A Study of Covariance Cleaning Methods in Hourly Frequency Ideal Mean-Variance Trading

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Abstract—**The estimation of covariance matrices in modern portfolio theory is crucial for optimal investment strategies. This study investigates various covariance cleaning methods in hourly frequency ideal mean-variance trading. We analyze methods including empirical estimation, eigenvalue clipping, Ledoit-Wolf linear shrinkage, optimal approximate shrinkage, RIE optimal shrinkage, and BAHC filtering. Using a high-resolution dataset of 85 stocks, we assess the impact of these methods on portfolio optimization. The results show significant differences in portfolio performance and risk profiles across methods. L1 distance metrics reveal nuanced variations in portfolio compositions, highlighting the importance of effective risk management and strategic alignment. While statistical approaches offer valuable insights, limitations include reliance on historical data, Gaussian distribution assumptions, and computational instability. Future research may explore alternative methodologies to enhance portfolio optimization in real-world stock market scenarios.**

I. Introduction

In modern portfolio theory, the estimation of the covariance matrix of asset returns is a critical component of many investment strategies. In fact, even slightly inaccurate covariance misestimates can lead to awfully suboptimal portfolios because many optimization techniques and algorithms are very sensitive to the covariance matrix estimate. One such popular strategy is Mean-Variance optimization whose optimal solution involves the inverse of the covariance matrix.

Many techniques [1] [2] [3] [4] [5] [6] and others have been developped to improve the estimate of the covariance matrix. Many rely on the notion of covariance matrix shrinkage, which is basically cleaning the covariance matrix by correcting its eigenvalues, while others rely on clustering and heuristics.

In this paper, we study the impact of some covariance matrix cleaning techniques using empirical covariance estimation as baseline. Our study is performed on a very high resolution dataset of 85 stocks between early 2005 and late 2008. We process the data in a way that complies with the goals of our study, utilizing in the process big data concepts and frameworks such as multiprocessing, distributed and cluster computing, adapted data formats and compression, optimizations for compute-intensive tasks, and topped by numerical cleaning. We justify and use ideal Mean-Variance optimization as investment strategy, ideal in the sense that we do not account for liquidity, transaction costs, or risk aversion.

We conduct a series of experiments aimed at evaluating the efficacy of various covariance cleaning methods in the context of hourly frequency ideal mean-variance trading. These experiments involve the application of different techniques, including empirical covariance estimation, eigenvalue clipping, Ledoit-Wolf linear shrinkage, optimal approximate shrinkage, RIE optimal shrinkage, and BAHC filtering, to assess their impact on portfolio optimization. Through meticulous analysis of a high-resolution dataset comprising 85 stocks over a specific time frame, we seek to answer fundamental questions regarding the suitability and performance of each method in improving the accuracy and stability of covariance matrix estimation. Our experiments focus on examining the resulting portfolio compositions, risk profiles, and performance metrics under different cleaning methodologies, providing insight into the comparative effectiveness and practical implications of each approach.

II. Data: Sources and Processing

A. Raw Dataset

We use a dataset of very high resolution stock prices of 85 assets from the S&P100 between January 2005 and December 2008.

The source data contains the following columns of interest:

- *xltime*: the time instant based on the Excel format and epoch.
- *bid-price*: the bid price at the given instant.
- *ask-price*: the ask price at the given instant.

The source data format is a hierarchy of TAR archives containing GZIP compressed files. The compressed raw dataset size is 24GB.

B. Data Processing

The data processing can be summarized in the following steps:

1) *Time*: We converted Excel time instants to pythonic date and time objects for ease of use.
2) *Missing values*: We estimate the sequences of missing values using linear interpolation, forward filling, and backward filling. Linear interpolation is used when there exists at least one known value before and after the sequence of missing values. Forward filling is used to estimate the sequence of missing values that occurs at the end of a time series in accordance with the principle of propagating the last known value. Backward filling is used to estimate the sequence of missing values that occurs at the start of a time series in accordance with the principle of propagating the first known value.
3) *Subsampling*: Very high frequency data exhibits little to no change over a big number of observations, and would only add on unnecessary computational complexity and numerical issues such as matrix singularity. Consequently, we subsample the data by averaging the mid-prices until we get hourly frequency which we deem sufficiently high frequency for the purpose of our study. The mid prices are computed as the middle point between the bid price and the ask price.
4) *Asset Returns*: We compute the return of any asset between two consecutive instants based on the mid prices.

The processing is done using a distributed computation framework as discussed in section V-A.

III. Covariance Matrix Estimation and Cleaning

We study a variety of covariance cleaning methods. Some covariance cleaning methods require an estimate of the covariance matrix in which case we use the empirical estimator.

A. Empirical Covariance Estimation (Baseline)

The empirical covariance estimation without any cleaning is used as baseline.

B. Eigenvalue Clipping

Eigenvalue clipping is a cleaning method derived from random matrix theory and precisely from the Marcenko-Pastur distribution [7] that identifies the random bulk, a component assumed to have an extremely low SNR, which is eliminated to obtain the clean matrix. It does so by clipping the eigenvalues of the original matrix. In our case, we apply this method to the correlation matrix and recover the clean covariance matrix from it. It is indeed better to apply this theory to the correlation matrix because its entries are bounded. For the mathematical formulation, we encourage the reader to check the paper [1].

C. Ledoit-Wolf Linear Shrinkage

Ledoit-Wolf Linear Shrinkage [2] cleans the empirical covariance matrix by finding an optimal linear shrinkage factor λ^* typically chosen to minimize a Mean Squared Error (MSE) by cross-validation.

Mathematically, the shrinked covariance matrix $\hat{\Sigma}$ is given in terms of the empirical covariance estimate Σ and the shrinkage factor λ as follows:

$$\hat{\Sigma} = (1 - \lambda)\Sigma + \lambda \text{diag}(\Sigma) I$$

The optimal shrinkage factor λ^* is optimized over K cross-validation steps. In what follows, Σ_i is the empirical covariance matrix estimated on the validation partition at step i of the cross-validation, whereas $\hat{\Sigma}_i$ is the shrinked covariance matrix computed on the estimation partition at step i of the cross-validation using the candidate λ:

$$\lambda^* = \text{argmin}_\lambda \frac{1}{K} \sum_{i=1}^{K} E[||\Sigma_i - \hat{\Sigma}_i||_F^2]$$

Note that we use an empirical estimate of the expectation above.

D. Optimal Approximate Shrinkage (RBLW)

The Optimal Approximate Shrinkage method is an improvement over the Ledoit-Wolf method. It is approximately optimal in the minimal mean squared error (MMSE) sense. In fact, the estimator of this method goes a step further than LW by condtionning on a sufficient statistic. It is known the Rao-Blackwell Ledoit-Wolf estimator. Then, it uses an iterative algorithm that converges to the optimal estimate of the true covariance matrix. We encourage the reader to get familiar with this technique by reading the paper [3].

E. RIE Optimal Shrinkage

Optimal shrinkage using Rotation Invariant Estimators (RIE) [4] is interesing because estimating the true covariance matrix boils down to estimating the eigenvalues of that matrix. In fact, for an RIE Ξ such that $\hat{\Sigma} = \Xi(\Sigma)$, and given that Σ is Hermitian symmetric:

$$\hat{\Sigma} = \Xi(\Sigma) = \Xi(UDU^T) = U\Xi(D)U^T$$

Consequently, as long as $\Xi(D) \approx D^*$ we will have that $\hat{\Sigma} \approx \Sigma^*$ the true covariance matrix and independently of the unitary (rotation) matrix U. Clearly, this is also a shrinkage method since it acts on the eigenvalues.

The theory behind building such an estimator uses advanced Random Matrix Theory, algebraic, and probabilistic concepts that include the Stieltjes Transform. We leave it to the reader to read the papers [4] and [8].

F. BAHC Filtering

Bootstrap-Averaged Hierarchical Clustering is an enhanced hierarchical approach designed to better capture the structure of eigenvectors and that is inspired the similar HCAL method. The former method involves computing filtered hierarchical structures from multiple bootstrapped samples of the original data, resulting in probabilistic hierarchical structures. This procedure enhances the description of correlation and covariance matrices, ensuring improved structural understanding while maintaining the robustness of hierarchical clustering. We invite the reader to refer to [5] and [6] for a full description of this technique.

IV. Experiment Description

A. Overview

In our experiments, keeping everything else unchanged, we analyse the covariance matrix cleaning methods detailed in section III. Using a trading strategy that relies on covariance matrix estimation, namely Mean-Variance optimization as will be discussed in section IV-B, and in the spirit of backtesting, we fit the strategy to a long training window and evaluate it on a timely adjacent short testing window. Fitting the strategy mainly boils down to estimating the covariance matrix of asset returns. For each pair of training and testing windows, we obtain the following:

- an estimate of the mean vector of asset returns;
- an estimate of the asset covariance matrix based on the predetermined covariance cleaning method;
- the optimal portfolio, its return, and its standard deviation.

B. Trading Strategy: Mean-Variance Optimization

The choice of the trading strategy settled on Mean-Variance Optimization (MVO) for two crucial reasons.

Firstly, Mean-Variance portfolio optimization is extremely sensitive to the estimate of the covariance matrix of the asset returns. In fact, the strategy involves the inverse of that matrix. Numerically, inversion is a highly unstable operation due to the ill-conditioning of nearly singular matrices. Covariance matrices of asset returns are likely to be ill-conditionned due to the curse of dimensionality that arises when considering a large number of assets with a relatively low number of observations as is our case. Furthermore, due to the nature of the asset returns themselves which are known to have a very low Signal-to-Noise Ratio (SNR) which can lead to spurious correlations and colinearity between asset returns based on a limited set of observations, the covariance matrix is even more likely to be ill-conditionned. Therefore, we can conclude that Mean-Variance optimization is a good strategy to assess the impact of covariance cleaning methods as small changes to the covariance matrix can lead to potentially significant divergences in the estimated optimal portfolios at any given point in time.

Secondly, Mean-Variance portfolio optimization is a very simple technique whose effectiveness is entirely based on the goodness of estimation of the mean vector of asset returns but even more importantly the covariance matrix of asset returns. As there are no other factors involved, and given that the mean vector estimate is the same in any given time window and independently of the covariance cleaning method, we can attribute any divergences in optimal portfolios directly to the covariance estimation method. This reinforces MVO as our strategy of choice to better grasp the impact of different covariance cleaning methods.

In conclusion, Mean-Variance portfolio optimization is very convenient for the purpose of your study.

C. Training and Testing Windows

The training window is 90 days long with hourly observations, taking into account only the assets for which there is at least a single original observation within that window. By original observations, we mean those that do not result from missing value filling or interpolation. That is we do not consider an asset in a training window in which we have no true and only assumed or interpolated returns of that asset. It is also a requirement that the training window contains a minimum of 240 observations to avoid the potential scenario of very scarce observations, in which case the empirical covariance estimator has a rather very large variance.

The test window is 7 days long during which we evaluate the performance of the obtained optimal portfolio. The latter is held constant throughout the testing window.

In summary, we use around 90 days of observations to estimate an optimal portfolio that is held for the next 7 unobserved days. Afterwards, we shift both windows by 7 days, the size of the testing window, and repeat this process.

D. Asset Short Selling

In all of our experiments, we allow short selling up to 20%. In section VI, we will look at whether any given covariance cleaning method may disagree with the baseline in terms of whether to short or long any given asset.

E. Instability Handling

Throughout our experiments, and despite all efforts, it may still be possible that the clean covariance matrix is nearly singular in which case the optimization step is unstable. To overcome this issue, we extend the training window by at most a single day at a time until the clean covariance matrix is no longer nearly singular. We also put a small limit on the number of extensions allowed to ensure that the training windows always remain of very comparable size. If the limit of extensions is exceeded, the considered training window becomes just the original training window shifted forward by the number of extension days. This means that a number of extension days is skipped at the start of the original window while keeping the same original window size. This approach drastically reduced instabilities while providing some guarantees on comparable window sizes across different cleaning methods. We have not been able to find any other way to overcome this issue without giving up on sizable amounts of data.

V. Distributed Computing, Optimizations and Implementation Details

A. Distributed Data Processing

The raw data is tabular so we opt for Dask as our distributed data processing framework. It is a powerful framework that focuses on ease of use through the similarity of its API to the Pandas API, a library we are very familiar with. We utilized Dask locally to load, transform, clean and save the processed data in a GZIP compressed Parquet format. The Parquet format was chosen due to it's partitioned column-wise storage which makes it easy select and manipulate specific columns.

B. Compression

Since we had limited disk space, we opted for GZIP compression for data at rest given the high compression ratio of GZIP. However, there is no free lunch since it requires full file decompression to manipulate a single column in a Parquet file as opposed to other compression codecs such as Snappy which is block-wise and would allow us to decompress only the manipulated columns. Nevertheless, this trade-off is acceptable for us because we have enough memory resources and would rather prefer low storage space requirements.

C. Leveraging CUDA GPUs

We re-implemented many codebase blocks and functions, including covariance estimation and cleaning methods, from different libraries such as PyRMT and Sklearn in order to make them compatible with Pytorch tensors. Consequently, we were able to leverage CUDA GPUs on a SLURM cluster in order to significantly speed up our experiments and development lifecycle.

D. Multiprocessing

Our experiments required several IO operations to log our sizable results (up to 30MB/IO-op). IO operations are an inconvenient bottleneck in compute-intensive tasks such as our experiments because the program needs to stop until it goes through with the IO operation. To avoid the IO bottleneck, we leverage multiprocessing by queuing IO operations on a process pool separate from the main process, with dedicated CPU resources. Since most of our compute tasks are run on GPU, we allow the process pool to use most of our available CPU cores to go through with the IO operations, effectively allowing us to log previous results while building the next ones in parallel.

E. Numerical Cleaning

In the codebases that we re-implemented, we managed to add additional guarantees against the propagation of numerical inaccuracies. Those guarantees took the form of extra numerical cleaning steps as well as sanity checks. For instance, we enforced that the covariance matrix of asset returns always be hermitian symmetric and positive semi-definite.

VI. Results and Analysis

As previously discussed, covariance matrices are important in modern portfolio theory, as they provide information about the collective movement of asset prices and help to achieve efficient portfolio selection. The covariance matrix shows the interactions between assets, and indicates which combinations can yield maximum profits with minimum volatility (reduced risk). Particular asset allocations (portfolio weights) are typically found by optimization using the sample covariance matrix. The estimation of the sample covariance matrix however is problematic: when there are many stock price series of limited duration, their statistical characteristics can not be determined accurately as their computation accumulates significant errors. The reason is that financial data is typically contaminated by large amounts of noise, while standard estimation procedures cannot remove the noise and fail to adequately identify the structure of dependencies among the stocks in the market. Using a distorted matrix for optimization results in inaccurate weights and leads to portfolios with unsatisfactory performance. Robust portfolios can be designed after cleaning the covariance matrix.

Table I outlines the six different strategies, including their names and respective cleaning method.

TABLE I: Mapping of Strategies

Strategy name	**Cleaning method**
BASE	Baseline (no cleaning)
LWLS	Ledoit-Wolf Linear Shrinkage
OAS	RBLW Optimal Approximate Shrinkage (OAS)
RIE	RIE Optimal Shrinkage
EigClip	Eigenvalue Clipping
BAHC	Bootstrap-Averaged Hierarchical Clustering Filtering

A. Summary statistics

Table II provides the performance metrics achieved by the various cleaning methods. They are the return, risk, and simple Sharpe Ratio (i.e., SR without a risk-free asset). Note that since there is no risk-free asset, the simple SR isn't very representative. The Appendix A also contains the plots of the returns and standard deviation of each strategy over time. This proposes an intuitive and visual way of comparing the results of each strategy in each time window.

TABLE II: Summary Statistics

Strategy	**Returns**	**Std**	**Sharpe Ratio**
BASE	0.014	0.0030	4.755
LWLS	0.015	0.0040	3.774
OAS	0.016	0.0033	4.893
RIE	0.017	0.0035	4.890
EigClip	0.020	0.0034	5.931
BAHC	0.021	0.0036	5.911

B. Portfolio divergence with Manhattan distance

While the previous metrics offer valuable insights into the overall behavior of investment portfolios, they may present certain deficiencies when used in isolation.

One of the primary limitations of cumulative statistics lies in their inability to capture nuanced differences and structural variations between portfolios. Mean returns and standard deviations provide aggregate measures that obscure the individual dynamics of portfolio constituents, leading to a simplified representation of portfolio behavior. As a result, portfolios with similar cumulative statistics may exhibit distinct underlying compositions and risk profiles, rendering traditional metrics insufficient for comprehensive analysis.

Moreover, cumulative statistics fail to account for the relative importance and contribution of individual assets within a portfolio. By treating all assets equally in the calculation of summary statistics, these metrics overlook the differential impact of asset allocation decisions on portfolio performance and risk exposure. Consequently, portfolios with divergent asset compositions may yield identical cumulative statistics, masking the inherent differences in their investment strategies and asset allocations.

In contrast, L1 distance offers a robust alternative for quantifying portfolio divergence by measuring the absolute differences between corresponding elements of two portfolios. By focusing on the individual weights or returns of assets, L1 distance provides a granular assessment of portfolio dissimilarity, capturing the unique characteristics and structural nuances of each portfolio. This level of granularity enables investors to discern subtle variations in asset allocation strategies, identify potential sources of divergence, and tailor their investment decisions accordingly.

Let's start by looking at the portfolio differences that result from the different covariance cleaning methods by looking at how far away the portfolios are from the baseline 'BASE'. See table III.

Note that the strategy using OAS is quite close to the baseline, while eigenvalue clipping generates a big distance. However, larger distance doesn't necessarily equate to better overall performance as can be seen by RIE which is the second closest to the baseline yet has the 3rd highest aggregate return.

TABLE III: Distances from baseline strategy

Strategy	Distance
OAS	28.21
RIE	69.93
LWLS	98.25
BAHC	154.61
EigClip	364.29

Next up, we look at the average distances of each strategy compared to the rest. See IV. Interestingly, linear shrinkage is the technique that is closest to all others, and by a large margin and that. However no useful scientific conclusions can be made.

TABLE IV: Average Differences

Strategy	Average Difference
LWLS	66.75
OAS	112.27
RIE	115.19
BAHC	116.08
BASE	143.06
EigClip	202.21

We also look at the closest and furthest strategy pairs using the L1 distance. See tables V and VI. The linear shrinkage strategy is the closest to 4 others, which is in agreement with the fact that it has the smallest average differences as previously seen.

TABLE V: Closest Strategies

Strategy	Difference
RIE vs. LWLS	18.63
BAHC vs. LWLS	20.90
OAS vs. LWLS	27.77
BASE vs. OAS	28.21
EigClip vs. BAHC	55.74

TABLE VI: Furthest Strategies

Strategy	Difference
LWLS vs. EigClip	168.20
BAHC vs. RIE	192.71
RIE vs. BAHC	192.71
OAS vs. EigClip	238.52
BASE vs. EigClip	364.29
EigClip vs. BASE	364.29

With all of this in mind, the pairwise distances between the strategies are as follows (see table VII). There is almost a tenfold difference between the two closest and two furthest portfolios. Examining portfolios with exceptionally high or low L1 distances relative to others can help identify outliers and understand the factors contributing to their uniqueness. Outliers may represent unconventional strategies, extreme risk profiles, or significant deviations from market norms. Furthermore, grouping portfolios based on their L1 distances can facilitate cluster analysis, revealing patterns of similarity and dissimilarity among strategies. Clustering enables the identification of distinct investment styles, risk preferences, and performance characteristics across different segments of the market.

TABLE VII: L1 Distances Between Different Portfolios

Portfolio Comparison	L1 Distance
LWLS vs. OAS	27.77
OAS vs. BASE	28.21
RIE vs. LWLS	18.63
BAHC vs. LWLS	20.90
RIE vs. BASE	69.93
BAHC vs. BASE	154.61
LWLS vs. BASE	98.25
EigClip vs. BAHC	55.74
RIE vs. LWLS	18.63
BAHC vs. LWLS	20.90
BAHC vs. BASE	154.61
EigClip vs. LWLS	168.20
RIE vs. EigClip	184.28
OAS vs. RIE	110.42
OAS vs. BAHC	156.41
OAS vs. EigClip	238.52

C. Comparison of best and worst performing stocks according to their portfolio weight

In this section we look at the stocks that appear most consistently through the testing time periods across all 6 strategies. More specifically, we look at the top 5 stocks with the highest weight accorded, as well as the bottom 5 stocks (those with the lowest weight).

From tables VIII and IX, we note the following observations:

1) Common Stocks between Portfolios: Portfolios 'OAS' and 'LWLS' share the most common stocks, indicating similar investment preferences or sector exposures. Conversely, portfolios 'OAS' and 'RIE' have no common stocks, suggesting divergent investment strategies or sector allocations.

2) Consistency of Portfolio Composition: Stocks '$DOW', '$HD', '$HON' appear most frequently across top-performing portfolios, indicating consistency in portfolio composition and potential sector preferences. The absence of common stocks in bottom-performing portfolios suggests varying sector exposures or risk appetites among strategies.

3) Diversity in Holdings: Portfolios 'OAS' and 'RIE' have the most diverse holdings, encompassing a wide range of stocks, while 'OAS' and 'LWLS' share fewer stocks, highlighting potential sector concentration or alignment in investment approaches.

4) Risk Management and Diversification: The presence of common stocks across multiple portfolios may signify a collective focus on high-performing assets or strategic alignment in portfolio construction. Diversification benefits may be realized by incorporating stocks that appear most frequently across portfolios, potentially mitigating risk and enhancing overall portfolio stability.

TABLE VIII: Top 5 Performing Stocks

Comparison	Strategies	Count	Stocks
Closest strats	('OAS', 'LWLS')	3	'DOW', 'HD', 'HON'
Furthest strats	('OAS', 'RIE')	0	-
Popular Stocks	('DOW', 'HD', 'HON')	4	-
Unpopular Stocks	-	-	-

TABLE IX: Bottom 5 Performing Stocks

Comparison	Strategies	Count	Stocks
Closest strats	('OAS', 'RIE')	4	'COP', 'EMR', 'GE', 'UTX'
Furthest strats	('OAS', 'LWLS')	3	('COP', 'GE', 'UTX')
Popular Stocks	('COP', 'EMR', 'GE', 'UTX')	6	-
Unpopular Stocks	('COP', 'GE', 'UTX')	1	-

VII. Conclusion

Based on the in-depth analysis of covariance cleaning methods and their effects on portfolio optimization in hourly frequency ideal mean-variance trading, several key findings emerge:

Covariance matrices are pivotal in modern portfolio theory, aiding in efficient portfolio selection by capturing collective asset price movements. Cleaning these matrices is crucial for accurate estimation, as contaminated matrices can lead to suboptimal portfolio allocations.

We examined various covariance cleaning methods, including empirical estimation, eigenvalue clipping, Ledoit-Wolf linear shrinkage, optimal approximate shrinkage, RIE optimal shrinkage, and BAHC filtering. Each method offers distinct advantages and limitations, influencing portfolio performance and risk profiles differently.

L1 distance metrics facilitated portfolio composition comparisons, highlighting significant differences in asset allocations and risk exposures among strategies. Common stocks and consistent portfolio compositions emerged as indicators of effective risk management and strategic alignment.

While statistical approaches to covariance matrix cleaning offer valuable insights, they have limitations in real-world applications, such as reliance on historical data, Gaussian distribution assumptions, and computational instability. Future research may explore alternative methodologies to address these challenges and enhance portfolio optimization.

REFERENCES

[1] J. P. Bouchaud and M. Potters, "Financial applications of random matrix theory: a short review," 2009.

[2] O. Ledoit and M. Wolf, "A well-conditioned estimator for large-dimensional covariance matrices," *Journal of Multivariate Analysis*, vol. 88, no. 2, pp. 365–411, 2004.

[3] Y. Chen, A. Wiesel, Y. C. Eldar, and A. O. Hero, "Shrinkage algorithms for mmse covariance estimation," *IEEE Transactions on Signal Processing*, vol. 58, p. 5016–5029, Oct. 2010.

[4] J. Bun, R. Allez, J.-P. Bouchaud, and M. Potters, "Rotational invariant estimator for general noisy matrices," *IEEE Transactions on Information Theory*, vol. 62, no. 12, pp. 7475–7490, 2016.

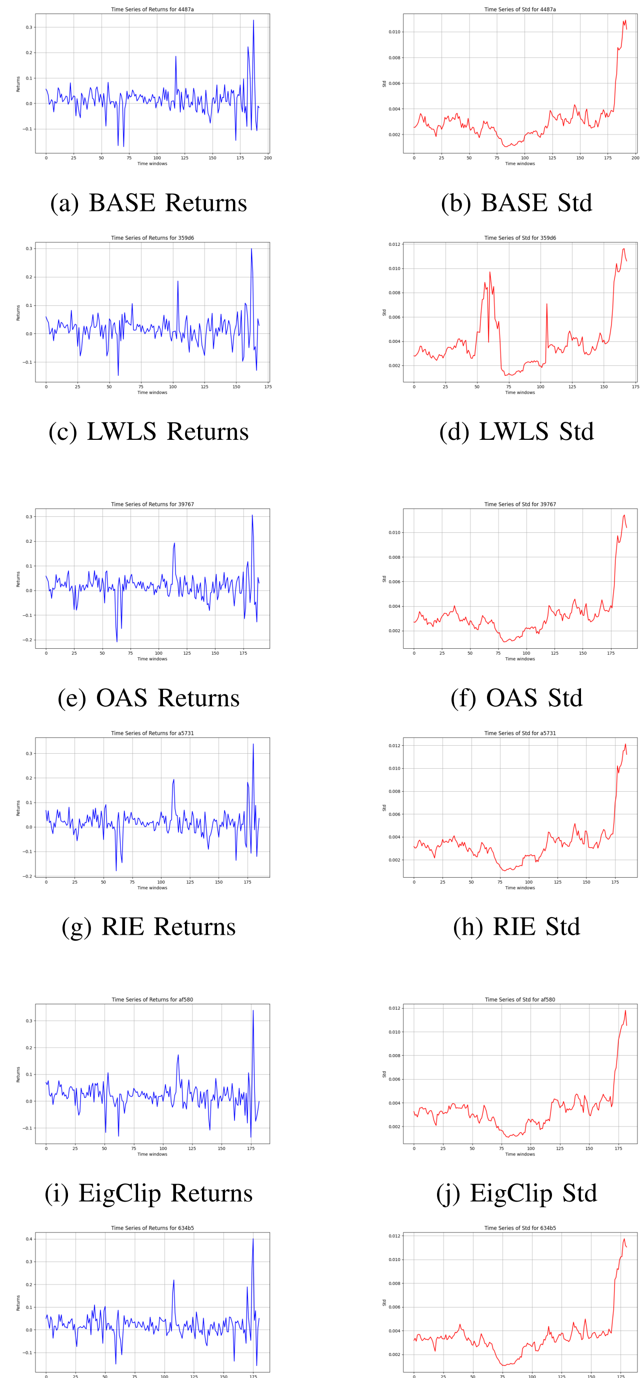
[5] C. Bongiorno and D. Challet, "Reactive global minimum variance portfolios with $k-$bahc covariance cleaning," 2023.

[6] C. Bongiorno and D. Challet, "Covariance matrix filtering with bootstrapped hierarchies," *PLOS ONE*, vol. 16, p. e0245092, Jan. 2021.

[7] V. A. Marčenko and L. A. Pastur, "Distribution of eigenvalues for some sets of random matrices," *Mathematics of the USSR-Sbornik*, vol. 1, p. 457, apr 1967.

[8] J. Bun, J.-P. Bouchaud, and M. Potters, "Cleaning large correlation matrices: Tools from random matrix theory," *Physics Reports*, vol. 666, p. 1–109, Jan. 2017.

APPENDIX



(a) BASE Returns (b) BASE Std

(c) LWLS Returns (d) LWLS Std

(e) OAS Returns (f) OAS Std

(g) RIE Returns (h) RIE Std

(i) EigClip Returns (j) EigClip Std

(k) BAHC Returns (l) BAHC Std

Fig. 1: Returns and Stds of the Different Strategies