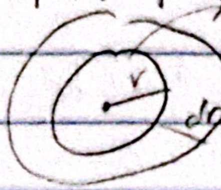
HOMEWORK 2

1. Since the charge lies off-center inside the cube, we cannot use Gauss' Law to calculate the flux as this situation's not symmetrical.

$\to$ The side of the cube closest to the charge will contain more flux. The total flux is unknown as Gauss' Law cannot be used but it will be independent from the charge's position (total flux will stay the same as long as charge still in box).

2. $\rho_{charge} = \rho_0 r^n / R^n$, $n > -3$. $q = \rho V = \rho 4\pi r^2 \to dq = 4\pi r^2 dr$

$\to dq = \rho 4\pi r^2 dr \quad \oint \vec{E} \cdot d\vec{s} = \dfrac{Q_{encl}}{\varepsilon_0} \to \vec{E} \cdot ds = \int \dfrac{Q_{encl}}{\varepsilon_0}$



$ds = 4\pi r^2$

$$\to E\, 4\pi r^2 = \frac{4\pi}{\varepsilon_0} \int_0^r \rho_{charge}\, r^2 dr = \frac{\cancel{4\pi}\, \rho_0}{\varepsilon_0 R^n} \int_0^r r^n r^2 dr$$

$$\Rightarrow E = \frac{\rho_0}{\varepsilon_0 R^n r^2} \int_0^r r^{n+2} dr = \frac{\rho_0}{\varepsilon_0 R^n r^2} \frac{r^{n+3}}{n+3}$$

$$\to E = \frac{\rho_0 r^{n+1}}{\varepsilon_0 R^n (n+3)} \quad \text{(radially outward).}$$

$\dfrac{dE}{dr} = 0$ for constant magnitude of electric field.

$$\Rightarrow \frac{dE}{dr} = (n+1) \frac{\rho_0 r^n}{\varepsilon_0 R^n (n+3)} = 0 \Leftrightarrow (n+1) r^n = 0$$

$$\Rightarrow n r^n + r^n = 0$$

$$\to n r^n = -r^n \to n = -1$$

Grading Space For 1 & 2:

3. $\oint \vec{E}\cdot ds = \frac{Q_{encl}}{\varepsilon_0} = E.2\pi r h \Rightarrow E = \frac{Q_{encl}}{2\pi r h \varepsilon_0}$

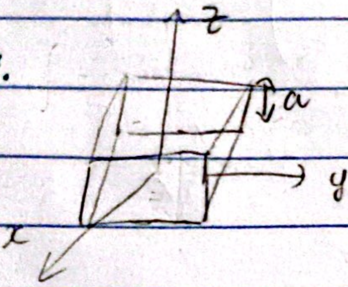
$Q_{encl} = \rho_{ch}\oint_V r\,dr\,d\phi\,dz = \rho_{ch}\int_0^{a}\int_0^{2\pi}\int_0^{h} r\,dr\,d\phi\,dz = \frac{\cancel{2}\pi h \rho_{ch} a^2}{\cancel{2}}$

$\Rightarrow E = \frac{\pi h \rho_{ch} a^2}{2\pi r h \varepsilon_0} = \frac{\rho_{ch} a^2}{2 r\varepsilon_0}$. Inside $\Rightarrow r<a \Rightarrow \frac{\rho_{ch} r^2}{2r\varepsilon_0} = \frac{\rho_{ch} r}{2\varepsilon_0}$

Outside $\Rightarrow r>a \Rightarrow \frac{\pi h\rho_{ch}a^2}{2\pi r h\varepsilon_0} = \frac{\rho_{ch}a^2}{2\varepsilon_0 r}$

$\Rightarrow \vec{E}_{in} = \frac{\rho_{ch} x}{2\varepsilon_0}\hat{x}$, $\Rightarrow \vec{E}_{out} = \frac{\rho_{ch}a^2}{2\varepsilon_0 x}\hat{x}$.

4.



$Q_{encl\,(in)} = \rho_{ch}\oint_V dx\,dy\,dz = \rho_{ch}\int_{-a/2}^{a/2}\int_{-y}^{y}\int_{-x}^{x} dx\,dy\,dz$
$= \rho_{ch}\,2x\,2y\,a$

$\oint \vec{E}\cdot dV = E.\frac{x}{2}\frac{y}{2}a = \frac{\rho_{ch}\,4xy\,a}{\varepsilon_0}$

$\Rightarrow \vec{E}(z\le a/2) = \frac{\rho_{ch}}{\varepsilon_0} z\hat{z}$

$Q_{encl\,(out)} = \rho_{ch}\int_a^z\int_{-y}^{y}\int_{-x}^{x} dx\,dy\,dz = \rho_{ch}\,4xy(z-a)$

$\Rightarrow \vec{E}(z\ge a/2) = \frac{\rho_{ch}\;(z-a)}{\varepsilon_0}\, z\hat{z}$

Grading Space For 3&4

5. $E_r = \frac{2A\cos\theta}{r^3}$, $E_\theta = A\sin\theta$, $E_\phi = 0$, $A = $ const for $r > a$

$$\oint E\cdot d\vec{l} = \int_a^r \frac{2A\cos\theta}{r^3}\,dr + \int_0^{2\pi} \frac{A\sin\theta}{r^3}\,r\,d\theta + 0$$

$$= \frac{2A\cos\theta}{2r^2}\Big|_a^r + \frac{A}{r^2}\overset{0}{\cancel{(-\cos\theta)_0^{2\pi}}}$$

$$= A\cos\theta\left(\frac{1}{r^2} - \frac{1}{a^2}\right)$$

$$\oint E\cdot d\vec{l} = (1) = \int E\cdot dr = \int_a^r \frac{Q}{4\pi\varepsilon_0}\frac{1}{r^2}\,dr = -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{a}\right)$$

$$\Rightarrow A\cos\theta\left(\frac{1}{r^2} - \frac{1}{a^2}\right) = -\frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r} - \frac{1}{a}\right)$$

$$\Leftrightarrow A\cos\theta\left(\frac{r^2 - a^2}{r^2 a^2}\right) = \overset{\to\sigma}{\boxed{\frac{Q}{4/3\,\pi a^3}}}\cdot\frac{a^3}{3\varepsilon_0}\,\frac{a-r}{ar}$$

$$\Leftrightarrow A\cos\theta\,\frac{\cancel{(r-a)}(r+a)}{r^2 a^2} = \frac{\sigma a^3}{3\varepsilon_0}\cdot\frac{\cancel{r-a}}{ar}$$

$$\Rightarrow \sigma = \frac{3A\cos\theta\,(r+a)\,\varepsilon_0\,a}{r}$$

6. $\vec{E} = (yz - 2x)\hat{x} + xz\hat{y} + xy\hat{z}$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ yz-2x & xz & xy \end{vmatrix} = (x - x)\hat{x} - (y - y)\hat{y} + (z - z)\hat{z} = 0$$

$\Rightarrow$ It is possible for the electrostatic field.

$$d\phi = -E\cdot d\vec{l} = -(yz - 2x\,dx + xz\,dy + xy\,dz)$$

$$\Rightarrow \phi = -\int(yz - 2x)\,dx - \int xz\,dy - \int xy\,dz$$

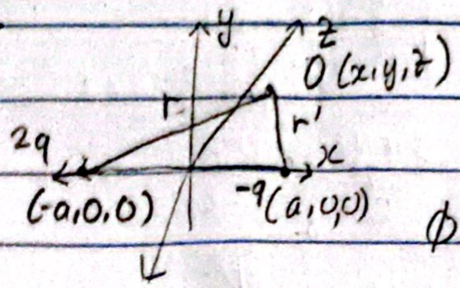
$$= -yzx + x^2 - xzy - xyz$$

$$= x^2 - 3xyz$$

Grading Space for 5&6:

Scanned with CamScanner

7. O: A random point on the surface



$r=\sqrt{(x+a)^2+y^2+z^2}$

$r'=\sqrt{(x-a)^2+y^2+z^2}$

$\phi=\frac{Q}{4\pi\varepsilon_0 R}$

$$\to \phi_P = \left(\frac{2q}{\underbrace{\sqrt{(x+a)^2+y^2+z^2}}_{r}} - \frac{q}{\underbrace{\sqrt{(x-a)^2+y^2+z^2}}_{r'}}\right)\frac{1}{4\pi\varepsilon_0}$$

$$\phi=0 \to \left(\frac{2\cancel{q}}{\sqrt{(x+a)^2+y^2+z^2}} - \frac{\cancel{q}}{\sqrt{(x-a)^2+y^2+z^2}}\right)\frac{1}{\cancel{4\pi\varepsilon_0}}=0$$

$\Rightarrow 2\sqrt{(x-a)^2+y^2+z^2} = \sqrt{(x+a)^2+y^2+z^2}$

$\Leftrightarrow 4[(x-a)^2+y^2+z^2] = (x+a)^2+y^2+z^2$

$\Leftrightarrow 4x^2-8xa+4a^2+4y^2+4z^2 = x^2+2xa+a^2+y^2+z^2$

$\Leftrightarrow 3x^2-10xa+3a^2+3y^2+3z^2=0$

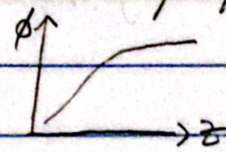
$\Leftrightarrow x^2+y^2+z^2+a^2-\frac{10}{3}xa=0 \to$ This surface is a sphere.

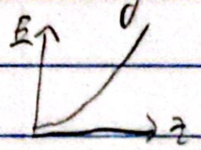
8. Circular disk with radius r & thickness dr: $d\phi = \frac{2\pi\sigma}{4\pi\varepsilon_0\sqrt{r^2+z^2}}r\,dr$

$\to \phi = \int d\phi = \int_0^a \frac{2\pi\sigma}{4\pi\varepsilon_0\sqrt{r^2+z^2}}r\,dr = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{a^2+z^2}-|z|\right)$

For $a \gg z$:

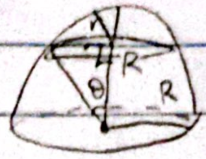
$\sqrt{a^2+z^2}=a \Rightarrow \phi=\frac{\sigma a}{2\varepsilon_0}$. This just means the potential in the cente of the disk as a point charge. I don't really know if this can use to calculate x & y, I assume we can't since by symmetry the potential should cancel out for x & y.





Grading space for 7 & 8!

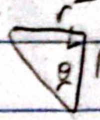
9.



$\Rightarrow \phi_{center} = \frac{\sigma}{4\pi\varepsilon_0} \int \frac{1}{R}\, da$ ↳ surface area $= 2\pi R^2$

$\Rightarrow \phi_{center} = \frac{\sigma}{4\pi\varepsilon_0} \frac{2\pi R^2}{R} = \frac{\sigma R}{2\varepsilon_0}$

$\Rightarrow \phi_{NP} = \frac{\sigma}{4\pi\varepsilon_0} \int \frac{1}{r}\, da$ ↳ $2\pi R^2 \sin\theta\, d\theta$



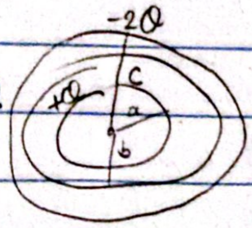
$\Rightarrow r^2 = R^2 + R^2 - 2R^2\cos\theta = 2R^2(1-\cos\theta)$

$= \frac{\sigma}{4\pi\varepsilon_0} \frac{2\pi R^2}{R\sqrt{2}} \int_0^{\pi/2} \frac{\sin\theta}{\sqrt{1-\cos\theta}}\, d\theta = \frac{\sigma R}{2\sqrt{2}\varepsilon_0} \left(2\sqrt{1-\cos\theta}\right)_0^{\pi/2}$

$= \frac{\sigma R}{\sqrt{2}\varepsilon_0}$

$\Rightarrow \phi_{NP} - \phi_{center} = \frac{\sigma R}{\sqrt{2}\varepsilon_0} - \frac{\sigma R}{2\varepsilon_0} = \frac{\sigma R}{\varepsilon_0}\left(\frac{1}{\sqrt{2}} - \frac{1}{2}\right)$

10.



a) Since a net charge +Q is placed on the sphere

$\Rightarrow$ +Q charge must be on the surface of a

$\Rightarrow$ −Q charge must be on the surface of b to cancel out the one on a.

$\Rightarrow -2Q - (-Q) = -Q$ must be on the surface of c to make up the net charge $-2Q$.

b) $\oint \vec{E} \cdot d\vec{S} = \frac{Q_{encl}}{\varepsilon_0} = E 4\pi r^2 \Rightarrow E = \frac{Q_{encl}}{\varepsilon_0 4\pi r^2}$

- $r < a$ & $b < r < c$: $E = 0$ (inside conductor)
- $a < r < b$: $Q_{encl} = Q 4\pi r^3 \Rightarrow \vec{E} = \frac{Qr}{\pi\varepsilon_0}\hat{r}$
- $r > c \Rightarrow Q_{encl} = Q - Q + Q - 2Q = -Q \Rightarrow \vec{E} = \frac{-Q}{4\pi r^2 \varepsilon_0}\hat{r}$

Grading Space for 9&10ab:

c. $a<r \Rightarrow \phi(r) = -\int_\infty^r \vec E\cdot d\vec r = -\int_\infty^c \vec E\cdot d\vec r - \cancel{\int_c^b \vec E\cdot d\vec r} - \int_b^a \vec E\cdot d\vec r - \cancel{\int_a^r \vec E\cdot d\vec r}$

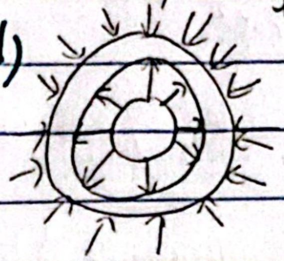
$$= \frac{-Q}{4\pi\varepsilon_0 c} - \frac{Q}{2\pi\varepsilon_0}(a^2-b^2)$$

- $a<r<b \Rightarrow \phi(r) = -\int_\infty^c \vec E\cdot d\vec r - \int_b^r \vec E\cdot d\vec r = \frac{-Q}{4\pi\varepsilon_0 c} - \frac{Q}{\pi\varepsilon_0}\int_b^r r\,dr$

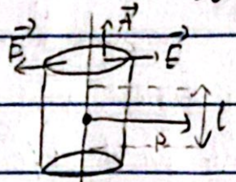
$$= \frac{-Q}{4\pi\varepsilon_0 c} - \frac{Q}{\pi\varepsilon_0}\left(\frac{r^2}{2}\right)_b^r = \frac{-Q}{4\pi\varepsilon_0 c} - \frac{Q}{2\pi\varepsilon_0}(r^2-b^2)$$

- $b<r<c \Rightarrow \phi(r) = -\int_\infty^c \vec E\cdot d\vec r = \frac{-Q}{4\pi\varepsilon_0 c}$
- $r>c \Rightarrow \phi(r) = \int_\infty^r \frac{Q}{4\pi\varepsilon_0 r^2}\,dr = \frac{Q}{4\pi\varepsilon_0}\left(\frac{-1}{r}\right)_\infty^r = \frac{Q}{4\pi\varepsilon_0}\left(\frac{-1}{r} - \frac{1}{\infty}\right)$

d)



11.



radius a. Charge per unit length q_e

perpendicular distant from axis ρ.

$$\oint \vec E\cdot d\vec s = \frac{Q_{enc}}{\varepsilon_0} = E\,2\pi\rho l = \frac{q_e l}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q_e}{2\pi\varepsilon_0 \rho}$$

$$E = \frac{-d\phi}{dr} \Rightarrow \int d\phi = -\int E\,dr = -\int_{\rho_0}^{\rho} \frac{q_e}{2\pi\varepsilon_0\rho}\,d\rho = \frac{-q_e}{2\pi\varepsilon_0}\ln\rho\Big|_{\rho_0}^{\rho}$$

$$\Rightarrow \phi(\rho) = \frac{q_e}{2\pi\varepsilon_0}\ln\left(\frac{\rho_0}{\rho}\right)$$

Since electric field inside cylinder $=0 \Rightarrow \phi = \text{const.}$

$\Rightarrow$ Potential inside = potential at surface of cylinder $\Rightarrow \rho = a$

$$\Rightarrow \phi(a) = \frac{q_e}{2\pi\varepsilon_0}\ln\left(\frac{\rho_0}{a}\right)$$

12. $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$. If a sheet thickness t is inserted $\Rightarrow C' = \frac{Q}{V'}$ where $V' = E(d-t) = \frac{Q}{\varepsilon_0 A}(d-t)$

$$\Rightarrow C' = \frac{\varepsilon_0 A}{d-t}$$

$$\Rightarrow \Delta C = C' - C = \frac{\varepsilon_0 A}{d-t} - \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A(d-d+t)}{d(d-t)}$$

$$\Rightarrow \Delta C = \frac{\varepsilon_0 A t}{d(d-t)}$$

I don't know if I can do this by combining 2 capacitors in some way but I think we might can? I don't know what way though.

Grading space for 10cd, 11, 12.

Scanned with CamScanner

13. a) charge density: $\sigma_b = \frac{q_b}{\underbrace{4\pi b^2}_{\text{Area}}}$ & $\sigma_c = \frac{q_c}{\underbrace{4\pi c^2}_{\text{Area}}}$

→ Charge density of the outer surface $= \frac{q_b + q_c}{4\pi a^2}$.

b) $E_a = \frac{q_b + q_c}{4\pi\varepsilon_0 a^2}$ for $r > a$

c) $E_b = \frac{q_b}{4\pi\varepsilon_0 b^2}$ $0 < r \leq b$ & $E_c = \frac{q_c}{4\pi\varepsilon_0 c^2}$ $0 < r \leq c$

d) The charges will not move as field caused by each charge doesn't effect other field → force $= 0$.

e) It would probably would. Charge will become $\frac{Q + q_b + q_c}{4\pi a^2}$

$\Rightarrow E = \frac{Q + q_b + q_c}{4\pi\varepsilon_0 a^2} > E_a \Rightarrow$ it would move?

Grading space for 13.