Introduction to Computer Vision

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Lecture 3


REVIEW

Images in Python (`import numpy`)

`N x M` grayscale image "`im`"

- `im[0,0]` = top-left pixel value
- `im[y,x]` = y pixels down, x pixels to right
- `im[N-1,M-1]` = bottom-right pixel

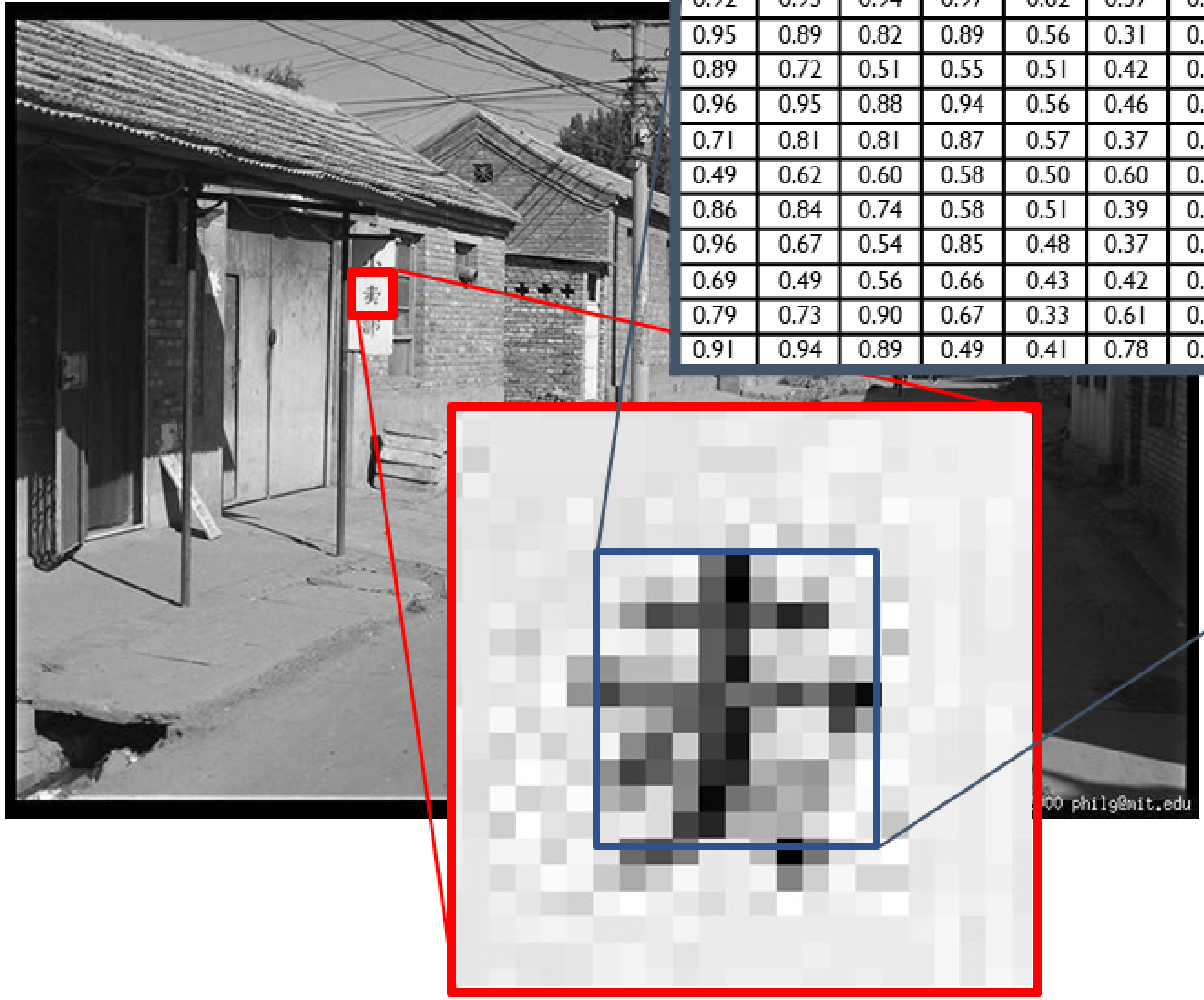
Column x

Row y

0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

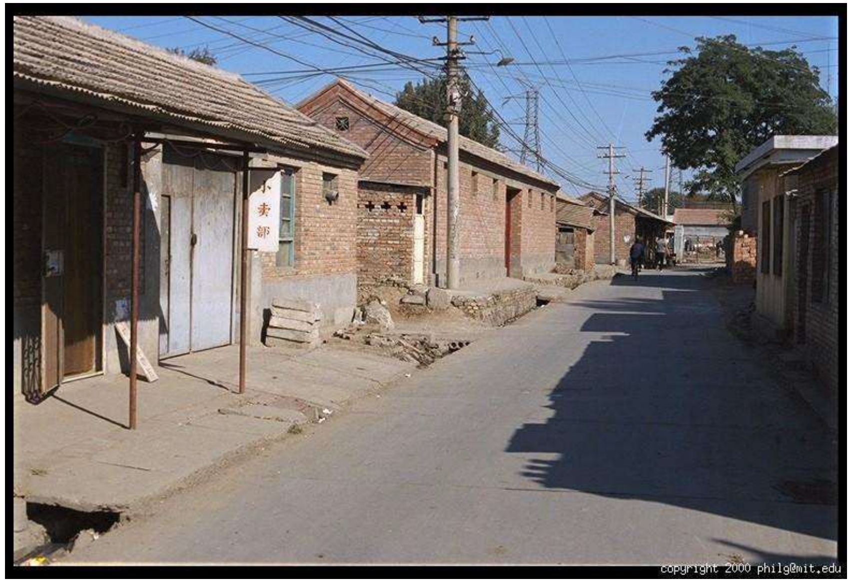
James Hays

Grayscale Intensity



0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

Color



Red intensity

Green

Blue

Images in Python (import numpy)

`N x M` grayscale image "`im`"

- `im[0,0,0]` = top-left pixel value, red channel
- `im[y,x,1]` = y pixels down, x pixels to right, green channel
- `im[N-1,M-1,2]` = bottom-right pixel, blue channel

R

0.92	0.93	0.94	0.97	0.62	0.37	0.85	0.97	0.93	0.92	0.99
0.95	0.89	0.82	0.89	0.56	0.31	0.75	0.92	0.81	0.95	0.91
0.89	0.72	0.51	0.55	0.51	0.42	0.57	0.41	0.49	0.91	0.92
0.96	0.95	0.88	0.94	0.56	0.46	0.91	0.87	0.90	0.97	0.95
0.71	0.81	0.81	0.87	0.57	0.37	0.80	0.88	0.89	0.79	0.85
0.49	0.62	0.60	0.58	0.50	0.60	0.58	0.50	0.61	0.45	0.33
0.86	0.84	0.74	0.58	0.51	0.39	0.73	0.92	0.91	0.49	0.74
0.96	0.67	0.54	0.85	0.48	0.37	0.88	0.90	0.94	0.82	0.93
0.69	0.49	0.56	0.66	0.43	0.42	0.77	0.73	0.71	0.90	0.99
0.79	0.73	0.90	0.67	0.33	0.61	0.69	0.79	0.73	0.93	0.97
0.91	0.94	0.89	0.49	0.41	0.78	0.78	0.77	0.89	0.99	0.93

G

B

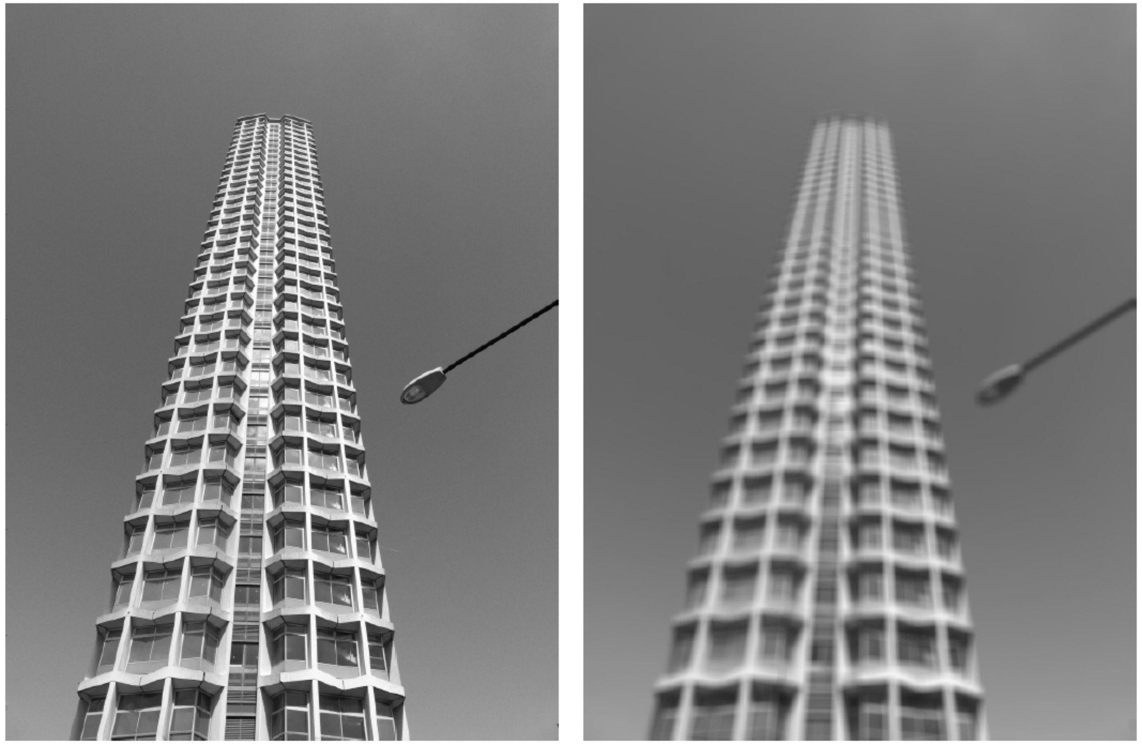
James Hays

Image Filtering

Image Filtering

Learning outcomes:

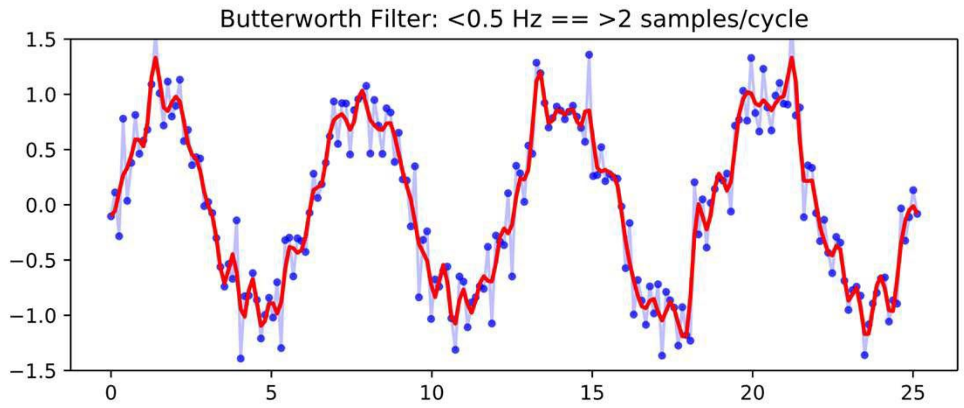
- Filter
- Image Filtering
- Low-pass Filter
- High-pass Filter
- Convolution
- Correlation
- Kernel
- Linearity and Shift Invariance
- Separability
- Sobel Filter
- Convolution
- Correlation & Template Matching
- Non-Linear Filters



Filtering

Definition

Filtering: An operation that modifies a (measured) signal.



https://ggbaker.ca/data-science/media/butter4.svg

- Modifications include
  - Removing undesirable components
  - Transforming signal in a desirable way
  - Extract specific components of a signal
- Can be analog or discrete signals
- Can be in any dimension

- mainly 2D for images (gray)
- 3D for colored

1D Filtering: Moving Average

$$I \in \mathcal{R}^{m\times 1}$$

filter signal

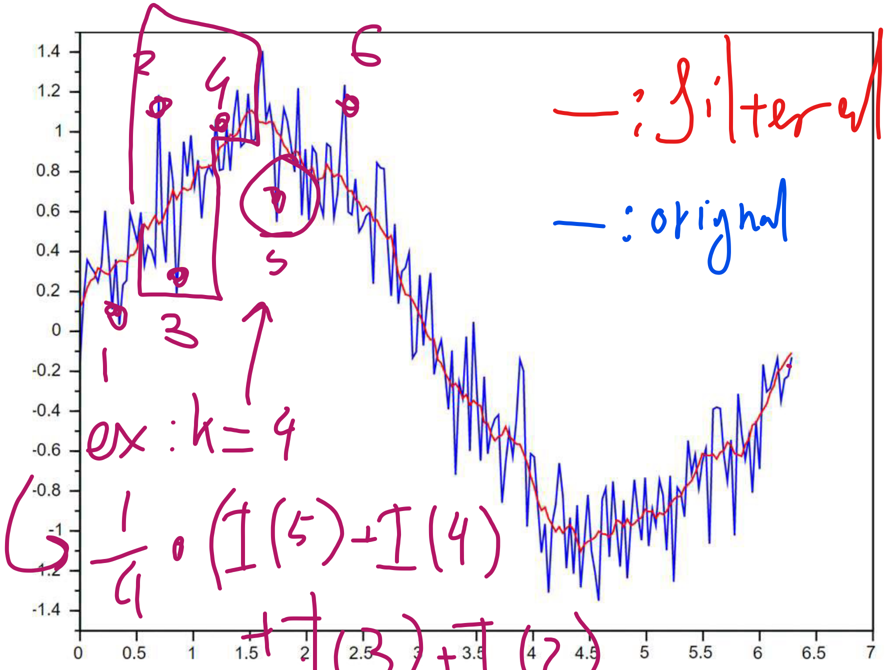
$$h[n] = \frac{1}{k}\sum_{i=n-k+1}^{n} (1)\, I[i]$$

Window size 'k'

↳ # of previous value

$$\underbrace{h[n]}_{1\times 1} = \frac{1}{k}\, \underbrace{\begin{bmatrix}1\\1\\ \vdots\\1\end{bmatrix}^T}_{1\times k} \underbrace{I[n-k+1:n]}_{\begin{bmatrix} I(n-k+1)\\ \vdots \\ I(n-1)\\ I(n)\end{bmatrix}_{k\times 1}}$$

https://en.wikipedia.org/wiki/Moving_average



— : filtered (moving average)

— : original

ex : $k = 4$

↳ $\frac{1}{4}\cdot\left(I(5)+I(4)+I(3)+I(2)\right)$

1D Filtering: Moving Average

$$I \in \mathcal{R}^{m \times 1}$$

$$h[n] = \frac{1}{k} \sum_{i=n-k+1}^{n} (1) I[i]$$

Window size ‘k’

ex: $I(n) = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ \vdots \end{bmatrix}$

$h(n) = [0001]$

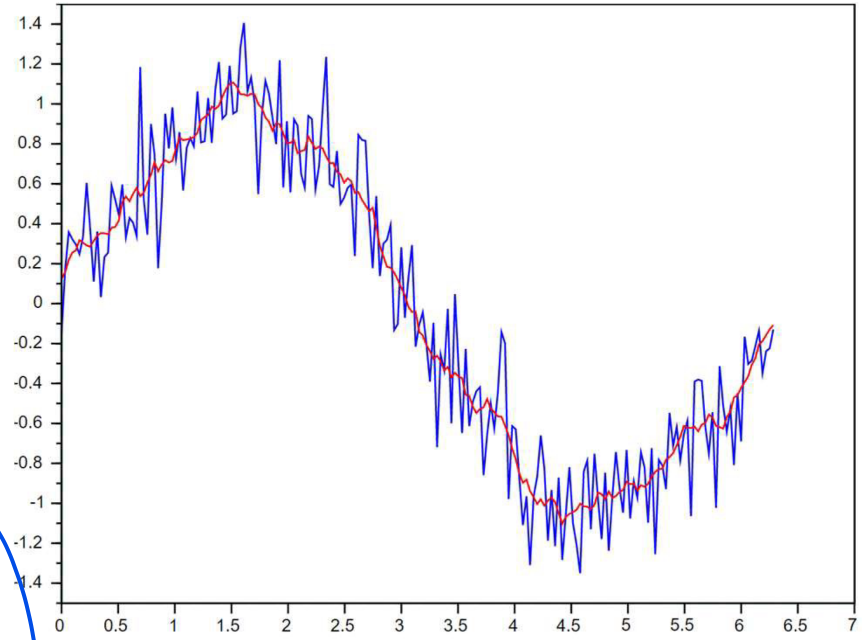
$\hookrightarrow I'(n) = \begin{bmatrix} 4 \\ 5 \\ \vdots \end{bmatrix}$ $n \geq 4$

(Identity)

$h(n) = [1000]$

$\hookrightarrow I'(n) = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(delay)



last k value of I(n)

$$h[n] = \frac{1}{k} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}^T I[n-k+1:n]$$

$I[\underset{k}{\leftarrow} o \;]$

$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$$

What is this filter? Identity

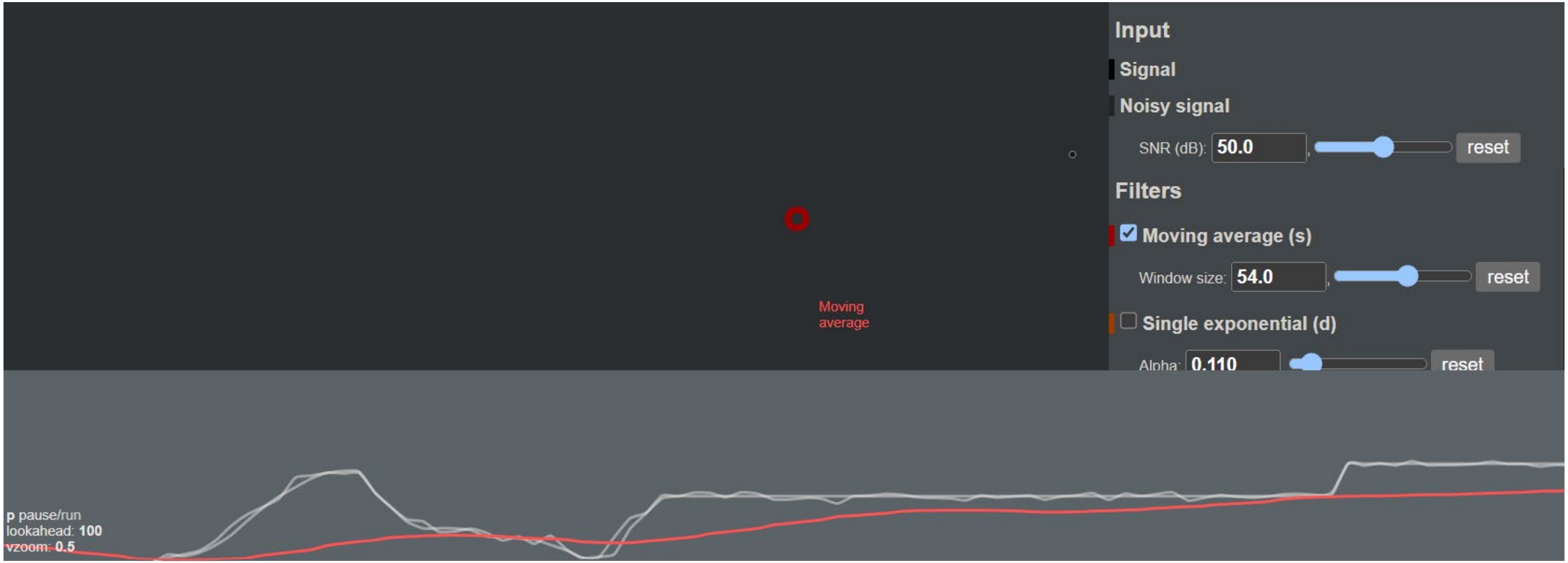
What is the difference?

$I[n-k/2 : n+k/2]$ delay filtered

$[\underset{k/2}{\leftarrow} o \underset{k/2}{\rightarrow}]$

https://en.wikipedia.org/wiki/Moving_average

Filtering Demo



https://gery.casiez.net/1euro/InteractiveDemo/

Image Filtering

Compute function of local neighborhood at each position:

$$h[m,n]=\sum_{k,l} \overset{\text{filter}}{f}[k,l]\; \overset{\text{image}}{I}[\underbrace{m+k}_{y},\underbrace{n+l}_{x}]$$

`Window size 'k' and 'l'`

Image Filtering

Compute function of local neighborhood at each position:

h=output I=image

$$h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$$

2d coords=k,l 2d coords=m,n

[ ] [ ] [ ]

Fundamental Equations of Computer Vision

1. Image Filtering $$h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$$

2. Optical Flow $$I(x,y,t)=I(x+\Delta x, y+\Delta y, t+\Delta t)$$

3. Camera Geometry $$\mathbf{x}=\mathbf{K}\left[\mathbf{R}\ \mathbf{t}\right]\mathbf{X}$$ $$x^T F x' = 0$$

4. Machine Learning $$\underset{\mathbf{S}}{\arg\min} \sum_{i=1}^{k} \sum_{\mathbf{x}\in S_i} \|\mathbf{x}-\boldsymbol{\mu}_i\|^2.$$ $$y=\varphi\left(\sum_{i=1}^{n} w_i x_i + b\right)=\varphi(\mathbf{w}^T\mathbf{x}+b)$$

Example: Box Filter / Kernel

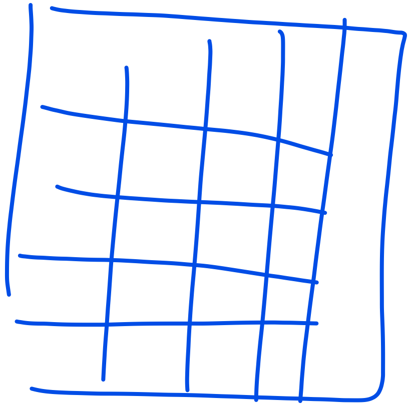
$f[\cdot,\cdot]$

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

3x3

5x5 $\frac{1}{25}$

7



- every values are 1

Step:

1. time by $\frac{1}{k}$ | 2. Sum all value in box | 3. put sum into filtered

David Lowe

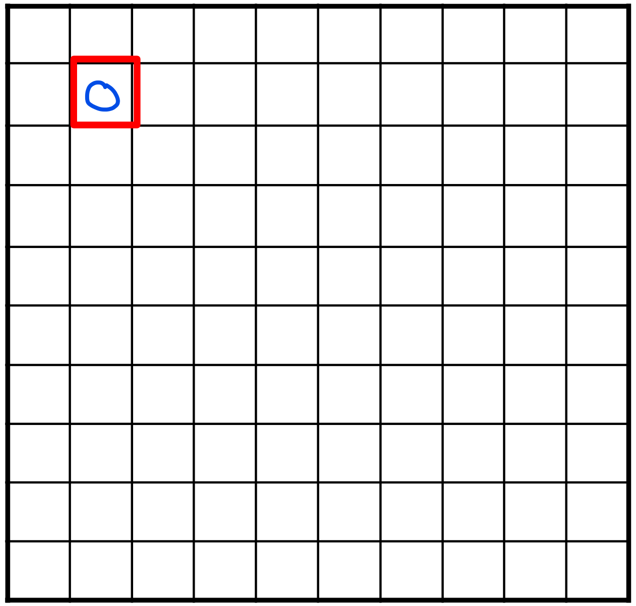
Image Filtering

$$f[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$



$$h[m,n]=\sum_{k,l} f[k,l]\ I[m+k,n+l]$$

$m = 1, n = 1$
$k,l = [-1,0,1]$

Image Filtering

$f[\cdot,\cdot]\ \frac{1}{9}$

1	1	1
1	1	1
1	1	1

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10							

$$h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$$

$m = 2, n = 1$

$k, l = [-1,0,1]$

Image Filtering

$$f[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	$\boxed{20}$						

$$h[m,n]=\sum_{k,l} f[k,l]\ I[m+k,n+l]$$

$m = 3, n = 1$
$k,l = [-1,0,1]$

Image Filtering

$$f[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30					

$$h[m,n]=\sum_{k,l} f[k,l]\ I[m+k,n+l]$$

$$m = 4, n = 1$$
$$k, l = [-1,0,1]$$

Image Filtering

$f[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				

$$h[m,n]=\sum_{k,l} f[k,l]\, I[m+k,n+l]$$

$m = 5, n = 1$

$k,l = [-1,0,1]$

Image Filtering

$$f[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90 1	0	90 3	90	90	0	0
0	0	0	90 2	90 50	90 4	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
				?					

$$h[m,n]=\sum_{k,l} f[k,l]\ I[m+k,n+l]$$

$m = 4, n = 6$
$k,l = [-1,0,1]$

Image Filtering

$$f[\cdot\,,\cdot\,]\ \frac{1}{9}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$$

$I[.,.]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$h[.,.]$

	0	10	20	30	30				
						?			
				50					

$$h[m,n]=\sum_{k,l} f[k,l]\ I[m+k,n+l]$$

$$m = 6, n = 4$$
$$k,l = [-1,0,1]$$

Image Filtering

$$f[\cdot,\cdot]\ \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

$$h[m,n]=\sum_{k,l} f[k,l]\ I[m+k,n+l]$$

→ 2D moving average

Box Filter / Kernel

What does it do?

- Replaces each pixel with an average of its neighborhood

- Achieve smoothing effect (remove 'sharp' features)

- Why does it sum to one?

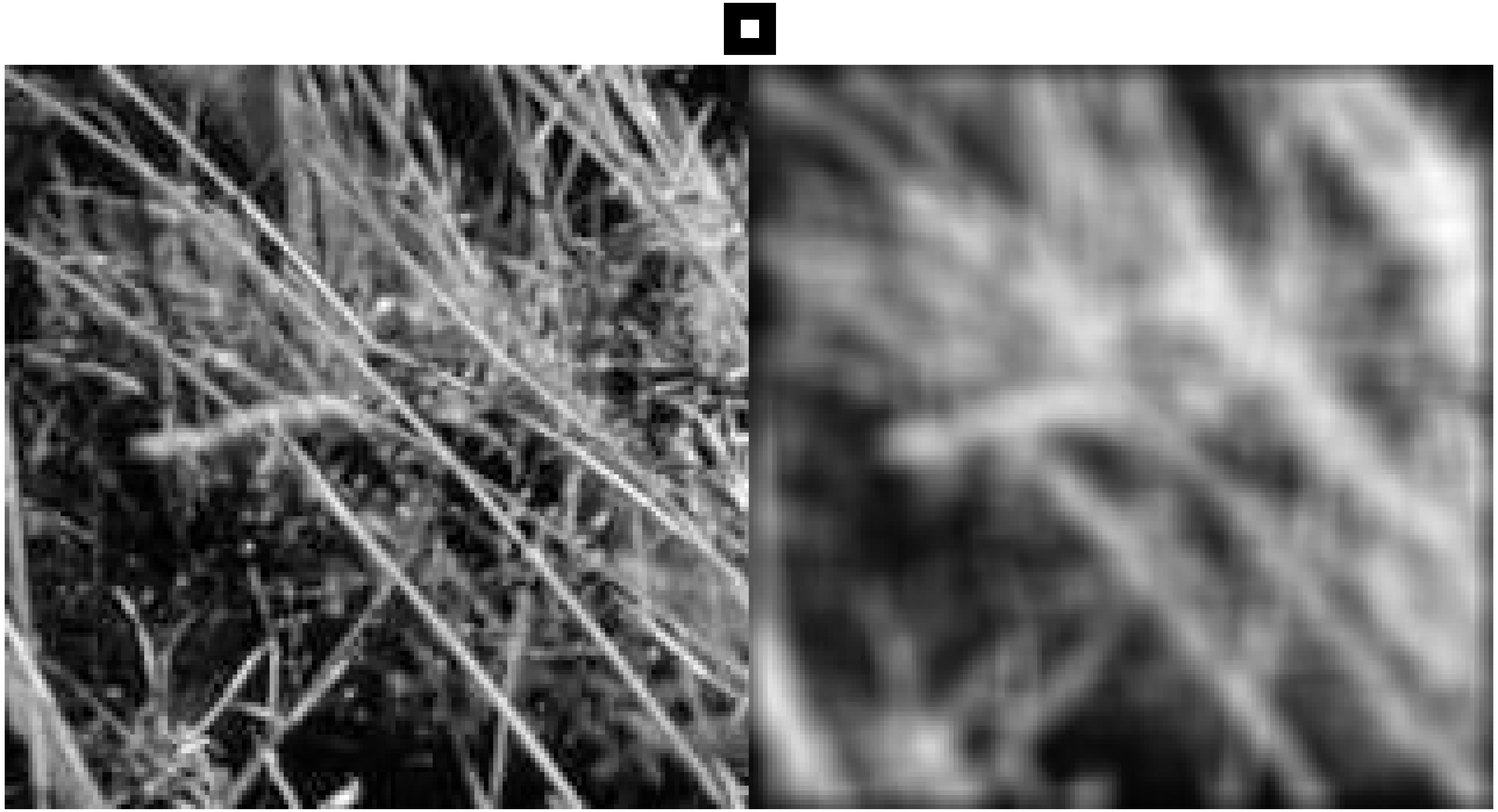
$$f[\cdot,\cdot] = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

filter size $\uparrow$ $\to$ blur $\uparrow$

$\to$ image size $\downarrow$

Smoothing with Box Filter

$$f[\cdot\,,\cdot\,]\,\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$



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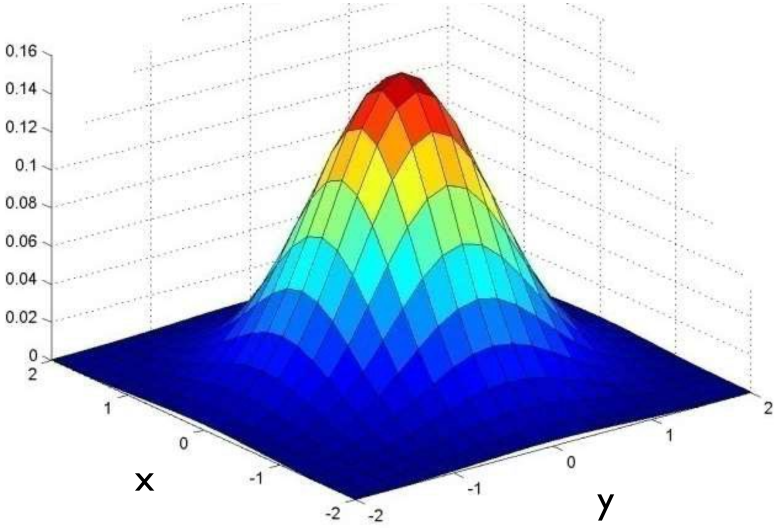
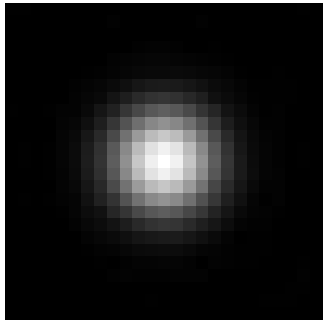
Gaussian Filter / Kernel



$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Viewed from top



y

x

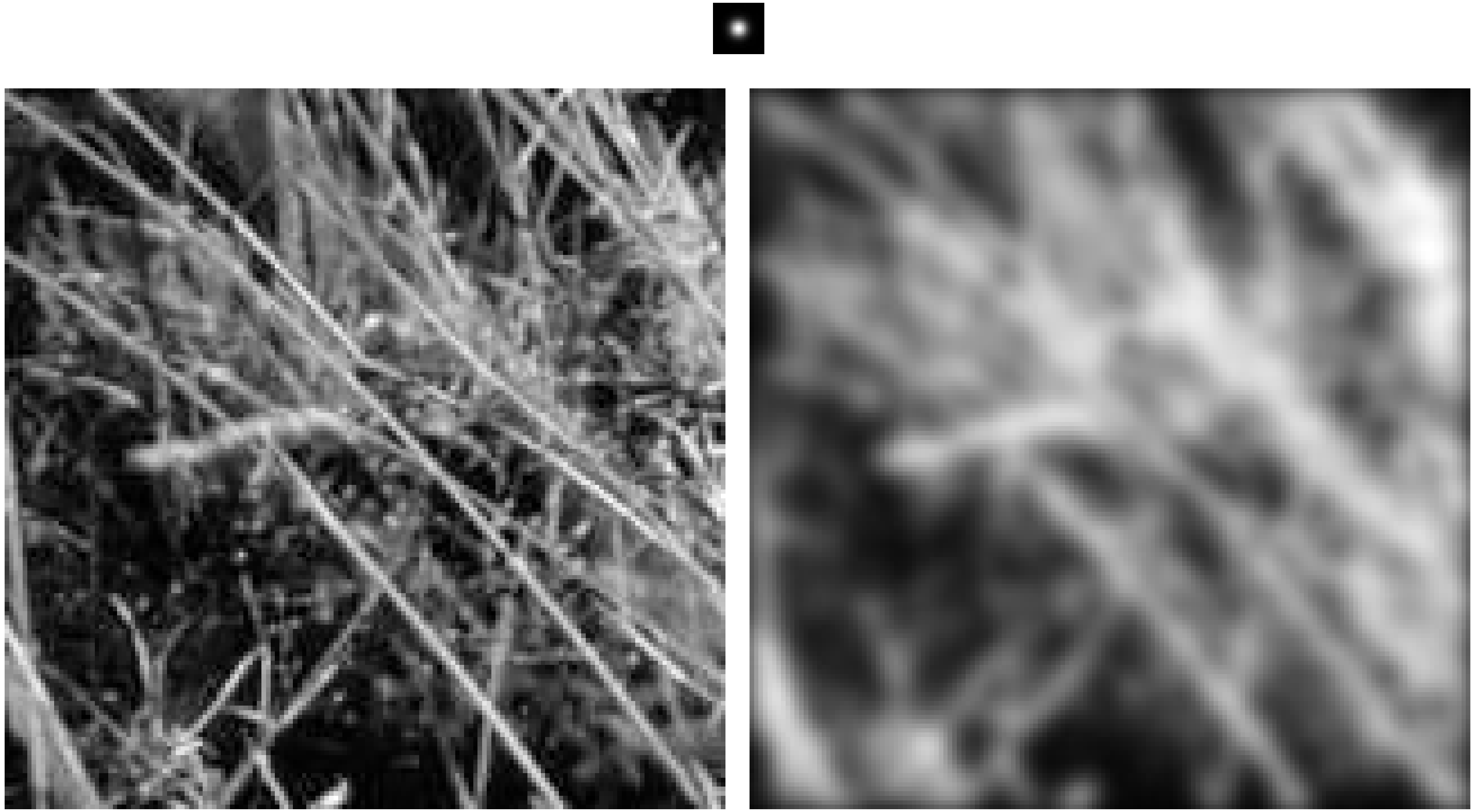
x

0.003	0.013	0.022	0.013	0.003
0.013	0.059	0.097	0.059	0.013
0.022	0.097	0.159	0.097	0.022
0.013	0.059	0.097	0.059	0.013
0.003	0.013	0.022	0.013	0.003

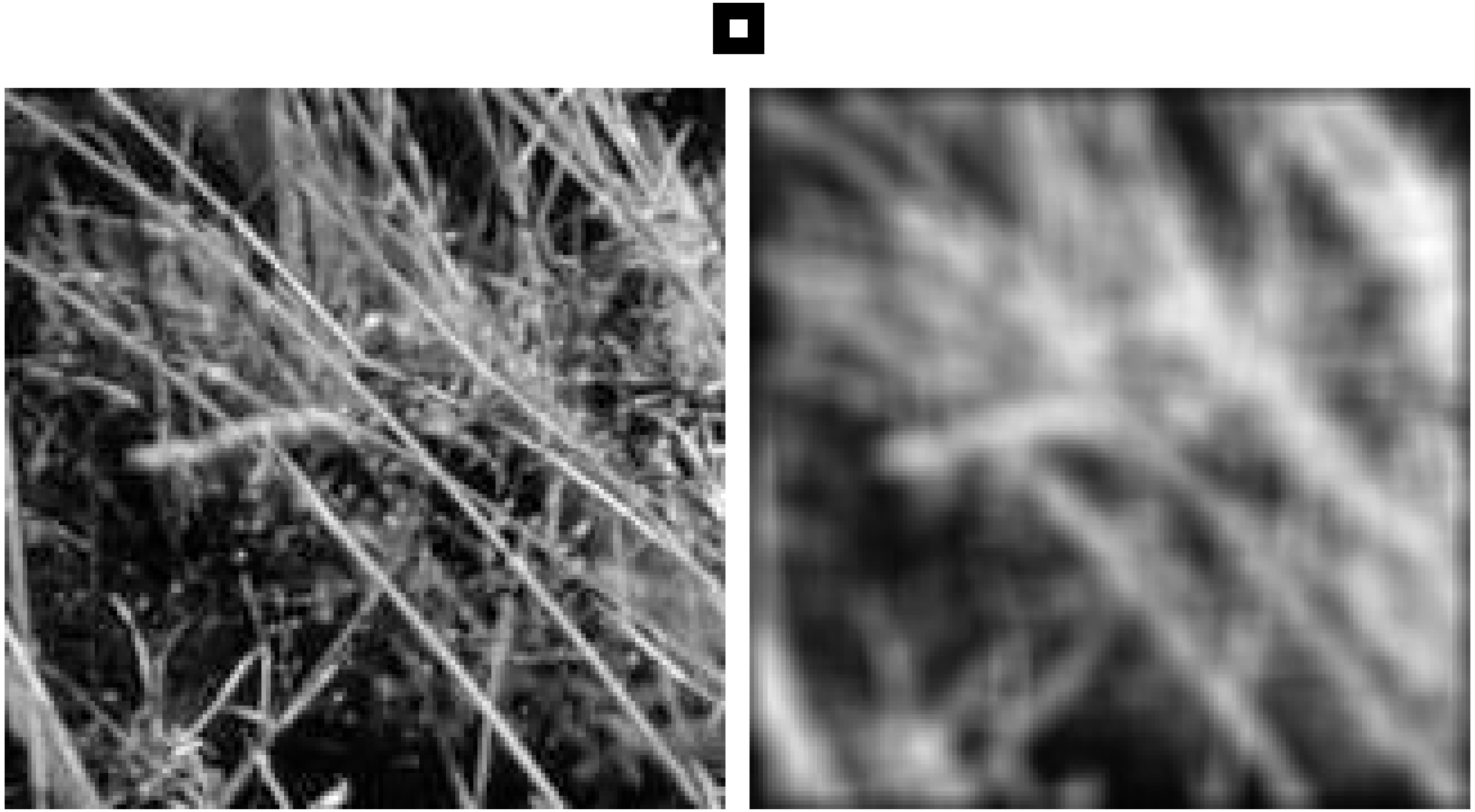
y

Filter/Kernel size 5 x 5,
Standard deviation σ = 1

Smoothing with Gaussian Filter



Smoothing with Box Filter



Linear Filter Properties

Linearity:

$$\text{imfilter}(I,\ f_1 + f_2) = \text{imfilter}(I, f_1) + \text{imfilter}(I, f_2)$$

Shift/translation invariance:

Same behavior given intensities regardless of pixel location m,n

$$\text{imfilter}(I, \text{shift}(f)) = \text{shift}(\text{imfilter}(I, f))$$

Svetlana Lazebnik

Gaussian Filter Properties

Gaussian convolved with **Gaussian**…

…is another **Gaussian**

- So can smooth with small-width kernel, repeat, and get same result as larger-width kernel
- Convolving twice with Gaussian kernel of width σ is same as convolving once with kernel of width $\sigma\sqrt{2}$

Separable kernel

- Factors into product of two 1D Gaussians

Image Filtering Applications

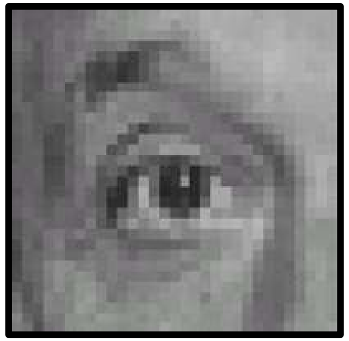
Compute function of local neighborhood at each position:

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

Really important!

- **Enhance images**
  - Denoise, resize, increase contrast, etc.
- **Extract information** from images
  - Texture, edges, distinctive points, etc.
- **Detect patterns**
  - Template matching

Practice with linear filters



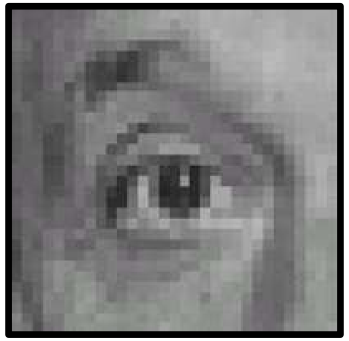
1. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$

3. $\begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix}$

4. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

Practice with linear filters

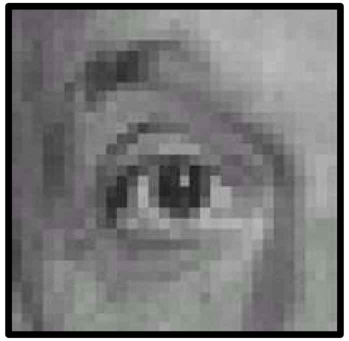


Original

0	0	0
0	1	0
0	0	0

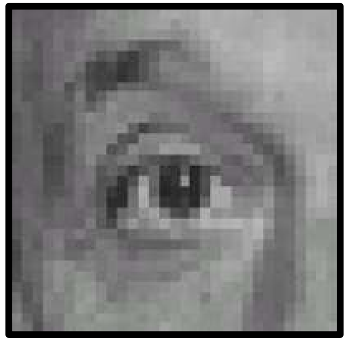
?

Practice with linear filters



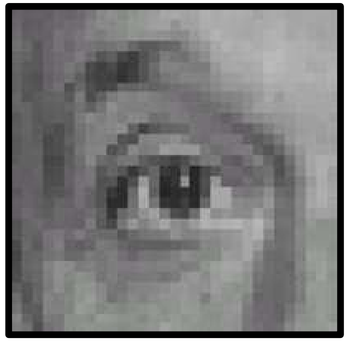
Original

0	0	0
0	1	0
0	0	0



Filtered
(no change)

Practice with linear filters

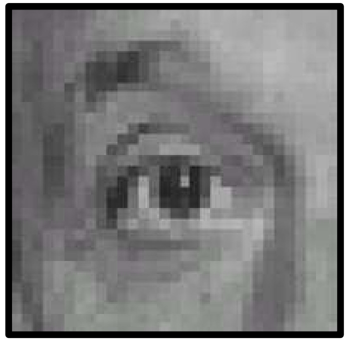


Original

0	0	0
0	0	1
0	0	0

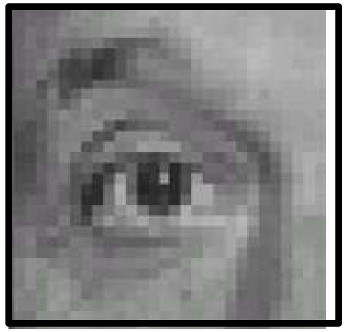
?

Practice with linear filters



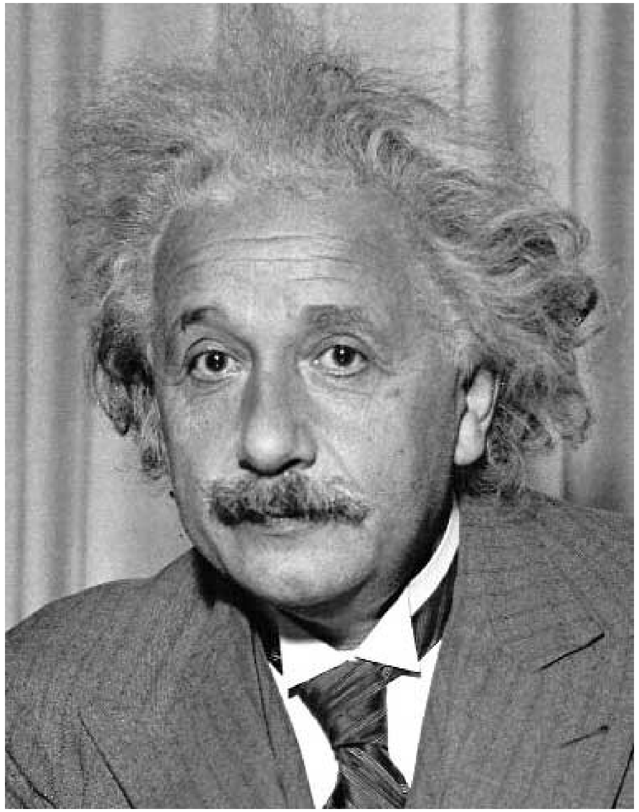
Original

0	0	0
0	0	1
0	0	0



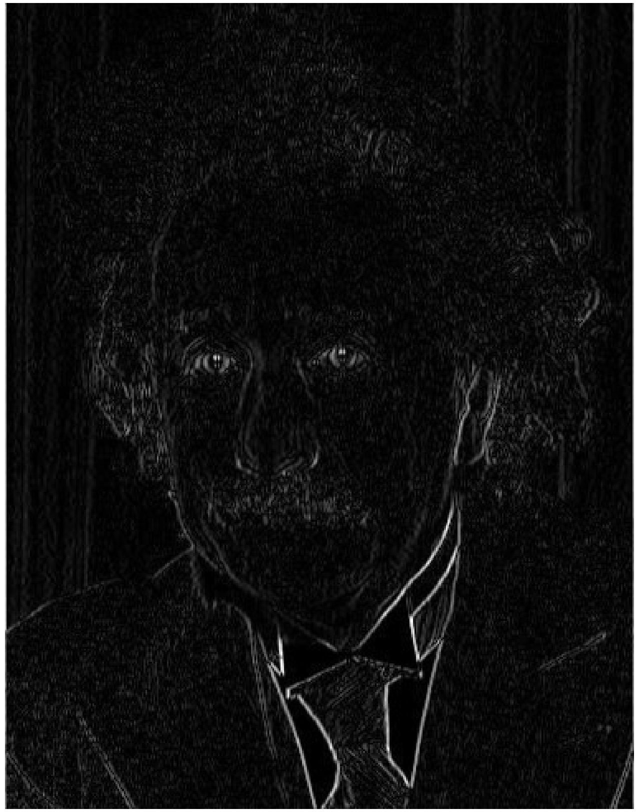
Shifted left
By 1 pixel

Practice with linear filters



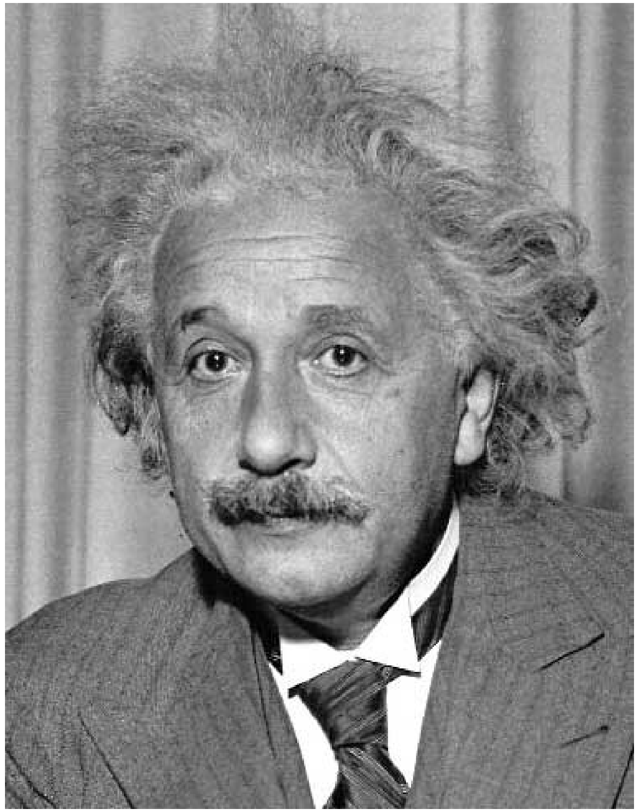
1	0	-1
2	0	-2
1	0	-1

Sobel



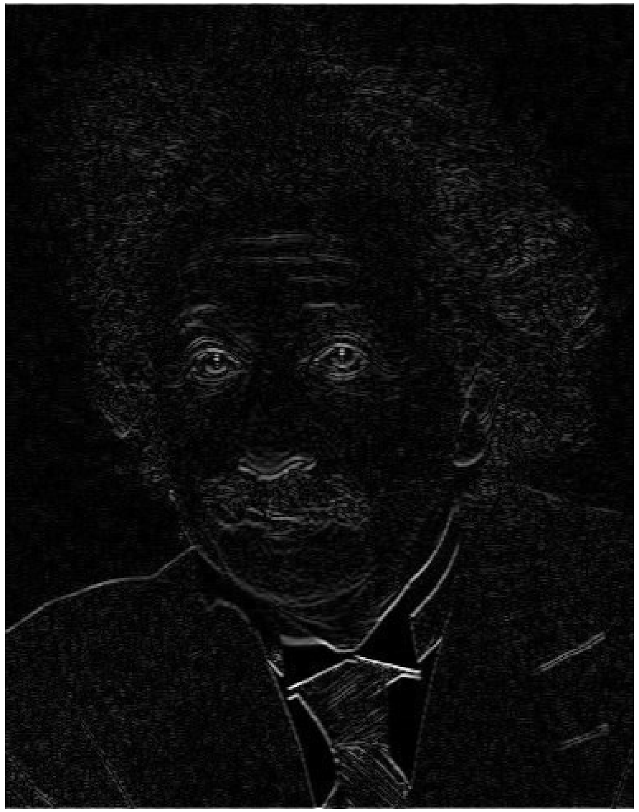
Vertical Edge
(absolute value)

Practice with linear filters



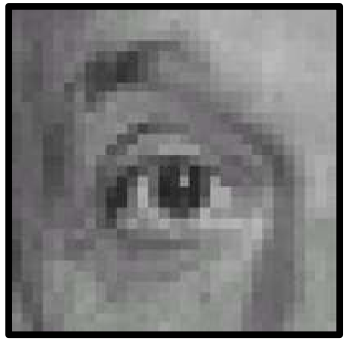
1	2	1
0	0	0
-1	-2	-1

Sobel



Horizontal Edge
(absolute value)

Practice with linear filters



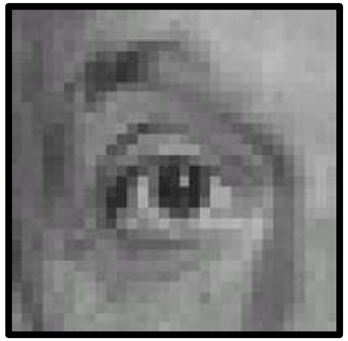
Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

?

(Note that filter sums to 1)

Practice with linear filters



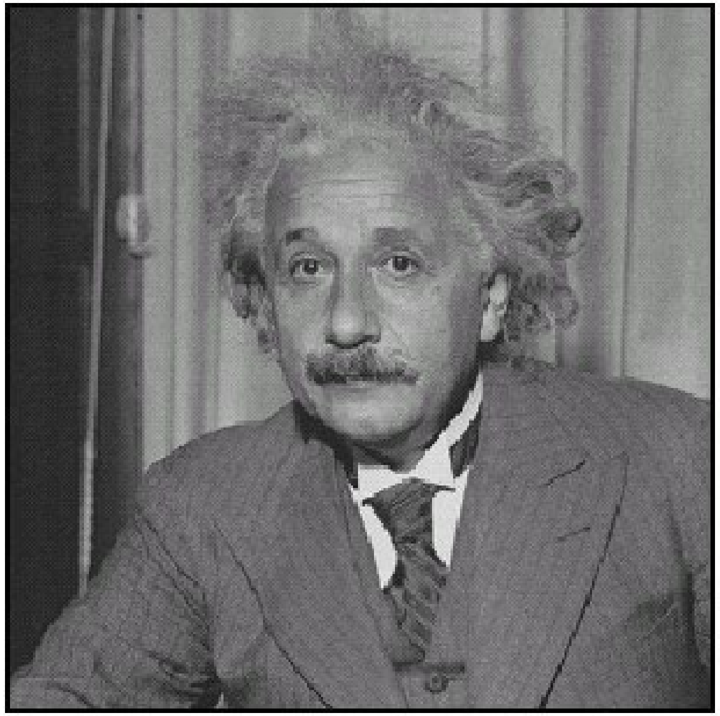
Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

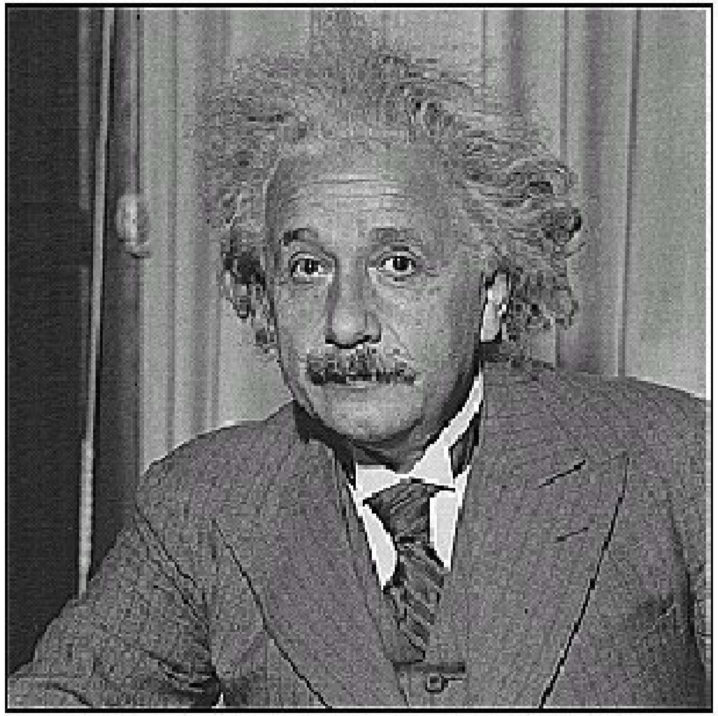
Sharpening filter

- Accentuates differences with local average

Practice with linear filters



before



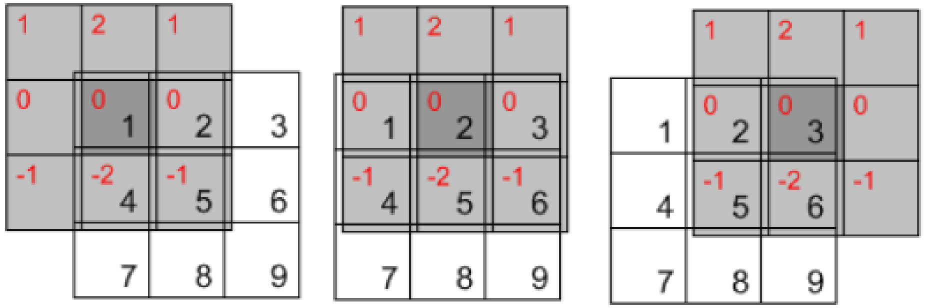
after

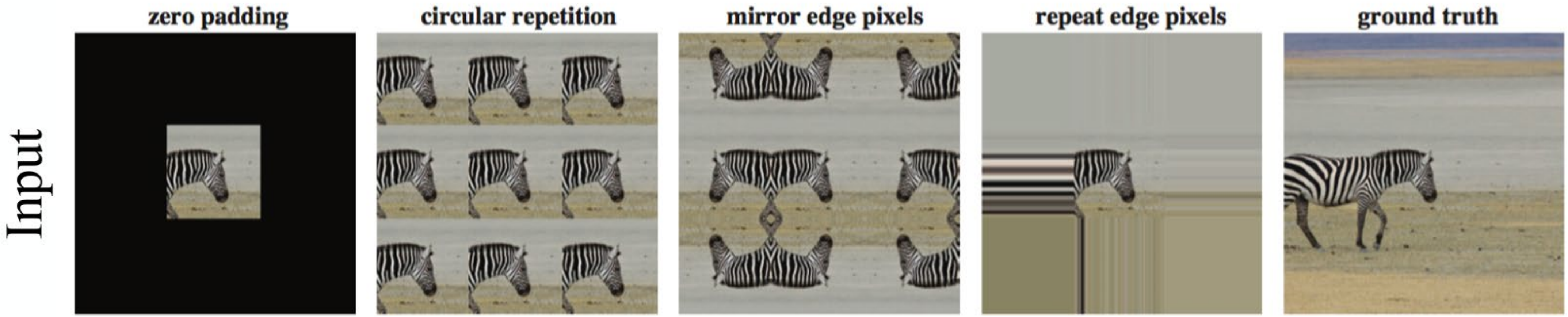
Filters in Practice

What about near the edge?

- The filter window falls off the edge of the image
- Need to extrapolate (padding)

- Methods:
  - clip filter (black / zero padding)
  - wrap around
  - copy edge
  - reflect across edge





Correlation and Convolution

Definition

- **2D correlation**

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

`e.g., h = scipy.signal.correlate2d(f,I)`

* Symmetric in the geometric sense, not in the matrix linear algebra sense.

Filtering

Definition

- 2D correlation

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

```
e.g., h = scipy.signal.correlate2d(f,I)
```

- **2D convolution**

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m-k, n-l]$$

```
e.g., h = scipy.signal.convolve2d(f,I)
```

Convolution is the same as correlation with a 180° rotated filter kernel.
Correlation and convolution are identical when the filter kernel is symmetric*.

* Symmetric in the geometric sense, not in the matrix linear algebra sense.

Convolution Properties

Commutative: $a * b = b * a$
- Conceptually no difference between filter and signal
- But filtering implementations might break this equality, e.g., image edges

Associative: $a * (b * c) = (a * b) * c$
- Often apply several filters one after another: $(((a * b_1) * b_2) * b_3)$
- This is equivalent to applying one filter: $a * (b_1 * b_2 * b_3)$
- Why important?
- Correlation is **NOT** associative (rotation effect)

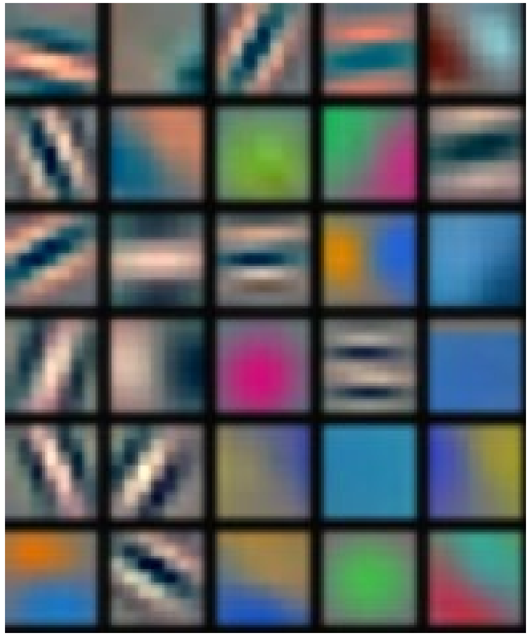
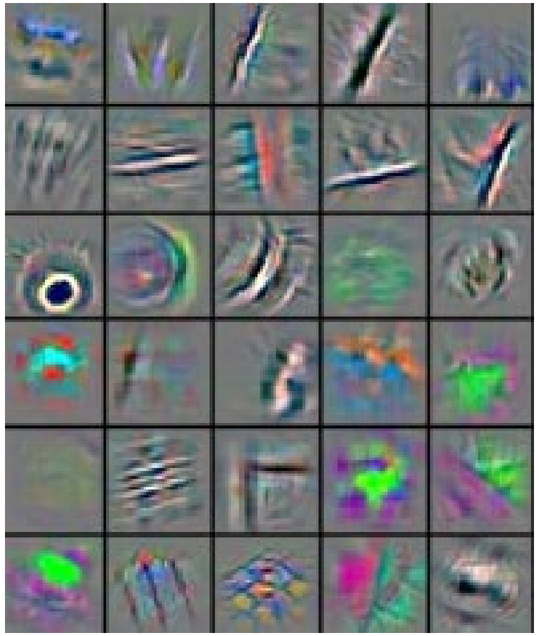
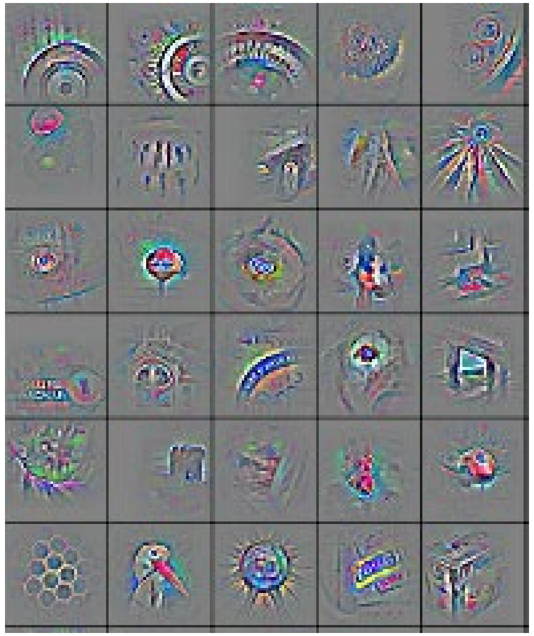
Distributes over addition: $a * (b + c) = (a * b) + (a * c)$

Scalars factor out: $ka * b = a * kb = k\,(a * b)$

Identity: unit impulse $e = [0, 0, 1, 0, 0]$, $a * e = a$

Convolution in Convolutional Neural Networks

- Convolution is the basic operation in CNNs
- Learning convolution kernels allows us to learn which `features’ provide useful information in images.

Linear Filters

Linearity:

$$\texttt{imfilter(I, } f_1 + f_2\texttt{)} = \texttt{imfilter(I,}f_1\texttt{)} + \texttt{imfilter(I,}f_2\texttt{)}$$

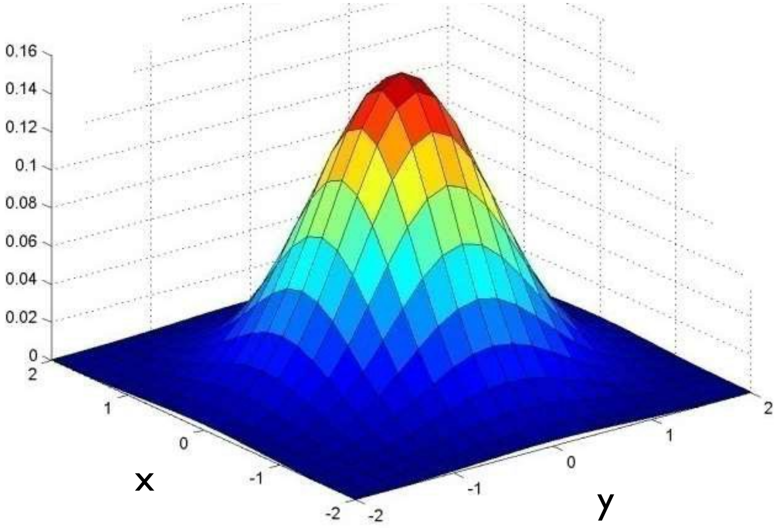
Shift/translation invariance:

Same behavior given intensities regardless of pixel location m,n

```
imfilter(I,shift(f)) = shift(imfilter(I,f))
```

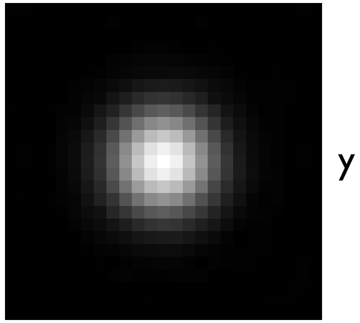
Any linear, shift-invariant operator can be represented as a convolution

Gaussian Filter / Kernel



$$G_\sigma = \frac{1}{2\pi\sigma^2} e^{-\frac{(x^2+y^2)}{2\sigma^2}}$$

Viewed from top



x

0.003	0.013	0.022	0.013	0.003	
0.013	0.059	0.097	0.059	0.013	
0.022	0.097	0.159	0.097	0.022	
0.013	0.059	0.097	0.059	0.013	y
0.003	0.013	0.022	0.013	0.003	

Filter/Kernel size 5 x 5,
Standard deviation σ = 1

Separability of the Gaussian Filter

- The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

- In this case, the two functions are the (identical) 1D Gaussian:

$$
\begin{aligned}
G_\sigma(x, y) &= \frac{1}{2\pi\sigma^2} \exp^{-\frac{x^2 + y^2}{2\sigma^2}} \\
&= \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{x^2}{2\sigma^2}} \right) \left(\frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{y^2}{2\sigma^2}} \right)
\end{aligned}
$$

Separability Example

2D convolution (center location only):

$$\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix} * \begin{bmatrix}2&3&3\\3&5&5\\4&4&6\end{bmatrix} \qquad \begin{aligned}&=2+6+3=11\\&=6+20+10=36\\&=4+8+6=18\\ &\overline{\qquad\qquad\quad 65}\end{aligned} \qquad \begin{bmatrix}\ &\ &\ \\ \ &65&\ \\ \ &\ &\ \end{bmatrix}$$

The filter factors into a product of 1D filters:

$$\begin{bmatrix}1&2&1\\2&4&2\\1&2&1\end{bmatrix} = \begin{bmatrix}1\\2\\1\end{bmatrix} \times \begin{bmatrix}1&2&1\end{bmatrix}$$

Perform convolution along rows:

$$\begin{bmatrix}1&2&1\end{bmatrix} * \begin{bmatrix}2&3&3\\3&5&5\\4&4&6\end{bmatrix} = \begin{bmatrix}\ &11&\ \\ \ &18&\ \\ \ &18&\ \end{bmatrix}$$

Followed by convolution along the remaining column:

$$\begin{bmatrix}1\\2\\1\end{bmatrix} * \begin{bmatrix}\ &11&\ \\ \ &18&\ \\ \ &18&\ \end{bmatrix} = \begin{bmatrix}\ &\ &\ \\ \ &65&\ \\ \ &\ &\ \end{bmatrix}$$

Separability Use

MxN image, PxQ filter

- 2D convolution: ~MNPQ multiply-adds
- Separable 2D: ~MN(P+Q) multiply-adds

Speed up = PQ/(P+Q) 9x9 filter = ~4.5x faster

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$$

Sobel Filter

```
>> I = img_to_float32( io.imread( 'luke.jpg' ) )
>> h = convolve2d( I, sobelKernel )

>> plt.imshow( h )
```

1	2	1
0	0	0
-1	-2	-1

Sobel

Sobel Filter

- What happens to negative numbers?

1	2	1
0	0	0
-1	-2	-1

Sobel

Sobel Filter

`h(:,:,1) < 0`

`h(:,:,1) > 0`

Sobel Filter

- For visualization:
  - Shift image + 0.5
  - If gradients are small, scale edge response

1	2	1
0	0	0
-1	-2	-1

Sobel

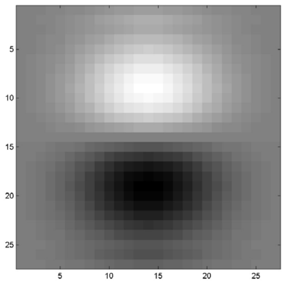
```
>> I = img_to_float32( io.imread( 'luke.jpg' ) )
>> h = convolve2d( I, sobelKernel )

>> plt.imshow( h )

>> plt.imshow( h + 0.5 )
```

Practice

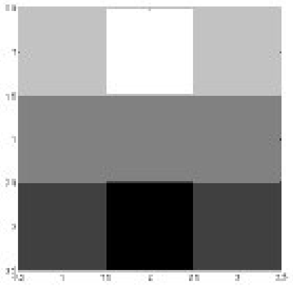
* = Convolution operator

1) _ = D * B
2) A = _ * _
3) F = D * _
4) _ = D * D

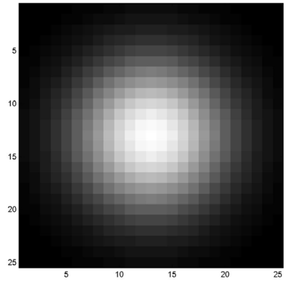
A



B

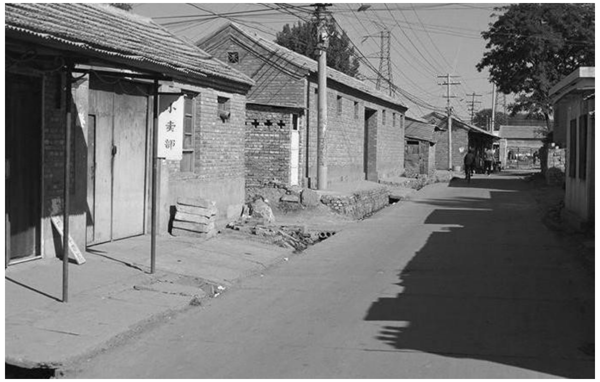


C



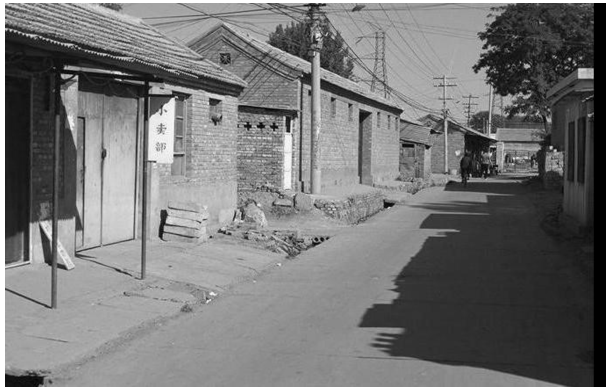
E

D

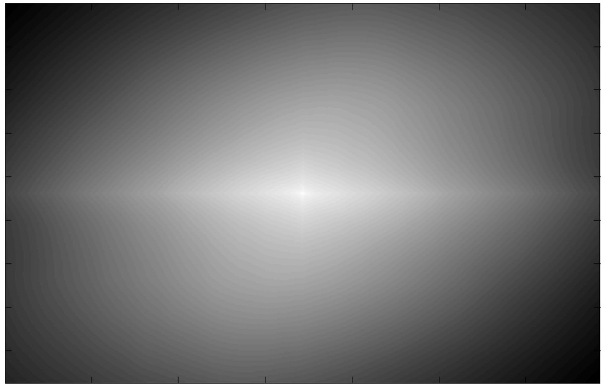


H

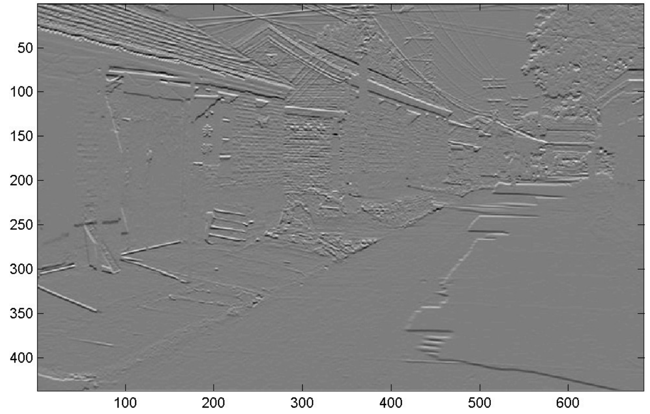
F



I



G

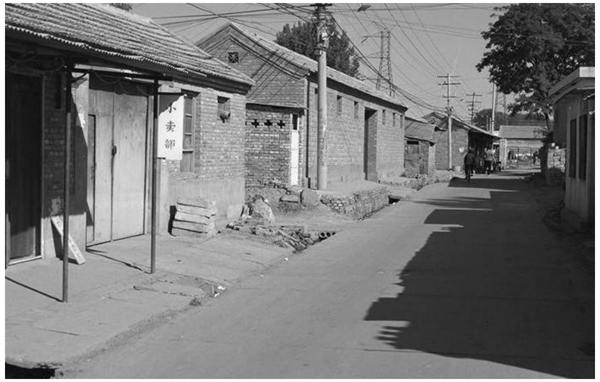


Derek Hoiem

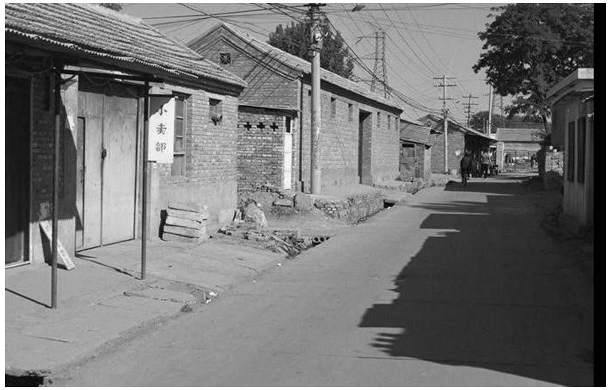
Practice

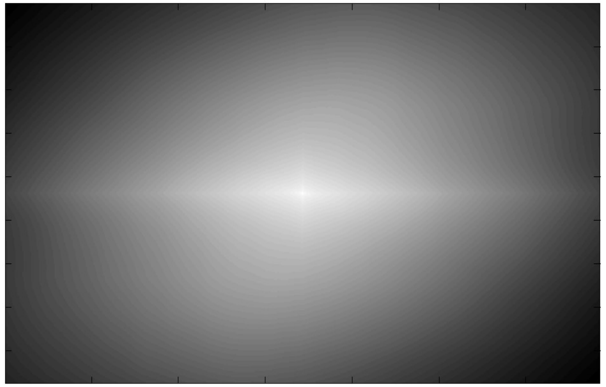
* = Convolution operator

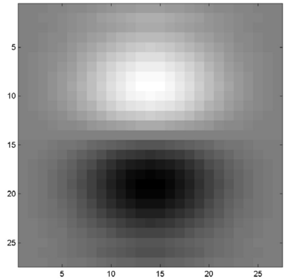
1) G = D * B
2) A = B * C
3) F = D * E
4) I = D * D

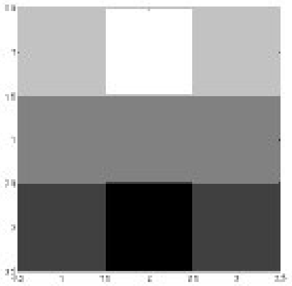
D


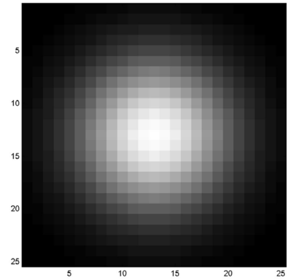
H


F


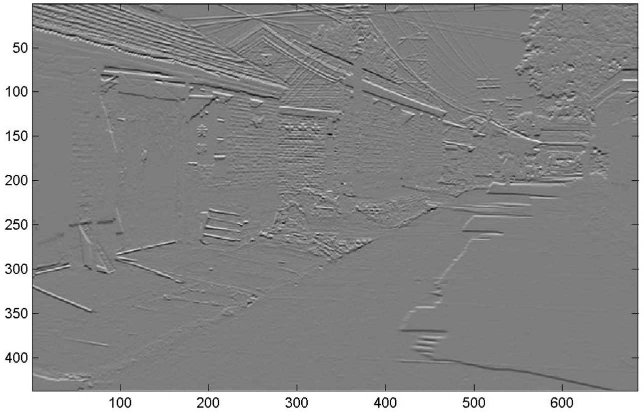
I


A


B


C


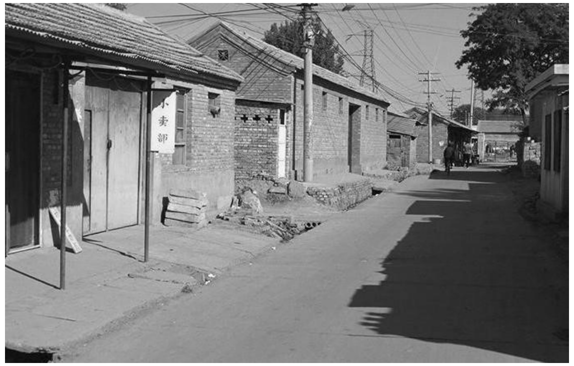
E


G


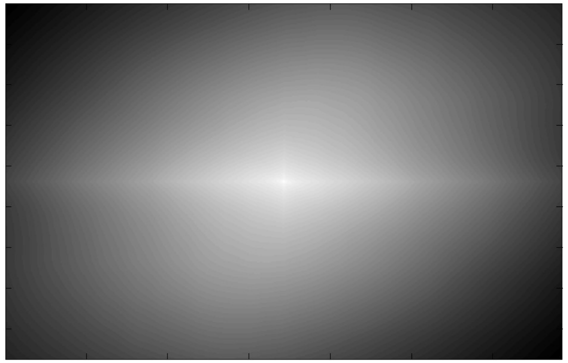
Derek Hoiem

Convolution

D (275 x 175 pixels)



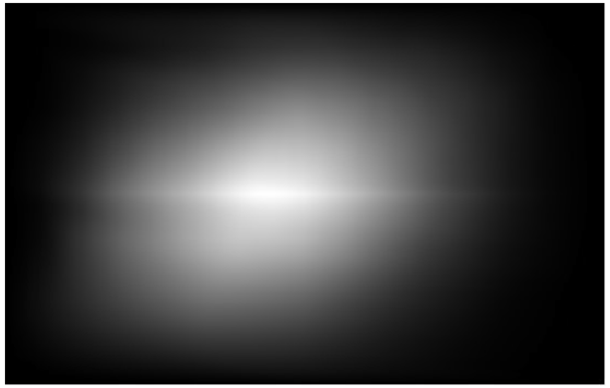
I (275 x 175 pixels)



```
>> D = img_to_float32( io.imread( ‘convexample.png’ ) )
>> I = convolve2d( D, D )
>> np.max(I) 1.1021e+04
```

```
# Normalize for visualization
>> I_norm = (I – np.min(I)) / (np.max(I) – np.min(I))
>> plt.imshow( I_norm )
```

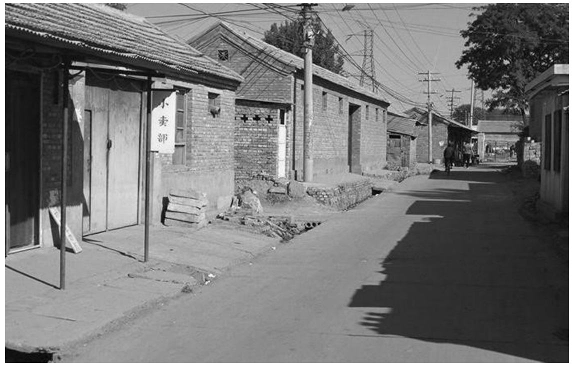
I_norm



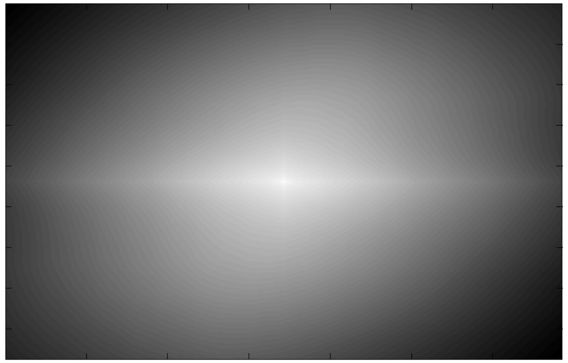
Zero padding causes black regions

Convolution

D (275 x 175 pixels)



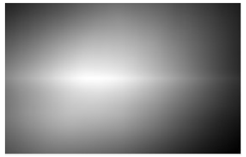
I (275 x 175 pixels)



```
>> I = convolve2d( D, D, mode='full' )
```
(Default; pad with zeros)

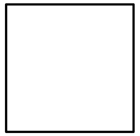
549 x 349

```
>> I = convolve2d( D, D, mode='same' )
```
(Return same size as D)



275 x 175

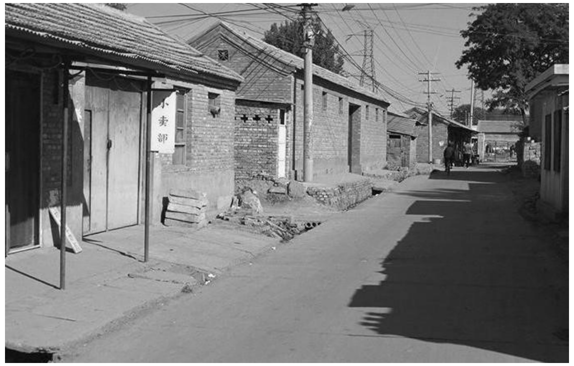
```
>> I = convolve2d( D, D, mode='valid' )
```
(No padding)



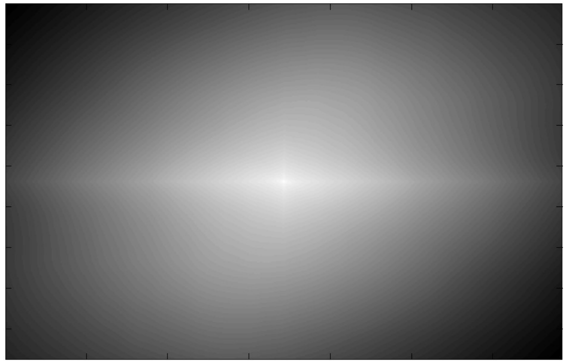
Value = 10528.3 1x1

Convolution

D (275 x 175 pixels)



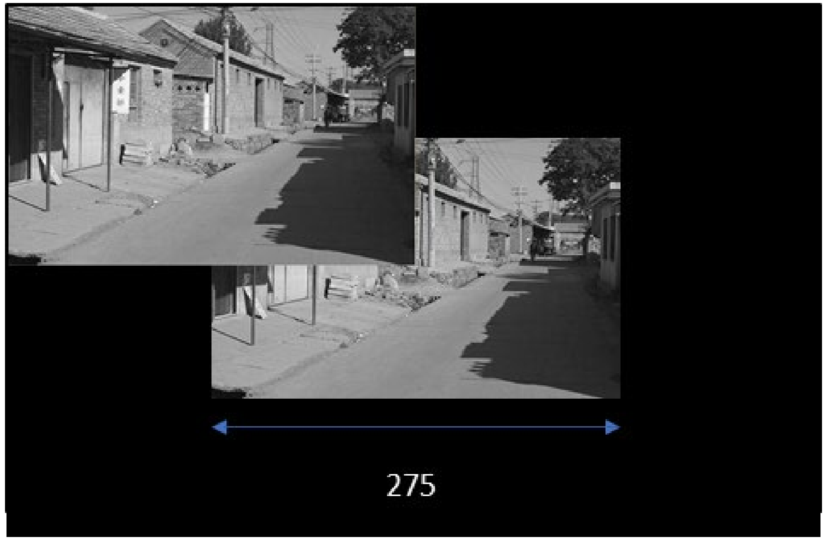
I (275 x 175 pixels)



```
>> I = convolve2d( D, D, mode='full' )
(Default; pad with zeros)
```

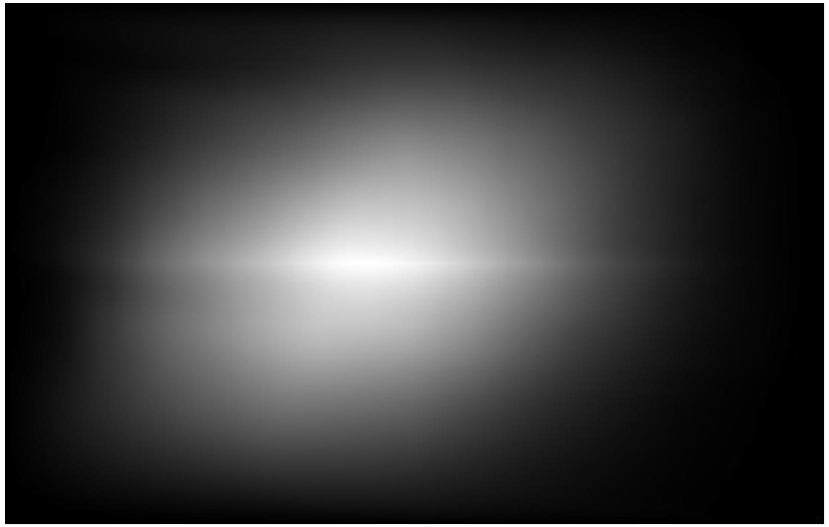
(275-1)/2

(275-1)/2



For x: 275 + (275-1)/2 + (275-1)/2 = 549

I_norm (549 x 349 pixels)



Correlation

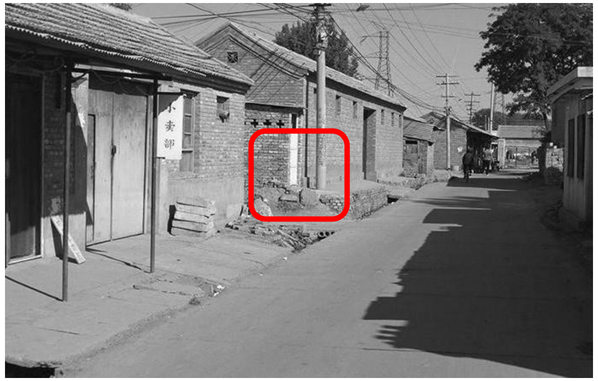
- When the filter 'looks like' the image = 'template matching'

- Filtering viewed as comparing an image of what you want to find against all image regions.

- For **symmetric** filters: use either convolution or correlation.
- For **nonsymmetric** filters: correlation is template matching.

$$h[m,n] = \sum_{k,l} f[k,l]\, I[m+k, n+l]$$

```
e.g., h = scipy.signal.correlate2d(f,I)
```

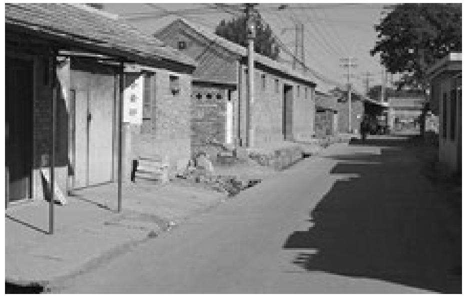


As brightness in I increases, the response in h will increase, as long as f is positive.

Correlation

Let's see if we can use correlation to 'find' the parts of the image that look like the filter

D (275 x 175 pixels).



```
>> f = D[ 57:117, 107:167 ]
```

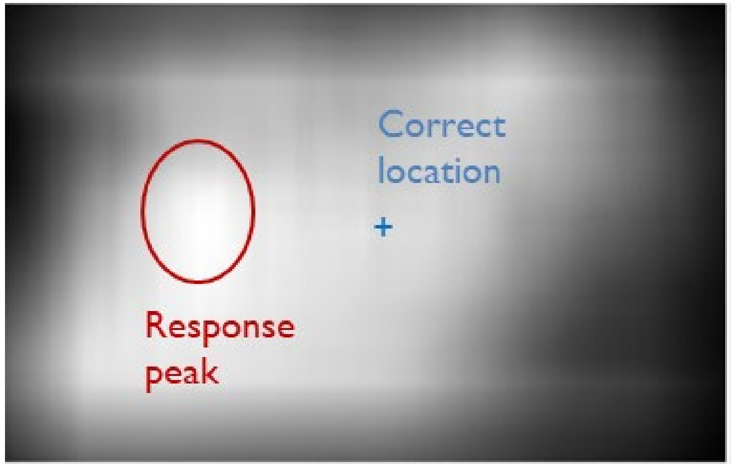
Expect response 'peak' in middle of I

f
61 x 61

```
>> I = correlate2d( D, f, 'same' )
```

Hmm…
That didn't work – why not?

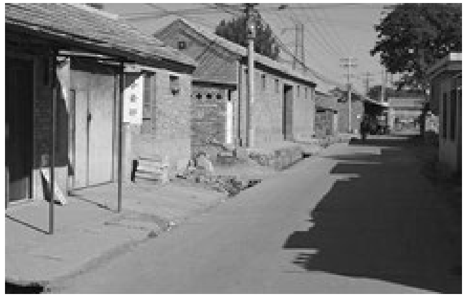
I



Correlation

Let's see if we can use correlation to 'find' the parts of the image that look like the filter

D (275 x 175 pixels).



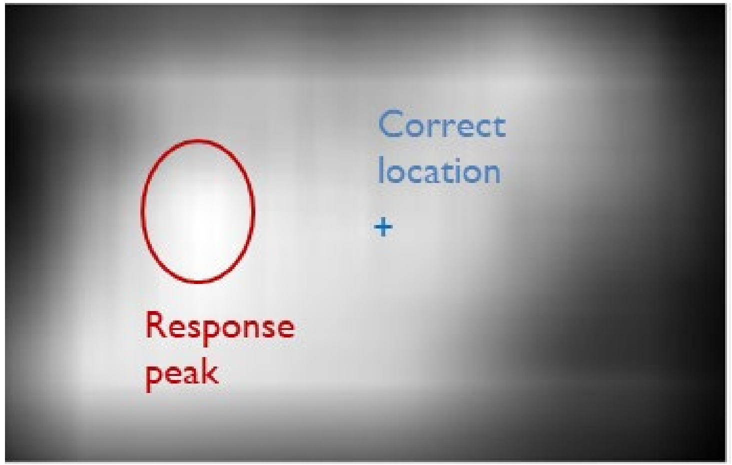
```
>> f = D[ 57:117, 107:167 ]
```

Expect response 'peak' in middle of I

f
61 x 61

```
>> I = correlate2d( D, f, 'same' )
```

I



Hmm…
That didn't work – why not?

Overall brighter regions will give higher correlation response
-> not useful!

Robert Collins

Correlation

OK, so let's subtract the mean

```
>> f = D[ 57:117, 107:167 ]
>> f2 = f – np.mean(f)
>> D2 = D – np.mean(D)
```

D2 (275 x 175 pixels)

f2
61 x 61

Remember – float data type– goes negative!

Now zero centered.
Score is higher only when dark parts match and when light parts match.

```
>> I2 = correlate2d( D2, f2, ‘same’ )
```

I2

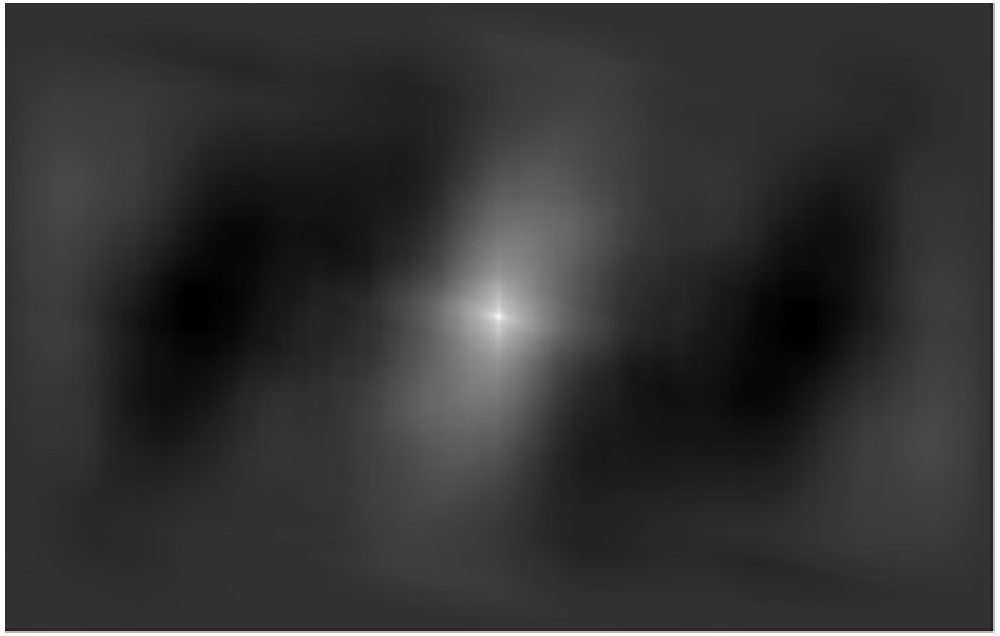
Correlation

Or our original example: D2 correlated with itself

```
>> I3 = correlate2d( D2, D2, ‘full’ )
```

D2 (275 x 175 pixels)

I3



Correlation vs. Convolution

What happens with convolution?

```
>> f = D[ 57:117, 107:167 ]
>> f2 = f – np.mean(f)
>> D2 = D – np.mean(D)
>> I2 = convolve2d( D2, f2, ‘same’ )
```

I2

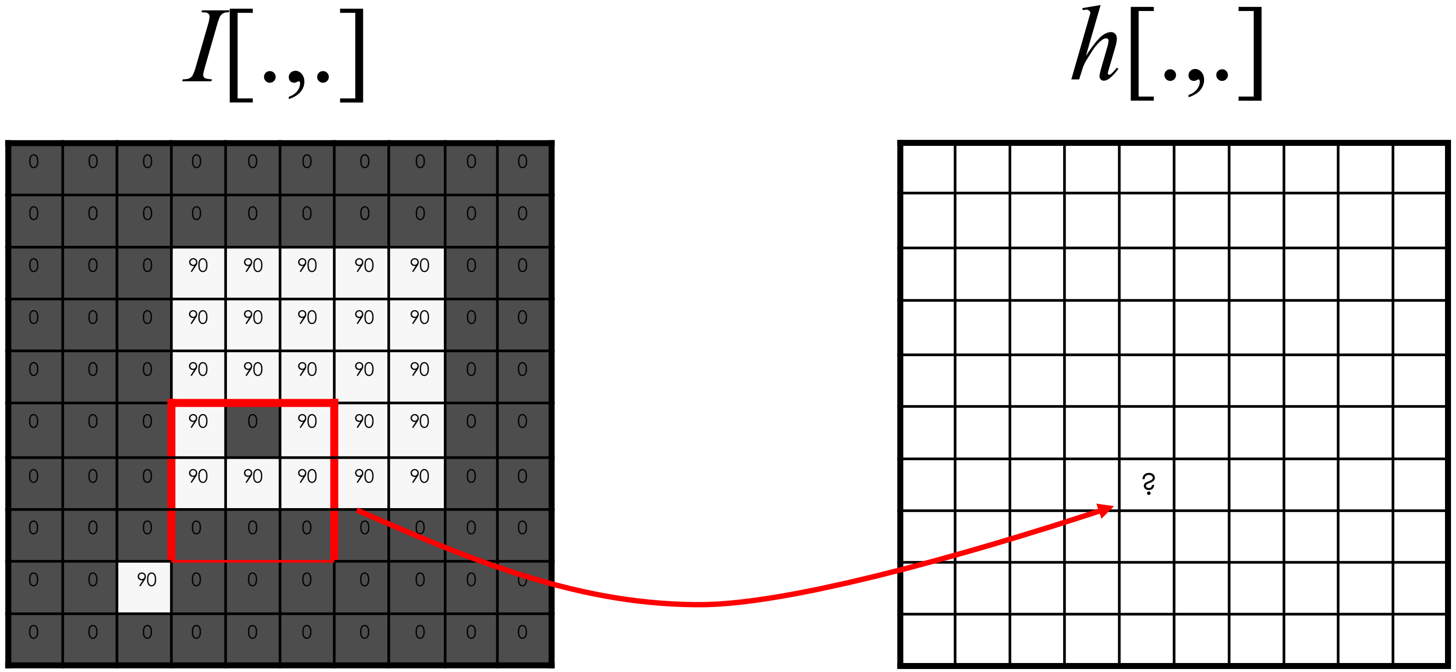
D2 (275 x 175 pixels)

f2
61 x 61

Non-Linear Filters

- **Rank filters** are non-linear filters based on ordering of gray levels
  - e.g., median, min, max, range filters

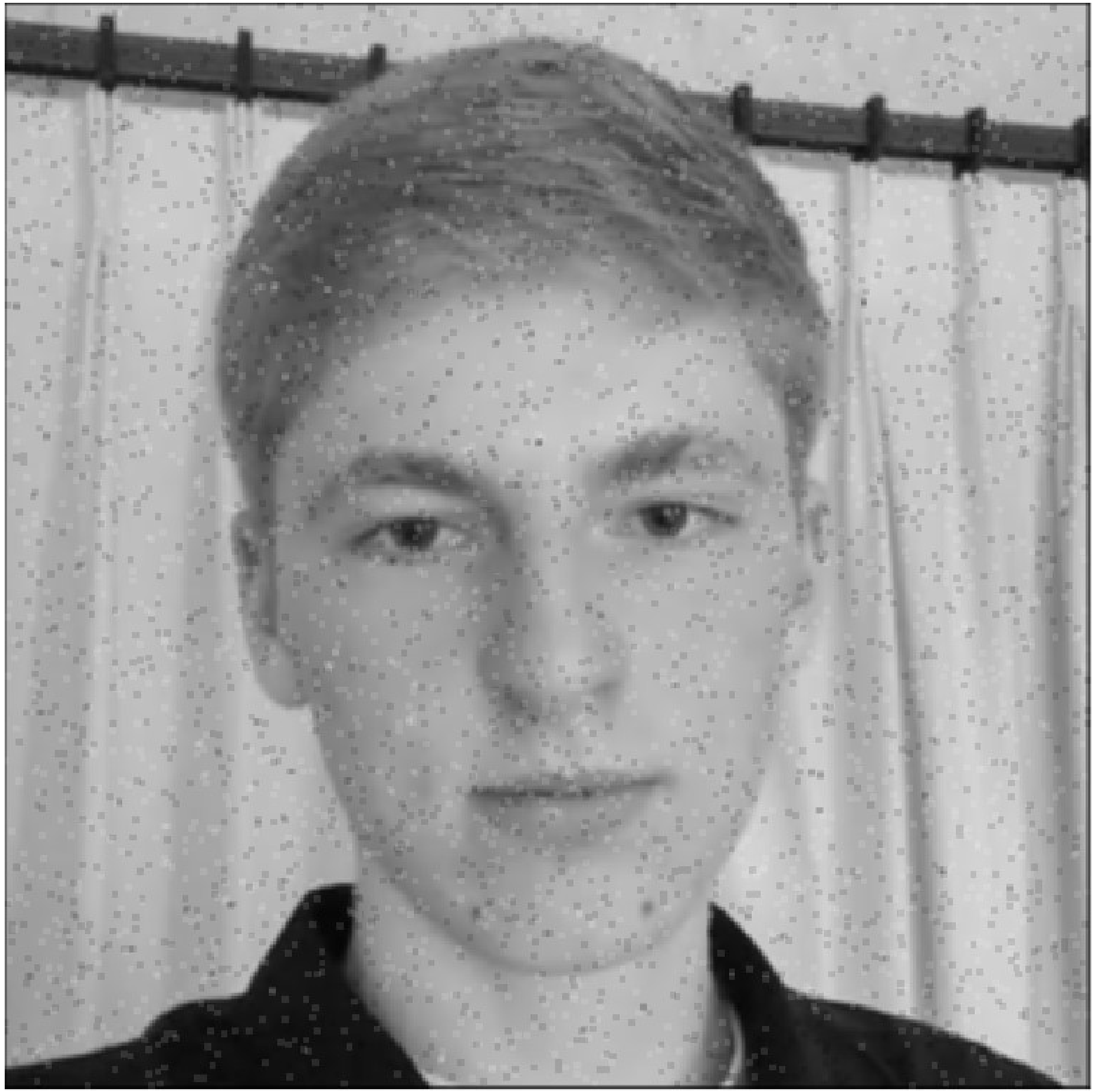
- **Median filter** operates over a window by selecting the median intensity in the window



Steve Seitz, Steve Marschner

Salt and Pepper Noise

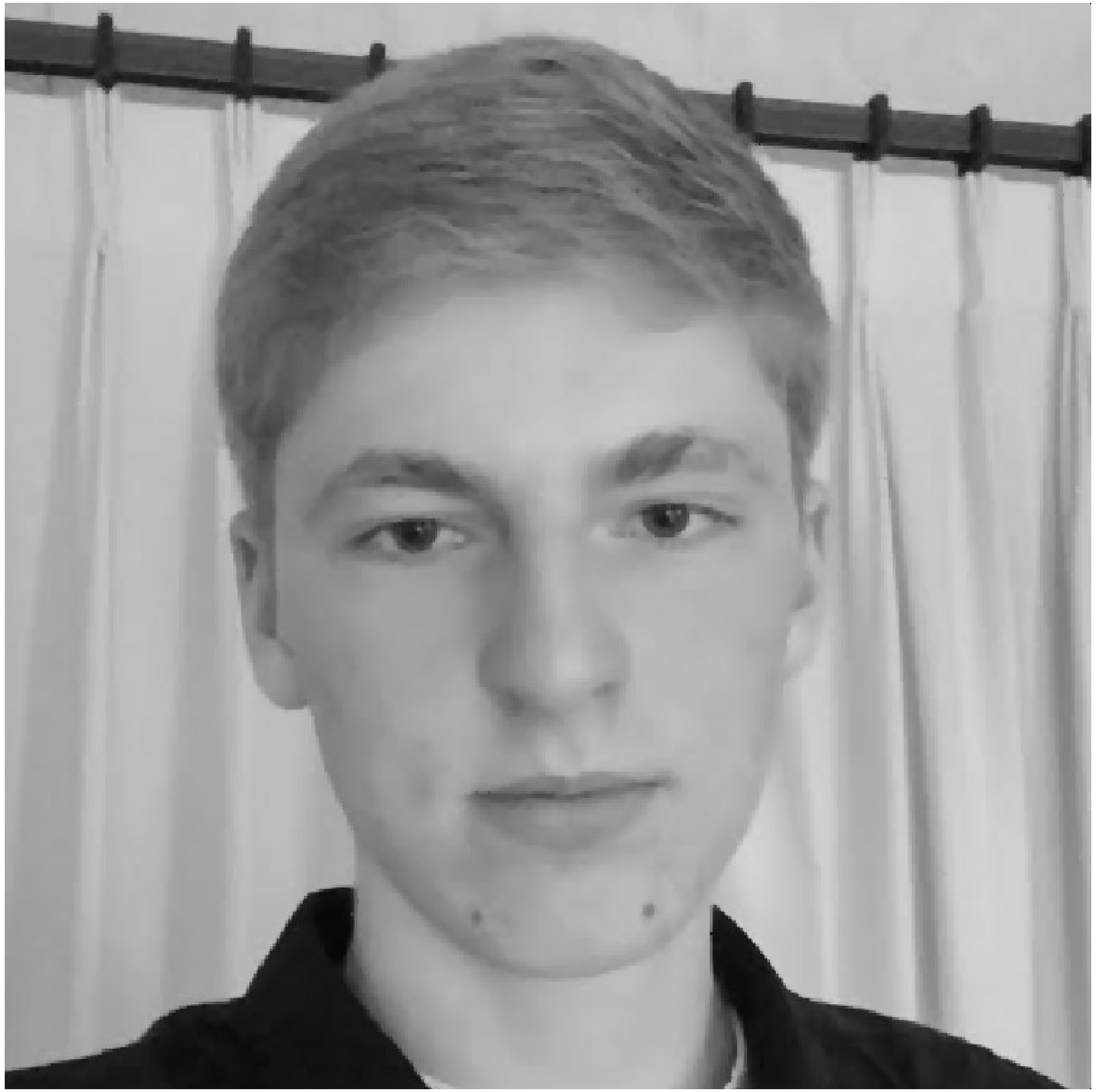
3 x 3 Mean Filter



11 x 11 Mean Filter

Salt and Pepper Noise

3 x 3 Median Filter



11 x 11 Median Filter

Median filters

- Operates over a window by selecting the median intensity in the window

- What advantage does a median filter have over a mean filter?

- Is a median filter a kind of convolution?

- Interpretation: Median filtering is sorting