LAST HOMEWORK!
(I survived...)

1. Wangsness 20-1:

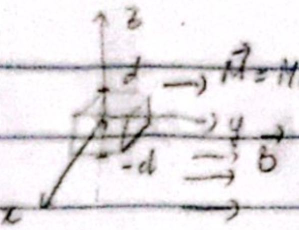
$m = 55.8 \, g/mole$

$\left.\begin{array}{l} \rho = 7870 \, kg/m^3 = 7.87 \, g/cm^3 \\ \mu_e = 9.27 \cdot 10^{-24} \, A\text{-}m^2 \\ l = 5cm \Rightarrow V = 125 \, cm^3 \end{array}\right\} \Rightarrow M = V\rho = 125 \cdot 7.87 = 983.75 \, g$

$\Rightarrow n = \dfrac{M}{m} = \dfrac{983.75}{55.8} \approx 17.63 \cdot 6.023 \cdot 10^{23} \approx 1.062 \cdot 10^{25}$ atoms

$\Rightarrow P_{max} = n\mu_e = 1.062 \cdot 10^{25} \cdot 9.27 \cdot 10^{-24} \approx 98.447 \, A\text{-}m^2$

$\Rightarrow B = \dfrac{P_{max}}{V} = \dfrac{98.447}{125 \cdot 10^{-6}} \approx 787579.2 \, \dfrac{A}{m}$

2. 

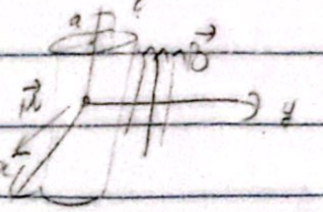
$\vec{J}_m = \vec{\nabla}\times\vec{M} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & M_0 & 0 \end{vmatrix} = 0$, $\vec{K}_m = \vec{M}\times\vec{n} = M_0\hat{x}$

$\vec{\nabla}\times\vec{H} = \vec{J}_f + \frac{\partial \vec{D}}{\partial t} = 0 \Rightarrow \vec{H} = 0$

$\Rightarrow \vec{B} = \vec{M}\mu_0 = \mu_0 M_0\hat{y}$ (in)

No magnetisation outside the slab $\Rightarrow \vec{B} = 0$ (out)

3. $\vec{J}_m = \vec{\nabla}\times\vec{M} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ M_0 & 0 & 0 \end{vmatrix} = 0$, $\vec{K}_m = \vec{M}\times\vec{n} = M_0\sin\phi\,\hat{z}$



$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{encl} = \mu_0\int_S \vec{J}\cdot d\vec{a}$

$\vec{B} = \frac{\mu_0 M_0}{2}\hat{x}$

4. 

a) $\vec{M}_{r<a} = \vec{M}_{r>b} = 0$

$\vec{B}_{a<r<b} = \frac{\mu}{2\pi r}\int \vec{J}\cdot d\vec{a} = \frac{\mu}{2\pi r}\int_0^{2\pi}\int_0^a J_0\, r\,dr\,d\phi$

$\rightarrow \vec{B}_{a<r<b} = \frac{\mu_0}{2\pi r} J_0\pi a^2 = \frac{\mu J_0 a^2}{2r} \Rightarrow \vec{H}_{a<r<b} = \frac{J_0 a^2}{2r}\hat{\phi}$

• $r<a$ $\rightarrow \vec{B}_{r<a} = \frac{\mu_0 J_0 r^2\pi}{2\pi r} = \frac{\mu_0 J_0 r}{2}\hat{\phi} \Rightarrow \vec{H}_{r<a} = \frac{J_0 r}{2}\hat{\phi}$

• $r>b$ $\rightarrow \vec{B}_{r>b} = \frac{\mu_0 J_0 a^2}{2r}\hat{\phi} \Rightarrow \vec{H}_{r>b} = \frac{J_0 a^2}{2r}\hat{\phi}$

$\Rightarrow \vec{M}_{a<r<b} = \frac{1}{\mu_0}\vec{B} - \vec{H} = \frac{J_0 a^2}{2r}\left(\frac{\mu}{\mu_0} - 1\right)\hat{\phi}$

*) $r<a$: $\vec{M} = 0$, $\vec{B} = \frac{\mu_0 J_0 r}{2}\hat{\phi}$, $\vec{H} = \frac{J_0 r}{2}\hat{\phi}$

*) $a<r<b$: $\vec{M} = \frac{J_0 a^2}{2r}\left(\frac{\mu}{\mu_0} - 1\right)\hat{\phi}$, $\vec{B} = \frac{\mu J_0 a^2}{2\rho}\hat{\phi}$, $\vec{H} = \frac{J_0 a^2}{2r}\hat{\phi}$

*) $r>b$: $\vec{M} = 0$, $\vec{B} = \frac{\mu_0 J_0 a^2}{2\rho}\hat{\phi}$, $\vec{H} = \frac{J_0 a^2}{2r}\hat{\phi}$

Grading Space for 1-4a: (No self-grading I forgot...)

4b) $\vec{J}_m = 0$ (for $r<a$ & $r>b$)

• $a<r<b$. $\vec{J}_m = \nabla\times\vec{M} = \frac{J_0 a^2}{2}\left(\frac{\mu}{\mu_0}-1\right)\underbrace{\left(\nabla\times\frac{1}{r}\hat{\phi}\right)}_{0} = 0$

$$\vec{K}_m = \vec{M}\times\hat{n} = \frac{J_0 a}{2}\left(\frac{\mu}{\mu_0}-1\right)\hat{z} \quad (r=a)$$
$$= \frac{J_0 a^2}{2b}\left(\frac{\mu}{\mu_0}-1\right)(-\hat{z}) \quad (r=b)$$

$\vec{M}$ is in $\hat{\phi}$ & certainly appears to circle around z axis? Using the right hand curl rule, $\vec{M}$ is in the $\hat{\phi}$ around the $\hat{z}$ axis.

Verify charge transfered across a circular cross section = 0? The circular cross section = 0 for $\vec{M} = 0$ at $r<a$ & $r>b$

c) At $r=a \rightarrow \vec{B} = \frac{\mu J_0 a}{2}\hat{\phi} \Rightarrow \vec{H} = \frac{J_0 a}{2}\hat{\phi} \neq \vec{H}_{a<r<b} = \frac{J_0 a^2}{2r}\hat{\phi}$

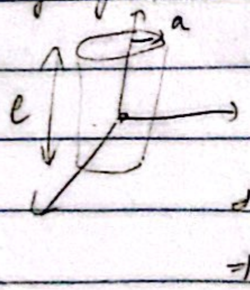
At $r=b \Rightarrow \vec{H} = \frac{J_0 a^2}{2r} = \vec{H}_{a<r<b} = \frac{J_0 a^2}{2r}$

⇒ Parallel component of $\vec{H}$ are discontinuous at $r=a$ & continuous at $r=b$ ⇒ boundary condition is satisfied.

d) $U_m = \frac{1}{2}\int \vec{B}\cdot\vec{H}\,d\tau = \frac{1}{2}\int_0^{2\pi}\int_0^{L}\int_a^{b}\left(\frac{J_0^2 a^4 \mu_0}{4r^2} + \frac{\mu J_0^2 a^4}{4r^2}\right) r\,dr\,dz\,d\phi$

$$= \frac{J_0^2 a^4 L\pi}{4}\left[\frac{\mu_0}{4} + \mu\ln\frac{b}{a} + \mu_0\ln\frac{r}{b}\right]$$

5. Center of cylinder is at the origin, aligning with the z-axis:



⇒ $\nabla\times H = 0$, $M=0$ outside cylinder

, $H = -\nabla\phi_M$, $M = M_0\hat{z}$ inside

& $\nabla\cdot B = 0 \Leftrightarrow \nabla\cdot(\mu_0 H + \mu_0 M) = 0$

⇒ $\nabla\cdot H = -\nabla\cdot M \Leftrightarrow \nabla^2\phi_M = -\nabla\cdot M = -\rho_M$

Magnetization = const inside ⇒ divergence = 0.

$\rho_M = M_0[\delta(z-l/2) - \delta(z+l/2)]$

⇒ $\phi_M = \frac{1}{4\pi}\int \rho_M\,d\tau$ (Poisson equation)

$$\Rightarrow \phi_M = \frac{1}{4\pi}\int_{-\infty}^{\infty}\int_0^{2\pi}\int_0^{\infty} \frac{\rho_M}{r^2+r'^2-2rr'\cos(\phi-\phi')+(z-z')^2}\, r'dr'd\phi'dz'$$

$$= \frac{1}{4\pi}\int_{-\infty}^{\infty}\int_0^{2\pi}\int_0^{\infty} \frac{M_0[\delta(z'-L/2)-\delta(z'+L/2)]}{\cdots}\, r'dr'd\phi'dz'$$

$$= \frac{M_0}{4\pi}\left[\int_0^{2\pi}\int_0^{a} \frac{r'dr'd\phi'}{\sqrt{r^2+r'^2-2rr'\cos(\phi-\phi')+(z-L/2)^2}} - \int_0^{2\pi}\int_0^{a}\frac{r'dr'd\phi'}{\cdots}\right]$$

For $r=0 \Rightarrow \phi_M = \frac{M_0}{4\pi}\left[\int_0^{2\pi}\int_0^{a}\frac{r'dr'd\phi'}{\sqrt{r'^2+(z-L/2)^2}} - \int_0^{2\pi}\int_0^{a}\frac{r'dr'd\phi'}{\sqrt{r'^2+(z+L/2)^2}}\right]$

$$\Rightarrow \phi_M = \frac{M_0}{2}\left[\sqrt{a^2+(z-L/2)^2} - \sqrt{a^2+(z+L/2)^2} - (z-L/2) + (z+L/2)\right]$$

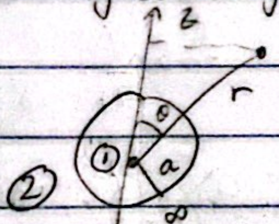
$$\Rightarrow H = -\frac{M_0}{2}\left[\frac{z-L/2}{\sqrt{a^2+(z-L/2)^2}} - \frac{z+L/2}{\sqrt{a^2+(z+L/2)^2}} - \frac{z-L/2}{z-L/2} + \cancel{\frac{z+L/2}{z+L/2}}\right]\hat{z}$$

$\Rightarrow B = \mu_0 H + \mu_0 M = -\frac{\mu_0 M_0}{2}\left[\frac{z-L/2}{\sqrt{a^2+(z-L/2)^2}} - \frac{z+L/2}{\sqrt{a^2+(z+L/2)^2}}\right]\hat{z}$

(For $r<a$) problem 3 answer!

(*) Demagnetizing factor: $\nabla\cdot\vec{H} = -4\pi\nabla\cdot\vec{M} \Rightarrow \nabla^2\phi = -4\pi\nabla\cdot\vec{M}$

6.



Sphere with permeability $\mu_1 = \mu$ in some material with permeability $\mu_2 = \mu_0$

Magnetic field strength $\vec{H_0} = H_0\hat{z}$

$$\phi_1(r,\theta) = \sum_{n=0}^{\infty}\left(A_n r^n + \frac{B_n}{r^{n+1}}\right)P_n(\cos\theta) \quad (1) \quad (r<a)$$

external field

$$\phi_2(r,\theta) = \sum_{n=0}^{\infty}\left(C_n r^n + \frac{D_n}{r^{n+1}}\right)P_n(\cos\theta) + \phi_0 \quad (2) \quad (r>a)$$

$\vec{H_0} = -\vec{\nabla}\phi_0 \Leftrightarrow H_0\vec{z} = -\frac{\partial\phi_0}{\partial z}\hat{z} \Leftrightarrow \phi_0 = -\int H_0 dz = -H_0 r\cos\theta$

- $\phi_1 = \infty$ inside the sphere $\Rightarrow B_n = 0$.
- $r\to\infty \Rightarrow \phi_2 \to \phi_0 \Rightarrow C_n = 0$

Boundary conditions:
- $\phi_1 = \phi_2$ at $r=a$
- $\mu_1\frac{\partial\phi_1}{\partial r} = \mu_2\frac{\partial\phi_2}{\partial r}$ at $r=a$

$\Leftrightarrow \begin{cases} A_n = 0 \\ D_n = 0 \end{cases}$ for $n \geq 2$

$\Rightarrow \phi_1 = A_0 + A_1 r\cos\theta \quad , \quad \phi_2 = \frac{D_1}{r^2}\cos\theta - H_0 r\cos\theta$

6. Wangsness 20-21 (cont.)

Boundary conditions: $A_0 + A_1 a\cos\theta = \frac{D_1}{a^2}\cos\theta - H_0 a\cos\theta$

$\mu_1 A_1 \cos\theta = \mu_2\left(-\frac{2D_1}{a^3} - H_0\right)\cos\theta$

$\Rightarrow \frac{D_1}{a^3} - H_0 = -\frac{\mu_2}{\mu_1}\left(\frac{2D_1}{a^3} + H_0\right) \Rightarrow D_1 = \frac{\mu_1-\mu_2}{\mu_1+2\mu_2} H_0 a^3$

$\Rightarrow A_1 = \frac{\mu_1-\mu_2}{\mu_1+2\mu_2} H_0 - H_0 = \frac{-3\mu_2}{\mu_1+2\mu_2} H_0$

$\Rightarrow \phi_1 = \frac{-3\mu_2}{\mu_1+2\mu_2} H_0 r\cos\theta \Rightarrow \vec{H_1} = -\frac{\partial\phi_1}{\partial z}\hat{z} = \frac{3\mu_2 H_0}{\mu_1+2\mu_2}\hat{z}$

$\phi_2 = \frac{\mu_1-\mu_2}{\mu_1+2\mu_2}\frac{H_0 a^3}{r^2}\cos\theta - H_0 r\cos\theta \Rightarrow \vec{H_2} = -\frac{\partial\phi_2}{\partial z}\hat{z}$

$= \frac{\mu_1-\mu_2}{\mu_1+2\mu_2}\frac{H_0 a^3}{r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}) + H_0\hat{z}$

We have the relative permability $K_m = \frac{\mu_1}{\mu_2} = \frac{\mu}{\mu_0}$

$\Rightarrow \vec{H_1} = \frac{3H_0}{K_m+2}\hat{z}, \quad \vec{H_2} = \frac{K_m-1}{K_m+2}\frac{H_0 a^3}{r^3}(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}) + H_0\hat{z}$

7. Wangsness 21-1 Area S, $d = d_0 + d_1\sin\omega t$

$S H = \mu_0 I_D(t) \Rightarrow \vec{H} = \frac{\mu_0 I_D(t)}{S}$, $C = \frac{\varepsilon_0 S}{d}$, $Q = C\varepsilon$

$\Rightarrow I_D(t) = \frac{dQ}{dt} = \varepsilon\frac{dC}{dt} = \varepsilon_0 S\frac{d}{dt}\frac{1}{d} = \varepsilon_0 S\frac{d}{dt}\frac{1}{d_0 + d_1\sin\omega t}$

$\Rightarrow I_D(t) = -\frac{\varepsilon_0 S d_1 \omega\cos(\omega t)}{2d^2} \Rightarrow \vec{H} = \frac{-\mu_0\varepsilon_0\varepsilon d_1\cos(\omega t)\,\omega}{2d^2}(-\hat{\phi})$

Capacitor disconnect from battery $\Rightarrow \vec{H} = 0$ (no stored charges)

8. a) $\vec{J} = \sigma\vec{E}$, $\vec{\nabla}\cdot\vec{J} + \frac{\partial\rho_{ch}}{\partial t} = 0 \Leftrightarrow \vec{\nabla}\cdot(\sigma\vec{E}) + \frac{\partial\rho_{ch}}{\partial t} = 0$

$\vec{\nabla}\cdot\vec{E} = \frac{\rho_{ch}}{\varepsilon_0} \Leftrightarrow \frac{\sigma\rho_{ch}}{\varepsilon_0} + \frac{\partial\rho_{ch}}{\partial t} = 0 \Leftrightarrow \frac{\partial\rho_{ch}}{\partial t} = -\frac{\sigma\rho_{ch}}{\varepsilon_0}$

$\Rightarrow \int\frac{\partial\rho_{ch}}{\rho_{ch}} = \int\frac{\sigma}{\varepsilon_0}dt \Rightarrow \log\rho_{ch} = -\frac{\sigma}{\varepsilon_0}t + C$

$\Rightarrow \rho_{ch} = Ce^{-\sigma/\varepsilon_0 t}$

At $t=0 \Rightarrow \rho_{ch} = \rho_{ch_0} \Rightarrow C = \rho_{ch_0}$

$\Rightarrow \rho_{ch} = \rho_{ch_0} e^{-\sigma/\varepsilon_0 t}$

b) No displacement current outside the plate $\Rightarrow \oint\vec{H}\cdot d\vec{l} = 0 \Rightarrow \vec{H} = 0$.

Scanned with CamScanner