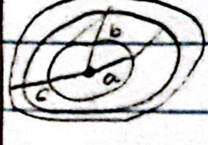
HOMEWORK 7

1. Wangsness 15-7.

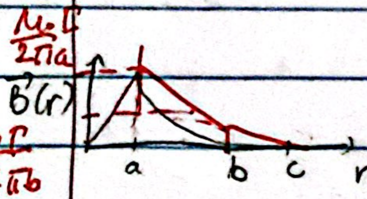


Using Ampere's circular law: $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl}$

$\vec{B}(r) = \frac{\mu_0 I r}{2\pi a^2}\hat{\phi}$ (for $r<a$) ✓

$\vec{B}(r) = \frac{\mu_0 I}{2\pi r}\hat{\phi}$ (for $a<r<b$) ✓ contribution from 2 cylinder

$\vec{B}(r) = 0$ ✓ (for $r>b$) $= \frac{\mu_0 I}{2\pi r} - \frac{\mu_0 I}{2\pi r}$



$b<\rho<c$: $\vec{B}(2\pi r) = \mu_0\left(I - I\frac{\rho^2-b^2}{c^2-b^2}\right) \Rightarrow \vec{B} = \frac{\mu_0 I}{2\pi r}\frac{c^2-r^2}{c^2-b^2}\hat{\phi}$

2. Wangsness 15-8. $\vec{B}=0$ for $0<\rho<a$, $\vec{B} = \frac{\mu_0 I}{2\pi\rho}\frac{\rho^2-a^2}{b^2-a^2}\hat{\phi}$ for $a<\rho<b$, and $\vec{B} = \frac{\mu_0 I}{2\pi\rho}\hat{\phi}$ for $b<\rho$.

$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$. In cylindrical coord: $\frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho\vec{B})\hat{z} = \mu_0\vec{J}$

$a<\rho<b$: $\frac{1}{\rho}\left(\vec{B} + \rho\frac{\partial\vec{B}}{\partial\rho}\right)\hat{z} = \mu_0\vec{J} = \frac{\mu_0 I}{\pi(b^2-a^2)} \Rightarrow \vec{J} = \frac{I}{\pi(b^2-a^2)}\hat{z}$

$\vec{B} = \frac{\mu_0 I}{2\pi\rho}\frac{\rho^2-a^2}{b^2-a^2}\hat{\phi}$ for $a<\rho<b$

$\Rightarrow \vec{J} = \frac{1}{\mu_0}(\vec{\nabla}\times\vec{B}) = \hat{z}\frac{I}{2\pi\rho}\frac{\partial}{\partial\rho}\left[\frac{\rho^2-a^2}{b^2-a^2}\right] = \hat{z}\frac{I}{\pi(b^2-a^2)}$ ✓

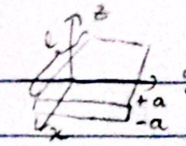
$b<\rho$: $\vec{J}=0$ because $\vec{B} = \frac{\mu_0 I}{2\pi\rho}$

$\Rightarrow \frac{\partial}{\partial\rho}(\rho\vec{B}) = \frac{\partial}{\partial\rho}\left(\frac{\mu_0 I}{2\pi}\right) = 0$ ✓

there is current I evenly spred in glindrical region from r=a to r=b

You could produce this kind of field by having coaxial cables carrying current in the opposite direction.

Grading space for 1 & 2: I got 1 wrong at the part that I thought there's no field beyond b but there are 1 between b&c. 2 is correct.

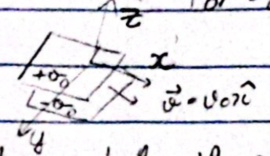
3.  $\vec{J} = J_0 \hat{x}$ $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$

Using the right hand rule → $\vec{B}$ in $-\hat{y}$ for $z > 0$

$I_{encl} = \int \vec{J} \cdot d\vec{a} = J_0 z l$ $\vec{B}$ in $\hat{y}$ for $z < 0$

→ $\oint \vec{B} \cdot d\vec{l} = B \cdot l = \mu_0 J_0 z l$ → $B = \mu_0 z J_0$ → $\vec{B} = \frac{z}{|z|} - \mu_0 J_0 a \hat{y}$ $(z > a)$ ✓

For $-a < z < a$: $\vec{B} = -\mu_0 z J_0 \hat{y}$ $\quad z \mu_0 J_0 a \hat{y}$ $(z < -a)$ ✓

4.  $\vec{K} = \sigma \vec{v}$

a) → $\oint \vec{B} \cdot d\vec{l} = 2Bl = \mu_0 I_{encl} = \mu_0 K l$ → $B = \frac{\mu_0 K}{2}$

• Above & below the capacitors: B-field = 0 since K points to opposite directions from the 2 plates. into the page

• In between: $\vec{B} = \frac{\mu_0}{2} \sigma_0 v_0 (-\hat{y}) - \frac{\mu_0}{2} \cdot (-\sigma_0) v_0 (-\hat{y}) = \mu_0 \sigma_0 v_0 (+\hat{y})$ ✓

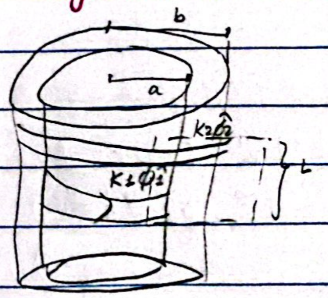
b) $\vec{f} = \vec{K} \times \vec{B} = \sigma v_0 \cdot \frac{\mu_0 \sigma_0 v_0}{2} = \left(\frac{\mu_0}{2} \sigma_0^2 v_0^2 \hat{z} \right)$ (using right hand rule) ✓

c) Electric field @ bottom plate: $E = \frac{\sigma_0}{2\varepsilon_0}$; Force on the upper plate:

$f = \frac{\sigma_0^2}{2\varepsilon_0}$ (force/unit area) $\quad F = \frac{\sigma_0^2 A}{2\varepsilon_0}$

$= \frac{\mu_0 \sigma_0^2 v_0^2}{2}$ ⇒ $\boxed{v_0 = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = c}$ → forces would never balance ✓

5. Wangsness 15-10.



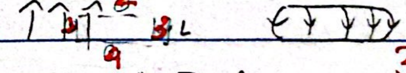
$\oint B \cdot dl = \mu_0 I_{inc} = \mu_0 (K_1 + K_2) L$

⇒ $\vec{B} = \mu_0 (K_1 + K_2) \hat{z}$ ✓ (right-hand rule)

→ Magnetic field between 2 cylinders: $\vec{B} = \mu_0 K_2 \hat{z}$ ✓

⇒ Magnetic field outside $= 0$. ✓

Magnetic field from loop

6. Wangsness 15-11.  $\oint \vec{B} \cdot d\vec{s} = 0$

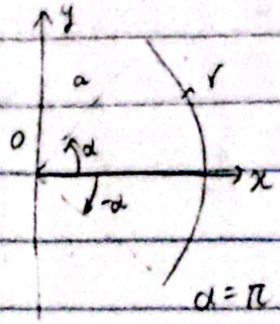
$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$ ⇒ $B = \frac{\mu_0 I_{encl}}{l}$ At 2, 3, 4 → $\cancel{B} = 0$ $B < 0$, $B_1 > 0$

⑦ ⇒ $B_{loop} = B_1 + B_2 = \frac{2\mu_0 I_{encl}}{l}$ ⇒ induction cannot drop to 0.

Change in B is also consistent with capacitor image as current move parallel in the same direction → Additive B → not drop to 0.

Grading space for 3→6: I forgot z for 3. Correct 4 & 5. I think I am wrong for 6, I got the sign for B incorrectly & I think my explaination is also wrong.

7. Wangsness 16-10.



$r = z\hat{z},\ r' = a\hat{\rho} \Rightarrow R = z\hat{z} - a\hat{\rho} = \sqrt{z^2+a^2}$

$\vec{A} = \frac{\mu_0 I}{4\pi}\int \frac{ds}{R} = \frac{\mu_0 I}{4\pi}\int \frac{a\,d\phi\,d\theta}{\sqrt{z^2+a^2}}$

$= \frac{\mu_0 I a}{4\pi\sqrt{z^2+a^2}}\, 2\pi \cdot \sin\alpha\, \hat{y} = \boxed{\frac{\mu_0 I a \sin\alpha}{2\pi\sqrt{z^2+a^2}}\hat{y}}$ ✓

$\alpha = \pi \Rightarrow \vec{A} = 0$ that doesn't make sense, I just don't know why.

8. Wangsness 16-14.

Uniform ∞ sheet of current in xy plane, carrying current in y-direction:

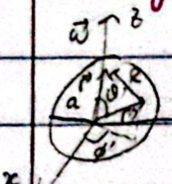
$B = \begin{cases} -\frac{\mu_0}{2}k\hat{x}\ (z>0) ✓ \\ \frac{\mu_0}{2}k\hat{x}\ (z<0) ✓ \end{cases}$

$\vec{\nabla}\times\vec{A} = \vec{B}$, using Stock's Theorem: $\oint A\cdot dl = \int_A B\cdot dA$

$\Rightarrow A = \frac{\vec{B}\times\vec{r}}{2} = -\frac{\mu_0 k}{2}|z|\cancel{(\hat{x}\times\vec{r})}\ \hat{y}$

$\Rightarrow \begin{cases} A = -\frac{\mu_0 k}{4}(z\hat{y} - k\hat{y}) \text{ for } z>0 \\ A = \frac{\mu_0 k}{4}(z\hat{y} - k\hat{y}) \text{ for } z<0 \end{cases}$ ✗

9. Wangsness 16-15.



Assuming $\vec{\omega} = \omega\hat{z}$

$\vec{r}' = r\sin\theta'\cos\phi'\hat{x} + r\sin\theta'\sin\phi'\hat{y} + r\cos\theta'\hat{z}$

$\vec{v} = \vec{\omega}\times\vec{r} = (-\omega r\sin\theta'\sin\phi')\hat{x} - (-\omega r\sin\theta'\cos\phi')\hat{y}$

$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')}{R}\,d\tau$, where $J(\vec{r}') = \rho\vec{v} = \rho(\vec{\omega}\times\vec{r}')$

$= \frac{\mu_0}{4\pi}\int_0^R\int_0^\pi\int_0^{2\pi}(-\omega r'\sin\theta'\sin\phi'\hat{x} + \omega r\sin\theta'\cos\theta'\hat{y})\ldots r^2\sin\theta'\,dr\,d\theta'\,d\phi'$

But since $\int_0^{2\pi}\sin\phi'\,d\phi' = 0 = \int_0^{2\pi}\cos\phi'\,d\phi'$ $d\phi$ vanishes.

$\Rightarrow \vec{A} = 0$ ✓ on axis.

Grading space for 7→9: I got 7 & 9 correct. I am correct for finding $\vec{B}$ in 8, but I messed up A. I did the reverse order for compared to the answer. I should find $\vec{A}$ first so $\vec{\nabla}\times\vec{A} = \vec{B}$ can be easier.

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10. $L = \int \frac{dL}{dt} dt = \int T dt = \int (r \times F) dt$ (Angular momentum of particle).
Initial velocity: $\vec{v_0} = v_0 \hat{\rho}$. Since the particle is moving in B-field
$\rightarrow \vec{F} = q(\vec{v_0} \times \vec{B}) \rightarrow L = \int r \times q(v \times B) dt = \int r \times q(dl \times B) = qI \int (r \cdot B) dl - \int B (r \cdot dl)$
0 since $r \perp B$ ✓
$r \cdot dl = r \cdot dr = \frac{1}{2} d(r^2) = r dr = \frac{1}{2\pi} (2\pi r dr)$
$\rightarrow L = \frac{-q}{2\pi} \int_0^R B 2\pi r dr = \frac{-q}{2\pi} \int B da = \boxed{\frac{-q}{2\pi} \cdot \phi}$ $\rightarrow$ flux $= \int B da$
$\rightarrow$ If $\phi = 0 \rightarrow L = 0 \rightarrow \iint B \cdot da = 0$. Therefore, we can conclude that the charge leave the region pretty radially.

11. Wangsness 16-5.
$\vec{B} = B\hat{z} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$ $\begin{cases} \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = 0 & (1) \\ \frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} = 0 & (2) \\ \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = B & (3) \end{cases}$

From section 16-3

(a) $A_x = 0, A_y = Bx, A_z = \text{const}$. From (1) $\rightarrow \frac{\partial A_z}{\partial y} = 0, \frac{\partial A_y}{\partial z} = 0$
$\frac{\partial A_x}{\partial z} = 0, \frac{\partial A_z}{\partial x} = 0$; $\frac{\partial A_y}{\partial x} = B\frac{\partial x}{\partial x} = B$; $\frac{\partial A_x}{\partial y} = 0$
$\Rightarrow \frac{\partial A_y}{\partial x} - \frac{\partial A_y}{\partial y} = B - 0 = B \rightarrow \vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = 0 + 0 + 0 = 0$ $\begin{cases} A_x = 0 \\ A_y = Bx \\ A_z = \text{const} \end{cases}$

I just proof some textbook stuffs in section 16-3

(b) $A_x = -By, A_y = 0, A_z = \text{const}. \Rightarrow \frac{\partial A_z}{\partial y} = -B, \frac{\partial A_x}{\partial z} = 0, \frac{\partial A_y}{\partial z} = 0 = \frac{\partial A_y}{\partial x}$
$\frac{\partial A_z}{\partial y} = 0, \frac{\partial A_z}{\partial x} = 0 \Rightarrow$ (1) & (2) satisfied $\rightarrow$ (3) $: \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \Rightarrow 0 - (-B) = B$
$\rightarrow \vec{\nabla} \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} = -B\frac{\partial y}{\partial x} + 0 + \frac{\partial \text{const}}{\partial z} = 0$

(c) $A_x = -\frac{1}{2}By, A_y = \frac{1}{2}Bx, A_z = \text{Const}$
$\Rightarrow \vec{\nabla} \cdot \vec{A} = -\frac{1}{2}B\frac{\partial y}{\partial x} + \frac{1}{2}B\frac{\partial x}{\partial y} + \frac{\partial}{\partial z}\text{const} = 0$
$\Rightarrow \frac{\partial A_x}{\partial A_y} = -\frac{1}{2}B, \frac{\partial A_y}{\partial x} = \frac{1}{2}B$, else $= 0$. Satisfied (1) & (2) & (3)

From Gauge transformation $\vec{A_1} = (0, Bx, \text{const})$
$\vec{A_2} = (-By, 0, \text{const}) = \vec{A_1} + \vec{\nabla}\chi_{12}$
$\rightarrow (-By, 0, \text{const}) = (0, Bx, c) + \left(\frac{\partial \chi_{12}}{\partial x}, \frac{\partial \chi_{12}}{\partial y}, \frac{\partial \chi_{12}}{\partial z}\right) = \left(\frac{\partial \chi_{12}}{\partial x}, Bx + \frac{\partial \chi_{12}}{\partial y}, c + \frac{\partial \chi_{12}}{\partial z}\right)$
$\Rightarrow \begin{cases} \frac{\partial \chi_{12}}{\partial x} = -By \Rightarrow \chi_{12} = -Bxy + c' \\ \frac{\partial \chi_{12}}{\partial y} = -Bx \\ \frac{\partial \chi_{12}}{\partial z} = 0 \end{cases}$ ✓

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• $\vec{A_2} \to \vec{A_3} \to \vec{A_3} = \vec{A_2} + \vec{\nabla}\chi_{23}$

$\Rightarrow \left(-\frac{1}{2}By, \frac{1}{2}Bx, c\right) = (-By, 0, c) + \left(\frac{\partial \chi_{23}}{\partial x}, \frac{\partial \chi_{23}}{\partial y}, \frac{\partial \chi_{23}}{\partial z}\right)$

$\Rightarrow \left(-\frac{1}{2}By, \frac{1}{2}Bx, c\right) = \left(-By + \frac{\partial \chi_{23}}{\partial x}, \frac{\partial \chi_{23}}{\partial y}, c + \frac{\partial \chi_{23}}{\partial z}\right)$

$\Rightarrow -\frac{1}{2}By = -By + \frac{\partial \chi_{23}}{\partial x} \Rightarrow \frac{\partial \chi_{23}}{\partial x} = \frac{1}{2}By \Rightarrow \chi_{23} = \frac{1}{2}Bxy + c''$ ✓

• $\vec{A_3} \to \vec{A_2} \Rightarrow \vec{A_2} = \vec{A_3} + \vec{\nabla}\chi_{23}$

$\Rightarrow (0, Bx, c) = \left(-\frac{1}{2}By + \frac{\partial \chi_{23}}{\partial x}, \frac{1}{2}Bx + \frac{\partial \chi_{23}}{\partial y}, c + \frac{\partial \chi_{23}}{\partial z}\right)$

$\Rightarrow -\frac{1}{2}By + \frac{\partial \chi_{23}}{\partial x} = 0 \Rightarrow \frac{\partial \chi_{23}}{\partial x} = -\frac{1}{2}By \Rightarrow \chi_{23} = -\frac{1}{2}Bxy + c''$ ✓

Grading space for 10+11: All correct for 10+11.

Conclusion: I guess I did relatively well this homework. My mistakes would be about not be able to find correct $\vec{A}$ in defined the region (sign) of $\vec{B}$ wrong. That just proof I didn't understand what was going on (which I admit, I didn't) so I need to work on my understanding more.