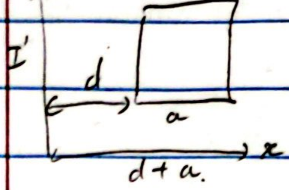
HOMEWORK 8

1. Wangsness 17-3



$b \quad I' = I_0 e^{-\lambda t}$

$B = \frac{\mu_0 I'}{2\pi x} \Rightarrow \phi = \int d\phi = \int_d^{d+a} B\,dA = \int_d^{d+a} \frac{\mu_0 I'}{2\pi x} b\,dx$

$\Rightarrow \phi = \frac{\mu_0 I'}{2\pi} b\left[\ln(a+d) - \ln d\right] = \frac{\mu_0 I'}{2\pi} b \ln\left(\frac{a+d}{d}\right)$

$\Rightarrow \mathcal{E} = \frac{d\phi}{dt} = \frac{d\left(\frac{\mu_0 I'}{2\pi}\right) b \ln\left(\frac{a+d}{d}\right)}{dt} = \frac{\mu_0}{2\pi} b \ln\left(\frac{a+d}{d}\right) \frac{dI'}{dt}$

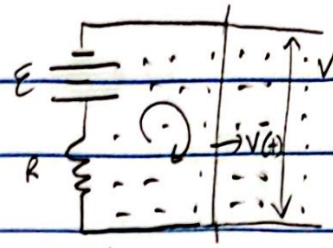
$\Rightarrow \mathcal{E} = -\frac{\mu_0}{2\pi} b \ln\left(\frac{a+d}{d}\right) I_0 e^{-\lambda t} \cdot \lambda$. Induced current direction: CW

2. a) Current develop in loop 2 only in $-\hat{y}$ direction. Since $\vec{B} \perp d\vec{a} \Rightarrow$ if $K\uparrow$, there will be an induced current.

Flux = 0 in loop 1 & 3 when $K\uparrow \Rightarrow$ no induced current.

b) $P = \frac{\mathcal{E}^2}{R}$, $|\mathcal{E}| = \left|\frac{d\phi}{dt}\right|$

Grading Space for 1 & 2:

3. 

Induced emf: $\mathcal{E} = \frac{d\phi}{dt} = \frac{d}{dt}(Blx) = BlV$

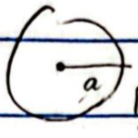
Effective current: $\frac{V}{R} = I = \frac{\mathcal{E} - BlV}{R}$

Force on the wire from B: $F = BIl = m\frac{dV}{dt} = \left(\frac{\mathcal{E}-BlV}{R}\right)Bl$

$$\Rightarrow \int_0^V \frac{dV}{\frac{\mathcal{E}Bl}{R} - \frac{B^2l^2V}{R}} = \int_0^t \frac{dt}{m} \Rightarrow -\frac{R}{B^2l^2}\ln\left|\frac{\frac{\mathcal{E}Bl}{R}-\frac{B^2l^2V}{R}}{\mathcal{E}Bl/R}\right| = \frac{t}{m}$$

$$\Rightarrow 1-\frac{BlV}{\mathcal{E}} = e^{-\frac{B^2l^2t}{Rm}} \Rightarrow V = \frac{\mathcal{E}}{Bl}\left(1-e^{-\frac{B^2l^2t}{mR}}\right)$$

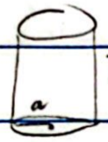
4. $Q=\int I\,dt$, $I = \frac{\mathcal{E}}{R} \Rightarrow Q = \int \frac{1}{R}\frac{d\phi}{dt}dt = \frac{\phi_f-\phi_i}{R}$ (Wangsness 17-6).

5. 

$\vec{B} = B_0\hat{z}$

$\vec{F}_B = q\vec{v}\times\vec{B} = q\rho\omega\hat\phi \, B_0\hat z = qa\omega\sin\theta B_0\hat\rho \Rightarrow \vec{F}_E = -\vec{f}_B = -a\omega B_0\sin\theta\hat\rho$

$\Rightarrow \phi_{equator} - \phi_{North} = -\int_0^{\pi/2}(-a\omega B_0\sin\theta\hat\rho)\,a\,d\theta\,\hat\theta = \frac{1}{2}\omega B_0 a^2$

6. 

$\vec{F}_B = q\vec v\times\vec B = q\rho\omega B\hat\rho$ $\quad \sigma_b = (K_e-1)\epsilon_0 a\omega B$

$\vec P = \chi_e\epsilon_0\vec E = (K_e-1)\epsilon_0\rho\omega B\hat\rho$ $\quad \rho_b = -\vec\nabla\cdot\vec P = -(K_e-1)\epsilon_0\omega B(2)$

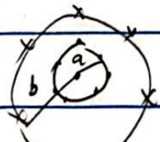
$Q = \sigma_0(2\pi a L) + \rho_b(\pi a^2 L) = 0$ (Wangsness 17-7).

7. a) $\oint \vec E\cdot d\vec s = -\frac{d\phi_E}{dt} \Rightarrow E\, 2\pi\rho = -\frac{d}{dt}(\mu_0 n I_0 e^{-t/\tau}\pi\rho^2)$

$\Rightarrow \vec E = \frac{\mu_0 n I_0 \rho e^{-t/\tau}}{2\tau}\hat\phi$

b) $E(2\pi\rho) = -\frac{d}{dt}(\mu_0 I_0 e^{-t/\tau} n\pi R^2) \Rightarrow \vec E = \frac{\mu_0 n I_0 R^2 e^{-t/\tau}}{2\rho\tau}\hat\phi$

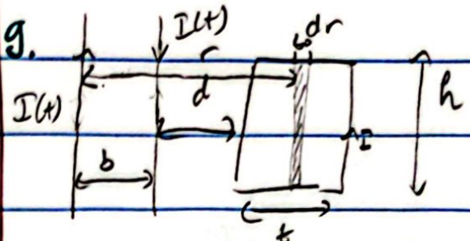
$\Rightarrow \vec a_e = \frac{e\vec E}{m} = \frac{e\mu_0 I_0 R^2}{2m\rho\tau}(-\hat\phi)$

8. (Wangsness 17-24). 

$\oint \vec B\cdot d\vec s = \mu_0 I_{encl} \Rightarrow \vec B = \begin{cases}0 & \text{for } \rho<a\\ \frac{\mu_0 I}{2\pi\rho} & \text{for } a<\rho<b \\ 0 & \text{for }\rho>b\end{cases}$

Grading Space for 3-8:

9.



Let current goes up in left & down in right wire. $B = \frac{\mu_0 I}{2\pi r}$

Magnetic field due to left wire: $B_L = \frac{\mu_0 I}{2\pi r}$

" right wire: $B_R = \frac{\mu_0 I}{2\pi(r-b)}$

$\Rightarrow B_{net} = B_R - B_L$ (opposite direction)

$= \frac{\mu_0 I}{2\pi}\left(\frac{1}{r-b} - \frac{1}{r}\right)$

Magnetic flux through the strip: $d\phi = B_{net} \times dA = B_{net}\, h\, dr$

$$\Rightarrow \phi = \int d\phi = \frac{\mu_0 I h}{2\pi}\int_{b+d}^{b+d+t}\left(\frac{1}{r-b} - \frac{1}{r}\right)dr = \frac{\mu_0 I h}{2\pi}\left[\ln\left(\frac{d+t}{b+d+t}\right) - \ln\left(\frac{d}{b+d}\right)\right]$$

$$= \frac{h\mu_0 I_0 \sin\omega t}{2\pi}\ln\frac{(d+t)(b+d)}{d(b+d+t)} \quad (I = I_0\sin\omega t)$$

$$\Rightarrow \mathcal{E} = \frac{d\phi}{dt} = \frac{d}{dt}\left[\frac{h\mu_0 I_0 \sin\omega t}{2\pi}\ln\frac{(d+t)(b+d)}{d(b+d+t)}\right.$$, $\mathcal{E}_{max}$ when $\omega t = \pi/2$

$$\Rightarrow \mathcal{E}_{max} = \frac{\mu_0 I_0 h\omega}{2\pi}\ln\frac{(d+t)(d+b)}{d(b+d+t)}$$

Grading Space for 9: