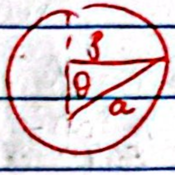
HOMEWORK6

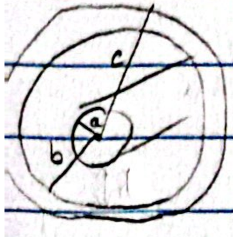
1. Wangsness 12-3 $|\vec{v}| = \omega r = \omega a$ ✓

$$I = \frac{dq}{dt} = \underbrace{\frac{Q}{2\pi a}}_{\lambda}\frac{dr}{dt} = \boxed{\frac{Q}{2\pi}\omega} \checkmark$$



$= r\sin\theta$
$\rho = a\sin\theta$

$$\vec{J} = \rho\vec{v} = \frac{Q}{4\pi a^3}\omega\vec{r} = \frac{Q}{4\pi a^3}3\omega r\sin\theta\,\hat{\phi} \checkmark$$



2. Wangsness 12-10. $E_{r>c} = 0$ (No charge enclose)

$E_{r<a} = E_{c>r>b} = 0$ (conductor)

$\rightarrow E_{b>r>a} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$ ✓, $\Delta\phi = -\int \vec{E}\cdot d\vec{s}$

$\rightarrow \phi_b - \phi_a = \int_a^b \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}\,dr = \frac{\lambda}{2\pi\varepsilon_0}[\ln(b) - \ln(a)]$ ✓ $= \Delta\phi \Rightarrow V$

$\rightarrow C = \frac{Q}{V} = \frac{\lambda l}{\frac{\lambda\ln(b/a)}{2\pi\varepsilon_0}} = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}$ $\oint\vec{D}\cdot d\vec{a} = Q_f$

$\rightarrow I = \sigma\frac{\lambda}{\varepsilon}$ ↑

$\rightarrow I = C\frac{dV}{dt} = \frac{2\pi\varepsilon_0 l}{\ln(b/a)}\Delta\phi$ $\lambda = \frac{2\pi\Delta\phi\sigma L}{\ln(b/a)}$

$I = \iint \vec{J}\cdot d\vec{a} = \iint \sigma\vec{E}\cdot d\vec{a}$ $\vec{J} = \sigma\vec{E} = \int_0^L\int_0^{2\pi}\frac{\sigma\lambda\hat{\rho}}{2\pi\varepsilon\rho}\cdot\rho\,d\phi\,dz\,\hat{\rho}$

Grading Space for 1&2: I got 1 correct, 2 is a little bit wrong because I actually didn't know how to get to I using E & Δφ, but now I understand we can use the charge density and $\vec{D}\cdot d\vec{a}$ for this.

3. $\vec{E} = E_0\hat{z}$, $\vec{B} = B_0\hat{x}$, $\omega = \frac{qB}{m}$

Lorentz force: $\vec{F} = q\vec{E} + q(\vec{v}\times\vec{B}) = qE_0\hat{z} + q(\vec{v}\times B_0\hat{x})$

$$\vec{v}\times B_0\hat{x} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ v_x & v_y & v_z \\ B_0 & 0 & 0 \end{vmatrix} = v_z B_0\hat{y} - v_y B_0\hat{z} \checkmark$$

$$\Rightarrow \vec{F} = qE_0\hat{z} + qB_0 v_z\hat{y} - qB_0 v_y\hat{z} = (qE_0 - qB_0v_y)\hat{z} + qB_0v_z\hat{y} = m\frac{d\vec{v}}{dt}$$

$m\frac{dv_x}{dt} = 0$ (1), $m\frac{dv_y}{dt} = qB_0v_z$ (2), $m\frac{dv_z}{dt} = qE_0 - qB_0v_y$ (3)

Initial Con: $t=0 \Rightarrow v_x = v_y = v_z = 0$.

$\frac{dv_x}{dt} = 0 \Rightarrow v_x = C_1 = 0$ at initial con.

(3): $\frac{d^2v_z}{dt^2} = \frac{q\cancel{E_0}^0}{m} - \frac{qB_0}{m}\frac{dv_y}{dt} = \frac{-q^2B_0^2}{m^2}v_z$ (from (2))

$= -\omega^2 v_z \Rightarrow v_z = A\sin(\omega t + C_2)$

At $t=0$, $v_z = 0 = A\sin C_2 \Rightarrow C_2 = 0 \Rightarrow v_z = A\sin\omega t$

Substitute v_z to (2) $\Rightarrow m\frac{dv_y}{dt} = qB_0A\sin\omega t \Rightarrow \frac{dv_y}{dt} = \omega A\sin\omega t$

$\Rightarrow v_y = -A\cos\omega t + C_3$. Initial Con $\Rightarrow 0 = -A + C_3 \Rightarrow C_3 = A$

$\Rightarrow v_y = A(1-\cos\omega t)$

Substitute v_z, v_y to (3) $\Rightarrow m\frac{d}{dt}A\sin\omega t = qE_0 - qB_0A(1-\cos\omega t)$

$\Leftrightarrow mA\omega\cos\omega t = qE_0 - qB_0A(1-\cos\omega t)$

$t=0 \Rightarrow mA\omega = qE_0 \Rightarrow A = \frac{qE_0}{m\omega} = \frac{qE_0}{m}\cdot\frac{m}{qB_0} = \frac{E_0}{B_0}$

$\Rightarrow v_x = 0$, $v_y = \frac{E_0}{B_0}(1-\cos\omega t)$, $v_z = \frac{E_0}{B_0}\sin\omega t$

Initial Con: $t=0 \Rightarrow x=y=z=0$ (particle @ origin)

$\Rightarrow \frac{dx}{dt} = 0 \Rightarrow x = C_4 = 0$.

$\Rightarrow \frac{dy}{dt} = \frac{E_0}{B_0}(1-\cos\omega t) \Rightarrow y = \frac{E_0}{B_0}\left(t - \frac{\sin\omega t}{\omega}\right) + C_5$

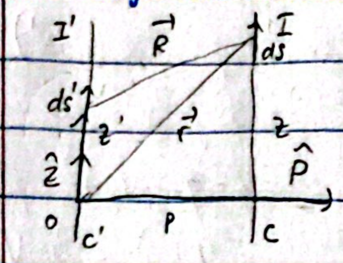
At $t=0 \Rightarrow C_5 = 0 \Rightarrow \boxed{y = \frac{E_0}{B_0}\left(t - \frac{\sin\omega t}{\omega}\right)} \checkmark$

$\Rightarrow \frac{dz}{dt} = \frac{E_0}{B_0}\sin\omega t \Rightarrow z = -\frac{E_0}{B_0}\frac{\cos\omega t}{\omega} + C_6$. At $t=0 \Rightarrow C_6 = \frac{E_0}{B_0\omega}$

$$\boxed{\Rightarrow z = \frac{E_0}{B_0\omega}(1-\cos\omega t)} \checkmark$$

Grading Space for 3: I did it all correct.

4. Wangsness 13-1.



According to Biot-Sawarts law, magnetic field due to I is: $d\vec{B} = \frac{\mu_0}{4\pi} \frac{I d\vec{s}' \times \hat{R}}{R^2}$

since $\vec{B} = \frac{\mu_0}{4\pi} \oint \frac{I d\vec{s} \times \hat{R}}{R^2}$

$d\vec{F} = \vec{J} \times \vec{B} d\tau = I(d\vec{s} \times \vec{B})$ for current I

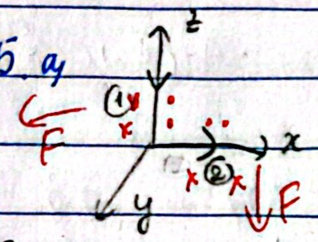
$\Rightarrow$ The force between 2 wires: $\vec{F} = \frac{\mu_0}{4\pi} I I' \oint_c \oint_{c'} \frac{d\vec{s} \times d\vec{s}' \times \vec{R}}{R^3}$

$d\vec{s} = dz\,\hat{z}$ ✓ & $d\vec{s}' = dz'\,\hat{z}$ ✓ & $R = \sqrt{\rho^2 + (z - z')^2}$,

$\vec{R} = \rho\hat{\rho} + (z - z')\hat{z}$ ✓

$\Rightarrow \vec{F} = -\frac{\mu_0 I I' \rho \hat{\rho}}{4\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{dz\,dz'}{[\rho^2 + (z - z')^2]^{3/2}}$ ✓

5. a)



$\vec{F} = q\vec{v} \times \vec{B}$

(1) Magnetic field is along $\hat{y}$

Current is along $-\hat{z}$

$\Rightarrow$ Force is along $-\hat{z} \times \hat{y} = -\hat{x}$

(?)

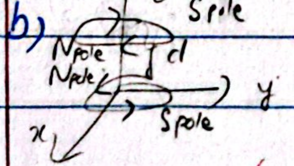
(2) Magnetic field is along $\hat{y}$

Current is along $\hat{x}$ (?)

$\Rightarrow$ Force is along $\hat{x} \times \hat{y} = -\hat{z}$ ✓

(?) The answer does seem to contradict the conservation law since the forces are not equal & opposite to each other. However, I don't think it's the case since we did not account for electric force, and the net force between (1) & (2) would obey the conservation law.

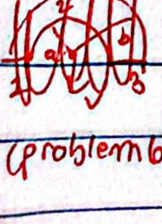
b)



$\vec{F} = I\vec{l} \times \vec{B}$

The current of 2 loops are repelling each other for CCW N-pole & CW S pole. (?)

Since the 2 N poles repell $\Rightarrow$ force on top ring will be in $\boxed{\hat{z}}$ ✓ (the bottom should be in $-\hat{z}$).



(problem 6)

Grading Space for 4 & 5: My 4 is correct but for 5, I don't really know if my logic holds.

6. Wangsness 14-15 (wrong Hw, I did 12-15 insted).

Charge q on the capacitor as a function of time is: $q(t) = \mathcal{E}C \cdot (1 - e^{-\frac{t}{RC}})$

$$I(t) = \frac{dq}{dt} = \mathcal{E}C \frac{d}{dt}(1 - e^{-\frac{t}{RC}}) = \mathcal{E}C \cdot \frac{-1}{RC} \cdot (-e^{-t/RC})$$

$$= \frac{\mathcal{E}}{R} \cdot e^{-t/RC}$$

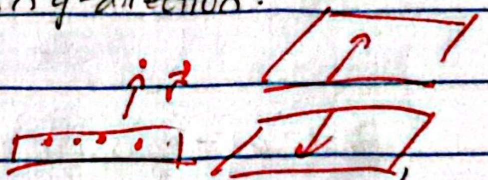
$$\Rightarrow E = \int_0^\infty \mathcal{E} I\,dt = \frac{\mathcal{E}^2}{R} \cdot e^{-t/RC}\,dt = \frac{\mathcal{E}^2}{R}\int_0^\infty e^{-t/RC}\,dt \quad u = \frac{-t}{RC},\ du = \frac{-1}{RC}dt$$

$$= \frac{\mathcal{E}^2}{R}\left(-RCe^{-t/RC}\right)_0^\infty = \frac{\mathcal{E}^2}{R}RC = \mathcal{E}^2 C$$ - final electrostatic energy.

7. Wangsness 14-9. $\vec{B} = \frac{\mu_0}{4\pi}\iint \frac{\vec{K}' \times \vec{R}\,da}{R^3} = \frac{\mu_0}{4\pi}\int_0^\infty \int_0^{2\pi} \frac{K'\hat{y} \times (z\hat{z} - \rho'\hat{\rho}')\,d\rho'\,d\varphi'\,\rho'}{(z^2 + \rho'^2)^{3/2}}$

Sheet of current lie in xy plane, current in y-direction:

$$\Rightarrow \vec{B} = \frac{\mu_0 K'}{2}\begin{cases}\hat{x} & \text{for } z > 0\\ -\hat{x} & \text{for } z < 0\end{cases}$$



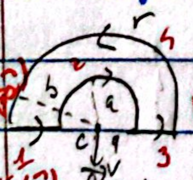
Sheet of current at $z = d \Rightarrow \vec{B}' = -\frac{\mu_0 K'}{2}\begin{cases}\hat{x} & \text{for } z > d\\ -\hat{x} & \text{for } z < d\end{cases} = \frac{\mu_0 K'}{2}\begin{cases}-\hat{x} & (z > d)\\ \hat{x} & (z < d)\end{cases}$

$\Rightarrow \vec{B}_{net} = \vec{B} + \vec{B}' = \frac{\mu_0 K'}{2}\hat{x} - \frac{\mu_0 K'}{2}\hat{x} = 0$ (for $z > d$) ✓

$\Rightarrow \vec{B}_{net} = \vec{B} + \vec{B}' = -\frac{\mu_0 K'}{2}\hat{x} + \frac{\mu_0 K'}{2}\hat{x} = 0$ (for $z < 0$) ✓

$\Rightarrow \vec{B}_{net} = \vec{B} + \vec{B}' = \frac{\mu_0 K'}{2}\hat{x} + \frac{\mu_0 K'}{2}\hat{x} = \mu_0 K'\hat{x}$ (for $0 < z < d$) ✓

6. Wangsness 14-15 (correct this time).



$\int_{0\pi}^{0} \frac{a\,d\varphi\,\hat{\varphi} \times (-a\hat{\rho})}{a^3} + \int_0^{\pi} \frac{b\,d\varphi\,\hat{\varphi} \times (-b\hat{\rho})}{b^3}$

$\frac{\mu_0 I}{4\pi}$

✓ Magnetic field at c due to a: $\vec{B} = \frac{\mu_0 I}{4a} - \hat{z}$

" b: $\vec{B}' = \frac{\mu_0 I}{4b} - \hat{z}$

No horizontal field b/c they're along the point c.

$\Rightarrow \vec{B}_{net} = \vec{B} - \vec{B}' = \left(\frac{\mu_0 I}{4a} - \frac{\mu_0 I}{4b}\right)\hat{z} = \frac{\mu_0 I}{4ab}(b-a) - \hat{z}$

$\Rightarrow \vec{F} = q\vec{v} \times \vec{B}_{net} = q\,v(-\hat{y}) \times \frac{\mu_0 I}{4ab}(b-a) \cdot (-\hat{z}) = \frac{qv\mu_0 I}{4ab}(b-a)\hat{x}$ ✓

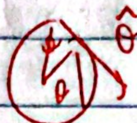
Grading Space for 6&7: I got it correct for 6&7.

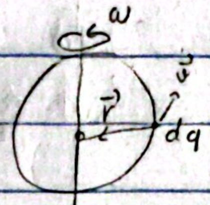
I jump the gun on 6 though. I got to write out everything more clearly.

Scanned with CamScanner

$$\vec{B} = \frac{\mu_0}{4\pi} \iiint \frac{\vec{J}(\vec{r}\,') \times \hat{R}}{R^2}\, d\tau' = \frac{\mu_0}{4\pi} \int_0^a \int_0^\pi \int_0^{2\pi} \frac{3a\omega r' \sin\theta' \hat{\phi} \times \vec{r}}{4\pi a^3}\, d\phi' \sin\theta'\, d\theta'\, r'^2 dr'$$

8. Wangsness 14-12





$$d\vec{B} = \frac{\mu_0}{4\pi} dq \frac{\vec{v} \times \vec{r}}{a^3}$$

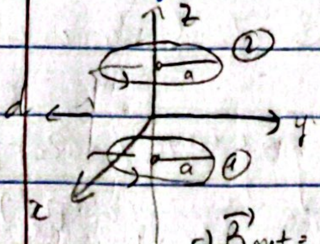
$$\vec{v} \perp \vec{r} \Rightarrow dB = \frac{\mu_0}{4\pi} dq \frac{v a \sin 90^\circ}{a^3}$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi} dq \frac{v}{a^2} = \frac{\mu_0}{4\pi} dq \frac{\omega}{a}$$

$$\Rightarrow \vec{B} = \int d\vec{B} = \frac{\mu_0 \omega}{4\pi a} \int dq\, \hat{z} = \frac{\mu_0 \omega Q}{4\pi a} \hat{z}$$

$$\vec{B} = \frac{\mu_0}{4\pi} \int_0^{2\pi} \frac{I a\, d\phi' \hat{\phi}' \times ((z - \frac{d}{2})\hat{z} - a\hat{\rho}')}{((z - d/2)^2 + a^2)^{3/2}}$$

9. Wangsness 14-6



$$\vec{B_1} = \frac{\mu_0}{4\pi} \frac{2\pi a^2 I}{[a^2 + (z - d/2)^2]^{3/2}} \hat{z} \text{ (loop 1)}$$

$$\vec{B_2} = \frac{\mu_0}{4\pi} \frac{2\pi a^2 I}{[a^2 + (z + d/2)^2]^{3/2}} \hat{z} \text{ (loop 2)}$$

$$\Rightarrow \vec{B}_{net} = \frac{\mu_0}{2} a^2 I \left\{ \frac{1}{[a^2 + (z - d/2)^2]^{3/2}} + \frac{1}{[a^2 + (z + d/2)^2]^{3/2}} \right\} \hat{z}$$

$z = 0$ at origin $\Rightarrow \vec{B}_{net} = \dfrac{\mu_0 a^2 I}{2} \left\{ \dfrac{1}{[a^2 + d^2/4]^{3/2}} + \dfrac{1}{[a^2 + d^2/4]^{3/2}} \right\} \hat{z}$

$$\Rightarrow \vec{B}_{net} = \frac{\mu_0 a^2 I}{(a^2 + d^2/4)^{3/2}}$$

Generally: $B_{net} = \dfrac{\mu_0 a^2 I}{2} \left\{ \dfrac{1}{[a^2 + (z+\ell)^2]^{3/2}} + \dfrac{1}{[a^2 + (z-\ell)^2]^{3/2}} \right\}$

where ℓ is the distance from z.

$$\Rightarrow \frac{dB_z}{dz} = \frac{\mu_0 a^2 I}{2} \left\{ -\frac{3}{2} [a^2 + (z+\ell)^2]^{-5/2}\, 2(z+\ell) - \frac{3}{2} [a^2 + (z-\ell)^2]^{-5/2} \cdot 2(z-\ell) \right\}$$

$$= -\frac{3\mu_0 a^2 I}{2} \left\{ \frac{z+\ell}{[a^2 + (z+\ell)^2]^{5/2}} + \frac{z-\ell}{[a^2 + (z+\ell)^2]^{5/2}} \right\}$$

$z = 0$ at origin

$$\Rightarrow \frac{dB_z}{dz} = -3\mu_0 a^2 I \cdot 0 = 0$$

$$\Rightarrow \frac{d^2 B_z}{dz^2} = \frac{d}{dz} \frac{dB_z}{dz} = -\frac{3\mu_0 a^2 I}{2} \left\{ [a^2 + (z+\ell)^2]^{-5/2} - 5(z+\ell)^2 [a^2 + (z+\ell)^2]^{-7/2} + [a^2 + (z-\ell)^2]^{-5/2} - 5(z-\ell)^2 [a^2 + (z-\ell)^2]^{-7/2} \right\}$$

$$\ell = d/2,\ z = 0 \Rightarrow \frac{d^2 B_z}{dz^2} = -3\mu_0 a^2 I \left(a^2 + \frac{d^2}{4}\right)^{-5/2} \left(\frac{a^2 - d^2}{a^2 + \frac{d^2}{4}} \right)$$

$$\Rightarrow \frac{d^2 B_z}{dz^2} = 0 \text{ when } a = d$$

when $d = a$, $\vec{B}_{z=0} = \dfrac{\mu_0 I a^2 \hat{z}}{(a^2 + a^2/4)^{3/2}} = \dfrac{\mu_0 I a^2 \hat{z}}{(5/4)^{3/2} a^3} \Rightarrow \vec{B}_{z=0} = \left(\dfrac{4}{5}\right)^{3/2} \dfrac{\mu_0 I \hat{z}}{a}$

Grading space for 8&9: I got it correct for both 8 & 9. ~~I~~
~~just write it too much is a shortcut~~ Nothing else to say I guess.
Conclusion: I think this is one of the easier homework for me, I understand the material farely well. The only big problem I have during this homework is to identify $\vec{R}'$, which I should have more practice on.

Scanned with CamScanner