"In words":

$\hat{n}\times\vec{F} \equiv \perp$ to $\hat{n}$ & F_t lie in tangent plane.

$(\hat{n}\times\vec{F})\hat{n} \equiv \perp$ to $\hat{n}$ & $\perp F_t$.

HOMEWORK 4

1. Equation 9-17: $(\hat{n}\times\vec{F})\times\hat{n} = \vec{F} - F_n\hat{n} = \vec{F_t}$

Equation 1-23: $\vec{B}\times\vec{A} = -(\vec{A}\times\vec{B})$

Equation 1-30: $\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}\cdot\vec{C}) - \vec{C}(\vec{A}\cdot\vec{B})$

Equation 1-17: $\vec{A}^2 = \vec{A}\cdot\vec{A} = A^2$

Equation 9-14: $\vec{F} = \vec{F_n} + \vec{F_t} = F_n\hat{n} + \vec{F_t} \Rightarrow \vec{F} - F_n\hat{n} = \vec{F_t}$

$\Rightarrow (\hat{n}\times\vec{F})\times\hat{n} = -\hat{n}\times(\hat{n}\times\vec{F})$ (1-23) $= -\hat{n}(\hat{n}\cdot\vec{F}) + \vec{F}(\underbrace{\hat{n}\cdot\hat{n}}_{|\hat{n}|^2 = 1 \text{ (1-17)}})$ (1-30)

$= \vec{F} - \hat{n}(\hat{n}\cdot\vec{F})$

$\Rightarrow \hat{n}\cdot\vec{F} = \hat{n}\cdot(F_n\hat{n} + \vec{F_t}) = F_n|\hat{n}|^2 + \underbrace{\cancel{\hat{n}\vec{F_t}}}_{\hat{n}\perp\vec{F_t}}^{\to 0} = F_n$

Miss "in words".

$\Rightarrow \vec{F} - \hat{n}(\hat{n}\cdot\vec{F}) = \vec{F} - \hat{n}F_n = \vec{F_t}$. ✓

2. Solution to 5-17: $\phi = \frac{\sigma}{2\varepsilon_0}\left[(a^2+z^2)^{1/2} - |z|\right]$, $\vec{E} = \hat{z}\frac{\sigma}{2\varepsilon_0}\left(\frac{z}{|z|}\right)\left[1 - \frac{|z|}{(a^2+z^2)^{1/2}}\right]$

$\Rightarrow A = 2\pi r\,dr,\quad dq = \sigma 2\pi r\,dr$

$\Rightarrow d\phi = \frac{1}{4\pi\varepsilon_0} = \frac{dq}{\sqrt{r^2+|z|^2}} = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi r\,dr}{\sqrt{r^2+|z|^2}} = \frac{\sigma r\,dr}{2\varepsilon_0\sqrt{r^2+|z|^2}}$

$\Rightarrow \phi = \int_0^a \frac{\sigma r\,dr}{2\varepsilon_0\sqrt{r^2+|z|^2}} = \frac{\sigma}{2\varepsilon_0}\int_0^a \frac{r\,dr}{\sqrt{r^2+|z|^2}} = \frac{\sigma}{2\varepsilon_0}\sqrt{r^2+|z|^2}\,\Big|_0^a$

$\Rightarrow \phi = \frac{\sigma}{2\varepsilon_0}\left[(a^2+z^2)^{1/2} - |z|\right]$ ✓

$E_{2n} - E_{1n} = \frac{\sigma}{\varepsilon_0}$ (?) $\lim_{z\to 0}\vec{E} = \frac{\sigma}{2\varepsilon_0}\frac{z}{|z|}\hat{z}$

$\Rightarrow \lim_{z\to 0^+}\vec{E} = \frac{\sigma}{2\varepsilon_0}\hat{z} = E_{2n}$, $\lim_{z\to 0^-}\vec{E} = \frac{-\sigma}{2\varepsilon_0}\hat{z} = E_{1n}$

$\Rightarrow E_{2n} - E_{1n} = \sigma/2\varepsilon_0$ (?)

$\Rightarrow \vec{E} = \frac{-dV}{da} = \frac{\sigma}{2\varepsilon_0}\left(\frac{z}{|z|}\right)\left[1 - \frac{|z|}{(a^2+z^2)^{1/2}}\right]\hat{z}$ (No tangential field due to symmetry) ✓ ✓

3 (Wagsness 9-1). $2x+y+z=1$, $\vec{E_1} = 4\hat{x}+\hat{y}-3\hat{z}$

$\Rightarrow \vec{n} = 2\hat{x}+\hat{y}+\hat{z} \Rightarrow \hat{n_1} = \frac{2\hat{x}+\hat{y}+\hat{z}}{\sqrt{4+1+1}} = \frac{2\hat{x}+\hat{y}+\hat{z}}{\sqrt{6}}$ ✓

$\Rightarrow \vec{E_{1n}} = (\vec{E_1}\cdot\hat{n_1})\hat{n_1} = \left(4\hat{x}+\hat{y}-3\hat{z} \cdot \frac{2\hat{x}+\hat{y}+\hat{z}}{\sqrt{6}}\right)\left(\frac{2\hat{x}+\hat{y}+\hat{z}}{\sqrt{6}}\right)$

$= \frac{(8+1-3)}{\cancel{\sqrt{6}}}\left(\frac{2\hat{x}+\hat{y}+\hat{z}}{6}\right) = 2\hat{x}+\hat{y}+\hat{z}$ ✓

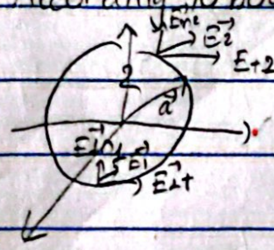
$\Rightarrow |\vec{E_n}| = \sqrt{6}$ or $E_n = \sqrt{6}\,\hat{n}$ (still correct)

$\Rightarrow \vec{E_{1t}} = \vec{E_1} - \vec{E_{1n}} = (4\hat{x}+\hat{y}-3\hat{z}) - (2\hat{x}+\hat{y}+\hat{z}) = 2\hat{x}-4\hat{z}$ ✓

$\Rightarrow \vec{E_{1n}}\cdot\vec{E_{1t}} = (2\hat{x}+\hat{y}+\hat{z})\cdot(2\hat{x}-4\hat{z}) = 4-4=0 \Rightarrow \vec{E_{1n}} \perp \vec{E_{1t}}$ ✓

4. (Wangsness 9-3). $\vec{E_1} = \alpha\hat{x}+\beta\hat{y}+\gamma\hat{z}$, $\alpha=\beta=0$, $\gamma=E_0$, $\sigma=\sigma_0\cos\theta$.

According to boundary con: $\vec{E_{2n}} - \vec{E_{1n}} = \frac{\sigma}{\varepsilon_0}\hat{n}$ & $\vec{E_{2t}} - \vec{E_{1t}} = 0$



$\Rightarrow \vec{E_2} - \vec{E_1} = \frac{\sigma}{\varepsilon_0}\hat{n} = \frac{\sigma}{\varepsilon_0}\frac{\vec{a}}{|\vec{a}|} \Rightarrow \vec{E_2} = \vec{E_1} + \frac{\sigma}{\varepsilon_0}\frac{\vec{a}}{|\vec{a}|}$

$\Rightarrow \vec{a} = x\hat{x}+y\hat{y}+z\hat{z} \Rightarrow |\vec{a}| = \sqrt{x^2+y^2+z^2}$

$z = |\vec{a}|\cos\theta \Rightarrow \cos\theta = \frac{z}{|\vec{a}|}$ I use spherical → cartesian convert early here.

$\Rightarrow \vec{E_2} = (\alpha\hat{x}+\beta\hat{y}+\gamma\hat{z}) + \frac{\sigma_0\cos\theta}{\varepsilon_0}\times\frac{\vec{a}}{|\vec{a}|} = (\alpha\hat{x}+\beta\hat{y}+\gamma\hat{z}) + \frac{\sigma_0}{\varepsilon_0}\times\frac{z}{|\vec{a}|}\times\frac{\vec{a}}{|\vec{a}|}$

$= (\alpha\hat{x}+\beta\hat{y}+\gamma\hat{z}) + \frac{\sigma_0}{\varepsilon_0}\times\frac{z}{|\vec{a}|^2}\vec{a} = E_0\hat{z} + \frac{\sigma_0}{\varepsilon_0}\frac{z}{\underbrace{x^2+y^2+z^2}_{=a^2}}(x\hat{x}+y\hat{y}+z\hat{z})$

$\Rightarrow \vec{E_2} = \frac{\sigma_0 xz}{\varepsilon_0 a^2}\hat{x} + \frac{\sigma_0 yz}{\varepsilon_0 a^2}\hat{y} + \left(E_0 + \frac{\sigma z^2}{\varepsilon_0 a^2}\right)\hat{z}$ ✓

Grading Space for 1-4: ① I missed the "in words" explaination for 1. ② I missed $E_{2n}-E_{1n}$ stuffs, I just write what $\vec{E}$ is & not what the normal component equals to b/c I thought I already matched the answer for 5-17. ③ & ④ Should be all correct.

5. $l = 0.001m$, $V = 500V$, $R = 0.529 \mathring{A} = 0.529 \cdot 10^{-10} m$, $\frac{\alpha}{4\pi\epsilon_0} = 0.66 \cdot 10^{-30} m^3$

Magnitude of dipole moment of atom: $p = ed$

Magnitude of dipole momentum of the hydrogen atom: $p = \alpha E$ $\Big\} \Rightarrow ed = \alpha E$, $E = V/l$

$\Rightarrow$ Separation: $d = \frac{\alpha E}{e} = \frac{\alpha V}{el} = \frac{0.66 \cdot 10^{-30} \cdot 4\pi\epsilon_0 \cdot 500}{1.6 \cdot 10^{-19} \cdot 0.001} \approx 2.29 \cdot 10^{-16} m$

$\Rightarrow \frac{d}{R} = \frac{2.29 \cdot 10^{-16}}{0.529 \cdot 10^{-10}} \approx 4.34 \cdot 10^{-6} m$ ✓

∞ line charge

$\Rightarrow \vec{E}_{\infty line} = \frac{\lambda}{2\pi\epsilon_0 \rho}$ ✗

$\lambda = \sigma a = Pa \cos\phi a$

$\Rightarrow \vec{E} = \int d\vec{E} = \frac{-P}{2\pi\epsilon_0 a} \int_0^{2\pi} \cos^2\phi' a \, d\phi' \hat{x} = -\frac{P\hat{x}}{2\epsilon_0}$

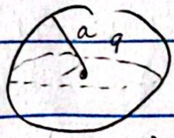
6. (Wangsness 10-12) $\vec{P} = P\hat{x}$

$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0} = \frac{\sigma \pi r^2 h}{\epsilon_0} = \frac{P \pi r^2 h}{\epsilon_0}$ (h is height of Gaussian cylinder) (wrong approach)

$\oint \vec{E} \cdot d\vec{A} = \int \vec{E} 2\pi r^2 h dz = \vec{E} 2\pi r^2 h = \frac{\vec{P} \pi r^2 h}{\epsilon_0} \Rightarrow \vec{E} = \frac{-P\hat{x}}{2\epsilon_0}$

$\sigma = P$ since the cylinder is uniformly polarized.

The total bound charge $= 0$. ✓



7. (Wangsness 10-17).

Electric Displacement: $\oint \vec{D} \cdot da = D 4\pi r^2 = q \Rightarrow \vec{D} = \frac{q}{4\pi r^2} \hat{r}$ ✓

Electric Field: $\vec{D} = K_e \epsilon_0 \vec{E} = (1 + \chi_e) \epsilon_0 \vec{E}$, $\chi_e = 0$ when $r > a$

$\Rightarrow \vec{E}_{r>a} = \frac{q}{4\pi r^2 \epsilon_0} \hat{r}$ ✓ $\Rightarrow \vec{E}_{r<a} = \frac{q}{4\pi r^2 \epsilon_0 k_e} \hat{r}$ ✓ $(\chi_e > 0 \rightarrow (1 + \chi_e) = K_e)$ $\underset{\epsilon}{}$

Polarization: $\vec{P} = \epsilon_0 \chi_e \vec{E} = (K_e - 1) \epsilon_0 \vec{E}$

$\chi_e = 0$ when $r > a$ & $\vec{P}_{r>a} = 0$ ✓

$\Rightarrow \vec{P}_{r<a} = (K_e - 1) \cdot \frac{q}{4\pi r^2 \epsilon_0 k_e} \hat{r}$ ✓

Grading Space for 5 & 6: ⑤ All correct. ⑥ I got the right magnitude but my approach is wrong.

In answer: $\frac{Ke-1}{Ke}\frac{q}{4\pi r^2}\hat{r}\cdot\hat{r}$
where does ϵ goes for $\vec{P}$? what is a?

Bound charge on surface: $Q = \sigma_b A$

$\sigma_b = \vec{P}\cdot\hat{n}A = (K_e-1)\frac{q}{4\pi r^2 \epsilon_0 K_e}\hat{r}\cdot\hat{r} = (K_e-1)\frac{q}{4\pi r^2\epsilon_0 K_e}$?

$\Rightarrow Q = (K_e-1)\frac{q}{4\pi r^2 \epsilon_0 K_e}\cdot 4\pi r^2 = \frac{q}{\cancel{\epsilon_0} K_e}(K_e-1)$?
remove ϵ_0

b) $\rho_b = -\nabla\cdot\vec{P} = -\nabla\cdot (K_e-1)\frac{q}{4\pi r^2\epsilon_0 K_e}\hat{r} = \frac{-q}{K_e}\delta^3(\vec{r})(K_e-1)$ ✓

The total bound charge is 0 ✗ $Q_{bound} = \int_V \rho_b\, d\tau = -q\,\frac{Ke-1}{Ke}$.

a) $\vec{E}_n = (\vec{E}\cdot\hat{n})\hat{n} = \frac{q}{4\pi r^2\epsilon_0}\hat{r}$ $=E_{2n}$ (for $r>a$) & $= \frac{q}{4\pi r^2\epsilon_0 K_e}\hat{r}$ $=E_{1n}$ (for $r<a$)

$\Rightarrow E_{2n}-E_{1n} = \frac{Ke-1}{ke}\frac{q}{4\pi a^2\epsilon_0}$

$\vec{D}_n = (\vec{D}\cdot\hat{n})\hat{n} = \frac{q}{4\pi r^2}\hat{r}$ $\Rightarrow$ satisfy the boundary conditions.

8. (Wangeness 10-19).

$\vec{E}_{b>\rho>a} = \frac{\lambda}{2\pi\epsilon_0\rho}\hat{\rho}$ & $\vec{E}=0$ else where

$= \frac{\lambda f}{2\pi\epsilon_0 \alpha\rho^{n+1}}\hat{\rho} \Rightarrow \vec{D} = K_e\epsilon_0\vec{E} = \frac{\lambda f}{2\pi\rho^{n+1}}\hat{\rho}$

$\rho_b = -\nabla\cdot\vec{P}$, $\vec{P} = (K_e-1)\epsilon_0\vec{E} = \frac{\lambda f}{2\pi\rho^n}\hat{\rho}$

$\Rightarrow \rho_b = -\nabla\cdot\frac{\lambda f}{2\pi\rho^n}\hat{\rho} = \frac{\lambda f}{2\pi\rho^{n+1}}$ $\frac{\lambda f}{2\pi\rho}$ $= -1/\rho\ \frac{\lambda f}{2\pi\alpha}\ \frac{-n}{\rho^{n+1}}$ $\Rightarrow \rho_b = \frac{-\lambda\rho n}{2\pi\alpha\rho^{n+2}}$

When $n = -1$, $\vec{E}$ would be constant. $\vec{D} = \frac{\lambda f}{2\pi\rho}$ & $\rho_b = \frac{\lambda\rho}{2\pi\rho^2}$

$U_e = \frac{1}{2}\int_0^l\int_0^{4\pi}\Delta\phi = \frac{1}{2}\int_0^l\int_0^{2\pi}\frac{\lambda f}{\alpha n 2\pi\epsilon_0}\left(\frac{1}{a^n}-\frac{1}{b^n}\right)\frac{\lambda f}{2\pi a}\,a\,dr\,d\theta$

$\Rightarrow U_e = \frac{l\lambda f^2}{4\pi\epsilon_0\alpha n}\left(\frac{1}{a^n}-\frac{1}{b^n}\right)$ ✓

9. (Wangsness 10-25). $C = \frac{2\pi(\epsilon_1+\epsilon_2)ab}{b-a}$ (Answer of 10-25)

$C = Q/V$ Let l be the distance from K_{e1} plate & d be the distance between 2 plates

$\Rightarrow K_e = K_{e1} + (K_{e2}-K_{e1})\frac{l}{d}$ The E field @ distance l from plate, $E = Q/\epsilon_0$

$\Rightarrow \epsilon = \epsilon_1 + (\epsilon_2-\epsilon_1)\frac{l}{d} \Rightarrow \sigma = \epsilon\cdot E \Rightarrow \sigma = \frac{\epsilon_1+(\epsilon_2-\epsilon_1)l}{d}\cdot\frac{\sigma_1+(\sigma_2-\sigma_1)l}{\epsilon_0 d}$

$Q = \int A\sigma\, dl = \frac{Al(\epsilon_1+\epsilon_2)(\sigma_1+\sigma_2)}{2\epsilon_0}$

$\Rightarrow C = Q/V = \frac{2\pi(\epsilon_1+\epsilon_2)ab}{b-a}$?

$V = \int_a^b E\,dl = \frac{(\sigma_1+\sigma_2)l}{2\epsilon_0}\ \ln\left(\frac{b-a}{K_{e2}-K_{e1}}\right)$

$C = \frac{\epsilon_0 A(K_{e2}-K_{e1})}{d\ln\frac{K_{e2}}{K_{e1}}}$ (answer sheet) ?

Grading Space for 7, 8, 9: ⑦ I got it right the first part but messed up part a & b. I also confused about σ_b & ρ parts. ⑧ I got it wrong completely because I rely on my previous homework's answer (I was lazy).
⑨. Actually when I did the Q/V, My answer is $\frac{A\ell(\varepsilon_1+\varepsilon_2)}{\ln\left(\frac{b-a}{ke_2-ke_1}\right)}$, $A=ab$
so I guess it's closer to the answer sheet than wrong guess.

10. Boundary Condition: $\frac{ke_1}{ke_2} = \frac{\tan\theta_1}{\tan\theta_2} \Rightarrow \tan\theta_2 = 1/4 \Rightarrow \theta = \tan^{-1}(1/4) = 14°$ ✓

E is at 45° to the surface $\rightarrow E_0 \cos 45° = \frac{E_0}{\sqrt{2}} = E$

$\sigma = \pm \frac{ke-1}{ke} \sigma_{free} = \pm \frac{ke-1}{ke} \varepsilon_0 E \underset{bot}{\overset{top}{=}} \pm \frac{ke-1}{ke} \varepsilon_0 \frac{E_0}{\sqrt{2}}$ ✓

Grading space for 10: All correct.

Overall Performance: I did this one pretty bad, part of it is also because I was lazy and just skip stuffs are rely on previous hw's answer (I'm sorry). I still need to work on my $\vec{D}, \vec{E}, \vec{P}$, ρ_b & σ_b finding, as well as my $\vec{E}_n$ & $\vec{E}_t$ (lots of time I'm not sure in theory how they works).