_{net} = n\vec{B} = n\frac{\mu_0 I}{4\pi R}\left[\sin\frac{\pi}{n} - \sin\left(\frac{-\pi}{n}\right)\right] = \frac{\mu_0 I n}{\cancel{2}4\pi R}\cancel{2}\sin\left(\frac{\pi}{n}\right)$

c) $n\to\infty \Rightarrow$ small angle approx $\Rightarrow \sin\frac{\pi}{n} \approx \frac{\pi}{n} \Rightarrow \vec{B}_{net} = \frac{n\mu_0 I}{2\pi R}\cdot\frac{\pi}{n} = \frac{\mu_0 I}{2R}$

$\Rightarrow$ The field at the center of the n-sided polygon is equal to the field at the center of circular loop when $n\to\infty$ (makes sense, an ∞ sides polygon is just a circle anyway).

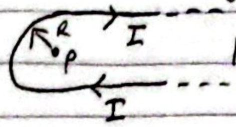
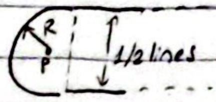
$\vec{B_a}$: out of page, $\vec{B_b}$: in the page : $B_a > B_b \Rightarrow \vec{B}_{net}$ out of page



7. (5.9). a) We already know for a full circle: $B = \frac{\mu_0 I}{2r}$

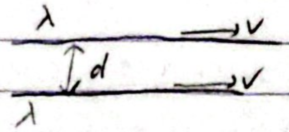
$\Rightarrow$ For a quarter of a circle: $B = \frac{\mu_0 I}{8r} \Rightarrow B_a = \frac{\mu_0 I}{8a}$ & $B_b = \frac{\mu_0 I}{8b}$

$\Rightarrow \vec{B}_{net} = \frac{\mu_0 I}{8a} - \frac{\mu_0 I}{8b} = \frac{\mu_0 I}{8}\left(\frac{1}{a} - \frac{1}{b}\right)$ (out of the page)

b)  Kind of similar to:  1/2 a circle + a full line

$\Rightarrow B = \frac{\mu_0 I}{4R}$ for half a circle. According to (5.8), $B = \frac{\mu_0 I}{2\pi R}$ for a line length R.

$\Rightarrow \vec{B}_{net} = \frac{\mu_0 I}{4R} + \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 I}{4R}\left(1 + \frac{2}{\pi}\right)$ (into the page)

8. (5.13). 

Force per unit length: $f = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d}$

$I_1 = I_2 = \lambda v \Rightarrow f = \frac{\mu_0 \lambda^2 v^2}{2\pi d} = F_{attractive}$

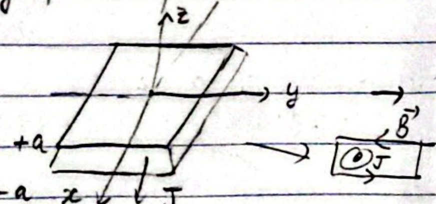
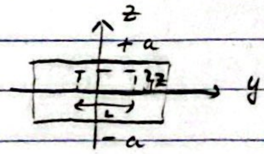
Electric field at distance d away due to ∞ straight wire: $E = \frac{\lambda}{2\pi\varepsilon_0 d}$

$\Rightarrow F_{repulsive} = qE = \frac{\lambda q}{2\pi\varepsilon_0 d} = \frac{\lambda^2}{2\pi\varepsilon_0 d}$ (per length) $\Rightarrow F_{repulsive} = \frac{\lambda^2}{2\pi\varepsilon_0 d}$

$\Rightarrow F_{attractive} = F_{repulsive} \Leftrightarrow \frac{\mu_0 \lambda^2 v^2}{2\pi d} = \frac{\lambda^2}{2\pi\varepsilon_0 d} \Rightarrow v = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{1}{(4\pi\cdot 10^{-7}\cdot 8.85\cdot 10^{-12})^{1/2}}$

$\Rightarrow v \approx 299863380.5 \approx 0.9995c \Rightarrow$ very close to the speed of light

$\Rightarrow$ Unreasonably fast speed

9. (5.15).   Draw an Amperian loop length l, height z

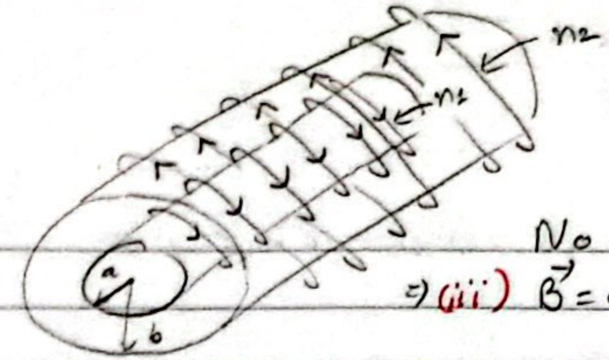
$\Rightarrow \oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc} = \mu_0 J A$ Using the RHR:

(Inside): $-a < z < a \Rightarrow Bl = \mu_0 J z l \Rightarrow \vec{B} = \mu_0 J z\,(-\hat{y})$

(Outside): $z > a \Rightarrow Bl = \mu_0 J a l \Rightarrow \vec{B} = \mu_0 J a\,(-\hat{y})$

$z < -a \Rightarrow Bl = -\mu_0 J a l \Rightarrow \vec{B} = \mu_0 J a\,\hat{y}$

10. (5.17).



No magnetic field outside of the solenoid

$\Rightarrow$ (iii) $\vec{B} = 0$ outside of both solenoid.

Magnetic field of a solenoid: $B = \mu_0 n I$ $\Rightarrow$ Magnetic field inside the smaller solenoid: $B_1 = \mu_0 n_1 I$ & $B_2 = -\mu_0 n_2 I$ for the mag. field inside the bigger solenoid (negative sign since the current is opposite for both solenoids).

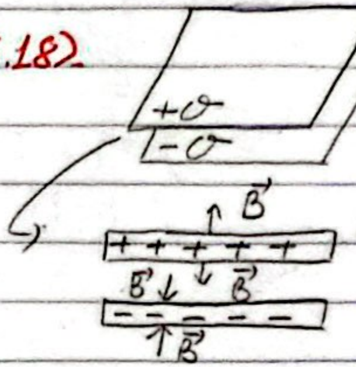
(i). Inside both:

$B = B_1 + B_2 = \mu_0 I (n_1 - n_2)$

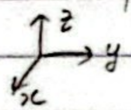
(ii). Between both: (magnitude)

$B = B_2 + 0 = -\mu_0 I n_2 = \mu_0 I n_2$

11. (5.18).



Head-on pov



a) Magnetic field of an ∞ uniform surface $\vec{K}$ is: $\vec{B} = \dfrac{\mu_0 \vec{K}}{2}$ (5.58)

$\Rightarrow$ $\vec{B}$ field outside canceled $\Rightarrow$ Add up the $\vec{B}$ field in the middle $\Rightarrow B_{net} = \dfrac{\mu_0 K}{2} + \dfrac{\mu_0 K}{2} = \mu_0 K$

$\Rightarrow \vec{B}_{net} = \mu_0 \sigma v \, \hat{x}$

b) Magnetic force acting on the upper plate due to the lower plate:

$\vec{F} = \int \vec{K} \times \vec{B} \, da \Rightarrow f$ (force/unit area) $= \vec{K} \times \vec{B} = \sigma v \hat{y} \times \dfrac{\mu_0 \sigma v}{2} (\hat{x}) = \dfrac{\mu_0 \sigma^2 v^2}{2} (\hat{z})$

c) Electric field for lower plate: $E = \dfrac{\sigma}{2\varepsilon_0} \Rightarrow f = \dfrac{\sigma^2}{2\varepsilon_0}$

To balans: $f_{up} = f_{low}$

$\Rightarrow \dfrac{\mu_0 \sigma^2 v^2}{2} = \dfrac{\sigma^2}{2\varepsilon_0} \Rightarrow v = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}} \approx 3 \cdot 10^8 \, m/s$