UNIVERSITY OF DELHI
DEPARTMENT OF MATHEMATICS
GENERIC ELECTIVE (GE) Courses for Hons. Courses
(For students other than Bachelor of Mathematics Hons.)

(Effective from Academic Year 2018-19)

PROGRAMME BROCHURE

XXXXX Revised Syllabus as approved by Academic Council on XXXX, 2018 and

Executive Council on YYYY, 2018

GENERIC ELECTIVE (GE) COURSES OFFERED TO
B.Sc. (H)/ B. A. (H)/ B. Com (H)
(Other than B.Sc. (H) Mathematics)

Semester	Core Course (14)	Ability Enhancement Compulsory Course (AECC)(2)	Skill Enhancement Course (SEC) (2)	Discipline Specific Elective (DSE)(4)	Generic Elective (GE) (4) Credits: 6 each
I					**GE-1** Calculus OR Analytic Geometry and Theory of Equations
II					**GE-2** Linear Algebra OR Discrete Mathematics
III					**GE-3** Differential Equations (with practicals) OR Linear Programming and Game Theory
IV					**GE-4** Numerical Methods (with practicals) OR Elements of Analysis
V					
VI					

Semester I

GE-1: Calculus OR GE-1: Analytic Geometry and Theory of Equations

GE-1: Calculus

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures (per week), 1 Tutorial (per week per student)
Duration: 14 Weeks (70 Hrs.) **Examination**: 3 Hrs.

Course Objectives: The main aim of this course is to introduce the concept of differentiation of functions, points of inflection, curve sketching etc. Concepts of functions of several variables, their partial derivatives and saddle points is also discussed.

Course Learning Outcomes: This course will enable the students to:
  i)   Visualize three dimensional figures and calculating their volumes.
  ii)   Draw the graph of functions in polar coordinates and level curves of functions of several variables.
  iii) Understand Limits, continuity and partial derivatives of functions of several variables.

Course Contents:

Unit 1: Applications of Derivatives and Limits   (Lectures: 20)
The first derivative test, Concavity and inflection points, Second derivative test, Curve sketching using first and second derivative test; Limits at infinity, Horizontal asymptotes, Vertical asymptotes, Graphs with asymptotes; L'Hôpital's rule.

Unit 2: Applications of Definite Integrals   (Lectures: 15)
Volumes by slicing, Volumes of solids of revolution by the disk method, Volumes of solids of revolution by the washer method, Volume by cylindrical shells, Length of plane curves, Arc length of parametric curve, Area of surface of revolution.

Unit 3: Conics, Vector-Valued Functions and Partial Derivatives   (Lectures: 35)
Techniques of sketching conics, Reflection properties of conics; Polar coordinates, graphing in polar coordinates; Vector-valued functions**:** Limits, Continuity, Derivatives, Integrals, Arc length, Unit tangent vector, Curvature, Unit normal vector; Functions of several variables**:** Graphs and level curves, Limits and continuity, Partial derivatives and differentiability, The chain rule, Directional derivatives and gradient vectors, Tangent plane and normal line, Extreme values and saddle points.

References:

1. Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). John Wiley & Sons Singapore Pte. Ltd. Reprint (2016) by Wiley India Pvt. Ltd. Delhi.

2. Strauss, M. J., Bradley, G. L., & Smith, K. J. (2007). *Calculus* (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Sixth impression 2011.

Teaching Plan (GE-1: Calculus):

Weeks 1 and 2: The first derivative test, Concavity and inflection points, Second derivative test, Curve sketching using first and second derivative test.
[2] Chapter 4 (Section 4.3)

Weeks 3 and 4: Limits at infinity, Horizontal asymptotes, Vertical asymptotes, Graphs with asymptotes; L'Hôpital's rule.
[2] Chapter 4 (Sections 4.4, and 4.5)
[1] Chapter 3 (Section 3.3), and Chapter 6 (Section 6.5)

Weeks 5 and 6: Volumes by slicing, Volumes of solids of revolution by the disk method, Volumes of solids of revolution by the washer method, Volume by cylindrical shells.
[1] Chapter 5 (Sections 5.2, and 5.3)

Week 7: Length of plane curves, Arc length of parametric curves, Area of surface of revolution.
[1] Chapter 5 (Sections 5.4, and 5.5)

Week 8: Techniques of sketching conics, Reflection properties of conics.
[1] Chapter 10 (Section 10.4)

Week 9: Polar coordinates, Graphing in polar coordinates.
[1] Chapter 10 (Section 10.2)

Week 10: Vector-valued functions**:** Limit, continuity, Derivatives, Integrals, Arc length, Unit tangent vector, Curvature, Unit normal vector.
[1] Chapter 12 (Sections 12.1 to 12.5)

Weeks 11 and 12: Functions of several variables**:** Graphs, Level curves, Limits and continuity, Partial derivatives and differentiability.
[1] Chapter 13 (Section 13.1 to 13.4)

Week 13: Functions of several variables**:** The chain rule, Directional derivatives and gradient vectors.
[1] Chapter 13 (Sections 13.5, and 13.6)

Week 14: Functions of several variables**:** Tangent plane and normal line, Extreme values and saddle points.
[1] Chapter 13 (Sections 13.7, and 13.8)

GE-1: Analytical Geometry and Theory of Equations

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures (per week), 1 Tutorial (per week per student)
Duration: 14 Weeks (70 Hrs.) **Examination**: 3 Hrs.

Course Objectives: The goal of this paper is to acquaint students with certain idea about conic sections, vectors in coordinate system with their geometrical properties, general properties of polynomials and equations with some applications.

Course Learning Outcomes: After completion of this paper, the students will be able to:
  i)   Link their mathematical skills with the real-world.
  ii)   Visualize cross sections of various 3D objects like sphere, ellipsoid etc.

Course Contents:

Unit 1: Conic Sections, Parametrized Curves, and Polar Coordinates **(Lectures: 25)**
Conic sections and quadratic equations**:** Circle, Parabola, Ellipse, and Hyperbola; Techniques for sketching**:** Parabola, Ellipse, and Hyperbola; Reflection properties of Parabola, Ellipse, and Hyperbola, Classifying conic sections by eccentricity, Classification of quadratic equations representing Lines, Parabola, Ellipse, and Hyperbola; Parameterization of plane curves, Conic sections in polar coordinates and their sketching.

Unit 2: Three-Dimensional Space: Vectors **(Lectures: 20)**
Rectangular coordinates in 3-space, Spheres and Cylindrical surfaces, Vector viewed geometrically, Vectors in coordinates system, Vectors determine by length and angle, Dot product, Cross product and their geometrical properties, Parametric equations of lines in 2-space and 3-space.

Unit 3: Theory of Equations **(Lectures: 25)**
General properties of polynomials and equations, Descartes' rule of signs for positive and negative roots, Relation between the roots and the coefficients of equations, Applications, Depression of an equation when a relation exists between two of its roots, Symmetric functions of the roots and its applications, Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms; Graphical representation of derived function, Rolle's theorem, Multiple roots of the equation.

References:
1.   Anton, Howard, Bivens, Irl, & Davis, Stephen (2013). *Calculus* (10th ed.). John Wiley & Sons Singapore Pte. Ltd. Reprint (2016) by Wiley India Pvt. Ltd. Delhi.
2.   Burnside, W.S., & Panton, A.W. (1954), *The Theory of Equations*, Vol. 1. Eleventh Edition, Fourth Indian Reprint. S. Chand & Co. New Delhi.
3.   Thomas, Jr. George B., Weir, Maurice D., & Hass, Joel (2014). *Thomas' Calculus* (13th ed.). Pearson Education, Delhi. Indian Reprint 2017.

Additional Reading:

1. Ferrar, W. L. (1956). *Higher Algebra*: *A Sequel to Higher Algebra for Schools*. Oxford

Teaching Plan (GE-I: Analytical Geometry and Theory of Equations):

Weeks 1 and 2: Conic sections and quadratic equations**:** Circle, Parabola, Ellipse, and Hyperbola; Techniques for sketching**:** Parabola, Ellipse, and Hyperbola; Reflection properties of Parabola, Ellipse, and Hyperbola.
[3] Chapter 11 (Section 11.6)
[1] Chapter 10 (Section 10.4)

Week 3: Classifying conic sections by eccentricity.
[3] Chapter 11 (Section 11.7)

Weeks 4 and 5: Classification of quadratic equations representing Lines, Parabola, Ellipse, and Hyperbola; Parameterization of plane curves, Conic sections in polar coordinates and their sketching.
[1] Chapter 10 (Section 10.2)
[3] Chapter 11 (Sections 11.1, and 11.4)

Weeks 6 and 7: Rectangular coordinates in 3-space, Spheres and cylindrical surfaces, Vector viewed geometrically, Vectors in coordinates system, Vectors determine by length and angle.
[1] Chapter 11 (11.1, and 11.2)

Weeks 8 and 9: Dot product, Cross product and their geometrical properties, Parametric equations of lines in 2-space and 3-space.
[1] Chapter 11 (Sections 11.3 to 11.5)

Weeks 10 and 11: General properties of polynomials and equations, Descartes' rule of signs for positive and negative roots, Relation between the roots and the coefficients of equations, Applications.
[2] Chapter 2 (Sections 12 to 22), Chapter 3 (Sections 23 and 24)

Weeks 12 and 13: Depression of an equation when a relation exists between two of its roots, Symmetric functions of the roots and its applications, Transformation of equations (multiplication, reciprocal, increase/diminish in the roots by a given quantity), Removal of terms.
[2] Chapter 3 (Sections 25 to 28), Chapter 4 (Sections 29 to 34)

Week 14: Graphical representation of derived function, Rolle's theorem, Multiple roots of the equation.
[2] Chapter 7 (Sections 69, 71, 73 and 74)
[1] Chapter 3 (Section 3.8.1)

Semester II

GE-2: Linear Algebra OR GE-2: Discrete Mathematics

GE-2: Linear Algebra

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures (per week), 1 Tutorial (per week per student)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The objective of the course is to introduce the concept of vectors in $\mathbb{R}^n$. The concepts of linear independence and dependence, rank and linear transformations has been explained through matrices. Various applications of vectors in computer graphics and movements in a plane has also been introduced.

Course Learning Outcomes: This course will enable the students to:
   i)   Visualize the space $\mathbb{R}^n$ in terms of vectors and the interrelation of vectors with matrices.
   ii) Learn about linear transformations, transition matrix and similarity.

Course Contents:

Unit 1: Euclidean space $\mathbb{R}^n$ and Matrices   (Lectures: 35)
Fundamental operation with vectors in Euclidean space $\mathbb{R}^n$, Linear combination of vectors, Dot product and their properties, Cauchy-Schwarz inequality, Triangle inequality, Projection vectors, Some elementary results on vectors in $\mathbb{R}^n$; Matrices: Gauss–Jordan row reduction, Reduced row echelon form, Row equivalence, Rank, Linear combination of vectors, Row space, Eigenvalues, Eigenvectors, Eigenspace, Characteristic polynomials, Diagonalization of matrices; Definition and examples of vector space, Some elementary properties of vector spaces, Subspace, Span of a set, a spanning set for an eigenspace, Linear independence and linear dependence of vectors, Basis and dimension of a vector space, Maximal linearly independent sets, Minimal spanning sets; Application of rank: Homogenous and non-homogenous systems of linear equations; Coordinates of a vector in ordered basis, Transition matrix.

Unit 2: Linear Transformations and Computer Graphics   (Lectures: 25)
Linear transformations: Definition and examples, Elementary properties, The matrix of a linear transformation, Linear operator and similarity; Application: Computer graphics, Fundamental movements in a plane, Homogenous coordinates, Composition of movements; Kernel and range of a linear transformation, Dimension theorem, One to one and onto linear transformations, Invertible linear transformations, Isomorphism, Isomorphic vector spaces (to $\mathbb{R}^n$).

Unit 3: Orthogonality and Least Square Solutions   (Lectures: 10)
Orthogonal and orthonormal vectors, Orthogonal and orthonormal bases, Orthogonal complement, Projection theorem, Orthogonal projection onto a subspace; Application: Least square solutions for inconsistent systems, Non-unique least square solutions.

References:

1. Andrilli, S., & Hecker, D. (2016). *Elementary Linear Algebr*a (5th ed.). Academic Press, Elsevier India Private Limited.
2. Kolman, Bernard, & Hill, David R. (2001). *Introductory Linear Algebra with Applications* (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.

Teaching Plan (GE-2: Linear Algebra):

Week 1: Fundamental operation with vectors in Euclidean space $\mathbb{R}^n$, Linear combination of vectors, dot product and their properties, Cauchy-Schwarz inequality, Triangle inequality, Projection vectors.
[1] Chapter 1 (Sections 1.1 and 1.2)

Week 2: Some elementary results on vectors in $\mathbb{R}^n$; Matrices**:** Gauss–Jordan row reduction, Reduced row echelon form, Row equivalence, Rank.
[1] Chapter 1 [Section 1.3 (Pages 34 to 44)]
[1] Chapter 2 [Sections 2.2 (up to page 111), 2.3 (up to page 122, Statement of Theorem 2.5)].

Week 3: Linear combination of vectors, Row space, Eigenvalues, Eigenvectors, Eigenspace, Characteristic polynomials, Diagonalization of matrices.
[1] Chapter 2 [Section 2.3 (Pages 122 to 132, Statements of Lemma 2.8 and Theorem 2.9)]
[1] Chapter 3 (Section 3.4)

Week 4: Definition and examples of vector space, Some elementary properties of vector spaces.
[1] Chapter 4 (Section 4.1)

Week 5 and 6: Subspace, Span of a set, a spanning set for an eigenspace, Linear independence and dependence, Basis and dimension of a vector space, Maximal linearly independent sets, Minimal spanning sets.
[1] Chapter 4 (Sections 4.2 to 4.5, Statements of technical Lemma 4.10 and Theorem 4.12)

Week 7: Application of rank**:** Homogenous and non-homogenous systems of linear equations; Coordinates of a vector in ordered basis, Transition matrix.
[2] Chapter 6 [Sections 6.6 (pages 287 to 291), and 6.7 (Statement of Theorem 6.15 and examples)]

Week 8: Linear transformations**:** Definition and examples, Elementary properties.
[1] Chapter 5 (Section 5.1)

Week 9: The matrix of a linear transformation, Linear operator and similarity.
[1] Chapter 5 [Section 5.2 (Statements of Theorem 5.5 and Theorem 5.6)]

Week 10: Application**:** Computer graphics, Fundamental movements in a plane, Homogenous coordinates, Composition of movements.
[1] Chapter 8 (Section 8.8)

Week 11: Kernel and range of a linear transformation, Statement of the dimension theorem and examples.
[1] Chapter 5 (Sections 5.3)

Week 12: One to one and onto linear transformations, Invertible linear transformations, isomorphism, isomorphic vector spaces (to $\mathbb{R}^n$)
[1] Chapter 5 [Sections 5.4, 5.5 (up to 378, Statements of Theorem 5.15 and Theorem 5.16)]

Week 13 and 14: Orthogonal and orthonormal vectors, orthogonal and orthonormal bases, orthogonal complement, statement of the projection theorem and examples. Orthogonal projection onto a subspace. Application: Least square solutions for inconsistent systems, non-unique least square solutions.
[1] Chapter 6 [Sections 6.1 (up to Example 3, page 416, Statement of Theorem 6.3),
    6.2 (up to 435, 439 to 443, and Statement of Theorem 6.12)]
[1] Chapter 8 [Section 8.9 (up to 593, Statement of Theorem 8.13)

GE-2: Discrete Mathematics

Total Marks: 100 (Theory: 75, Internal Assessment: 25)
Workload: 5 Lectures (per week), 1 Tutorial (per week per student)
Duration: 14 Weeks (70 Hrs.) **Examination**: 3 Hrs.

Course Objectives: The course introduces the concept of logic, statements, GCD, LCM, Mathematical Induction and Fermat primes. Lattice theory and Boolean Algebra is also discussed along with their applications in real-world.

Course Learning Outcomes: Students will get to learn the techniques of proving mathematical theorems. The concept of Karnaugh diagrams and Quinn –McCluksey method gives an aid to apply truth tables in real-world problems.

Course Contents:

Unit 1: Logical Mathematics **(Lectures: 30)**
Compound statements (and, or, implication, negation, contrapositive, quantifiers), Truth tables, Basic logical equivalences and its consequences, Logical arguments, Set theory, Operation on sets, Types of binary relations, Equivalence relations, Congruences and its properties, Partial and total ordering, Lattices, Properties of integers, Division algorithm, Divisibility and Euclidean algorithm, GCD, LCM, Relatively prime.

Unit 2: Applications of Numbers **(Lectures: 15)**
Prime numbers, Statement of fundamental theorem of arithmetic, Fermat primes, Mathematical induction, Recursive relations and its solution (characteristics polynomial and generating function), Principles of counting (inclusion/exclusion, pigeon-hole), Permutation and combinations (with and without repetition).

Unit 3: Lattices and its Properties **(Lectures: 25)**
Duality principle, Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms, Distributive lattices, Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials, Quinn-McCluskey method, Karnaugh diagrams, Switching circuits and applications of switching circuits.

References:

1. Davey, B A., & Priestley, H. A. (2002). *Introduction to Lattices and Order* (2nd ed.). Cambridge University Press. Cambridge. 7th Printing 2012.
2. Goodaire, Edgar G., & Parmenter, Michael M. (2003). *Discrete Mathematics with Graph Theory* (2nd ed.). Pearson Education (Singapore) Pte. Ltd. Indian Reprint.
3. Lidl, Rudolf & Pilz, Günter. (1998). *Applied Abstract Algebra* (2nd ed.). Undergraduate Texts in Mathematics. Springer (SIE). Indian Reprint 2004.

Additional Reading:
1. Rosen, Kenneth H. (2012) *Discrete Mathematics and its applications* (7th ed.). McGraw-Hill Education Private Limited.

Teaching Plan (GE-2: Discrete Mathematics):

Week 1: Compound Statements (and, or, implication, negation, contrapositive, quantifiers), Truth tables.
[2] Chapter 1 (Sections 1.1, and 1.3)

Week 2: Basic logical equivalences and its consequences, Logical arguments, Set theory,
[2] Chapter 1 (Sections 1.4, and 1.5), and Chapter 2 (Section 2.1)

Week 3: Operation on sets, types of binary relations, Equivalence relations, Congruences and its properties.
[2] Chapter 2 [Sections 2.2, 2.3, and 2.4 (left for convergence)], and Chapter 4 (Section 4.4)

Week 4: Partial and total ordering, Lattices.
[2] Chapter 2 (Section 2.5)

Week 5: Properties of integers, Division algorithm, Divisibility.
[2] Chapter 4 (Sections 4.1 to 4.1.6)

Week 6: Euclidean algorithm, GCD, LCM, Relatively prime.
[2] Chapter 4 (Section 4.2)

Week 7: Prime numbers, statement of fundamental theorem of arithmetic, Fermat primes.
[2] Chapter 4 (Sections 4.3 up to 4.3.11, page 119)

Week 8: Mathematical induction, Recursive relations and its solution (characteristics polynomial and generating function).
[2] Chapter 5 (Sections5.1, 5.3, and 5.4)

Week 9: Principles of counting (inclusion /exclusion, pigeon-hole), permutation and combinations (with and without repetition).
[2] Chapter 6 (Section 6.1), Chapter 7 (Sections 7.1 to 7.3)

Week 10: Duality principle, lattices as ordered sets.
[1] Chapter 1 (Section 1.20), Chapter 2 (Sections 2.1 to 2.7)

Week 11: Lattices as algebraic structures, Sublattices, Products and Homomorphisms, Distributive lattices.
[1] Chapter 2 (Sections 2.8 to 2.19), Chapter 4 (Sections 4.1 to 4.11)

Week 12: Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials.
[3] Chapter 1 (Section 2)

Weeks 13 and 14: Quinn-McCluskey method, Karnaugh diagrams, Switching circuits and applications of switching circuits.
[3] Chapter 2 (Section 1)

Semester III

GE-3: Differential Equations (with practicals)
OR
GE-3: Linear Programming and Game Theory

GE-3: Differential equations

Total Marks: 150 (Theory: 75, Internal Assessment: 25 and Practical: 50)
Workload: 4 Lectures (per week), 4 Practicals (per week per student)
Duration: 14 Weeks (56 Hrs. Theory + 56 Hrs. Practical) **Examination:** 3 Hrs.

Course Objectives: This course includes a variety of methods to solve ordinary and partial differential equations with basic applications to real life problems. It provides a solid foundation to further in mathematics, sciences and engineering through mathematical modeling.

Course Learning Outcomes: This course will enable the students to learn:
  i)   To analyze real-world scenarios to recognize when ordinary (or systems of) or partial differential equations are appropriate for creating an appropriate model.
  ii) To reduce a higher order equation to a system of first order simultaneous equations.
  iii) Explicit methods of solving higher-order linear differential equations.

Course Contents:

Unit 1: Ordinary Differential Equations and Applications (Lectures: 20)
First order exact differential equations. Integrating factors, rules to find integrating factor. Linear equations and Bernoulli equations, Orthogonal trajectories and oblique trajectories. Basic theory of higher order linear differential equations, Wronskian and its properties. Solving differential equation by reducing its order.

Unit 2. Explicit Methods of Solving Higher-Order Linear Differential Equations (Lectures: 16)
Linear homogenous equations with constant coefficients. Linear non-homogenous equations. The method of undetermined coefficients. The method of variation of parameters, The Cauchy-Euler equation. Simultaneous differential equations.

Unit 3. First and Second Order Partial Differential Equations (Lectures: 20)
Partial differential equations: Basic Concepts and definitions. Mathematical problems, First order equations: Classification, Construction, Geometrical interpretation. Method of characteristics, General solutions of first order partial differential equations. Canonical forms and method of separation of variables for first order partial differential equations. Classification of second order partial differential equations. Reduction to canonical forms. Second order partial differential equations with constant coefficients, General solutions.

References:

1. Kreyszig, Erwin. (2011). *Advanced Engineering Mathematics* (10th ed.). Wiley India.
2. Myint-U, Tyn and Debnath, Lokenath (2007). *Linear Partial Differential Equations for Scientist and Engineer*s (4th ed.). Birkkäuser Boston. Indian Reprint.
3. Ross, Shepley. L. (1984). *Differential equations* (3rd ed.). John Wiley & Sons.

Additional reading:
1. Sneddon I. N. (2006). *Elements of partial differential equations*. Dover Publications.

Practical / Lab work to be performed in a Computer Lab:

Use of Computer Algebra Systems (CAS), for example MATLAB/Mathematica /Maple/Maxima/Scilab etc., for developing the following programs:

1) Solution of first order differential equation.
2) Plotting of second order solution family of differential equation.
3) Plotting of third order solution family of differential equation.
4) Solution of differential equation by variation of parameter method.
5) Solution of system of ordinary differential equations.
6) Solution of Cauchy problem for first order partial differential equation.
7) Plotting the characteristics for the first order partial differential equation.
8) Plot the integral surfaces of a given first order partial differential equation with initial data.

Teaching Plan (GE-3: Differential Equations):

Weeks 1 and 2: First order ordinary differential equations: Basic concepts and ideas, First order exact differential equation, Integrating factors and rules to find integrating factors.
[3] Chapter 1 (Sections 1.1, and 1.2), and Chapter 2 (Sections 2.1, and 2.2)
[1] Chapter 1 (Sections 1.1, 1.2, and 1.4)

Week 3: Linear equations and Bernoulli equations, Orthogonal trajectories and oblique trajectories.
[3] Chapter 2 (Sections 2.3, and 2.4), and Chapter 3 (Section 3.1)

Weeks 4 and 5: Basic theory of higher order linear differential equations, Wronskian and its properties, Solving a differential equation by reducing its order.
[3] Chapter 4 (Section 4.1)

Weeks 6 and 7: Linear homogenous equations with constant coefficients, Linear non-homogenous equations, The method of undetermined coefficients.
[3] Chapter 4 (Sections 4.2, and 4.3)
[1] Chapter 2 (Section 2.2)

Weeks 8 and 9: The method of variation of parameters, The Cauchy-Euler equation, Simultaneous differential equations.
[3] Chapter 4 (Sections 4.4, and 4.5), and Chapter 7 (Sections 7.1, and 7.3)

Week 10: Partial differential equations**:** Basic Concepts and definitions, Mathematical problems; First order equations**:** Classification and construction.
[2] Chapter 2 (Sections 2.1 to 2.3)

Weeks 11 and 12: Geometrical interpretation, Method of characteristics, General solutions of first order partial differential equations.
[2] Chapter 2 (Sections 2.4, and 2.5)

Week 13: Canonical forms and method of separation of variables for first order partial differential equations.
[2] Chapter 2 (Sections 2.6, and 2.7)

Week 14: Second order partial differential equations**:** Classification, Reduction to canonical forms, With constant coefficients, General solutions.
[2] Chapter 4 (Sections 4.1 to 4.4)

GE-3: Linear Programming and Game Theory

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)
Workload: 5 Lectures (per week), 1 Tutorial (per week per student)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objective: This course develops the ideas underlying the Simplex method computational techniques for linear programming and game theory, having applications in management, social science, industry, warfare, economics and financial sectors, etc.

Course Learning Outcomes: This course will enable the students to learn:
  i) The optimal solution for linear optimization problems subject to certain constraints.
  ii) The dual to a production problem with profits to be maximized to keep total cost down.
  iii) The transportation and Hungarian algorithm specially designed to solve the transportation and assignment problems, respectively.
  iv) The strategies for two-person, zero-sum game are obtained by solving two dual linear programming problems.

Course Contents:

Unit 1. Linear Programming Problem, Simplex Method and Duality   (Lectures: 35)
Introduction to Linear Programming Problem**:** Graphical method of solution, Basic Feasible Solutions, Linear programming and convexity; Introduction to the Simplex method**:** Theory of the Simplex method, Optimality and unboundedness; The Simplex tableau and examples, Artificial variables; Introduction to Duality, Formulation of the Dual problem with examples and interpretations, The Duality theorem.

Unit 2. Transportation and Assignment Problems   (Lectures: 15)
Definition and mathematical formulation of Transportation problem, Methods of finding initial basic feasible solutions, North West corner rule, Least‐ cost method, Vogel's approximation method, Algorithm for solving Transportation problem; Mathematical formulation and Hungarian method of solving Assignment problem.

Unit 3. Two-Person, Zero-Sum Games   (Lectures: 20)
Introduction to Game theory, Formulation of two-person, Zero-sum rectangular game, Solution of rectangular games with saddle points, Mixed strategies, Dominance principle, Rectangular Games without saddle points, Graphical and linear programming solution of rectangular games.

References:

  1. Taha, Hamdy A. (2010). *Operations Research*: *An Introduction* (9th ed.). Pearson.
  2. Thie, Paul R., & Keough, G. E. (2014). *An Introduction to Linear Programming and Game Theory*. (3rd ed.). Wiley India Pvt. Ltd.

Additional Readings:

1. Hadley, G. (1997). *Linear Programming*. Narosa Publishing House. New Delhi.
2. Hillier, F. S., & Lieberman, G. J. (2010). *Introduction to Operations Research-Concepts and Cases* (9th ed.). Tata McGraw Hill.
3. Kolman. B. and Hill, D.R. (2003). *Introductory Linear Algebra with Applications*. (7th ed.). Pearson Education. First Indian Reprint 2003.

Teaching Plan (GE-3: Linear Programming and Game Theory):

Week 1: Introduction to Linear Programming Problem**:** Graphical method of solution, Basic Feasible Solutions, Linear programming and convexity.
[2] Chapter 2 (Section 2.2), and Chapter 3 (Sections 3.1, 3.2, and 3.9).

Weeks 2 and 3: Introduction to the Simplex method**:** Theory of the Simplex method, Optimality and unboundedness.
[2] Chapter 3 (Sections 3.3, and 3.4).

Weeks 4 and 5: The Simplex tableau and examples, Artificial variables.
[2] Chapter 3 (Sections 3.5, and 3.6).

Weeks 6 and 7: Introduction to Duality, Formulation of the Dual problem with examples and Interpretations, Statement of the Duality theorem with examples.
[2] Chapter 4 (Sections 4.1 to 4.4).

Weeks 8 and 9: Definition and mathematical formulation of Transportation problem, Methods of finding initial basic feasible solutions, North West corner rule, Least- cost method, Vogel's approximation method, Algorithm for solving Transportation problem.
[1] Chapter 5 (Sections 5.1, and 5.3)

Week 10: Mathematical formulation and Hungarian method of solving Assignment problem.
[1] Chapter 5 (Section 5.4)

Weeks 11 and 12: Introduction to Game theory, Formulation of two-person, Zero-sum rectangular game, Solution of rectangular games with saddle points.
[2] Chapter 9 (Sections 9.1 to 9.3)

Weeks 13 and 14: Mixed strategies, Dominance principle, Rectangular games without saddle points, Graphical and linear programming solution of rectangular games.
[2] Chapter 9 (Sections 9.4 to 9.6)

Semester IV

GE-4: Numerical Methods   **OR**   **GE-4: Elements of Analysis**
      (with practicals)

GE-4: Numerical Methods

Total Marks: 150 (Theory: 75, Internal Assessment: 25 and Practical: 50)
Workload: 4 Lectures (per week), 4 Practicals (per week per student)
Duration: 14 Weeks (56 Hrs. Theory + 56 Hrs. Practical) **Examination:** 3 Hrs.

Course Objectives: The goal of this paper is to acquaint students for the study of certain algorithms that uses numerical approximation for the problems of mathematical analysis. Also, the use of Computer Algebra Systems (CAS) by which the intractable problems can be solved both numerically and analytically.

Course Learning Outcomes: After completion of this course, students will be able to:
  i)   Find the consequences of finite precision and the inherent limits of numerical methods.
  ii)   Appropriate numerical methods to solve algebraic and transcendental equations.
  iii)   How to solve first order initial value problems of ODE's numerically using Euler methods.

Course Contents:

Unit 1: Errors and Roots of Transcendental and Polynomial Equations   **(Lectures: 16)**
Floating point representation and computer arithmetic, Significant digits; Errors**:** Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions; Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method.

Unit 2: Algebraic Linear Systems and Interpolation   **(Lectures: 20)**
Gaussian elimination method (with row pivoting), Gauss-Jordan method; Iterative methods**:** Jacobi method, Gauss-Seidel method; Interpolation**:** Lagrange form, Newton form, Finite difference operators, Gregory-Newton forward and backward difference interpolations, Piecewise polynomial interpolation (Linear and Quadratic).

Unit 3: Numerical Differentiation, Integration and ODE   **(Lectures: 20)**
Numerical differentiation**:** First and second order derivatives; Numerical integration**:** Trapezoid rule, Simpson's rule; Extrapolation methods**:** Richardson extrapolation, Romberg integration; Ordinary differential equation**:** Euler's method, Modified Euler's methods **(**Heun and Mid-point).

References:

1. Chapra, Steven C. (2018). *Applied Numerical Methods with* MATLAB *for Engineers and Scientists* (4th ed.). McGraw-Hill Education.
2. Fausett, Laurene V. (2009). *Applied Numerical Analysis Using* MATLAB. Pearson. India.

3. Jain, M. K., Iyengar, S. R. K., & Jain R. K. (2012). *Numerical Methods for Scientific and Engineering Computation* (6th ed.). New Age International Publishers. Delhi.

Additional Reading:

1. Bradie, Brian (2006). *A Friendly Introduction to Numerical Analysis*. Pearson Education India. Dorling Kindersley (India) Pvt. Ltd. Third Impression, 2011.

Practical /Lab work to be performed in the Computer Lab:

Use of Computer Algebra System (CAS), for example MATLAB/Mathematica/Maple/Maxima/ Scilab etc., for developing the following Numerical Programs:

1) Bisection Method
2) Secant Method and Regula-Falsi Method
3) Newton-Raphson Method
4) Gaussian elimination method and Gauss-Jordan method
5) Jacobi Method and Gauss-Seidel Method
6) Lagrange Interpolation and Newton Interpolation
7) Trapezoid and Simpson's rule.
8) Romberg integration
9) Euler methods for solving first order initial value problems of ODE's.

Teaching Plan (Theory of GE-4: Numerical Methods):

Weeks 1 and 2: Floating point representation and computer arithmetic, Significant digits; Errors**:** Roundoff error, Local truncation error, Global truncation error; Order of a method, Convergence and terminal conditions.
[2] Chapter 1 (Sections 1.2.3, 1.3.1, and 1.3.2)
[3] Chapter 1 (Sections 1.2, and 1.3)

Week 3 and 4: Bisection method, Secant method, Regula-Falsi method, Newton-Raphson method.
[2] Chapter 2 (Sections 2.1 to 2.3)
[3] Chapter 2 (Sections 2.2 and 2.3)

Week 5: Gaussian elimination method (with row pivoting), Gauss-Jordan method;
Iterative methods**:** Jacobi method, Gauss-Seidel method.
[2] Chapter 3 (Sections 3.1, and 3.2), Chapter 6 (Sections 6.1, and 6.2)
[3] Chapter 3 (Sections 3.2, and 3.4)

Week 6: Interpolation**:** Lagrange form, and Newton form.
[2] Chapter 8 (Section 8.1)
[3] Chapter 4 (Section 4.2)

Weeks 7 and 8: Finite difference operators, Gregory-Newton forward and backward difference interpolations.
[3] Chapter 4 (Sections 4.3, and 4.4)

Week 9: Piecewise polynomial interpolation**:** Linear, and Quadratic.
[2] Chapter 8 [Section 8.3 (8.3.1, and 8.3.2)]
[1] Chapter 18 (Sections 18.1 to 18.3)

Weeks 10 and 11: Numerical differentiation**:** First and second order derivatives;
Numerical integration**:** Trapezoid rule, Simpson's rule.
[2] Chapter 11 [Sections 11.1 (11.1.1, and 11.1.2), and 11.2 (11.2.1, and 11.2.2)]

Weeks 12 and 13: Extrapolation methods**:** Richardson extrapolation, Romberg integration;
Ordinary differential equations**:** Euler's method.
[2] Chapter 11 [Section 11.1 (11.1.4), and 11.2 (11.2.4)]
[1] Chapter 22 (Sections 22.1, and 22.2)

Weeks 14: Modified Euler's methods**:** Heun's method, The Midpoint method.
[1] Chapter 22 (Section 22.3)

GE-4: Elements of Analysis

Total Marks: 100 (Theory: 75 and Internal Assessment: 25)
Workload: 5 Lectures (per week), 1 Tutorial (per week per student)
Duration: 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

Course Objectives: The main aim of this course is to introduce the real number line, its completeness property and sequences. The relation between the convergent and Cauchy sequences has been explained. The concept of power series and its convergence has also been introduced.

Course Learning Outcomes: The students will be familiar with the concept of sequences, series. They will be able to test the convergence and divergence of series using the ratio test, Leibnitz test. The concept of power series will help them to enrich their knowledge of elementary functions like $\exp(x)$, $\sin x, \cos x$ and $\log(1+x)$.

Course Contents:

Unit 1. Real Numbers and Sequences **(Lectures: 35)**
Finite and infinite sets, Examples of countable and uncountable sets; Absolute value and the Real line, Bounded sets, Suprema and infima, The Completeness property of $\mathbb{R}$, Archimedean property of $\mathbb{R}$; Real sequences, Convergence, Sum and product of convergent sequences, Order preservation and squeeze theorem; Monotone sequences and their convergence; Proof of convergence of some simple sequences such as $\frac{(-1)^n}{n}, \frac{1}{n^2}, \left(1+\frac{1}{n}\right)^n, x^n$ with $|x| < 1$, a_n/n, where a_n is a bounded sequence. Subsequences and the Bolzano-Weierstrass theorem; Limit superior and limit inferior of a bounded sequence; Cauchy sequences, Cauchy convergence criterion for sequences.

Unit 2. Infinite Series of Real Numbers **(Lectures: 20)**
Definition and a necessary condition for convergence of an infinite series, Geometric series, Cauchy convergence criterion for series; Positive term series, The integral test, Convergence of *p*-series, Comparison test, Limit comparison test, D'Alembert's Ratio test, Cauchy's Root test; Alternating series, Leibniz test; Absolute and conditional convergence.

Unit 3. Power Series and Elementary Functions **(Lectures: 15)**
Definition of power series, Radius and interval of convergence, Cauchy-Hadamard theorem, Statement and illustration of term-by-term differentiation, Integration of power series, and Abel's theorem, Power series expansions for $\exp(x)$, $\sin x$, $\cos x, \log(1+x)$ and their properties.

References:
1. Bartle, Robert G., & Sherbert, Donald R. (2015). *Introduction to Real Analysis* (4th ed.). Wiley India Edition.
2. Denlinger, Charles G. (2015). *Elements of analysis*. Jones & Bartlett India Pvt. Ltd.
3. Ross, Kenneth A. (2013). *Elementary Analysis*: *The theory of calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

Teaching Plan (GE-4: Elements of Analysis):

Weeks 1 and 2: Finite and infinite sets, Examples of countable and uncountable sets; Absolute value of the real line, bounded sets, suprema and infima; Statement of order Completeness property of $\mathbb{R}$, Archimedean property of $\mathbb{R}$.
[1] Chapter 1 (Section 1.3), and Chapter 2 (Sections 2.2 to 2.4)

Weeks 3 and 4: Real sequences, Convergence, Sum and product of convergent sequences, Order preservation and squeeze theorem.
[1] Chapter 3 (Sections 3.1 and 3.2)

Week 5: Monotone sequences and their convergence, Proof of convergence of some simple sequences such as $\frac{(-1)^n}{n}, \frac{1}{n^2}, \left(1 + \frac{1}{n}\right)^n, x^n$ with $|x| < 1, a_n/n$, where a_n is a bounded sequence.
[1] Chapter 3 (Section 3.3)

Weeks 6 and 7: Subsequences and the Bolzano-Weierstrass theorem (statement and examples), Limit superior and limit inferior of a bounded sequence (definition and examples), Statement and illustrations of Cauchy convergence criterion for sequences.
[1] Chapter 3 (Sections 3.4, and 3.5)

Weeks 8 and 9: Definition and a necessary condition for convergence of an infinite series, Geometric series, Cauchy convergence criterion for series, positive term series, State the integral test and prove the convergence of *p*-series, Comparison test, Limit comparison test and examples.
[2] Chapter 8 (Section 8.1)
[1] Chapter 3 (Section 3.7)

Week 10: D'Alembert's Ratio test, Cauchy's Root test.
[2] Chapter 8 (Section 8.2)

Week 11: Alternating series, Leibnitz test; Absolute and conditional convergence.
[2] Chapter 8 (Section 8.3)

Week 12: Definition of power series, Radius and interval of convergence, Cauchy-Hadamard theorem.
[3] Chapter 4 [Article 23, 23.1 (without proof)]
[1] Chapter 9 [9.4.7 to 9.4.9 (without proof)]

Week 13: Statement and illustration of term-by-term differentiation, Integration of power series and Abel's theorem.
[3] Chapter 4 (Article 26)

Week 14: Power series expansions for $\exp(x), \sin x, \cos x, \log(1+x)$ and their properties.
[3] Chapter 7 (Article 37)
[1] Chapter 9 (9.4.14)