31/01/2055]

ALTERNATING SERIES

= series that alternates blue both the and re teamseg. $\sum_{n=1}^{\infty} \frac{(-1)^n+1}{n} = 1 - \frac{1}{2} + \frac{1}{3}$

eg.
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} = 1 - \frac{1}{2} + \frac{1}{3}$$

Leibnitz rule:-

An alternating series Eta) 2th \(\(\gamma\) (-1)^n+1 an where an >0 converges if the following two condition satisfies:

1) lun 9n=0 ii) can't is a decreasing sequence i.e. anti < an +n

Q.) Defermine the following series converges or directed.

 $i > \sum_{n=1}^{\infty} (-1)_{n+1} \frac{\pi u(u)}{\pi u(u)}$

of criven series s am alternating series, with an = Inn

{ or form) 1.> line lung

= lim Yn (by l'Hospital Rule)

ii) let Hm) = Inn

7/(N) = 1-7NN

i.e. finico for m>3

30, I'm in decreasing function for n >3.

<an> a decreasing function for n23

.. By alternating service test S(-1) the converger.

117 > (-1) "HI e-" This is alternating series with an = e-n lun an = lun = = 0 also, $f'(n) = \frac{d}{dn} (e^{-n}) = \frac{-1}{e^n} < 0 + n > 1$. :. f'(n) is decreasing tunction of x by libritz form, given series converger. ((1)) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2-1}{n^2+2}$ of this series alternating series with an = n2-1 (1+3)/n2) = 1 70 (1+3)/n2) = 1 70 Service diverges $\lim_{n \to \infty} \frac{n^2 - 1}{n^2 + 3} = \lim_{n \to \infty} \frac{\left(1 - \frac{1}{n^2}\right)^2}{\left(1 + \frac{3}{3}\right) \left(n^2\right)} = 1 \neq 0$ $\frac{8}{11}$ $\frac{8}{2}$ $\frac{(-1)^{n}}{3}$ $\frac{2^{n}}{3}$ $\frac{1}{3}$ $\frac{$

Prosolute and conditional convergence

I consider a series with both the L-re forms but not alternating series.

eq.
$$1 - \frac{1}{2} - \frac{1}{2^2} + \frac{1}{2^2} + \frac{1}{2^4} - \frac{1}{2^3} - \frac{1}{2^3} + \cdots$$

Aprolate convergence:-

A series Zan is said to be absolute convergence It the series of absolute values Z (an) a convertent

Conditionally conveyent:-

A social Ean is convergent but not absolutely convergent e.g. \(\frac{\sigma_{-1}}{\sigma_{-1}} \frac{(-1)^{n+1}}{n} \)

This is alternating series

$$f(N) = \frac{1}{N^2} < 0 + N > 0$$

f(n) is decreasing sequence.

.. By librartz test, the service is absolutely conveyent but Elan = { In is direugent harmonic senies

so given series convergent, but not completely conveyent.

Absolute convergence implies convergence)

Theoseam: -

If a service Ean u absolutely convergent, than it is convergent

i.e. if Slan) is convergent than Ean is convergent

Least

Let Epn/ be conveyent.

then, by cauchy cretimian of convergence of severy for any E70, 3 a tre integer M such that |sn-sm/ LE + n>m ZM

where sn= [ail+[az] + - + Am] + (ami)+ - + [an]

|Sn-Sm| = | 19 mt1 + 19 mt2 + - + 19 m]]

By caudry critory of convergence

| [] antil + - + | ani] CE + n>m 2 M

| amtil + |ant2| + - + |an| < E 7 n>m ≥ M - 0

since, late) < la1+161

To show Ean a convergence. [< |9mm| + |9mm] + ... + lan]

LE & nom 2 M [wing O) 1 aut + autz + . . tanl

Janti tantet - tanl + CE Ynome M

=> Ean is convergent. By couchy criterial of convergence. courses or not price.

e.g. 2 (4)mil is conveyent but not absolutely.

- " MEDSCORNI"

If a service Ean is absolutely convergent, then it is convergent.

i.e. if Elan is convergent then Ean is convergent.

Least

let Elen/ be convergent.

then, by cauchy cretistian of convergence of senter for any E70, I a tre integer M such that |Sn-Sm/ CE Tr>m ZM

Where sn= [911+[92] + - + 12m] + (9m1)+ -- + [9m]

N > M

|Sn-Sm| = | 19 mt1 + 19 mt2 + - + 19 m)

By cauchy critory of convergence

[[anti] + - + |ani] CE + n>m 2 M

| amtil + |94+2| + - + |911 | < E 7 n>m > M - 0

since, lated < late 161

To show Ean a convergence, [< |9m+1 + |9m+2 + - - + |an]

< & x nsm> [mmi @] 1 ant + antz + -- tan1

Jant + 9mt2+ - +9n/ -> LE + n>m>M

> Ean is convergent. by couchy criterial of convergence convoice is not true.

e.g. 2 (1)nfl is conveyant but not absolutely.